



A sequential remeshing framework for quasi-static propagation of large and curved delaminations using the virtual crack closure technique[☆]

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ABSTRACT

Delamination is a predominant failure mode in composite materials, needing advanced numerical techniques for accurate predictions. This paper presents a computational framework for simulating delamination growth under quasi-static loading conditions using the Virtual Crack Closure Technique (VCCT) in a sequential-simulation automated-remeshing approach to address the technique's inherent mesh dependence. The delamination propagation length is determined by substituting Strain Energy Release Rate (SERR) truncated Taylor's expansion series into the Griffith's energy-based propagation criterion. The evolving delamination front is updated through an automated remeshing algorithm, enabling accurate tracking of the delamination front advancement and mitigating the mesh dependency observed in conventional VCCT implementations. The framework is demonstrated through multiple models featuring large and curved delamination fronts under various and varying mixed-mode loading conditions.

1. Introduction

Delamination is a critical failure mechanism in composite structures [1], resulting from the breakdown of interlaminar bonds due to localised stresses [2]. Various Finite Element (FE) approaches have been developed to simulate delamination initiation and propagation under both quasi-static and cyclic loading conditions. The most commonly used ones are the Cohesive Zone Model (CZM) [3] and the Virtual Crack Closure Technique (VCCT) [4,5].

The CZM concept was initially introduced for elastic-plastic fracture in metals [6]. This was subsequently adopted to simulate the failure in bonded joints and later extended to model delamination in composite laminates by discretizing the interface between plies with thin cohesive elements [7]. The inter-ply behaviour is modelled with a Traction-Separation Law (TSL) to simulate progressive damage in the material. This allows the CZM to simulate a Fracture Process Zone (FPZ) at the delamination tip without explicitly modelling micromechanical non-linear phenomena like matrix cracking, fibre-matrix debonding, and fibre bridging, reducing the computational cost [8]. A wide range of TSLs have been proposed in the literature [9–13] requiring specific calibration of curve parameters on experimental data [14,15].

On the other hand, the VCCT is based on Linear Elastic Fracture

Mechanics (LEFM) [16]. This technique is formulated under the assumption that the energy required to propagate a crack is equal to the energy required to close it by the same length. Therefore, the SERR can be directly calculated by evaluating nodal forces, displacements, and a relevant crack extension area. Originally developed as Crack Closure Technique (CCT), the technique required two simulations. The first one was used to obtain the forces at the delamination front and the second one to calculate the displacement in the same location after a one-element-length delamination advancement. By introducing the assumption of self-similar propagating fronts, the nodal displacement can be obtained from the nodes in front of the delamination tip, reducing the number of simulations to one. Finally, the resulting SERR is directly compared to the fracture toughness to evaluate whether the front advances, without requiring the input of additional numerical values. Mode-Mixity (MM) formulations can be implemented by distinguishing the three orthogonal directions relative to the delamination front: mode I (normal to the delamination plane), mode II (normal to the delamination front), and mode III (tangent to the delamination front). The VCCT's direct output of SERR values allows for the integration of Paris-like propagation laws for cyclic loading conditions [17], but its accuracy, along with other LEFM based approaches, is limited by the assumptions of a perfectly brittle material with a FPZ of negligible size [18]. Additionally, the VCCT relies on an initial delamination to

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Nomenclature

a	Delamination length.
B	Bartlett function.
$da_{\max}$	Maximum delamination length increment between simulations.
G_I, G_{II}, G_{III}	SERR values associated with each fracture mode.
$G_{Ic}, G_{IIc}, G_{IIIc}$	Limit SERR values associated with each fracture mode.
G_{eq}	Equivalent SERR value calculated with a mode mixity formulation.
G_c	Equivalent limit SERR value calculated with a mode mixity formulation.
G_{eff}	Effective SERR: ratio between G_{eq} and G_c .
MM	Mode Mixity.
$\hat{n}$	Vector normal to a curve or a surface.
$N_{\max}$	Maximum number of simulations.
P	Applied load.
$P_{\max}$	Maximum applied load.
RF	Reaction forces.
W_f	Smoothing strength on the front shape.
W_s	Smoothing strength on the SERR distribution along the front.
x, y	Delamination front coordinates.
η	BK law exponent.

model propagation, not being able to detect the onset of damage in the pristine material, unlike the CZM.

As demonstrated from the first implementations of the VCCT [16], its accuracy is highly dependent on the mesh quality and regularity around the crack front. This sensitivity is especially evident along large or curved delamination fronts, where regular meshes introduce discontinuities along the front in the form of steps with respect to a conformal mesh (both shown in Fig. 1). These discontinuities lead to large spurious numerical peaks in the computed SERR values. To address these issues, several improvements have been proposed. Early modified formulations interpolate the displacement field away from the nodal values and adjust the crack extension area to create a virtual mesh perpendicular to the delamination front [19]. This approach was used by Xie and Biggers [20] to extend the VCCT applicability to arbitrarily shaped fronts. They demonstrate better results compared to the base implementation of the technique, but they did not achieve the accuracy of their reference conformal mesh. Similar results have later been obtained by Liu et al. [21,22]. Other proposed methods employ the eXtended Finite Element Method (XFEM) to enable the computation of the SERR values without explicitly introducing a discontinuity into the mesh [23]. This approach is particularly powerful with 3D crack propagations, but it becomes less

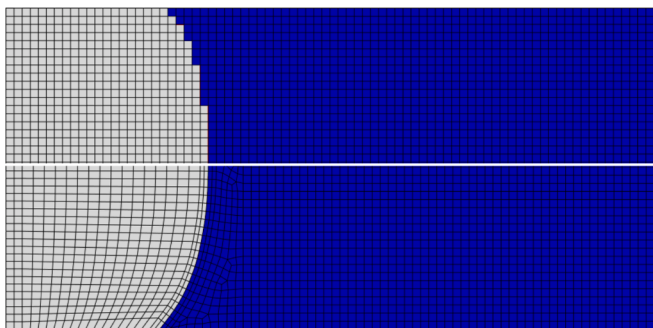


Fig. 1. Regular (top) and conformal (bottom) mesh on a curved front. The blue elements represent the undamaged interface.

relevant when the delamination surface can be defined a priori. Additionally, the XFEM does not directly calculate the SERR as it relays on additional computational schemes such as the J-integral [24,25] or the VCCT itself [26]. More recent publications regarding the VCCT introduce enriched interfaces, consisting of elastic elements with a cohesive law, in the damaged surface enabling a gradual transfer of tractions before the complete separation, bringing the VCCT's behaviour closer to the CZM. This approach is able to mitigate the limitations caused by the negligible FPZ and increase the technique's numerical stability through the gradual release of the nodes [27]. This approach was shown by Daniel et al. [28] to alleviate the mesh dependence and to remain accurate with large elements, but still, it achieves better results with a front conformal mesh. Alternative computational schemes have been explored beyond the VCCT, in conjunction with remeshing techniques, to model both fatigue and quasi-static loading conditions [29–33]. In particular, Davis et al. [33] presented an energy-based approach to calculate the delamination propagation length using the Virtual Crack Extension (VCE) method. This method computes the strain energy release rate (SERR) from the derivatives of the global stiffness matrix with respect to crack length through nodal perturbations. Despite these advancements, existing VCCT implementations still face challenges in handling arbitrarily shaped delamination fronts while maintaining numerical stability and accuracy. Furthermore, remeshing strategy have not been extensively explored to capture delamination growth with VCCT solutions.

In this paper, we present a VCCT-based sequential remeshing framework designed to simulate delamination growth under quasistatic loading conditions. The Sequential Remeshing Framework (SRF) is based on the sequential algorithm concept used by Martulli [17,34] and Safaei [35]. The SRF is implemented to model static propagation by subdividing the load in multiple simulations with progressively higher loads. The finite element mesh is updated between load steps as the delamination advances, aiming to address the VCCT inherent mesh dependency. Starting from the formulation presented by Davis et al. [33], the delamination increment at each step is determined using Griffith's energy criterion [36] and SERR truncated Taylor's expansion series. This formulation is further expanded by implementing a mode-mixity formulation with a varying mode-mixity correction. A comprehensive investigation of the algorithm's performance is conducted and presented, supported by numerical examples to illustrate the SRF's capability in modelling delamination growth under diverse and changing loading modes.

2. The sequential remeshing framework

The Sequential Remeshing Framework (SRF) is designed to simulate stable delamination propagation in composite laminates under quasi-static loads. Each analysis is based on an automated sequence of static VCCT simulations. The total load is divided in small increments: in each simulation, a static load ramp is applied up to an increasingly higher maximum load. Each simulation is followed by an automated procedure to extract the results and update the model by advancing the delamination front (Fig. 2). The delamination propagation length is calculated by applying Griffith's energy balance on a truncated Taylor's expansion

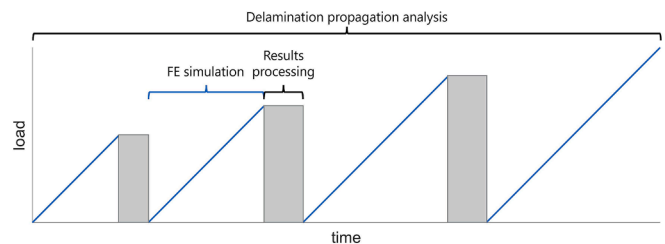


Fig. 2. Sequential simulation scheme.

series of the SERR values, approximating the advanced front. In Abaqus, a partition is a tool to subdivide geometric entities into smaller ones. The updated front is reintroduced in the FE model through a series of partitions: each front node and the segment connecting each couple of adjacent nodes is drawn on the specimen surface and extruded through the whole thickness. To guarantee smoother results, the VCCT simulations are run on front-conformal meshes. The algorithm keeps running new simulations until the required maximum load is reached. For the rest of this work, the term “analysis” will indicate the set of consecutive simulations that together reproduce the whole load ramp and

delamination advancement of the structure, while “simulation” will indicate the individual simulation applying the load up a certain increment (Fig. 2).

Although the load application is linear (Fig. 2), the framework is able to model geometrical non-linearities and large displacements, provided that the final equilibrium configuration is unique and not affected by the reloading procedure. Further work should be carried out to understand the framework's limitations in describing path-dependent problems such as buckling and snap-through instabilities, where multiple equilibrium configurations may exist and different final states may be

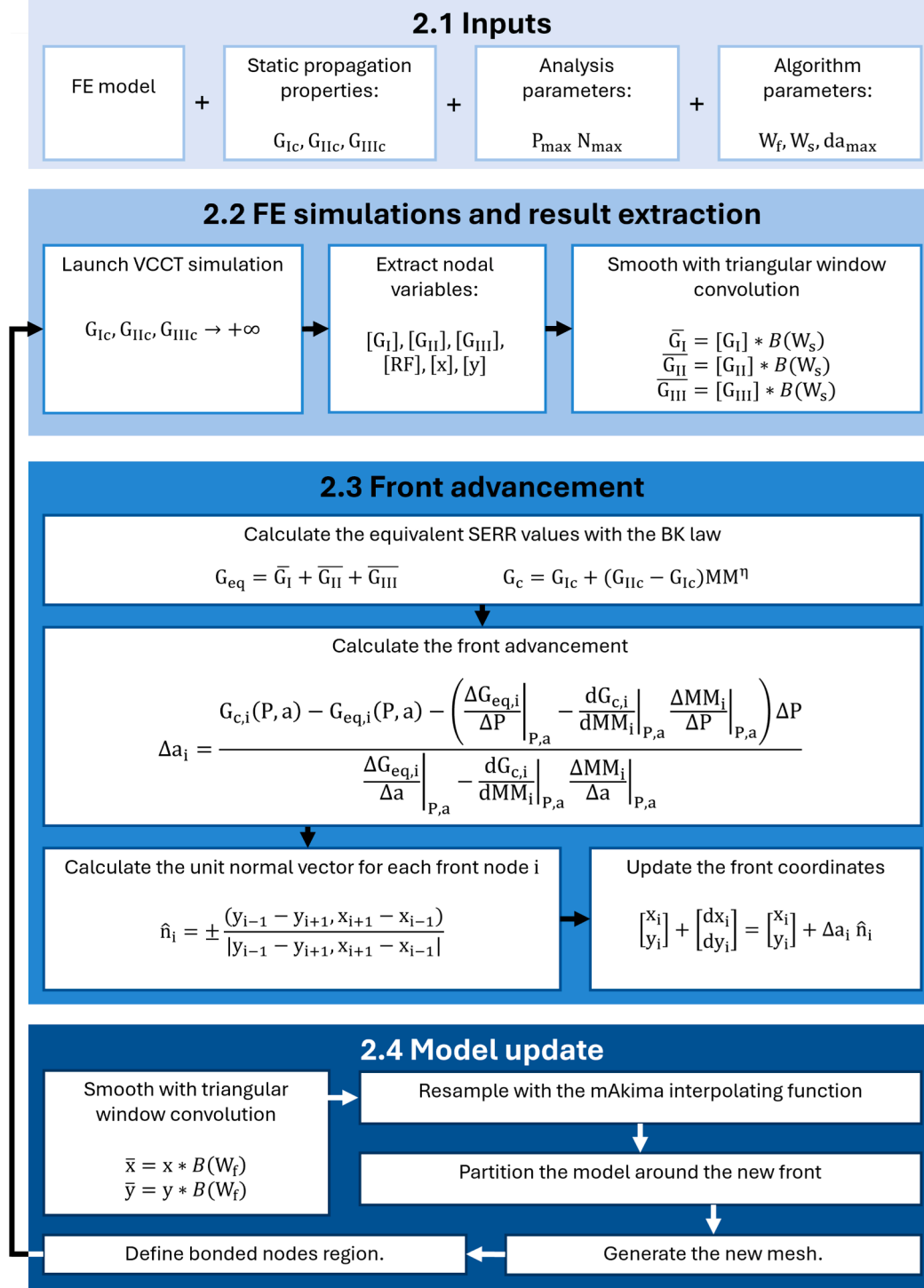


Fig. 3. SRF's structure.

reached depending on the loading path and the evolution of the delaminated surface.

Additional complexity comes from path-dependent material behaviours, for which a custom procedure should be developed. Therefore, this work limits the framework's implementation to purely elastic materials to prevent the accumulation of irreversible processes inside the material each time the load ramp restarts from zero.

Finally, the current implementation of the framework is incompatible with dynamic explicit simulations since the kinetic energy would be restored at the beginning of each simulation. An energy dissipation mechanism should be integrated to adjust the system state at the beginning of each simulation.

In the rest of this Section 2, the structure of the SRF will be explained as indicated in Fig. 3, which is adapted from the sequential algorithms previously published in [34].

2.1. Inputs

The SRF requires several user-defined inputs:

- The FE model, specifying the geometrical features, the boundary conditions, the initial delamination and the material properties. This model is used as reference for each subsequent simulation.
- Pure-modes fracture toughness G_{Ic} , G_{IIc} , G_{IIIc} and a mode-mixity law to correlate the fracture toughness to the mode-mixity. The BK law [37] is used, as described in Subsection 2.3.
- The analysis parameter which defines the stopping condition: i.e. the maximum load (P_{max}).
- Algorithm parameters: maximum advancement length (da_{max}), front smoothing intensity (W_f), and SERR smoothing intensity (W_s). These parameters will be further described in Subsection 2.2, Subsection 2.3.4, and Subsection 2.4.

2.2. FE simulation and result extraction

Static simulations employing the standard VCCT are used to evaluate the SERR value at each node along the delamination front. To prevent the front nodes from releasing after exceeding the material fracture energy, the fracture toughness is artificially set to non-physically high (virtually infinite) values.

The results are extracted and analysed automatically for each simulation after completion. The SERR for each fracture mode (G_I , G_{II} , G_{III}) is obtained for each node of the front and the front nodal coordinates are extracted.

To reduce the propagation of numerical noise, the SERR values are smoothed with a triangular (Bartlett) window [38]. The smoothing strength can be directly controlled by changing the window size (W_s).

2.3. Delamination advancement

2.3.1. Energy based delamination propagation

To calculate the front advancement between simulations, an energy-based analytical formulation can be derived from two simple hypotheses.

First, the SERR value G at the delamination tip is hypothesised to be solely a function of the globally applied load (P) and the delamination length (a) [39]:

$$G = G(P, a) \quad (1)$$

This allows to write the SERR Taylor's expansion series for any infinitesimal load and infinitesimal delamination length increments (dP , da):

$$G(P + dP, a + da) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{m!n!} \frac{\delta^{m+n} G(P, a)}{\delta P^m \delta a^n} dP^m da^n \quad (2)$$

It can be noted that different loads can be applied as long as they can

be represented as functions of a single independent variable. The formulation would use said variable instead of the load P , remaining otherwise unchanged. An example, with proportional loads, can be found in Subsection 3.3.

The second hypothesis is obtained by imposing the Griffith's energy balance during quasi-static stable propagation. Since the energy available for propagation exactly matches the surface energy required to create new fracture surfaces, the SERR value during said propagation is equal to the critical SERR value (G_c) [40]:

$$G(P + dP, a + da) = G_c \quad (3)$$

Eq. (3) can be substituted in Eq. (2), reducing the number of unknowns and obtaining a solvable relation between the infinitesimal delamination propagation and an imposed infinitesimal load increment. Different formulations can be obtained by truncating Taylor's expansion series at different orders. The 1st order produces a linear formulation in da :

$$G_c = G(P, a) + \left. \frac{\delta G}{\delta P} \right|_{P,a} dP + \left. \frac{\delta G}{\delta a} \right|_{P,a} da \quad (4)$$

$$da = \left(G_c - G(P, a) - \left. \frac{\delta G}{\delta P} \right|_{P,a} dP \right) \left(\left. \frac{\delta G}{\delta a} \right|_{P,a} \right)^{-1} \quad (5)$$

To prevent self-healing in the material, the calculated delamination length value is imposed to always be greater or equal than zero. This also implies that, if the load is too low, no propagation will occur: a SERR value ($G(P, a)$) sufficiently lower than G_c , divided by the SERR derivative (<0 for stability), would produce a negative propagation length and therefore it is rounded up to zero. If no propagation occurs on the whole front, the load is increased by a fixed amount for the next simulation.

2.3.2. Mixed-mode delamination formulation

The approach outlined earlier can be extended to diverse loading conditions by implementing a mixed-mode formulation, like the BK law [37] (Eq. (6) and Eq. (7)), to correlate the fracture toughness to the mode-mixity.

$$G_{eq}(P, a) = G_I(P, a) + G_{II}(P, a) + G_{III}(P, a) \quad (6)$$

$$G_c(P, a) = G_{Ic} + (G_{IIc} - G_{Ic}) \left(\frac{G_{II}(P, a) + G_{III}(P, a)}{G_I(P, a) + G_{II}(P, a) + G_{III}(P, a)} \right)^\eta = G_{Ic} + (G_{IIc} - G_{Ic}) MM(P, a)^\eta \quad (7)$$

where MM is the mode-mixity and η is the BK law fitting exponent. This formulation produces new equivalent and critical SERR values that can be used inside the propagation formulation. Since the SERR values are only functions of the applied load and delamination length, the equivalent and critical SERRs are functions of the same variables. Moreover, G_c is now also a function of the mode-mixity, which in turn depends on both the external load P and the delamination length a . By updating equation (3), a more accurate solution can be obtained for the delamination advancement, taking into account changes in fracture toughness caused by changes in mode-mixity:

$$G_{eq}(P + dP, a + da) = G_c(P + dP, a + da) \quad (8)$$

$$da = \frac{G_c(P, a) - G_{eq}(P, a) - \left(\left. \frac{\delta G_{eq}}{\delta P} \right|_{P,a} - \left. \frac{dG_c}{dMM} \right|_{P,a} \left. \frac{\delta MM}{\delta P} \right|_{P,a} \right) dP}{\left. \frac{\delta G_{eq}}{\delta a} \right|_{P,a} - \left. \frac{dG_c}{dMM} \right|_{P,a} \left. \frac{\delta MM}{\delta a} \right|_{P,a}} \quad (9)$$

Appendix A shows the mathematical steps required to obtain equation (9).

2.3.3. Discretised propagation formulation

To apply the SRF to a 3D discretised model, the propagation length

formulation is applied to each node (i) along the delamination front, and the propagation direction is considered instantaneously perpendicular to the front itself. This direction is calculated for each node as the perpendicular to the segment connecting the two adjacent nodes. The first and last nodes use the perpendicular to the segment connecting the node itself and the only other adjacent node.

To directly apply the present formulation to the FE model, Equation (9) has to be substituted with its discretised form:

$$\Delta a_i = \frac{G_{c,i}(P, a) - G_{eq,i}(P, a) - \left(\frac{\Delta G_{eq,i}}{\Delta P} \Big|_{P,a} - \frac{dG_{c,i}}{dMM_i} \Big|_{P,a} \frac{\Delta MM_i}{\Delta P} \Big|_{P,a} \right) \Delta P}{\frac{\Delta G_{eq,i}}{\Delta a} \Big|_{P,a} - \frac{dG_{c,i}}{dMM_i} \Big|_{P,a} \frac{\Delta MM_i}{\Delta a} \Big|_{P,a}} \quad (10)$$

The derivatives of the SERR are approximated using the finite difference method. Those relative to the applied load P can be calculated extracting the results from a single simulation at different load levels. However, the derivatives related to the delamination advancement require different front positions. A first attempt was carried out using consecutive simulations from the analysis. This approach proved ineffective due to changes in the local front shape leading to large G_{eff} variations and noisy derivatives. To overcome this limitation, a new model is created by offsetting the front along the local normal direction by a constant length. It is important to note that the direction chosen for this work—normal to the front—is a common choice in the literature [41]. However, this choice is not unique as any direction could be used, provided the propagation follows the same path the derivative is calculated on. Since movement tangential to the front does not contribute to propagation, the normal direction is the most efficient for calculating front advancement. While very effective, this second approach requires the use of double the number of simulations to complete the same analysis. This approach doubles the computational cost, but the total analysis time is not affected since the two simulations can be run in parallel.

2.3.4. Maximum delamination increment

To ensure the validity of the Taylor's expansion series, each load and delamination increments must remain within a maximum value. A maximum delamination advancement is imposed and translated in an equivalent applied load increment by inverting Equation (10). The minimum load increment across the front is then used to advance the delamination:

$$\overline{\Delta P} = \min_i \left(\Delta P_i \left(G_i, \frac{\delta G}{\delta P} \Big|_{P,a}, \frac{\delta G}{\delta a} \Big|_{P,a}, \Delta a_{max} \right) \right) \quad (11)$$

$$\Delta a_i = \Delta a_i \left(G_i, \frac{\delta G}{\delta P} \Big|_{P,a}, \frac{\delta G}{\delta a} \Big|_{P,a}, \overline{\Delta P} \right) \quad (12)$$

This method was chosen over directly imposing a constant load increment because it would result in larger delamination jumps at the beginning of the analysis and smaller towards the end, due to the compliance reduction in the simulated specimen.

2.4. Delamination front update

The updated delamination front is smoothed to even out minor discontinuities. A triangular (Bartlett) window [38] of width W_f is used to independently average the front nodal coordinates. Additionally, the front is mirrored around the first and last node to prevent the loss of nodes on the edges during the convolution. Finally, the front is interpolated with the modified Akima method [42] and resampled with constant spacing to obtain a uniform element size. This interpolating method was chosen for its ability to produce a continuous and smooth interpolating function, while avoiding the introduction of spurious oscillations.

The new front nodes are reintroduced in the FE model by partitioning the geometry provided by the user in the reference model. Two additional partitions are created by offsetting the front by 1 element length on both sides to generate a controlled region for a structured mesh orthogonal to the front itself. The whole part is then meshed with the software's built-in mesh-generating algorithm and submitted for a new static VCCT simulation.

3. Case studies

All presented simulations are run on Abaqus 2024 [43], and the SRF is implemented in a Python [44] master code. Different specimens are simulated under different loading conditions to explore the framework's ability to capture delamination under different and varying mode-mixity. Pure mode I and II are analysed with Double Cantilever Beam (DCB) and End Loaded Split (ELS) models, respectively; changing mode-mixity is simulated in a specimen with asymmetric loading conditions, designed to smoothly transition from mode I to mode II. Finally, the SRF is tested on a Reinforced DCB (R-DCB) proposed by Carreras et al. [45] to explore the framework's behaviour on highly curved delamination fronts.

All models are loaded under displacement control to obtain a stable delamination propagation. This allows for the observation of the delamination front evolution and the extraction of complete force-displacement curves that capture compliance changes due to damage evolution.

All specimens are composed by two arms made of unidirectional laminate of carbon-fibre/epoxy plies stacked at 0° . The material properties (Table 1) are taken from [45] and kept constant for all specimens.

The specimens are discretised with 1.5 mm continuum shell elements, with one element through the arm thickness. The results are compared against standard VCCT simulations with the same structured mesh as the initial SRF simulation.

3.1. DCB specimen

The first propagation analyses are carried out on a wide DCB specimen (Fig. 4) to study delamination under pure mode I propagation.

In contrast with the standard 25 mm wide DCB [46], designed to obtain straight delamination fronts, the wider width (b) of 60 mm configuration is chosen to accentuate the front curvature due to the edge effect. All dimensions are reported in Fig. 4, which also reports the applied boundary conditions. All material properties are reported in Table 1.

3.1.1. Sensitivity analysis and results

A sensitivity analysis to the algorithm parameters is first performed on the DCB specimen. A full factorial analysis is carried out on the three algorithm parameters introduced in Subsection 2.1, with three levels each (Table 2), for a total of 27 propagation analyses. The maximum delamination length da_{max} is expressed relative to the global mesh size: double (2), equal (1) or half (0.5) the global element size. In contrast, the smoothing strengths (W_f, W_s) are equal to the moving window's width.

Table 1
Lamina material parameters [45].

Lamina properties			Fracture properties		
E_{11}	154	[GPa]	G_{Ic}	0.305	$\left[\frac{mJ}{mm^2} \right]$
$E_{22} = E_{33}$	8.5	[GPa]	G_{IIc}	2.77	$\left[\frac{mJ}{mm^2} \right]$
$G_{12} = G_{13}$	4.2	[GPa]	G_{IIIc}	2.77	$\left[\frac{mJ}{mm^2} \right]$
G_{23}	3.036	[GPa]	η	2.0866	[-]
ν	0.35	[-]			

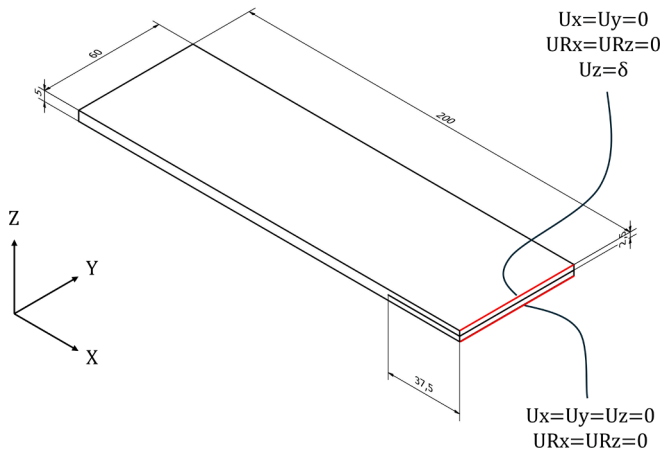


Fig. 4. DCB specimen.

Table 2
Sensitivity analysis variables.

da_{max}	W_f	W_s
2 – 1 – 0.5	7 – 13 – 19	7 – 13 – 19

A maximum of 30 simulations is imposed for each propagation analysis to keep the results manageable, while having enough data to evaluate the analyses quality.

The effective SERR ($G_{eff} = G_{eq}/G_c$) error is quantified relative to the objective value of 1, which is required along the entire front for simultaneous propagation. The Probability Distribution Function (PDF) of the error is extracted from all fronts in each analysis. The probability of errors smaller than 1% and 5% are computed and used as quality metrics (Fig. 5).

The sensitivity analysis shows clear trends: smaller increments and stronger smoothing on the SERR values lead to more accurate results. Instead, the effect of smoothing on the front shape is non-uniform. An intermediate optimal smoothing strength can be identified. Cross-hatched squares indicate analyses that stopped before completing all 30 simulations, as the low angle between the delamination front and the specimen edge leads to the failure of Abaqus's meshing algorithm.

To explain the observed dependencies, the results obtained from the combination $da_{max} = 1$, $W_f = 13$, $W_s = 7$ (marked with a red circle in

Fig. 5) are reported in Fig. 6 as an example of medium–low quality results ($e_{<1\%}=18\%$, $e_{<5\%}=66\%$).

As shown in Fig. 6.a, most of the analyses are affected by large-amplitude front oscillations. A ‘leapfrogging’ behaviour emerges, where groups of front nodes alternate between large and small advancements. This dynamic produces large G_{eff} oscillations (Fig. 6.b): nodes that lag behind experience an increased local SERR and subsequently propagate further in the next step. Despite these oscillations, the analysis generates a smoother force–displacement curve compared to the reference VCCT simulation (Fig. 6.c). These curves may also be compared to the analytical curve obtained by imposing $G = G_c$ in Equation (13) [47] and varying the delamination length; the force is calculated with the compliance of a cantilever beam.

$$G = \frac{9 E_{11} I \delta^2}{4 b a^4} \quad (13)$$

This shows that the framework coincides with the analytical solution, while the standard implementation drops below the critical value each time a new line of nodes is released. This increased accuracy comes with the drawback of a higher computational cost. Although the two analyses can not be directly compared since the SRF runs two simultaneous simulations in parallel while the standard VCCT is based on a single simulation; both analyses are run using the same number of CPU cores per simulation, on the same workstation. The standard simulation took 4'12" to complete while the SRF took 23'1".

The highest quality results ($e_{<1\%}=86\%$, $e_{<5\%}=100\%$) are obtained with the combination $da_{max} = 0.5$, $W_f = 7$, $W_s = 19$ (marked with a red square in Fig. 5).

These results show that small advancements and higher smoothing on the SERR are required to suppress the leapfrogging behaviour. The front evolves self-similarly (Fig. 7.a), producing a smoother load–displacement curve closer to the analytical solution (Fig. 7.c) with regular load increments (Fig. 7.d). This allows for a higher maximum final displacement in the same number of simulations and the same simulation time (22'55"), increasing the SRF's computational efficiency.

Excessive values of smoothing on the front distorts the front near the specimen edges, leading to non-physical shapes. This effect is visible in Fig. 7.b, where the edges show G_{eff} values above 1, while the central front remains close to the expected value.

Overall, the DCB results demonstrate that the SRF increases accuracy compared to standard VCCT in an easy-to-simulate configuration, but its computational cost is more than 5 times higher. Additionally, these analyses highlight the relation between the smoothing parameters and the delamination front shape. Although oscillations appear along the front, they remain within a ± 0.5 mm range, and such limited variations do not affect the global response, resulting in a force–displacement curve that remains largely insensitive to the tested range of smoothing parameters.

3.2. ELS specimen

The ELS specimen of Fig. 8 is analysed to assess the SRF under mode II conditions.

This specimen presents minor geometrical changes compared to the DCB. The arm thickness is increased to 5 mm to prevent excessive deformations under higher loads. Additionally, the length is increased by 30 mm to extend the contact surface and reduce dissimilar rotations in the top and bottom arms, thereby limiting the opening mode at the delamination tip. Finally, a longer initial delamination is introduced to overcome the inherent instability. According to ISO 15114:2014 [48], the minimum stable delamination length relative to the specimen length is $a/L > 0.55$, as shorter delamination lengths lead to unstable propagation. All material properties are reported in Table 1.

3.2.1. Results

This specimen exhibits stronger front oscillations compared to the

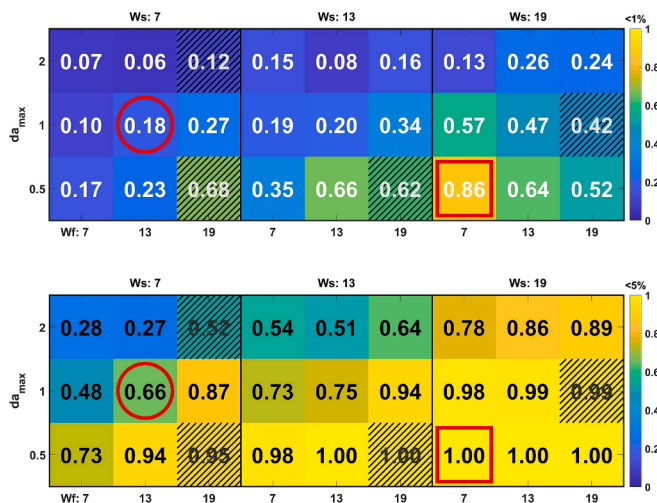


Fig. 5. Sensitivity analysis results (error probability). Crosshatched squares indicate analyses that stopped before completing all 30 simulations.

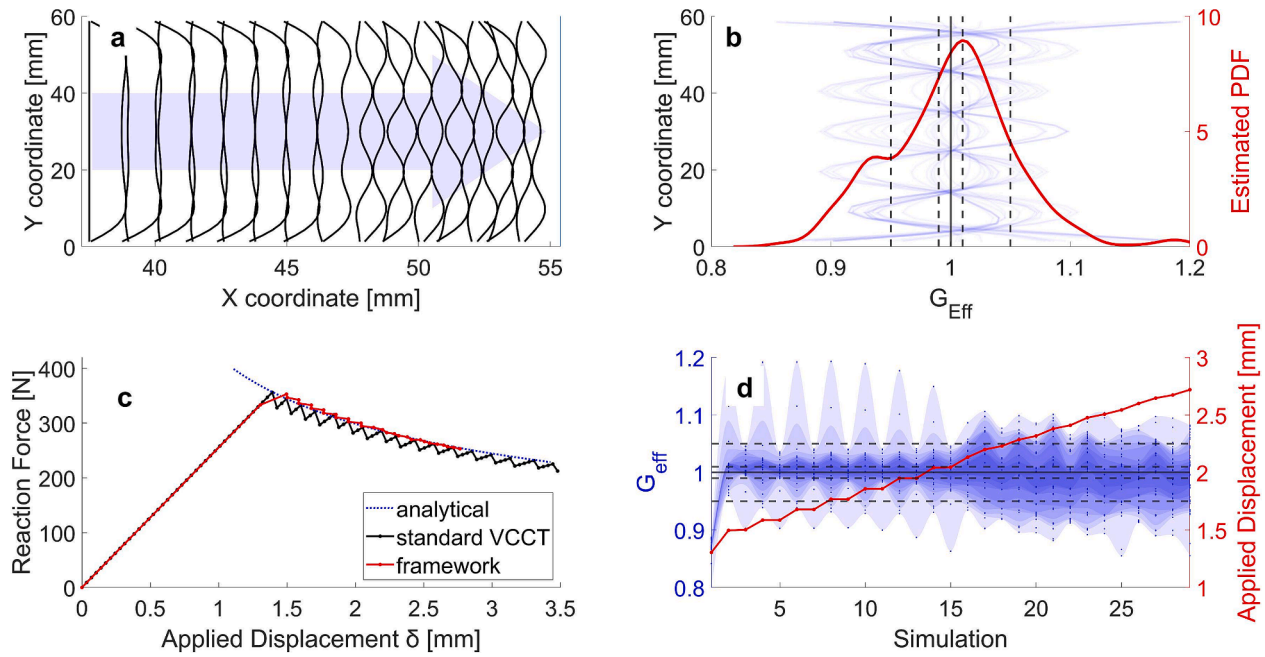


Fig. 6. Medium-low quality propagation analysis results. a) Front evolution showcasing a strong leapfrogging behaviour. b) G_{eff} distribution along the fronts (blue) and G_{eff} probability distribution (red). c) Force displacement graph. d) G_{eff} (blue) and load (red) evolution across the simulations.

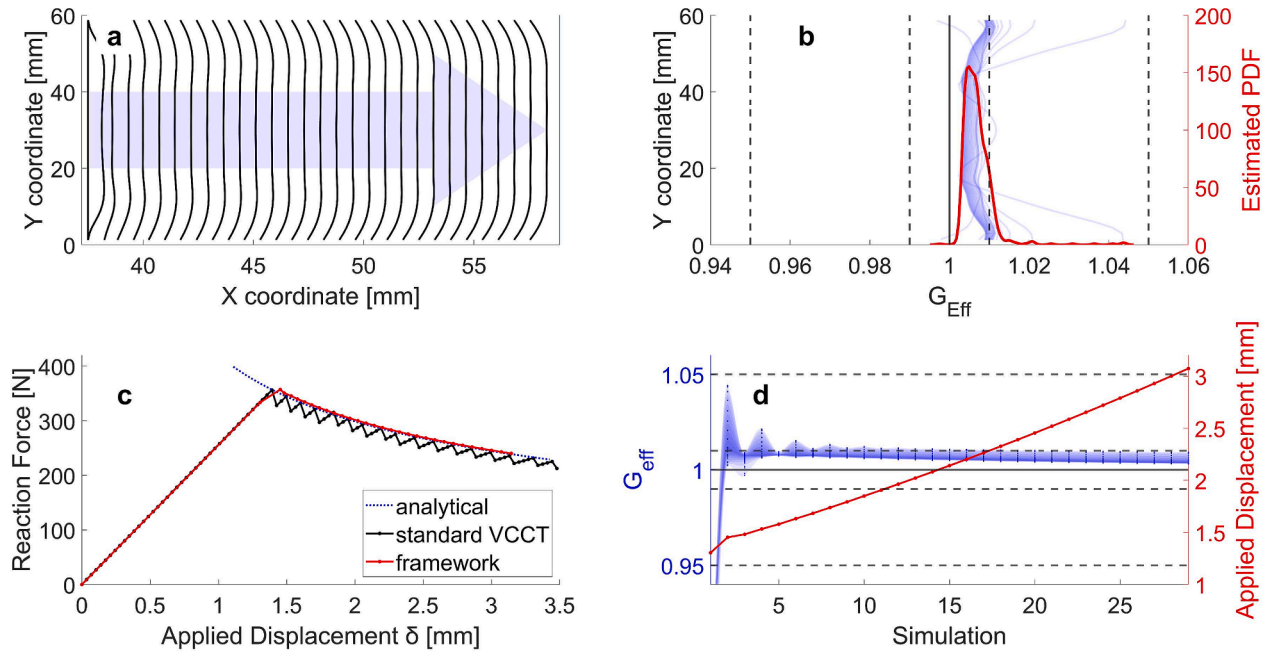


Fig. 7. High quality propagation analysis results. a) Front evolution showcasing no leapfrogging behaviour. b) G_{eff} distribution along the fronts (blue) and G_{eff} probability distribution (red). c) Force displacement graph. d) G_{eff} (blue) and load (red) evolution across the simulations.

DCB. To achieve an acceptable propagation speed, higher smoothing parameters ($da_{\text{max}} = 1$, $W_f = 27$, $W_s = 27$) are used. While yielding good SERR accuracy ($e_{<1\%} = 61.5\%$, $e_{<5\%} = 91\%$), some leapfrogging persists at the edges, as visible in Fig. 9.a. An optimized combination of parameters can be identified, although a trade-off between accuracy and computational cost would be inevitable. Delamination fronts are represented every 0.75 mm of propagation in Fig. 9.a to increase readability.

The leapfrogging effect is also visible in the load curve (Fig. 9.d), which shows a staircase shape: only a few simulations contribute to increasing the applied load, while the others are used to update the front

shape towards an *iso*- G_{eff} configuration.

The SRF produces a more stable force–displacement curve (Fig. 9.c) than the standard VCCT, which shows early signs of instability with important load drops. The premature failure of the interface in the VCCT simulation is caused by the spontaneous formation of a saw tooth pattern in the delamination front (Fig. 10). A similar sawtooth pattern was observed in mode II propagation in the literature [49,50]. The delamination initiation at a load lower than the theoretical prediction is instead caused by SERR peaks at the specimen edges. Several standard VCCT simulations were run trying to prevent these phenomena, but the results remained consistent with one another. In the end, this case study

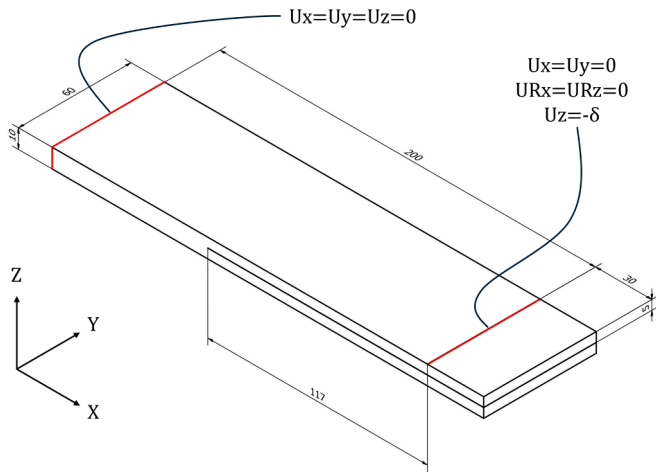


Fig. 8. ELS specimen.

was selected to highlight the potential impact of front discontinuities in mode II propagation and the importance of the conformal mesh in preventing a premature failure of the interface.

The high quality of the SRF result can also be appreciated by the SERR distribution being perfectly centred on the value of 1 (Fig. 9.b). The substantial increase in quality can therefore justify the higher computational time (SRF 3 h3'20", standard VCCT 19'33") as the standard VCCT implementation completely misses the analytical solution.

3.3. Mixed-mode specimen

Varying mixed-mode propagation is studied to assess the influence of shear contributions on both VCCT and the SRF. To this end, the ELS geometry is used with modified boundary conditions (Fig. 11).

An additional displacement is imposed on the lower arm, with twice the amplitude of the top one. This asymmetric loading produces a gradual transition from mode I to mode II, sweeping through different mode-mixities as the delamination grows. The initial delamination length is reduced to start from a near-zero mode-mixity. The smoothing

parameters are set to those identified as optimal in the DCB study ($da_{\max} = 0.5$, $W_f = 7$, $W_s = 19$).

3.3.1. Results

The analysis starts with an average mode-mixity of 0.011 with a delamination length of 36 mm and it ends at an average mode-mixity of 0.880 with an average delamination length of 174 mm. Fig. 12.a shows the delamination fronts every 2 mm of propagation demonstrating minimal leapfrogging behaviour across the analysis. Despite the G_{eff} scatter widens with the introduction of the shear component (Fig. 12.d), the SRF's results maintain good error probability ($e_{<1\%}=60\%$, $e_{<5\%}=98\%$). In contrast, the standard VCCT becomes highly unstable for mode-mixity values around 0.8 showing important load drops (Fig. 12.c), as previously observed in the ELS specimen. This analysis further demonstrates the VCCT limitations in modelling high mode-mixity. The SRF also shows a decrease in the result quality, but it maintains an average G_{eff} value of 1 across the whole analysis.

In this case study, the computational cost disparity is the highest (SRF 9 h19'44", standard VCCT 24'50") but the increased quality is still present, especially in the later part of the analyses.

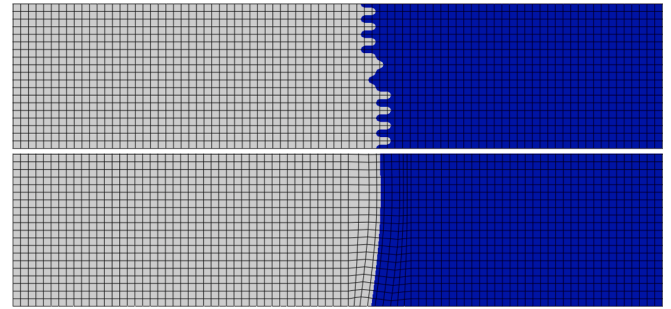


Fig. 10. Standard VCCT (top) and SRF (bottom) ELS simulation at similar delamination lengths. The blue elements represent the undamaged interface.

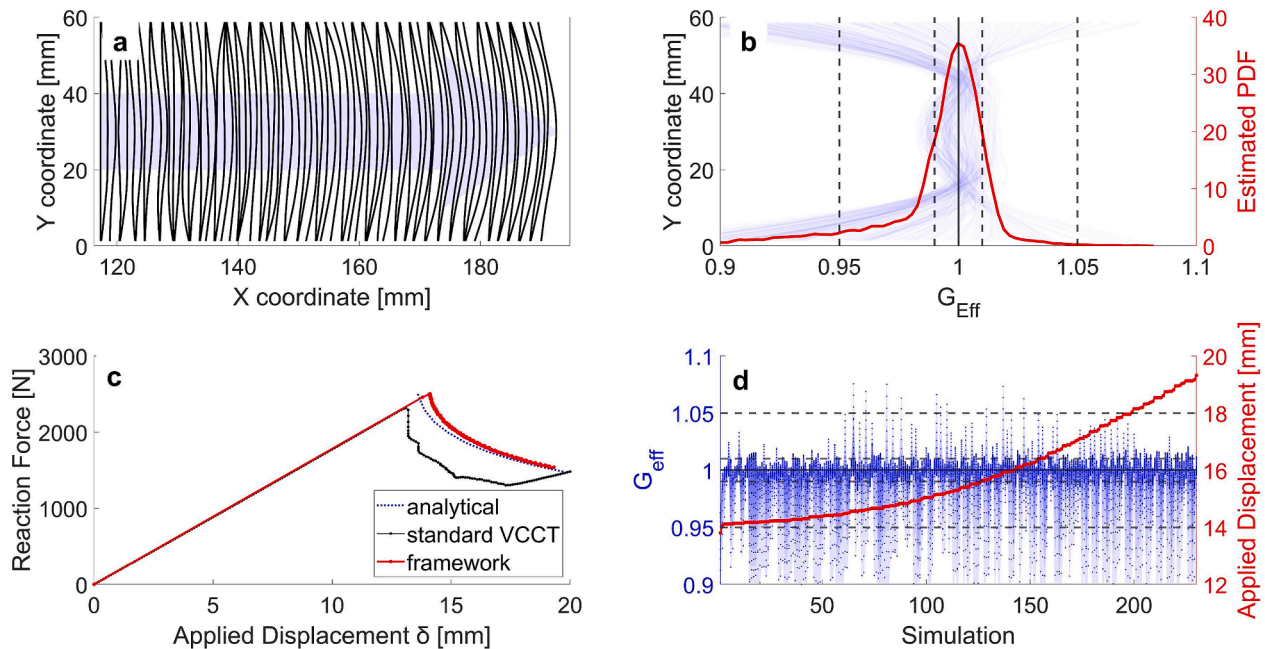


Fig. 9. ELS results. a) Front evolution. b) G_{eff} distribution (blue) and G_{eff} probability distribution (red) along the front. c) Force displacement graph. d) G_{eff} (blue) and load (red) evolution.

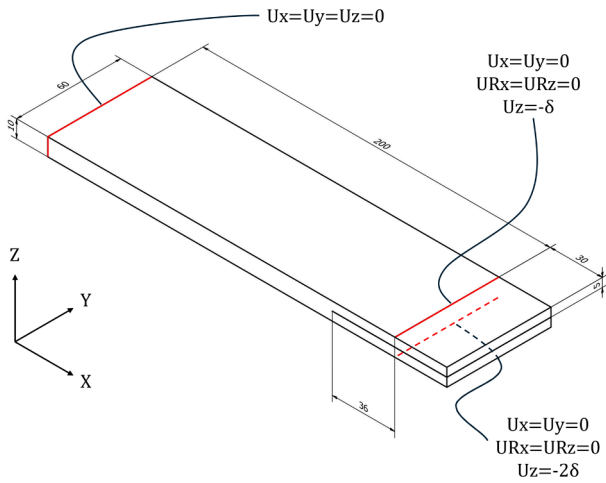


Fig. 11. Varying mixed mode specimen.

3.4. Reinforced DCB specimen

Finally, the SRF is assessed on the R-DCB benchmark presented by Carreras et al. [45] to explore its behaviour on highly curved fronts (Fig. 13).

The specimen is composed by two 1.51 mm thick arms with the addition of two 1.51 mm thick reinforcements. This geometry enables to generate strong front curvature during propagation while maintaining a purely opening loading mode. The initial delamination length is 35 mm. All material properties are reported in Table 1. Two different SRF analyses are run to compare the effect of the mesh size. The first model uses a 1.5 mm mesh with algorithm parameters: $da_{max} = 0.5$, $W_f = 11$, $W_s = 21$. The second one uses a 3 mm mesh with algorithm parameters: $da_{max} = 0.5$, $W_f = 5$, $W_s = 11$. The shorter maximum advancement was chosen to increase the analysis accuracy, as proven from the previous case studies. The low values of front smoothing are chosen to prevent the possible over correction of the front shape close to the reinforcement and the strong SERR smoothing is required to aide convergence. Smaller smoothing parameters are used in combination with the larger mesh to

obtain a similar smoothing effect between the two analyses.

This specimen is also simulated using the CZM to compare the result quality and computational cost. The additional material properties reported in Table 3 [51] are used to define the bilinear TSL, as required by Abaqus. Finer 0.5 mm elements are used to obtain an acceptable cohesive zone length [52]. A 0.02 mm thick layer of cohesive elements is used to model the interface.

3.4.1. Results

The results obtained with the SRF analyses are compared to the standard VCCT, the CZM, and the experimental data provided by the original authors [45]. The SRF, CZM, and the standard VCCT closely follow the experimental load–displacement response up to 10 mm of applied displacement. Fig. 14.c and Fig. 14.d report the G_{eff} values up to this limit. The delamination fronts in Fig. 14.a are represented every 0.5 mm of propagation to improve readability.

Additionally, Figure 14.a shows the experimental (green), the standard VCCT (blue), the CZM (orange), and the SRF (red and black) fronts obtained with the same applied displacement (4 mm, 6 mm, 10 mm).

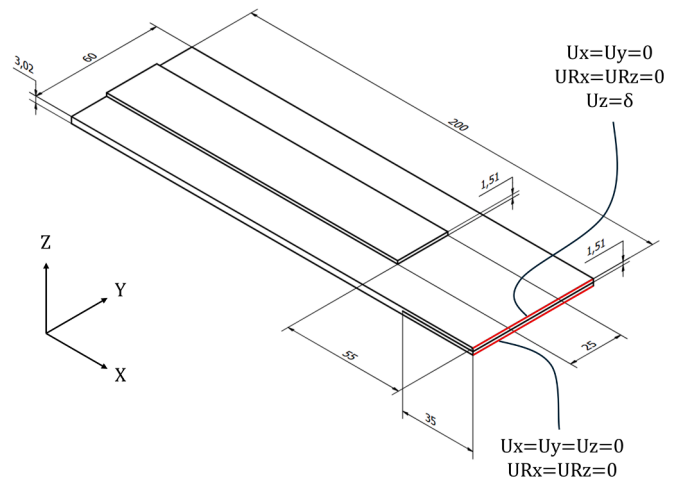


Fig. 13. R-DCB specimen.

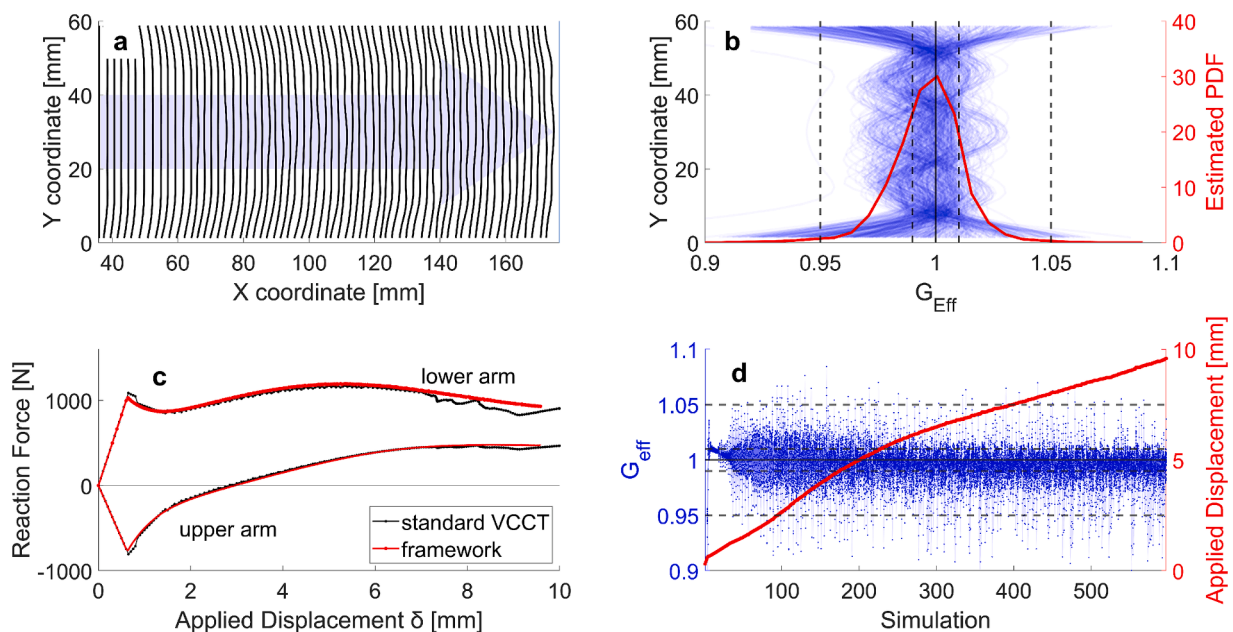


Fig. 12. MM results. a) Front evolution showcasing minimal leapfrogging behaviour in the later stages of the analysis. b) G_{eff} distribution along the fronts (blue) and probability distribution (red). c) Force displacement graph. d) G_{eff} (blue) and load (red) evolution across the simulations.

Table 3
CZM-TSL parameters [51].

τ_I	τ_{II}	E/Enn	G1/Ess	G2/ Ett
32.6 MPa	98 MPa	100 GPa	100 GPa	100 GPa

The position of the reinforcement is represented by a dashed line. The whole CZM cohesive zone, including damage values between 0 and 1, is reported. While slightly overestimating the delamination propagation, both SRF analyses are closer to the experimental data compared to the standard VCCT, especially at higher applied displacements. Additionally, the SRF fronts lie inside or right next to the CZM cohesive zone, demonstrating a good accordance between the two methods. The presence of the reinforcement strongly affected the quality of the SRF analyses, as visible in the SERR distribution and evolution (Fig. 14.b and Fig. 14.d). After the initial propagation up to 5 mm of applied displacement, the G_{eff} values begin oscillating around the value of 1 due to a non-self-similar front evolution. The shielding effect caused by the reinforcement reduces the SERR value in the central region of the front, reducing their propagation speed and increasing the front curvature. This process is alternated to minor instabilities, like the ones visible at 5.5 mm and 6 mm of applied displacement in Fig. 14.c, for the coarse and fine mesh respectively. Despite this, the SRF followed the load displacement closely up to the major load drop at 11 mm of applied displacement with both mesh sizes. This same behaviour was previously found by Daniel et al. [28], and is believed to be caused by a change in the material behaviour with the insurgence of strong non linearities at the delamination tip, such as fibre bridging. The same behaviour is also reproduced in our CZM simulation, demonstrating that this problem is not limited to LEFM-based approaches.

Despite the specimen's increased complexity compared to the previous three case studies, the SRF produces a more accurate response compared to the standard VCCT implementation, comparable with the CZM results. Similarly to the previous case studies, the simulation time of the SRF with the finer mesh (1 h 2'29") is higher than the standard VCCT (14'4"), but it becomes negligible when compared with the CZM (20 h 56'36"). In contrast, the SRF with the coarser mesh run with a total time of 30'24", further reducing the computational time, while maintaining the solution quality.

These results better demonstrate the potential of the SRF, which proves to be able to obtain results comparable to the CZM's in a fraction of the time.

4. Conclusions

This work introduces an algorithm, the Sequential Remeshing Framework (SRF), for simulating quasi-static delamination propagation in composite laminates using the Virtual Crack Closure Technique (VCCT). By combining Griffith's energy criterion with a truncated Taylor expansion of both the Strain Energy Release Rate (SERR) and the fracture toughness (G_c), the SRF produces an accurate approach to model delamination. The automated remeshing algorithm ensures accurate tracking of delamination growth under complex loading and geometrical configurations. While not fixing the mesh dependency that typically limits conventional VCCT implementations, the remeshing algorithm is able to mitigate its effects by generating a conformal mesh around the front. The SRF's capabilities are demonstrated through a comprehensive set of case studies showcasing an improved accuracy and stability compared to the standard VCCT implementation. In particular, the SRF produced smoother force–displacement curves and more physically accurate delamination fronts, even with sub-optimal smoothing parameters.

The sensitivity analysis highlighted the importance of finding the appropriate algorithm parameters, revealing that smaller propagation increments and stronger SERR smoothing are key to suppressing non-physical leapfrogging behaviours in the front evolution. In this work, these parameters are selected through an informed trial-and-error approach, therefore showing that non optimal parameters are also able to produce more accurate results compared to the standard VCCT implementation. However, developing a systematic and robust parameter identification strategy would further enhance the SRF's applicability and efficiency.

Overall, the proposed framework represents a versatile and reliable tool for quasi-static delamination modelling in composite structures. Future developments should focus on extending the SRF to cyclic loading conditions by incorporating Paris-like propagation laws, thereby broadening its applicability to fatigue-driven delamination scenarios. Additionally, new damage models are needed to capture the

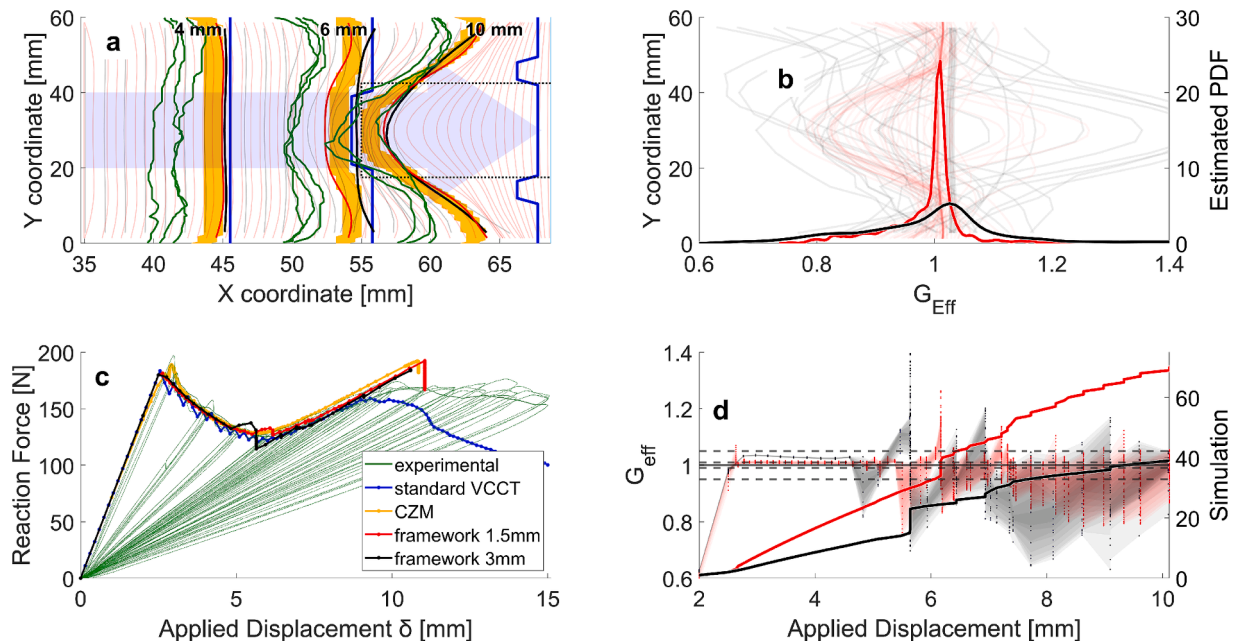


Fig. 14. R-DCB results. a) Front evolution. b) G_{eff} distribution along the fronts and probability distribution. c) Force displacement graph. d) G_{eff} and load evolution across the simulations.

non-linear behaviour of the materials close to failure. Finally, an intra-laminar fracture model is planned to capture fibre-bridging and delamination migrations, which are typical in angle-ply laminates.

CRedit authorship contribution statement

L.G. Salvi: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **A. Bernasconi:** Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition,

Conceptualization. **L.M. Martulli:** Writing – review & editing, Visualization, Supervision, Project administration, Methodology, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A:. Mixed-Mode formulation

Here is reported the complete derivation of the mixed-mode solution. The BK law is used to calculate the critical and equivalent SERR values:

$$G_{eq}(P, a) = G_I(P, a) + G_{II}(P, a) + G_{III}(P, a) \quad (A1)$$

$$G_c(P, a) = G_{Ic} + (G_{IIc} - G_{Ic}) \left(\frac{G_{II}(P, a) + G_{III}(P, a)}{G_I(P, a) + G_{II}(P, a) + G_{III}(P, a)} \right)^\eta = G_{Ic} + (G_{IIc} - G_{Ic}) MM(P, a)^\eta \quad (A2)$$

To take into account the changes of the critical value, a first order Taylor's expansion can be used:

$$G_c(P + dP, a + da) = G_c(P, a) + \left. \frac{\delta G_c}{\delta P} \right|_{P,a} dP + \left. \frac{\delta G_c}{\delta a} \right|_{P,a} da \quad (A3)$$

By exploiting the chain rule and the solution of the derivative of a fraction, the partial derivatives can be written underlining the relation to the MM, lightening the computation, since all terms can be calculated from variables P and a:

$$G_c(P + dP, a + da) = G_c(P, a) + \left. \frac{\delta G_c}{\delta MM} \right|_{P,a} \left. \frac{\delta MM}{\delta P} \right|_{P,a} dP + \left. \frac{\delta G_c}{\delta MM} \right|_{P,a} \left. \frac{\delta MM}{\delta a} \right|_{P,a} da \quad (A4)$$

$$\left. \frac{\delta G_c}{\delta MM} \right|_{P,a} = \eta (G_{IIc} - G_{Ic}) MM(P, a)^{\eta-1} \quad (A5)$$

$$\left. \frac{\delta MM}{\delta x} \right|_{P,a} = \frac{\left(\left. G_{eq} \frac{\delta(G_{II} + G_{III})}{\delta x} \right|_{P,a} - (G_{II} + G_{III}) \left. \frac{\delta G_{eq}}{\delta x} \right|_{P,a} \right)}{G_{eq}^2} \quad (A6)$$

By substituting the newly obtained critical value in Equation A(7), the 1st order formulation for varying MM can be obtained:

$$da = \left(G_c - G(P, a) - \left. \frac{\delta G}{\delta P} \right|_{P,a} dP \right) \left(\left. \frac{\delta G}{\delta a} \right|_{P,a} \right)^{-1} \quad (A7)$$

$$G_{eq}(P, a) - G_c(P, a) + \left(\left. \frac{\delta G_{eq}}{\delta P} \right|_{P,a} - \left. \frac{\delta G_c}{\delta MM} \right|_{P,a} \left. \frac{\delta MM}{\delta P} \right|_{P,a} \right) dP + \left(\left. \frac{\delta G_{eq}}{\delta a} \right|_{P,a} - \left. \frac{\delta G_c}{\delta MM} \right|_{P,a} \left. \frac{\delta MM}{\delta a} \right|_{P,a} \right) da = 0 \quad (A8)$$

Data availability

Data will be made available on request.

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