Cislunar orbit dynamics with an application to space situational awareness

Mathilda Bolis[1, a *], Elisa Maria Alessi[2, b] and Camilla Colombo[1, c]

[1]Department of Aerospace Science and Technology, Politecnico di Milano, Italy

[2]Istituto di Matematica Applicata e Tecnologie Informatiche "Enrico Magenes", CNR, Italy

[a]mathilda.bolis@polimi.it, [b]em.alessi@mi.imati.cnr.it, [c]camilla.colombo@polimi.it

Keywords: cislunar space, periodic orbits, stability, end-of-life, disposal, dynamical systems

Abstract. Given the growing number of planned missions and satellite launches, it is clear the cislunar orbital domain gained great interest from both industries and space agencies in the last few years. The chaotic dynamics that characterise this area of space set new challenges that have not been considered for near-Earth space. An increasing number of space objects orbiting in the chaotic cislunar domain could lead to a more complex debris problem than is found near-Earth today, making the design of effective End-of-Life (EoL) disposal strategies more important than ever. This Ph.D. research aims to develop a dynamic cartography of cislunar space through tools derived from dynamical systems theory. Thus, it will be possible to characterise the long-term stability of different families of orbits in cislunar space, creating a catalogue of stable and unstable orbits; to develop a tool to define effective and customisable EoL strategies; and to analyse the behaviour of possible fragmentations in the cislunar domain.

Introduction

Space is one of the final frontiers of scientific exploration: its importance as an economic and strategic asset has never been more pronounced than now. In a context in which hundreds of space missions are planned for the next few years and the number of in-orbit satellites exponentially increases, both industries and space agencies have shown a strong interest in the cislunar domain, a region strongly influenced by both the Earth and the Moon. The dynamics of this region cannot be modelled with the techniques developed for near-Earth space. The influence of the Moon, other celestial bodies, and multiple perturbations make the motion of an object in this region inherently chaotic. This means that, given the initial state of a satellite, it is not trivial to determine what its final state will be, especially when long-term simulations are considered. Consequently, if the number of space objects in the cislunar region increases, as has been observed lately in the vicinity of Earth [1], a potential debris issue will arise there as well, with additional complications given by the chaotic dynamics. In this context, one of the most effective mitigation measures is the careful design of the EoL phase of a mission. Creating a method to achieve simple and customizable disposal strategies is a matter of primary importance.

This Ph.D. research will therefore focus on the development of a dynamic cartography of cislunar space, thereby trying to define, through dynamical systems theory, parameters that will help characterize the state of a spacecraft. This will make it possible to develop, as a result, a tool capable of selecting the best disposal strategy for a spacecraft, given as input its operational orbit, end-of-life epoch, and amount of fuel still on board. Ideally, the legal aspects of the problem will also need to be considered in the development of this tool. The following sections present a description of the methodologies to be developed as well as of the planned activities.

Dynamic models

Many dynamical models can at least partially describe cislunar space dynamics. The Circular Restricted Three-Body Problem (CR3BP) [2] provides a good approximation of the qualitative behaviour of the motion of a spacecraft when its dynamic is mainly governed by the influence of two primaries with nearly circular orbits and a small inclination difference. Within it, it is possible to identify one energy integral of motion, the Jacobi Constant (JC), and five equilibrium points of the system, the Lagrangian points [3].

When considering the cislunar region, the main perturbation which is not included in the earlier model is the Sun's gravitational pull. To take it into account, the CR3BP can be modified into a Bicircular Restricted Four-Body Problem (BR4BP) (e.g. [4]). The major advantage of using this model is that, in general, it is simple to handle, implement, and use, although the effect of a third major body is added to the system. Moreover, despite in this case the motion of the three main bodies is not coherent, the model shows behaviours qualitatively similar to the ephemeris one, especially in some areas of the phase space.

The dynamic model that most closely matches reality is the N-body problem [5], which, however, comes with a greater computational effort. The influence of perturbations also plays a significant role in determining the motion of a spacecraft in cislunar space. For example, the effect of Solar Radiation Pressure (SRP) [6] becomes particularly relevant in the case of spacecraft with large area-to-mass ratios, while the lunar gravitational field influence has a not negligible effect when in proximity of the Moon [7].

Families of orbits in cislunar space

A particularly interesting aspect of CR3BP is that, by leveraging some of its properties, it is possible to define families of periodic and quasi-periodic orbits. Once initial conditions for possible periodic orbits are identified, differential corrections are exploited to refine them, and continuation methods are used to compute different orbits belonging to the same family [3]. These orbits can then be refined in more complex dynamical models. Families of periodic orbits can be found in cislunar space through continuation techniques, such as are Lyapunov orbits, Halo orbits, Near Rectilinear Halo orbits (NRHOs), Lissajous Orbits, Distant Retrograde orbits, etc. [8][9]. Cycler or horseshoe orbits (e.g. [10]), as well as orbits around the Moon, equivalent, for example, to Highly Elliptical Orbits (HEOs) around the Earth, can also be observed.

End-of-life phase design for spacecraft in cislunar space

The EoL phase of a mission is designed by performing disposal primarily with one of the following four strategies: insertion into a heliocentric orbit, impact on the Moon, destructive Earth re-entry, or insertion into a cislunar frozen graveyard orbit. Among these, the most complex, least studied, and exploited approach is the last one.

In [11], all four approaches are considered, and a dynamic cartography of cislunar space based on Monte Carlo (MC) simulations is proposed. Samples are generated along some cislunar orbits chosen as case studies and, after applying a manoeuvre to each of them, a random perturbation is introduced into the system and several MC simulations are performed, thus evaluating the robustness of each solution to possible disturbances. An analysis is also performed about the possibility for the satellite to interfere with the geostationary pseudo-region.

In [12] instead, the EoL design for GAIA, orbiting Sun-Earth L_2 (SEL2), is formulated as a multi-objective optimization problem. This approach is proposed for disposal with impact on the Moon, with destructive re-entry to Earth, and with insertion into a heliocentric orbit. For the latter case, an energy approach is also suggested, based on the variation of the problem JC to close the Hill curves in SEL2. An alternative approach, based on defining manoeuvres that ensure the constant increase of the minimum orbit intersection distance, the minimum distance between Earth and the disposal orbit, is proposed in [13]. Some of the cited studies refer to the design of the EoL phase of missions orbiting Lagrangian points of the SE system. Nevertheless, it is considered of interest to cite their contributions, as some strategies developed in those contexts could also be applied to cislunar space. Following the same logic, the case reported in [14] is also analysed. Possible disposal options at EoL for HEOs around Earth are studied, taking advantage of natural orbital perturbations and phase space to plan manoeuvres that ensure low fuel consumption.

Methodology and planned activities

Given what was discussed in the previous sections, this research work will be organized as outlined in the following paragraph.

First, a customized propagator for cislunar space will be developed, where the effect of perturbations will be included in the dynamic model only if considered truly non-negligible, exploiting both theoretical astrodynamics and long-term computer simulations. This will give the possibility to combine the simplicity of the CR3BP with the precision of an N-body perturbed model. Data recovered from past missions will be used to validate the developed tool.

It will be interesting then to define a catalogue of cislunar families of orbits, highlighting their characteristics and possible uses for missions with different objectives. An analysis of the stability properties of these families of orbits will follow. To do it, it will be necessary to identify stable and unstable areas of the phase space, exploiting tools such as Poincaré maps, finite-time Lyapunov exponents, and other tools of dynamical systems theory. The analyses developed will also be applied to the study of possible fragmentations in cislunar space.

Properties of HEO with large semi-major axes and eccentricities will be analysed, thanks to Keplerian map theory. In addition, a possible extension of the same approach to HEOs around the Moon and Lagrangian point orbits will be proposed. Also, strategies such as the ones developed in [14] can be exploited for the same application.

Finally, accurate design of EoL for cislunar missions will be performed, highlighting not how the single EoL scenario can be applied to the single mission, but identifying areas of space where a certain disposal strategy proves to be the optimal option, in terms of fuel savings, duration of the disposal phase, and possible risks to current or future missions. Passive disposal solutions, designed by taking advantage of the chaotic dynamics of cislunar space, would be preferred. Similar results have been obtained in the context of the ReDSHIFT project [15], for a region extending from low Earth orbit to the geostationary orbit.

The knowledge developed in this context could also be applied to the design of transfer trajectories and operational orbits for different missions, identifying frozen and quasi-frozen orbits or conditions of special interest.

As an outcome, an algorithm to select the best disposal strategy for a spacecraft will be developed, given as input its operational orbit, EoL epoch and amount of fuel remaining on board, avoiding performing for each disposal design hundreds of numerical simulations. Ideally, the algorithm will need to also consider the legal aspects of the problem, to represent, as accurately as possible, the complexities that characterize the cislunar domain. The developed tool will then be applied to specific cases, possibly ESA missions.

Conclusions

The research introduced in this paper focuses on the development of a dynamical cartography of cislunar space using tools derived from dynamical systems theory. This research aims to create a catalogue of orbit families in cislunar space, to study their stability properties, and to develop a tool useful for defining effective and customizable EoL disposal strategies.

First, dynamical models that are well suited to cislunar space are briefly described and presented, along with the periodic and quasi-periodic orbits that characterize it. Then a brief overview of the methods developed for disposal at EoL is given. Finally, planned activities within this Ph.D. research are described and briefly commented on.

Acknowledgments
This research has received funding as part of the work developed for the agreement n. 2023-37-HH.0 for the project "Attività tecnico-scientifiche di supporto a C-SSA/ISOC e simulazione di architetture di sensori per SST", established between the Italian Space Agency (ASI) and Politecnico di Milano (POLIMI).

References

[1] *ESA's Annual Space Environment Report*, 2021. URL: https://www.sdo.esoc.esa.int/environment_report/Space_Environment_Report_latest.pdf

[2] Szebehely, V. *Theory of Orbit: The restricted problem of three Bodies.* Academic Press, New York, 1967. doi:10.1016/B978-0-12-395732-0.X5001-6.

[3] Koon, W. S., Lo, M. W., Marsden, J. E. and Ross, S. D. *Dynamical Systems, the Three-Body Problem and Space Mission Design.* Marsden Books, 2011.

[4] Mingotti, G. P. *Trajectory Design and Optimization in Highly Nonlinear Astrodynamics.* PhD thesis, Politecnico di Milano, Milan, Italy, 2010.

[5] Meyer, K., Hall, G., and Offin, D. *Introduction to Hamiltonian Dynamical Systems and the N-Body Problem,* Second Edition, Vol. 90. Springer New York, 2009. doi: 10.1007/978-0-387-09724-4

[6] Jorba-Cuscó, M., Farrés, A., and Jorba, À. *On the stabilizing effect of Solar Radiation Pressure in the Earth-Moon system.* Advances in Space Research, 67(9), 2812–2822, 2021. doi: 10.1016/j.asr.2020.01.017

[7] Michael, W.H., Blackshear, W.T. *Recent results on the mass, gravitational field and moments of inertia of the moon.* The Moon 3, 388–402 (1972). doi: 10.1007/BF00562460

[8] Howell, K. C. *Three-dimensional, periodic, 'Halo' orbits*. Celestial Mechanics 32, 53–71, 1984.

[9] Hénon, M. *Generating Families In The Restricted Three-Body Problem*. Springer-Verlag, Springer Science Business Media, 1997.

[10] De Leo, L., Pontani, M. *Low-Thrust Orbit Dynamics and Periodic Trajectories in the Earth–Moon System.* Aerotec. Missili Spaz. 101, 171–183 (2022). doi:10.1007/s42496-022-00122-9

[11] Guardabasso, P., Skoulidou D. K., Bucci L., Letizia F., Lemmens S. and Lizy-Destrez S. *Cislunar debris mitigation development of a methodology to assess the sustainability of lunar missions*. in 72nd International Astronautical Congress (IAC), Dubai, 2021.

[12] Armellin, R., Rasotto, M., Di Lizia, P. and Renk, F. *End-of-life disposal of libration point orbit missions: The case of Gaia*. Advances in Space Research 56, 461–478, 2015. doi: 10.1016/j.asr.2015.03.014

[13] Alessi, E. M. and Sánchez, J.-P. *MOID-increasing disposal strategies for LPO missions*. in 65th International Astronautical Congress (IAC), Toronto, 2014.

[14] Colombo C., Alessi E. M., Van der Weg W., Soldini S., Letizia F., Vetrisano M., Vasile M., Rossi A., Landgraf M. *End-of-life disposal concepts for Libration Point Orbit and Highly Elliptical Orbit missions.* Acta Astronautica, Vol. 110, p. 298-312, 2015. doi: 10.1016/j.actaastro.2014.11.002

[15] Rossi A., Colombo C., Tsiganis K., Beck J., Rodriguez J. B., Walker S., Letterio F., Dalla Vedova F., Schaus V., Popova R., Francesconi A., Stokes H., Schleutker T., Alessi E. M., Schettino G., Gkolias I., Skoulidou D. K., Holbrough I., Bernelli Zazzera F., Stoll E. and Kim Y. *ReDSHIFT: A global approach to space debris mitigation.* Aerospace 5, 2018. doi: 10.3390/aerospace5020064