# Density-based debris cloud propagation and collision risk estimation through a binning approach

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# The model – Starling V2.0

## Block diagram

- Limit values in $A/M$ and ejection velocity
- Domain in Keplerian elements
- Binning and grid definition
- Density distribution in Keplerian elements

**Initial density estimation**

- Cloud sampling – initial characteristics
- Characteristics' propagation through MOC
- Density interpolation through binning

**Density propagation**

- Impact rate/number of impacts with targets
- Estimation of the probability of collision
- (Evaluation of a suitable sustainability index)

**Fragmentation effects**


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# Initial density estimation

## Limit values in $A/M$ and ejection velocity

- The fragments' characterisation relies on the probabilistic reformulation of the NASA Standard Breakup Model*, according to $3$ PDF:
    1. **Characteristic length**: $p_\lambda$
    2. **Area-to-mass ratio**: $p_{\chi|\lambda}$ (conditional)
    3. **Ejection velocity**: $p_{\nu|\chi}$ (conditional)

$\lambda = \log_{10}(L), \nu = \log_{10}(\Delta v), \chi = \log_{10}(A/M)$, $L$ = characteristic length, $\Delta v$ = ejection velocity, $A/M$ = area-to-mass ratio

➢ Frey S., Colombo C., "Transformation of Satellite Breakup Distribution for Probabilistic Orbital Collision Hazard Analysis," *Journal of Guidance, Control, and Dynamics*, vol. 44, pp. 88-105, 2021.

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  - Division of the domain in $\chi$ in $N_\chi$ bins
  - Cumulative density function $F_\chi, F_\nu$
  - User-defined accuracy level $\xi \in [0,1)$

Fraction of fragments probabilistically described by the model

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$N_\chi + 2$ **equations** solved numerically through:
- Root finding algorithms
- Function minimisation routines

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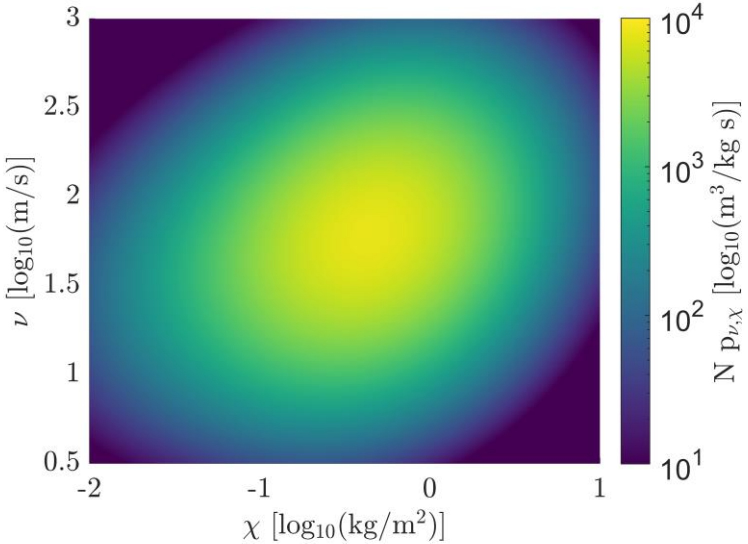


*Rocket body explosion, distribution in* $(\chi, \nu)$

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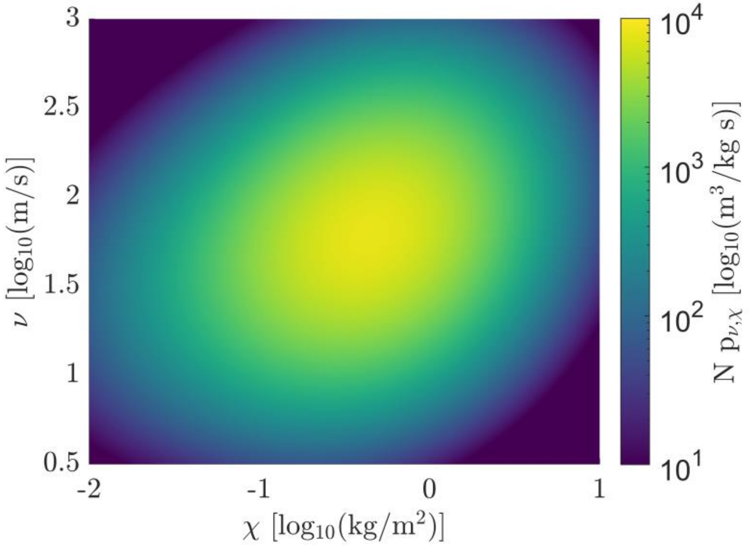


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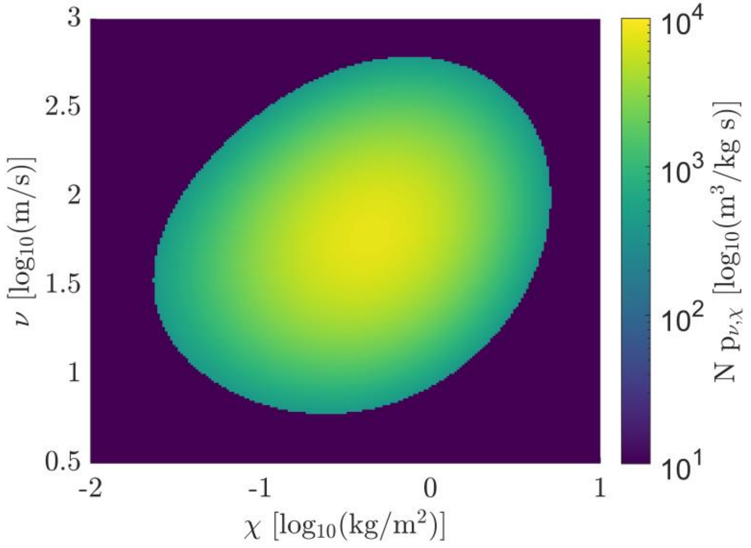
$$p_{\min} = \bar{p}_{\chi,\nu}(\xi) : \int_{\bar{p}_{\chi,\nu}(\xi)}^{\infty} \mathrm{d}\, p_{\chi,\nu} = \xi$$

*Rocket body explosion, distribution in $(\chi, \nu)$ containing 95 % of the fragments' population*



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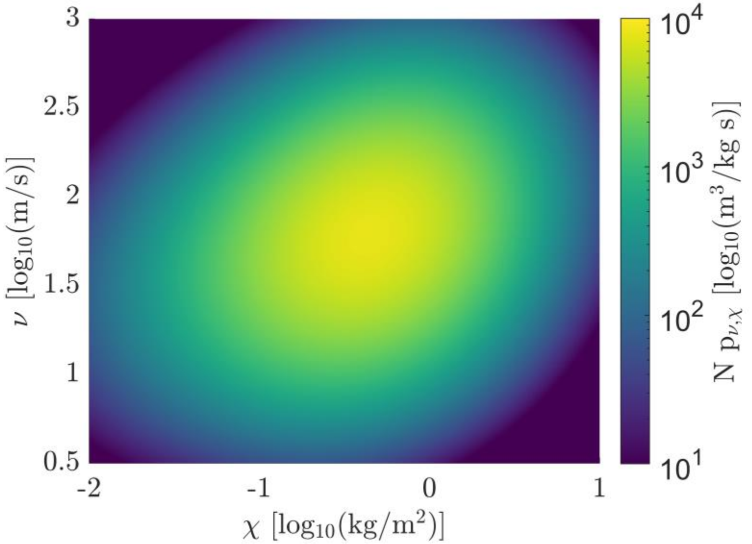


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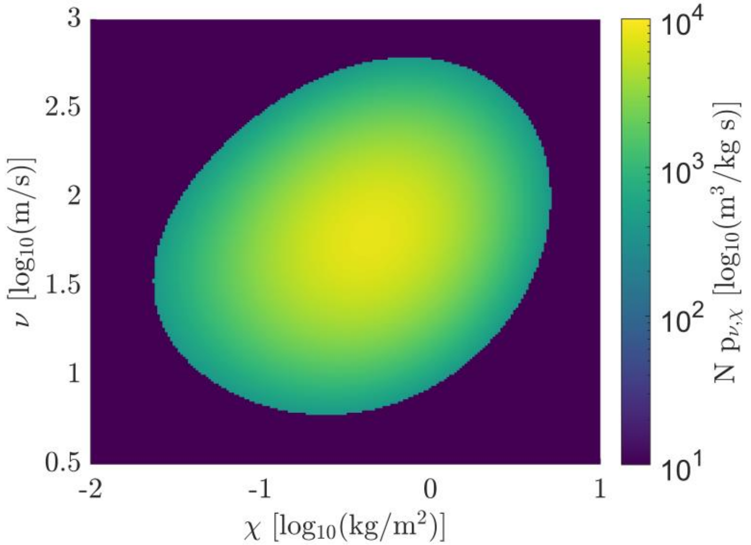
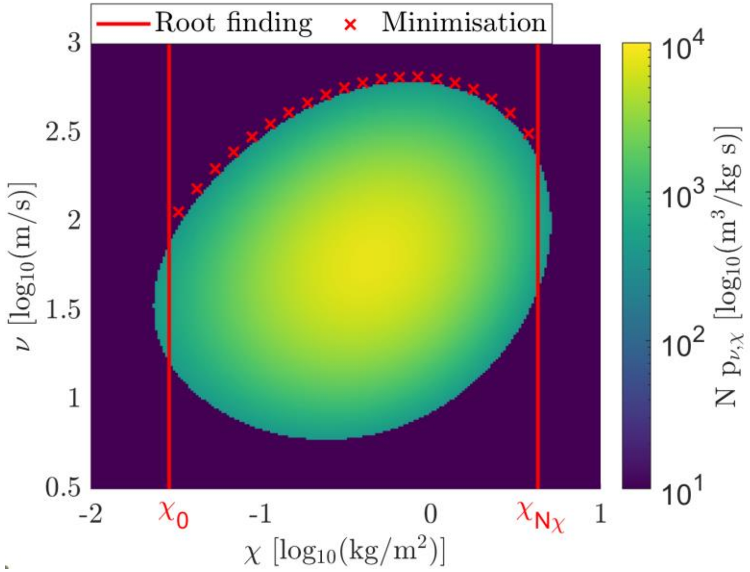



*Rocket body explosion, distribution in $(\chi, \nu)$ containing 95 % of the fragments' population – Predicted limits in $\chi$ and $\nu$ with $\xi = 0.95$*

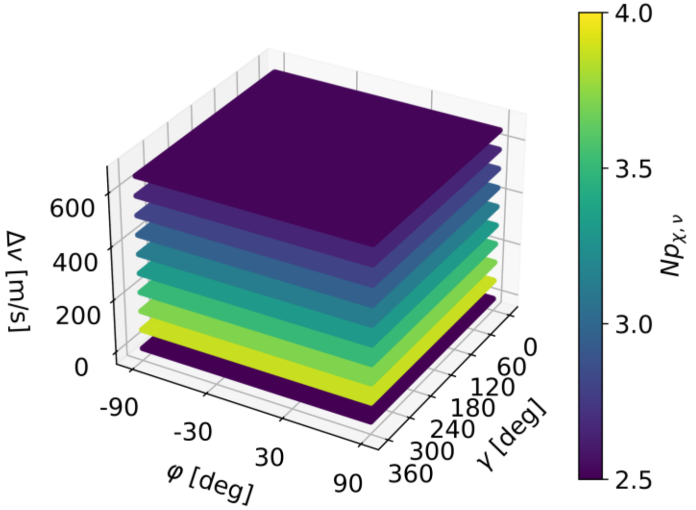
# Initial density estimation

## Dimensionality preserving transformation

**Cartesian coordinates** (+ $A/M$)

- Fragments share same initial position $\boldsymbol{r}_P$
- 3D isotropic distribution in ejection velocity vector $\boldsymbol{\Delta v}$

*3D density distribution in ejection velocity*



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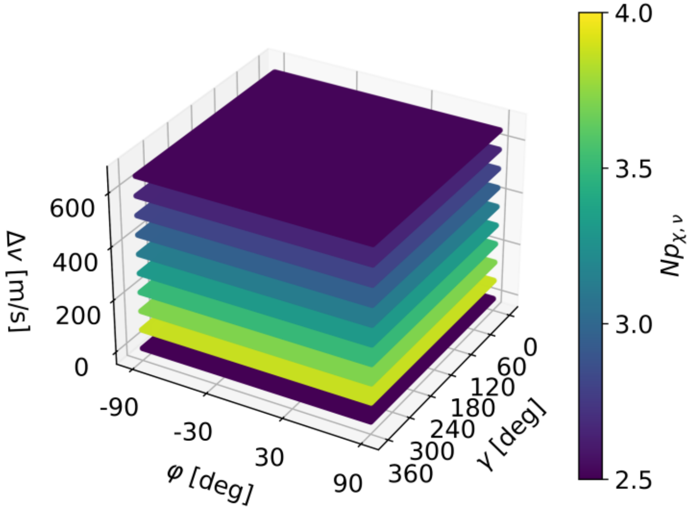
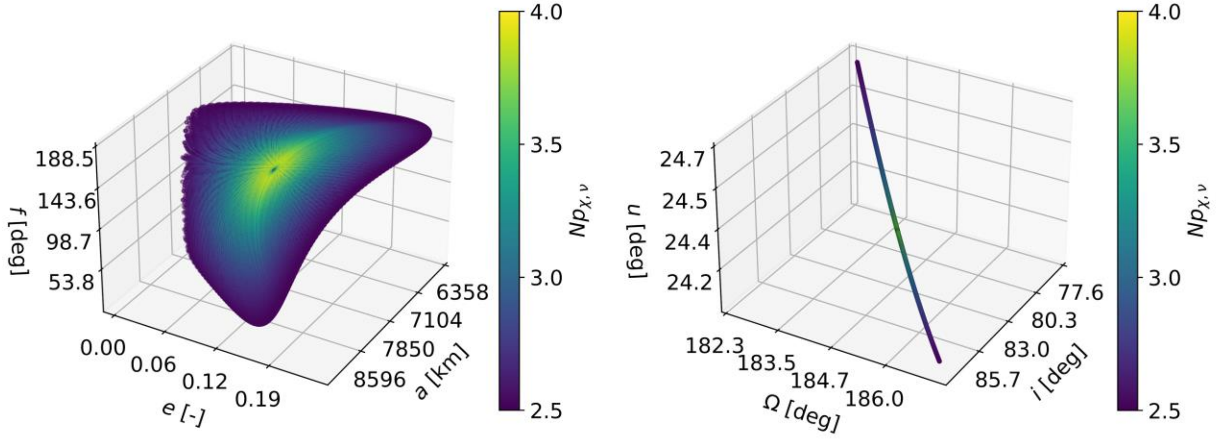



### Keplerian elements (+ $A/M$)

- Given $\boldsymbol{r}_P$, for each $(a, e, i)$ there exist 4 possible $(\Omega_k, \omega_k, f_k)$ that guarantee intersection

*3D density distribution in Keplerian elements, made up of a surface- and line-like distributions*



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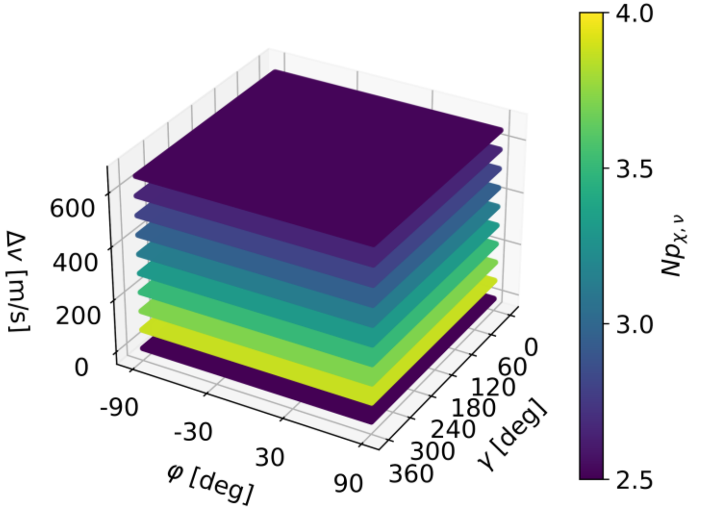


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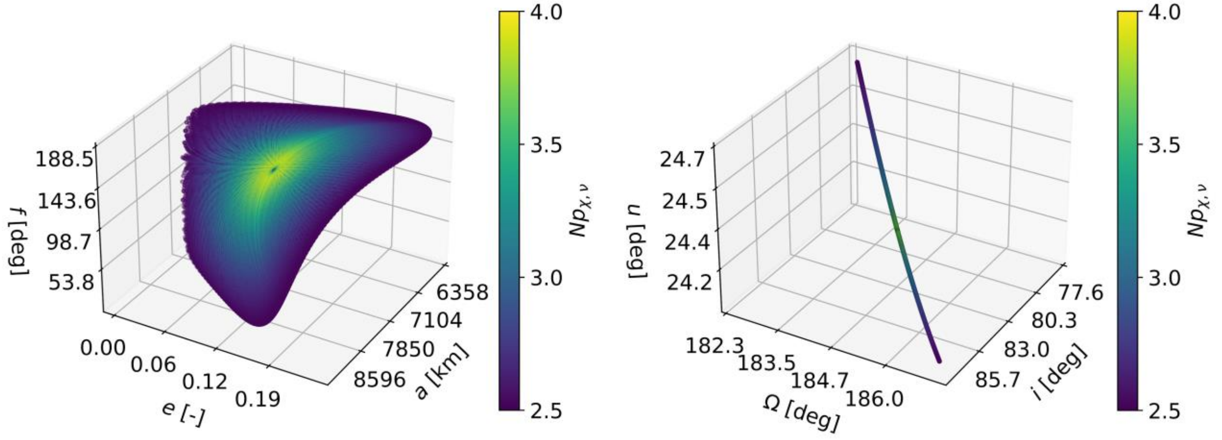


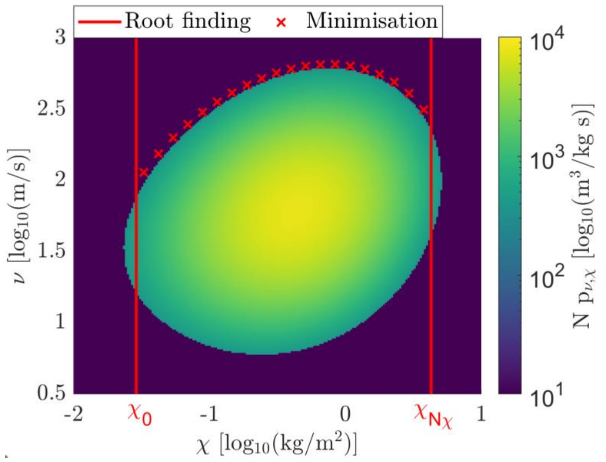

**Domain in Keplerian** elements defined in the **subset** $(\boldsymbol{a}, \boldsymbol{e}, \boldsymbol{i})$

# Initial density estimation

## Domain in Keplerian elements

*Fragmentation properties*

2D distribution in $A/M$ and $\Delta v$, bounded by $(\chi_0, \chi_{N_\chi}, \nu_j)$

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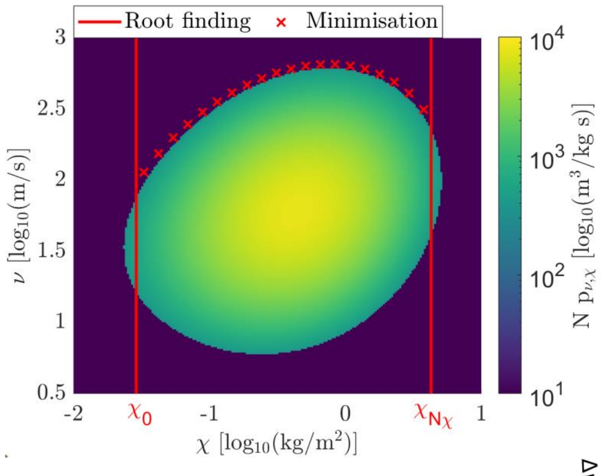
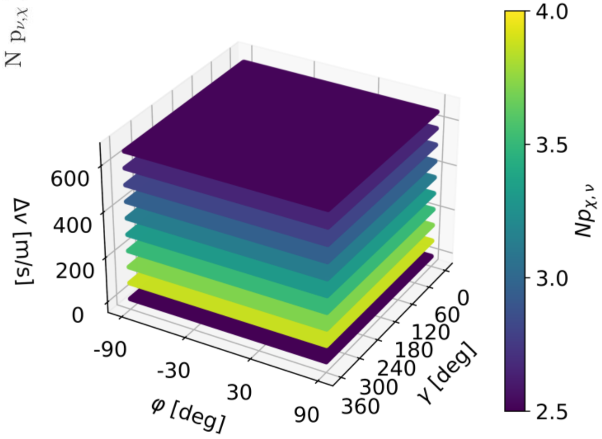
*Isotropic $\boldsymbol{\Delta v}$ distribution*

4D distribution in $A/M$, $\Delta v$, $\gamma$ and $\phi$

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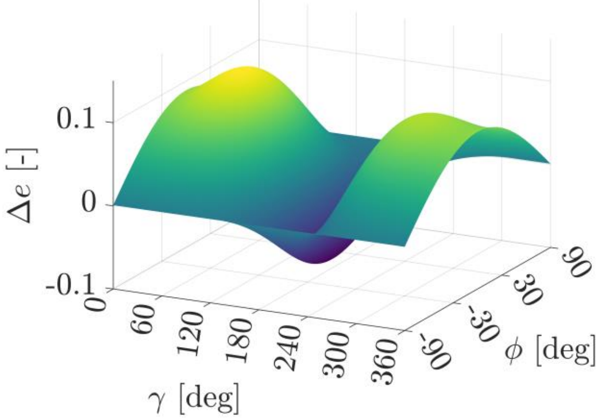
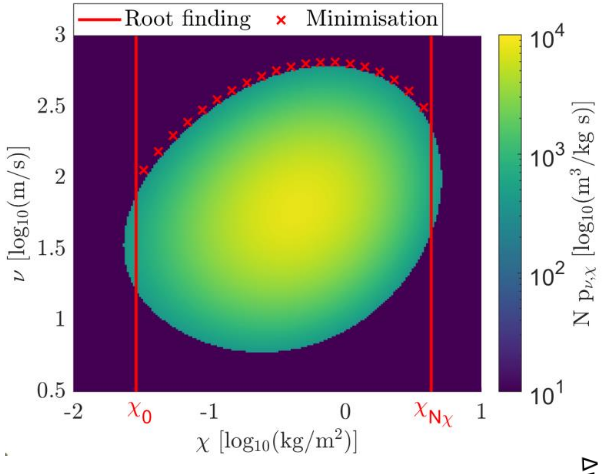
4D distribution in $A/M$, $\Delta v$, $\gamma$ and $\phi$

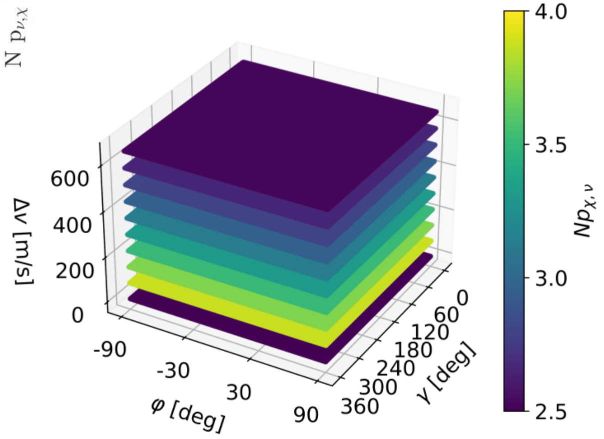
*Relations $\Delta\boldsymbol{\alpha} = f(\boldsymbol{\Delta v})$*

Variations $\Delta a, \Delta e, \Delta i$ as function of $\boldsymbol{\Delta v}$

Domain in Keplerian elements defined in the subset $(a, e, i)$



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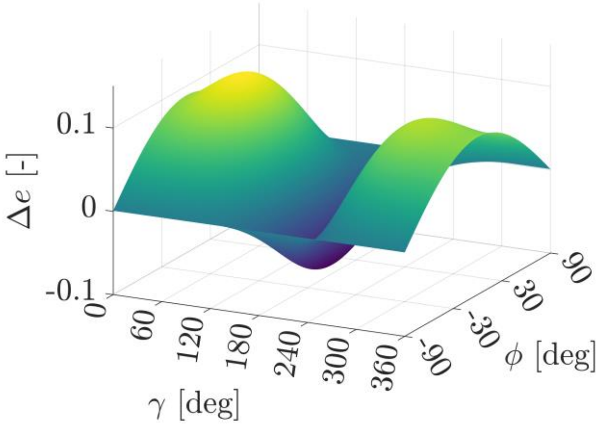


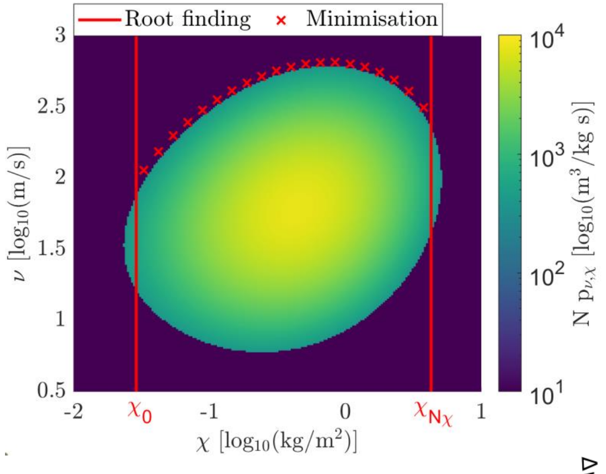


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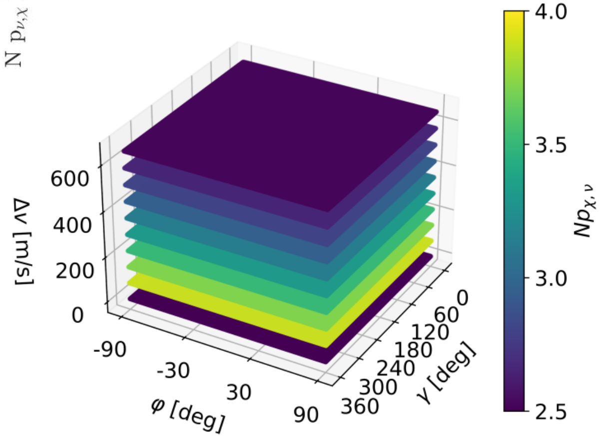



Variations $\Delta a, \Delta e, \Delta i$ as function of $\boldsymbol{\Delta v}$

If $\xi$ is high enough:

- Averaging over $\gamma, \phi$ $\rightarrow$ $\overline{\Delta\boldsymbol{\alpha}} = \overline{\Delta\boldsymbol{\alpha}}(\Delta v)$
- Maximisation over $\Delta v$ $\rightarrow \mathcal{D}_{\boldsymbol{\alpha}}$

Domain in Keplerian elements defined in the subset $(a, e, i)$

# Initial density estimation

## Density distribution in Keplerian elements

Density distribution approximated through a **binning approach:**

- Domain divided into bins with size proportional to the average density gradient:

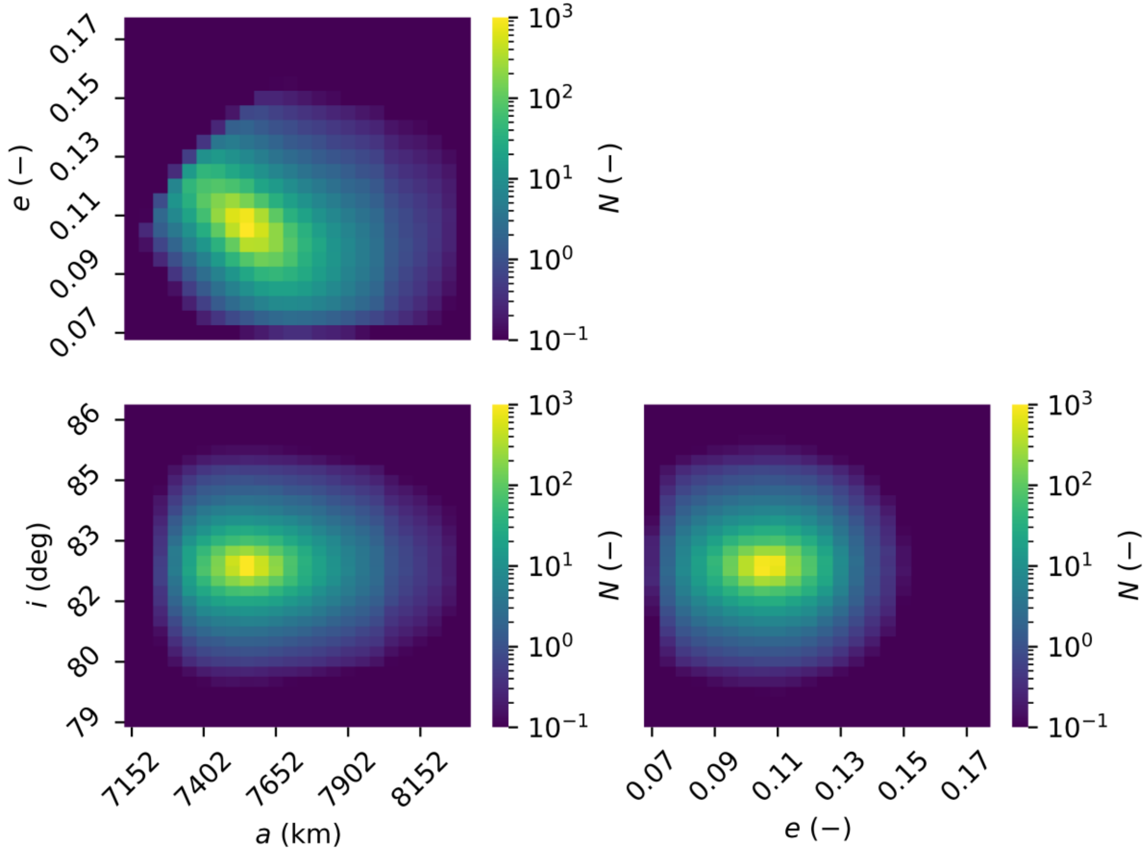
$$\delta \boldsymbol{\alpha} \propto \nabla_{\boldsymbol{\alpha}} \, p_{v,\chi}$$

- Density in Keplerian elements obtained through change of variable transformation:

$$p_{\boldsymbol{\alpha},A/M} = \frac{p_{\Delta v,A/M}\left(\psi^{-1}_{\boldsymbol{s}\to\boldsymbol{\alpha}}(\boldsymbol{\alpha})\right)}{|\det J_{\boldsymbol{s}\to\boldsymbol{\alpha}}|}$$

- Density in each bin averaged through Monte Carlo integration

*Initial density distribution in semi-major axis, eccentricity and inclination for the hypothetical explosion of satellite Cosmos-2292 in LEO*




$\psi_{\boldsymbol{s}\to\boldsymbol{\alpha}}$ = transformation from $(v_x, v_y, v_z)$ to $(a, e, i)$, $J_{\boldsymbol{s}\to\boldsymbol{\alpha}}$ = Jacobian of the transformation $\psi_{\boldsymbol{s}\to\boldsymbol{\alpha}}$

➢ Frey S., "Evolution and hazard analysis of orbital fragmentation continua," PhD thesis, Politecnico di Milano, 2020, Supervisors: Colombo C., Lemmens S., Krag., H.

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# Cloud propagation

## Sampling, Method Of Characteristics, and interpolation

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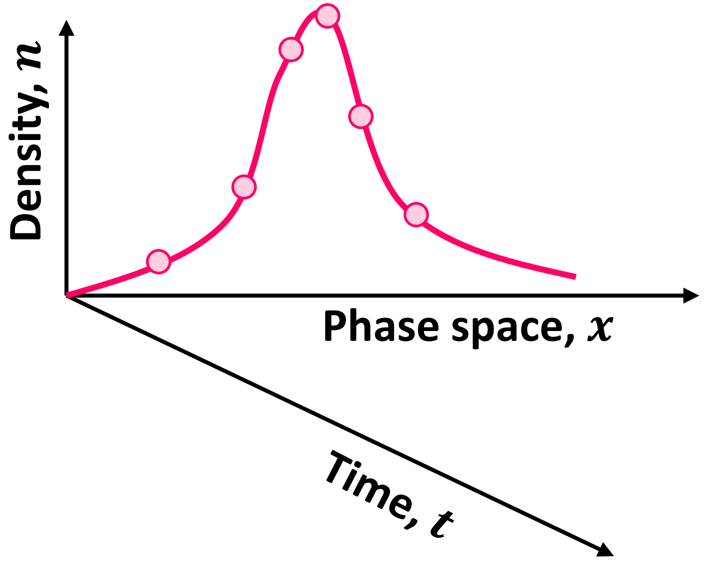
   - The samples are randomly extracted from the initial distribution
   - The samples are in the subset $\boldsymbol{x} \coloneqq \left(a, e, i, \frac{A}{M}\right)$



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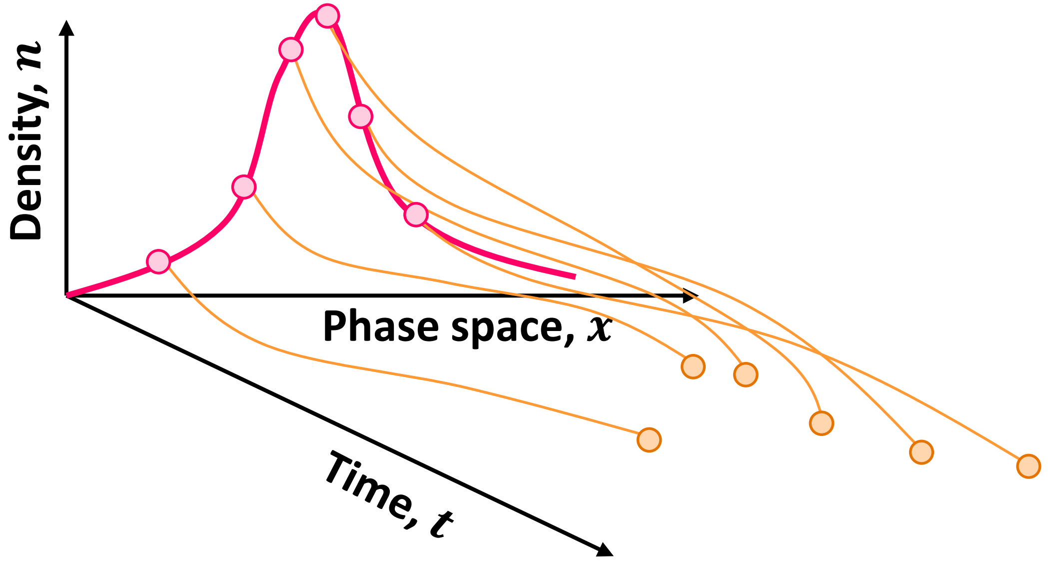
MOC: $\begin{cases} \dfrac{\mathrm{d}n}{\mathrm{d}t} = -n\, \boldsymbol{\nabla} \cdot \boldsymbol{F} \\ \dfrac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}t} = \boldsymbol{F} \end{cases}, \quad \boldsymbol{y} \coloneqq \left(a, e, i, \Omega, \omega, f, \frac{A}{M}\right)$

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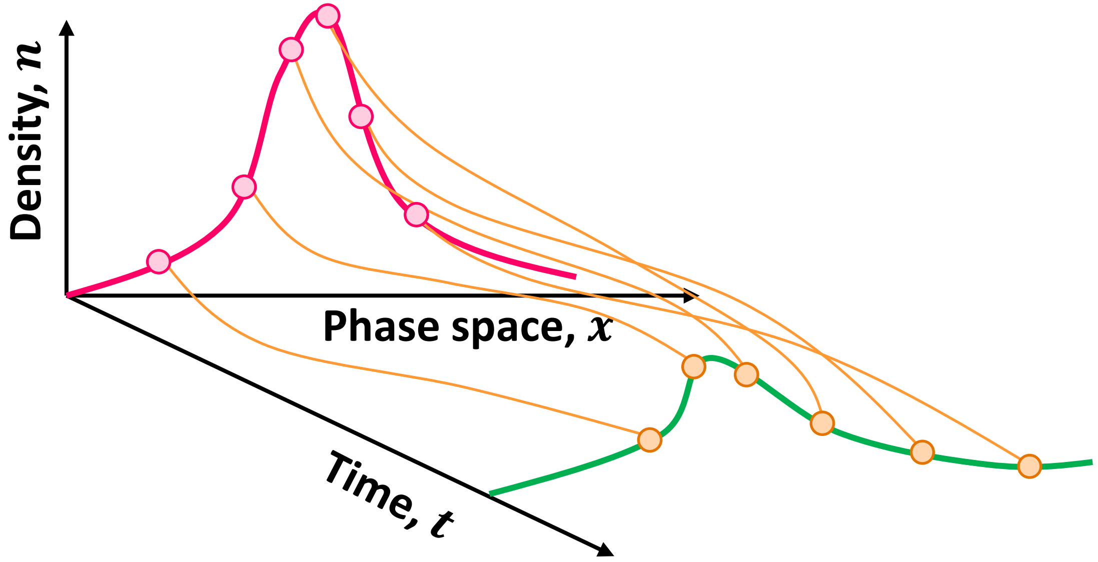
3. Density interpolation:
   - Binning in (up to) 6D phase space $\left(a, e, i, \Omega, \omega, \frac{A}{M}\right)$
   - Nearest-neighbour – like interpolation

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## Estimation of the impact rate

Impact rate: flux of fragments over the target area assuming

- Fixed target position $\boldsymbol{r}_t \rightarrow$ Computed $N$ times for $N$ target's mean anomaly $M \in [0,2\pi)$
- Area of the target $A_c \gg$ Area of the fragments $A_f$

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It can be computed from the phase space density:

1. In **Cartesian** coordinates

$$\dot{\eta}(\boldsymbol{r}_t, \boldsymbol{v}_t) = A_c \iiint_{\mathbb{R}^3} n_{\boldsymbol{s}}(\boldsymbol{r}_t, \boldsymbol{v}) \|\boldsymbol{v} - \boldsymbol{v}_t\| \, \mathrm{d}\boldsymbol{v}$$

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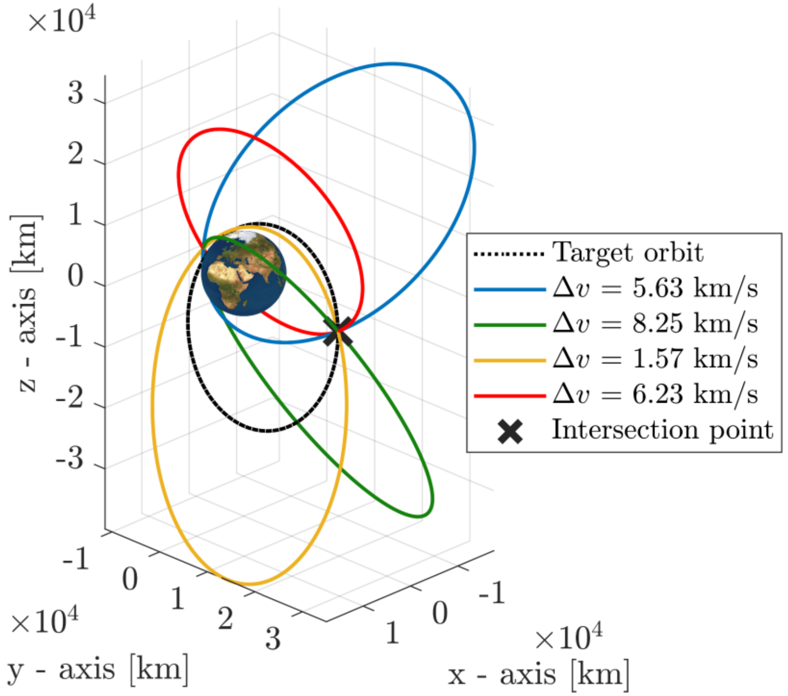
$$\dot{\eta}(\boldsymbol{r}_t, \boldsymbol{v}_t) = A_c \iiint_{\mathbb{R}^3} \sum_{k=1}^{4} \frac{n_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}, \boldsymbol{\beta}^k)}{\left|\det J^k_{\boldsymbol{r}\to\boldsymbol{\beta}}\right|} \left\|\boldsymbol{v}(\boldsymbol{\alpha}, \boldsymbol{\beta}^k) - \boldsymbol{v}_t\right\| \, \mathrm{d}\boldsymbol{\alpha}$$

*4 possible intersections, fixed $\boldsymbol{r}_t$ and $(a, e, i)$*




$\boldsymbol{\alpha} = (a, e, i)$, $\boldsymbol{\beta} = (\Omega, \omega, f)$, $J_{\boldsymbol{r}\to\boldsymbol{\beta}}$ = Jacobian of the transformation from $\boldsymbol{r}$ to $\boldsymbol{\beta}$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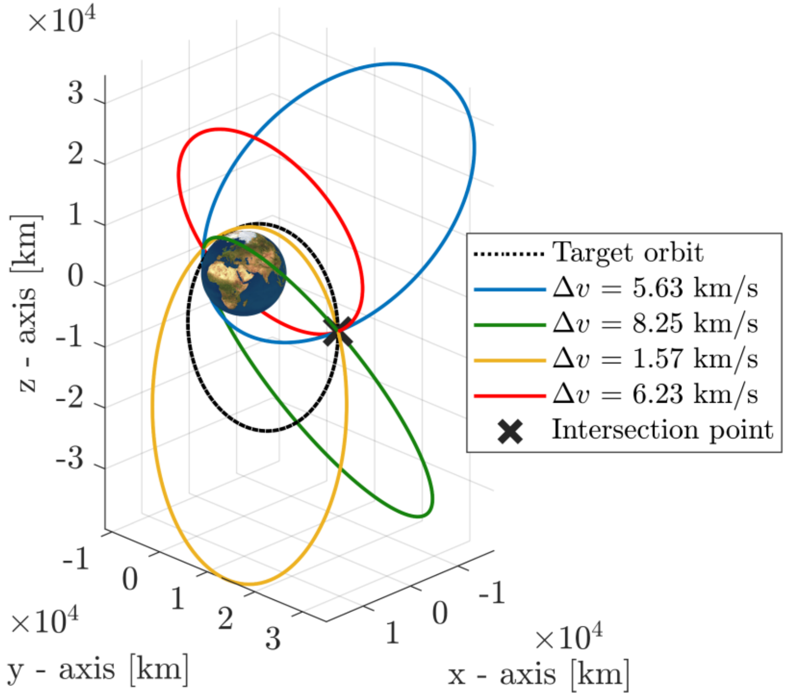


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Integrated semi-analytical in:

- Semi-major axis, eccentricity, inclination $(a, e, i)$
- Perigee radius, apogee radius, inclination $(r_p, r_a, i)$

$\boldsymbol{\alpha} = (a, e, i)$, $\boldsymbol{\beta} = (\Omega, \omega, f)$, $J_{\boldsymbol{r}\to\boldsymbol{\beta}}$ = Jacobian of the transformation from $\boldsymbol{r}$ to $\boldsymbol{\beta}$

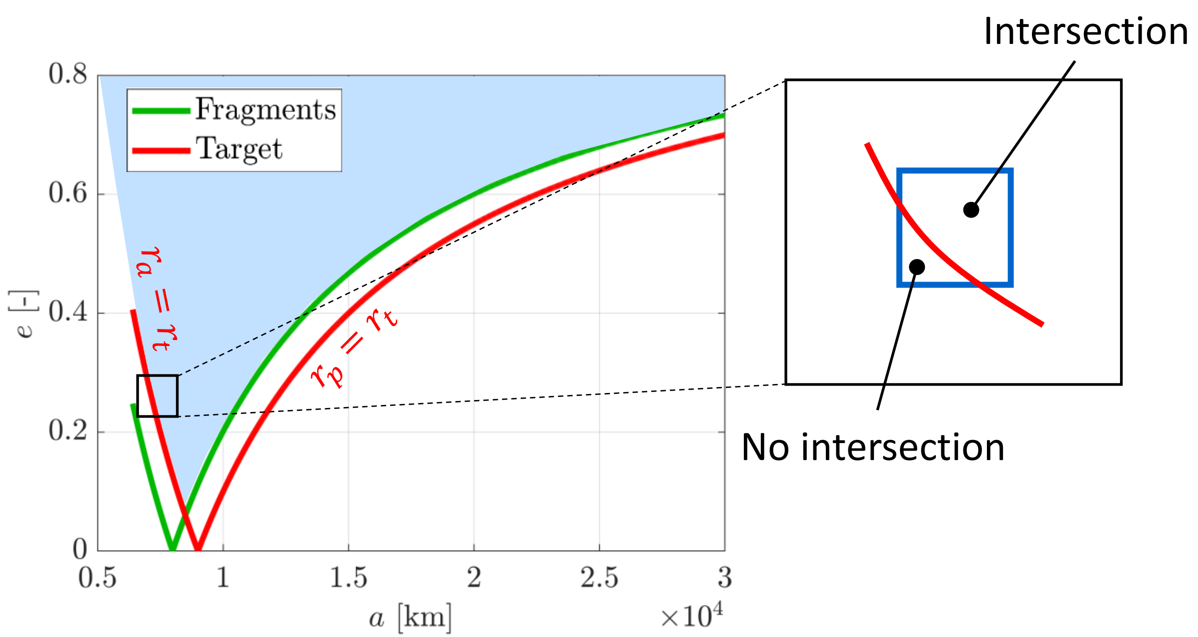

- Frey S., Colombo C., "Transformation of Satellite Breakup Distribution for Probabilistic Orbital Collision Hazard Analysis," *Journal of Guidance, Control, and Dynamics*, vol. 44, pp. 88-105, 2021.

# Fragmentation effects

## Impact rate with binning approach

**Integration in $a, e, i$**

- Elliptic integrals from the integration in $i$
- Easy solution of the integration in $a, e$
- Analytical solution not available in the entire domain




*Curves* $r_a = r_t$ and $r_p = r_t$ in the $(a, e)$ domain

# Fragmentation effects

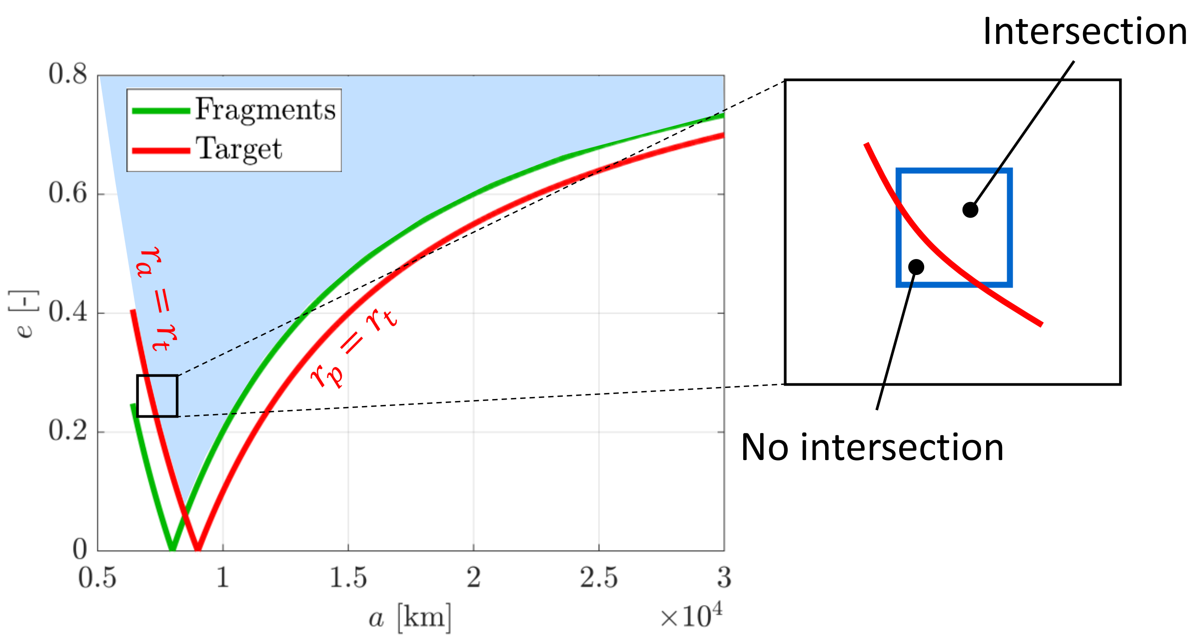
## Impact rate with binning approach

**Integration in $\boldsymbol{a, e, i}$**

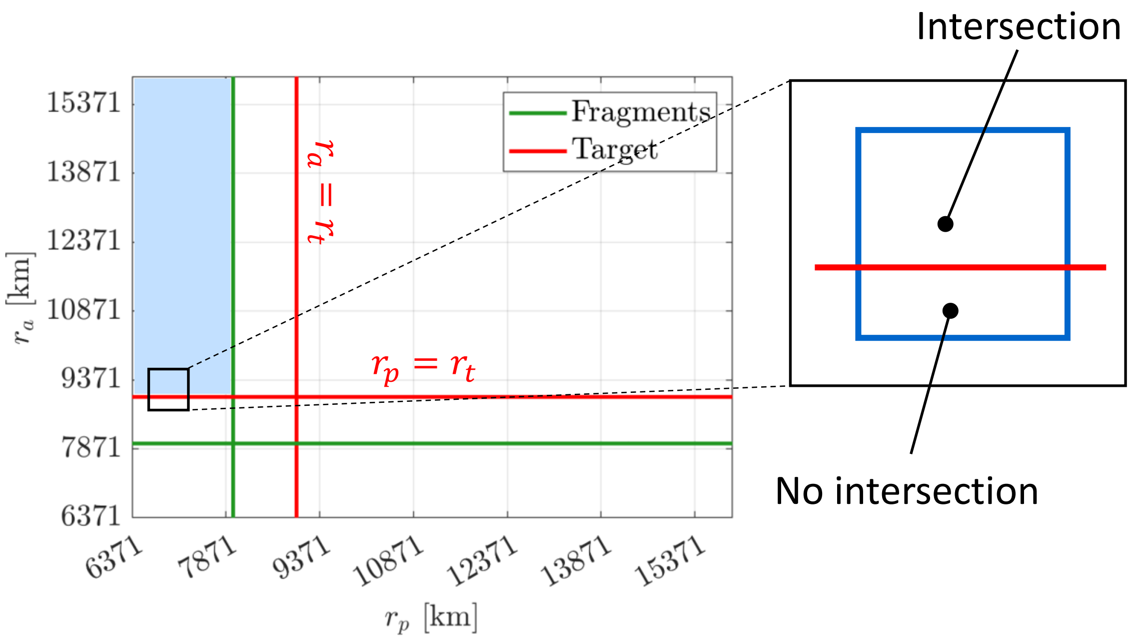
- Elliptic integrals from the integration in $i$
- Easy solution of the integration in $a, e$
- Analytical solution not available in the entire domain

**Integration in $\boldsymbol{r_p, r_a, i}$**

- Elliptic integrals from the integration in $i$
- Complex polylogarithms from the integration in $r_a, r_p$
- Analytical solution available in the entire domain



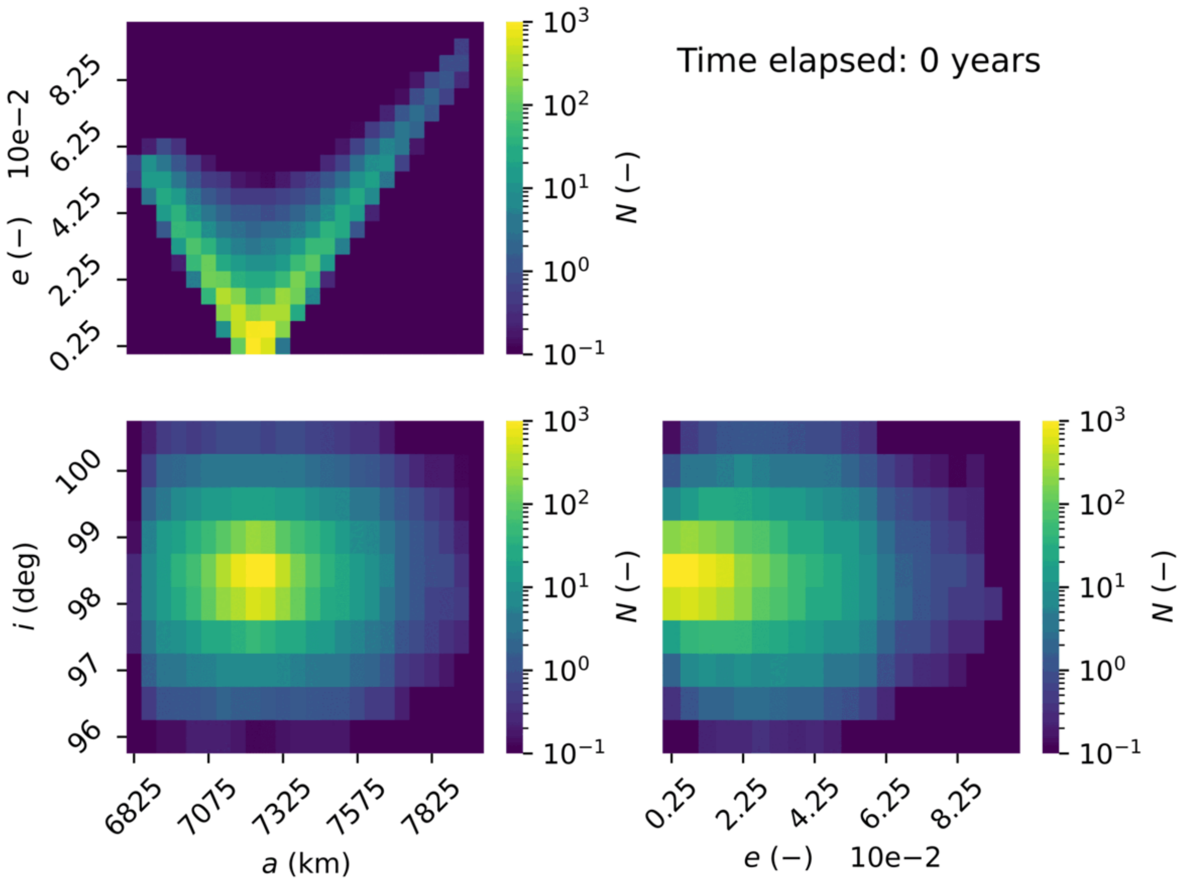

*Curves* $r_a = r_t$ and $r_p = r_t$ in the $(a, e)$ domain




*Curves* $r_a = r_t$ and $r_p = r_t$ in the $(r_p, r_a)$ domain

# Applications

## Fengyun-1C fragmentation under $J_2$ and drag



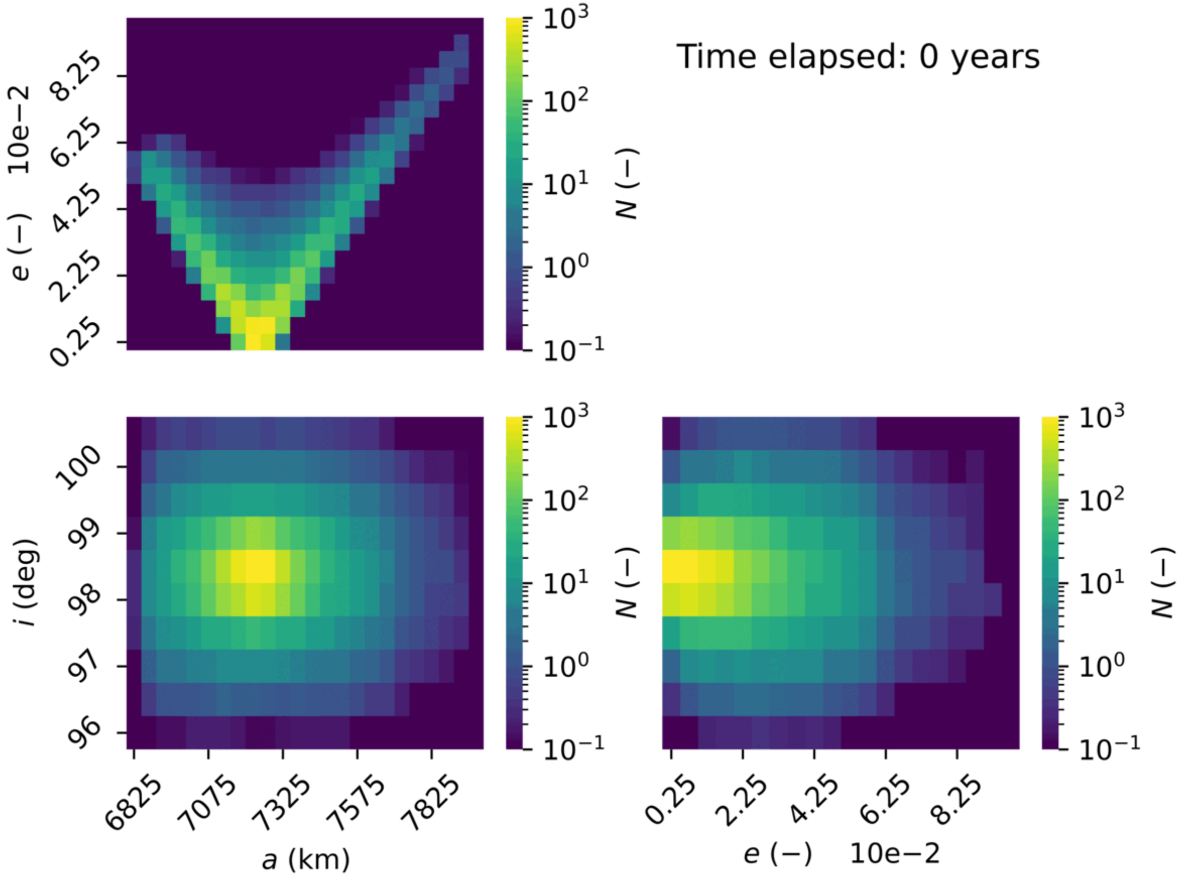

*Density distribution over time under J2 and drag perturbations*

**Parent orbit:** $a = 7231$ km, $e = 0.001$, $i = 98.6$ deg, $\Omega = 106.1$ deg, $\omega = 262.0$ deg, $f = 133.5$ deg

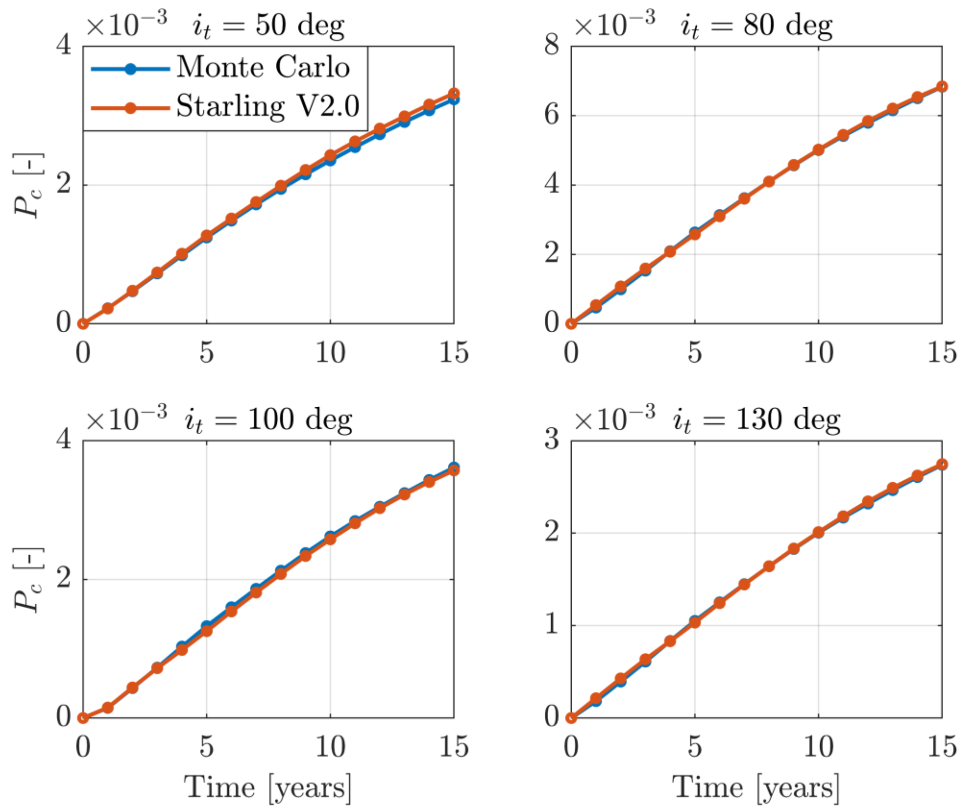
# Applications

## Fengyun-1C fragmentation under $J_2$ and drag

$$P_c = 1 - e^{-\int \dot{\eta}\,\mathrm{d}t}$$




*Density distribution over time under J2 and drag perturbations*

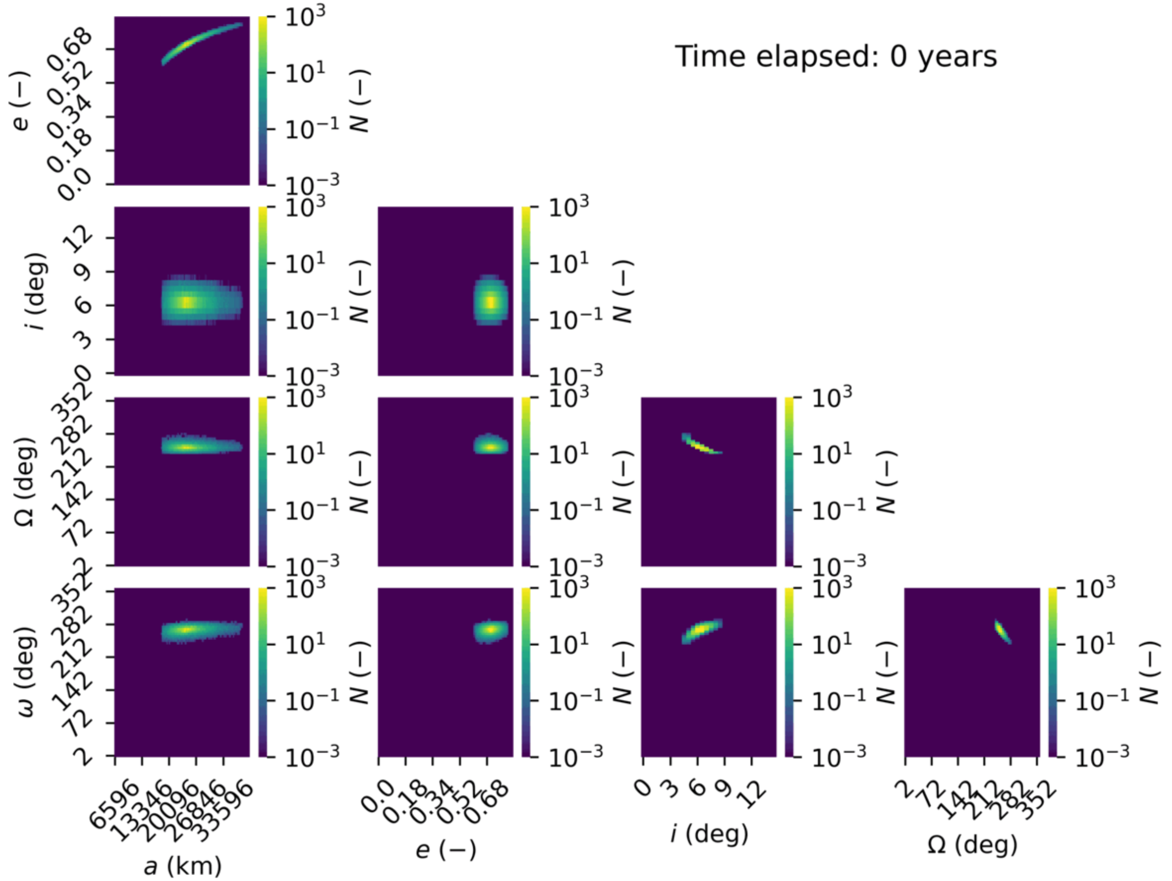



*Collision probability over time for different values of target inclination – Accuracy analysis against Monte Carlo*

**Target orbit:** $a = 7171$ km, $e = 0.0$, $i = 50/80/100/130$ deg, $\Omega = 0.0$ deg, $\omega = 0.0$ deg

# Application

## Ariane 5 explosion in GTO under $J_2$, drag, SRP, Moon + Sun

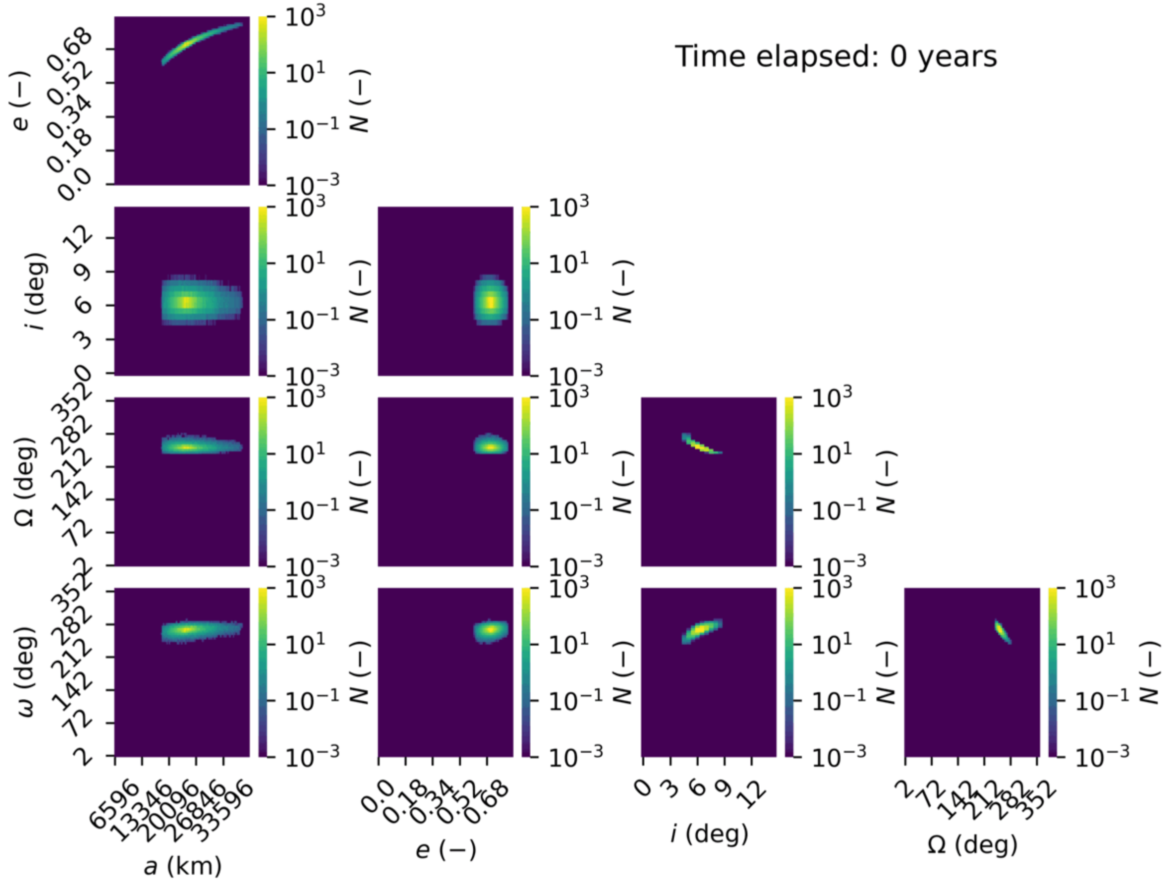



*Density distribution over time under J2, Drag, SRP, Moon and Sun perturbations*

**Parent orbit:** $a = 24443$ km, $e = 0.709$, $i = 6.5$ deg, $\Omega = 253.2$ deg, $\omega = 271.8$ deg, $f = 43.5$ deg

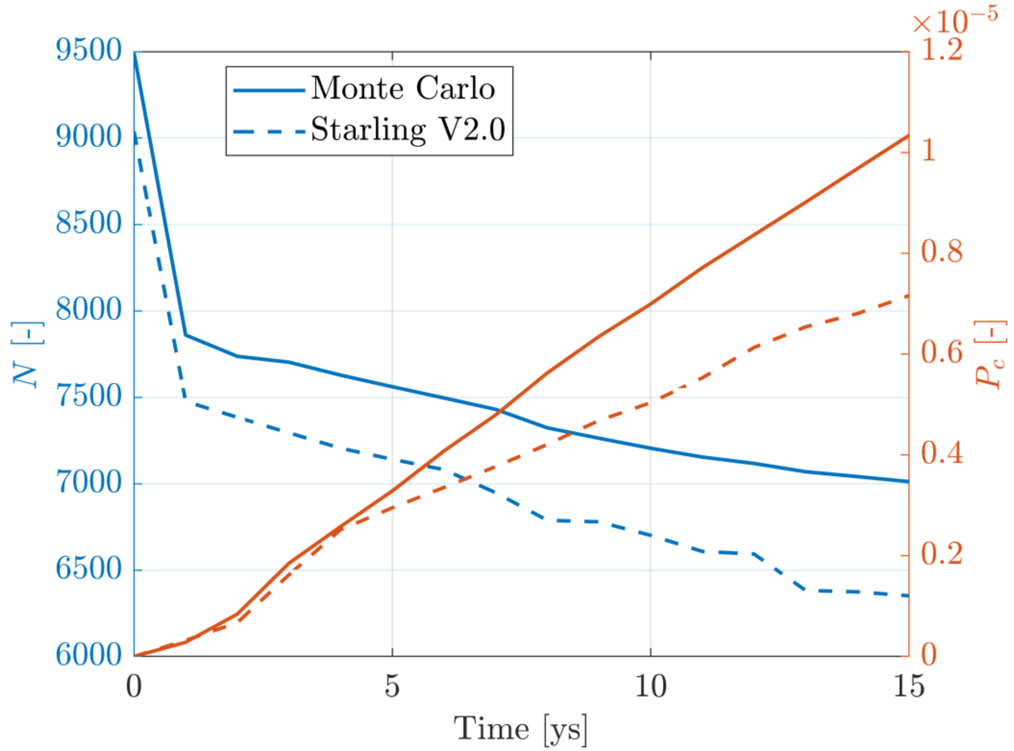
# Application

## Ariane 5 explosion in GTO under $J_2$, drag, SRP, Moon + Sun




*Density distribution over time under J2, Drag, SRP, Moon and Sun perturbations*

*Number of fragments and collision probability with Syracuse 4A over time – Accuracy analysis against Monte Carlo*




**Target orbit (Syracuse 4A):** $a = 24131$ km, $e = 0.725$, $i = 6.0$ deg, $\Omega = 264.8$ deg, $\omega = 167.3$ deg

# Coclusions & Future works

## Conclusions

- The **probabilistic definition of the phase space domain** allows to accurately target the phase space reachable by the ejected fragments
- The **automatised definition of the grid** in Keplerian elements reduces the user's responsibilities, granting accuracy independently of the fragmentation type
- The **semi-analytical computation of the impact rate** dramatically improved the accuracy and efficiency in the estimation of the fragmentation effects
- The **model**, being **agnostic to the force model**, proved validity under complex dynamical regimes

## Future works

- **Extensive validation** of the model against Monte Carlo simulations
- Application of the model for estimating **sustainability indices in any orbital region**

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# Density-based debris cloud propagation and collision risk estimation through a binning approach

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