

Vulnerability Assessment of Interdependent Infrastructures Based on a Cascading Failure Model

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Abstract—Road and power systems are the backbone of a city. With rapid urbanization, the two facilities are highly interconnected. The vulnerability of the Interdependent Road-Power Systems (IRPS) can cause large-scale failures and even catastrophic disasters with severe social and economic consequences. In this paper, the IRPS is composed of nodes, edges, and links between layers. Distance between junction nodes with traffic lights in the Road Network (RN) and power stations in the Power Network (PN) is used to construct the functional interdependence. A non-linear load-capacity model is proposed to describe the capacity and ability of IRPS to handle allocated loads. Considering the current ability of neighbor nodes, we propose an intelligent node loads redistribution rule. The vulnerability level of RN, PN, and IRPS are measured, respectively. A case study based on real road data in Xi'an, China is conducted to validate the model. Results show that: (1) The redistribution rule is an important tool to decrease the vulnerability of the RN, while the capacity tolerance degree is more influential for the PN. (2) Parameters for the global vulnerability, there is a cross-impact between the capacity tolerance parameters.

Keywords—vulnerability, road-power system, interdependent infrastructures, cascading failure, complex network

I. INTRODUCTION

Infrastructures play an important role in daily life, and the rapid development of cities makes infrastructures increasingly interconnected. The interdependence between different kinds of infrastructure systems brings more potential threats to the safety of cities and results in a large scale of failures. RN and PN are interdependent through traffic lights. Without electricity, the Road Network (RN) would be thrown into chaos. For example, in 2019, a power outage caused a large-scale traffic jam in London, UK. Such an accident also occurred in northeastern China in September 2021. Hence, it is essential to analyze the vulnerability of the Interdependent Road-Power Systems (IRPS).

Vulnerability, as an inherent property of a system, is the possible cause of disasters and accidents. Extensive research has focused on the vulnerability problem in infrastructure systems. Several methods, such as empirical approaches, agent-based models, system dynamics-based approaches, economic theory-based methods and network-based techniques are used to measure the vulnerability [1], [2]. Among these approaches, the network-based model is an efficient method to extract the physical features of real-world infrastructures. Farahmand et al. [3] and Ouyang et al. [4] used graph-based network measures to abstract topological features of infrastructures.

The vulnerability propagation process can be characterized as cascading failure based on complex networks, which have a devastating effect on the normal operation of infrastructures. The cascading failure is a mathematical presentation of the failure propagation process [5], and the most common methods used to simulate include the discrete dynamic system [6], [7], percolation theory [8], [9], the self-consistent probability method [10], [11], game theory, the load-capacity model. Considering the services made and delivered by infrastructures, the load-capacity model [12] can better describe the cascading failure process of infrastructures.

Cascading failures on the RN and PN mainly focus on a single system, and a few studies concern the interdependence between them. Interdependent network is a useful tool to characterize the complex interactions among infrastructures [13]–[16]. Primary studies of cascading failures on the interdependent network can be classified into two types: (1) Analyze the influence of different topological structures on the network vulnerability, which includes the dependent pattern [17] (e.g., partial interdependence [18], weak interdependence [8]), and the network type (e.g., Erdos-Renyi (ER) network, Barabasi-Albert (BA) scale-free network). (2) Study the overload behavior and construct the flow redistribution

mechanism. Pei et al. [13] proposed a new flow redistribution model with overlapping edges in multiplex networks. Wang et al. [9] investigated the effect of the load redistribution mechanism on network robustness and obtained the optimal values of the parameters that enable the network to achieve the best robust level through testing.

This paper focuses on constructing a cascading failure model on the interdependent network to assess the vulnerability of the IRPS, and finds out the influencing factors of system vulnerability. The rest of the paper is structured as follows. Section II gives the construction process of the IRPS. Section III presents the failure scenario, the intelligent redistribution rule, cascading failure process and the vulnerability assessment indicators. In Section IV, a case study based on real road data is conducted, and the results are analyzed. Section V concludes the main findings.

II. CONSTRUCTION OF THE INTERDEPENDENT ROAD-POWER SYSTEMS

The RN is the backbone of a city to transport essential goods and services. The normal operation of the RN depends on the traffic lights, which relies on the support of PN. For example, without electricity, the RN is likely to be blocked because of the failed traffic lights. It is necessary to analyze the inner structure of RN and PN and identify the interdependence between the RN and PN for measuring the vulnerability of the system precisely.

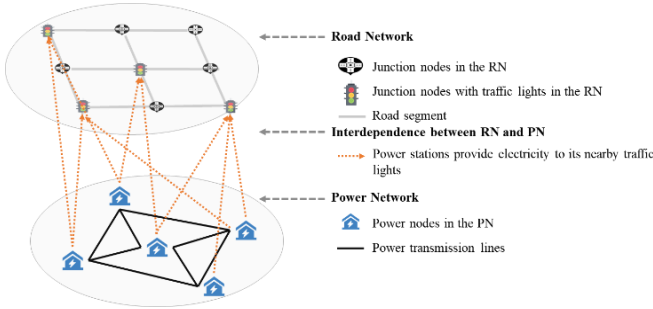


Fig. 1. The structure of the IRPS.

A. Road and Power Systems

RN consists of junctions and lines between them, which can be represented as nodes and edges of a network. In detail, according to whether the junctions have traffic lights or not, the nodes are divided into two categories: common junction nodes and junction nodes with traffic lights. The former is represented by $v_i^{R,Jun}$ and the latter is $v_i^{R,Tra}$. The node set of the RN is $V^R = (v_i^{R,Jun}, v_i^{R,Tra} | i = 1, 2, \dots, N)$, and $e_{ij}^R(i \neq j)$ denotes the road lines. If a road line exists between junctions, $e_{ij}^R(i \neq j) = 1$; otherwise, it is equal to 0. The edge set of the RN is $E^R = (e_{ij}^R(i \neq j) | e_{ij}^R(i \neq j) = e_{ji}^R(i \neq j), i, j = 1, 2, \dots, N)$. The RN is represented by $N^R = (V^R, E^R)$.

In the PN, the electricity is transformed from one station to another one. Similar to the RN, the PN is represented by $N^P = (V^P, E^P)$, where $V^P = (v_i^P | i = 1, 2, \dots, N)$ and $E^P = (e_{ij}^P(i \neq j) | e_{ij}^P(i \neq j) = e_{ji}^P(i \neq j), i, j = 1, 2, \dots, N)$ are the node set and edge set of the PN, respectively.

B. Functional Interdependence between Road and Power Systems

As the junction with traffic lights in the RN relies on the support of electricity, it means that the station in the PN also transports electricity to the junction with traffic lights. Functional interdependence is defined as the RN requiring the functional inputs of the power system. Links between two networks are proposed to describe the functional interdependence relationship between the RN and PN, which is represented by $link_{ij}, \forall i, j \in v_i^P, v_j^{R,Tra}$.

In reality, the junction with traffic lights is always supported by its nearby power transmission station. Hence, the Euclidean Distance (ED) based on the latitude and longitude of nodes is proposed, shown as follows:

$$ED_{ij}^{R-P} = \sqrt{(d_i^P - d_j^{R,Tra})^2 + (l_i^P - l_j^{R,Tra})^2}, \forall i, j \in v_i^P, v_j^{R,Tra} \quad (1)$$

$$link_{ij} = \begin{cases} 1, & \text{if } ED_{ij}^{R-P} < \alpha \\ 0, & \text{otherwise} \end{cases}, \forall i, j \in v_i^P, v_j^{R,Tra} \quad (2)$$

where α is the interdependent degree parameter, $\min(ED_{ij}^{R-P}) \leq \alpha \leq \max(ED_{ij}^{R-P})$.

C. Non-linear Load-Capacity Model

The degree $k_i^{R/P}$ is defined as the number of edges connected to the node $v_i^{R/P}$, and the initial load of the node $v_i^{R/P}$ is equal to its initial degree which is shown as follows:

$$L_{i,0}^{R/P} = k_{i,0}^{R/P} \quad (3)$$

The initial capacity of the node $v_i^{R/P}$ is represented by $C_{i,0}^{R/P}$, shown as follows:

$$C_{i,0}^{R/P} = \beta(L_{i,0}^{R/P} + (L_{i,0}^{R/P})^{1-\gamma}) \quad (4)$$

where β and γ are the capacity tolerance parameters, $\beta \geq 1$, $0 \leq \gamma \leq 1$.

III. CASCADING FAILURE MECHANISM

A. The Failure Scenario

Different reasons may result in the failure of the IRPS, such as natural disasters, human-made deliberate attacks, incorrect manipulation, and so on. As junction nodes with traffic lights need the support of electricity, the failure of the PN has a serious impact on the RN. What's more, compared with the RN, the power station is more vulnerable to human-made attacks. Therefore, the initial failure of the IRPS mainly focuses on the PN by removing parts of the important power station deliberately.

B. The Intelligent Load Redistribution Rule

After the failure occurs, the affected nodes may fail and need to transfer their loads to neighboring nodes. Considering the Current Load Capabilities (CLC) of neighbor nodes, a novel redistribution rule of node loads is proposed, defined as follows:

$$CLC_{j,t}^{R/P} = C_{j,t}^{R/P} - L_{j,t}^{R/P} \quad (5)$$

$$\Delta L_{j,t}^{R/P} = L_{i,t}^{R/P} * \frac{(CLC_{j,t}^{R/P})^\theta}{\sum_{\Gamma_i^{R/P}} (CLC_{j,t}^{R/P})^\theta} \quad (6)$$

$$L_{j,t+1}^{R/P} = L_{j,t}^{R/P} + \Delta L_{j,t}^{R/P} \quad (7)$$

where $\Gamma_i^{R/P}$ is the neighboring node set of node $v_i^{R/P}$, θ is a redistribution parameter, and the number of loads distribute from the node $v_i^{R/P}$ to $v_j^{R/P}$ is $\Delta L_{j,t}^{R/P}$.

C. Cascading Failure Process

The occurrence of cascade failures in the IRPS is triggered by destroying parts of power stations. After a deliberate attack, failed power stations are removed, which leads to a load redistribution among the remaining PN. The load of the failed power station is redistributed to its neighboring node according to (5), (6), and (7). The redistribution process continues until there are no overloaded power stations in the PN. As functional interdependence exists between the road and power systems, failed power stations cannot supply electricity to junction nodes with traffic lights. Therefore, junction nodes with traffic lights that lose power support are also removed. And the load redistribution process will continue also according to (5) to (7) until there are no overloaded junction nodes in the RN. The flow chart of the whole cascading failure process is shown in Fig. 2.

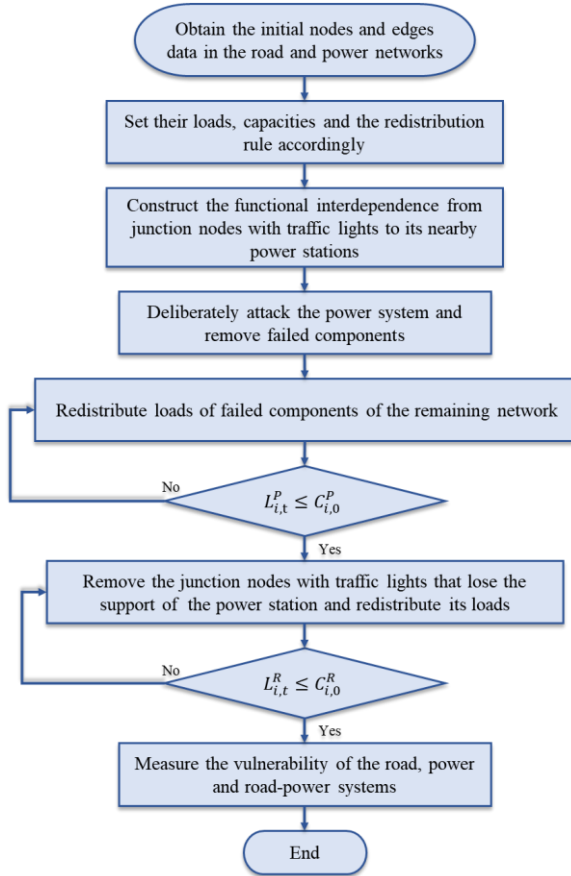


Fig. 2. The flowchart of cascading failure process.

D. Vulnerability Assessment Indicators

The failure in the IRPS can change the topology of the interdependent network and affect the service rate. The percentage of live nodes in the network is used to measure the vulnerability of the system.

1) The vulnerability assessment indicators for the RN and PN

The percentage of lived nodes in the network after cascading failures can be used to describe the vulnerability of the RN and PN, which is shown as follows:

$$V_{R/P} = 1 - \frac{N_{R/P}^{final}}{N_{R/P}} \quad (8)$$

where $N_{R/P}$ is the total number of nodes in the RN or PN, and $N_{R/P}^{final}$ is the total number of nodes in the RN or PN after cascading failures, $0 \leq V_{R/P} \leq 1$.

2) The global vulnerability of the IRPS

$$GV = (V_R + V_P)/2 \quad (9)$$

IV. CASE STUDY

To validate the performance of the proposed vulnerability assessment model, this paper applies it to the parts of the RN and PN in Xi'an, China. The road data is extracted from OpenStreetMap and converted into a network. The RN contains 774 road lines and 471 nodes including 41 junction nodes with traffic lights. The PN is generated by a BA scale-free network with 118 stations and 117 transition lines. The interdependent degree parameter α is set as 0.005. Hence, there are 298 functional interdependent links between the RN and the PN.

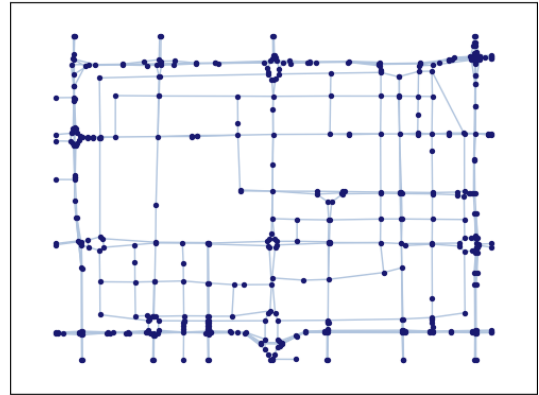


Fig. 3. Illustration of the RN in Xi'an.

A. Analysis of the Topological Characteristics of the IRPS

The analysis of the topology of the network provides the basis of empirical research on network science. According to the different performances of the network statistical index, the network characteristics of real networks like average degree connectivity, average node connectivity, average clustering, and diameter are obtained. Above mentioned statistical indexes for the RN and PN are shown in Table I.

TABLE I. STATISTICAL INDEXES OF THE RN AND PN

Network Indexes	Average degree connectivity	Average node connectivity	Average clustering	Diameter
Road	3.2927	2.0817	0.0359	30
Power	3.2885	1	0	14

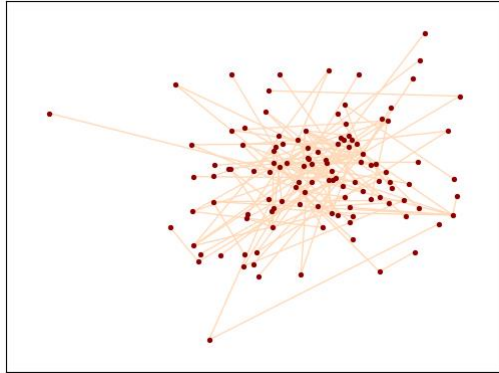


Fig. 4. The topology of the PN.

B. Results

1) The vulnerability of the RN

The RN vulnerability is shown in Fig. 5. The vulnerability of the RN depends greatly on the redistribution parameter θ . When the value of θ is smaller or bigger than 1, the RN is more robust when the capacity tolerance parameters β and γ are located in the left and upper regions of the color map. However, when θ equals 1, the RN is vulnerable even with the variation of the capacity of the junction nodes. This phenomenon demonstrates that we should carefully measure the proportion of the load to distribute to neighbor nodes and choose the redistribution rule differentiation.

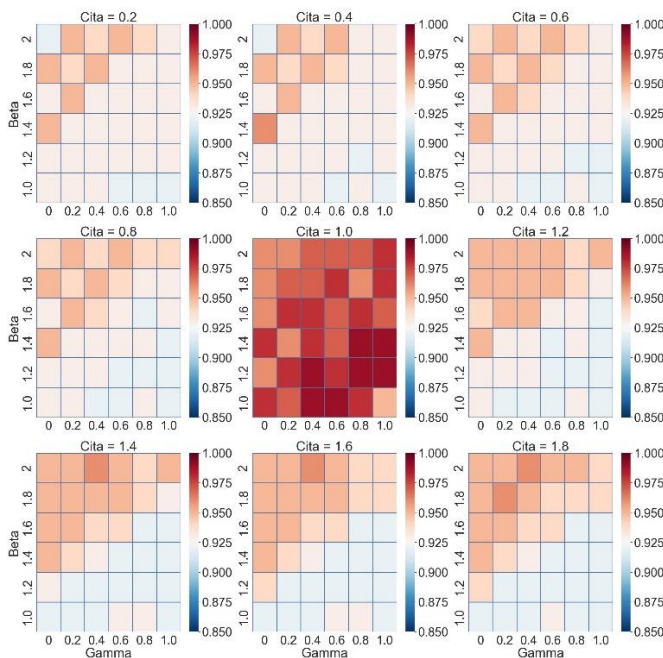


Fig. 5. The vulnerability of the RN.

2) The vulnerability of the PN

Fig. 6 gives the vulnerability of the PN. We can notice that the variation of the redistribution parameter θ does not have much impact on the redistribubility of the PN. What's more, when the capacity tolerance parameters β and γ are located in the right and upper regions of the color map, the PN is vulnerable, which is different from the RN. For the PN, the capacity tolerance parameter is the most important parameter of the system's vulnerability.

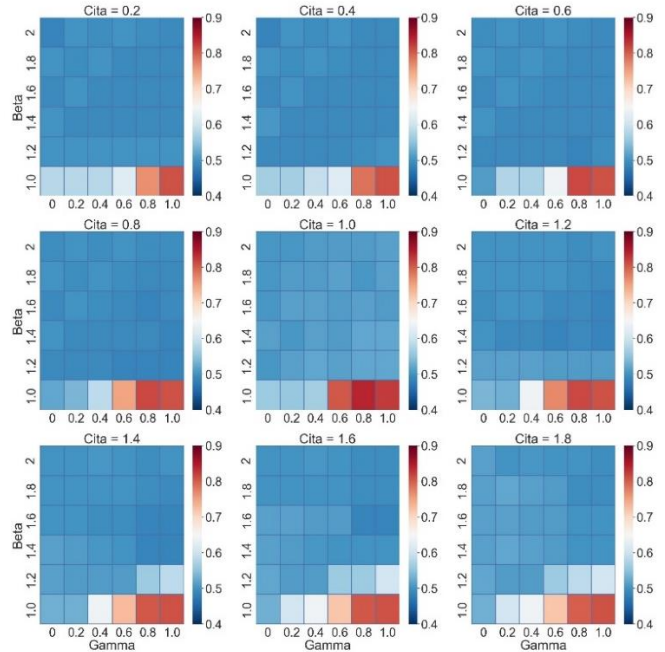


Fig. 6. The vulnerability of the PN.

3) The vulnerability of the IRPS

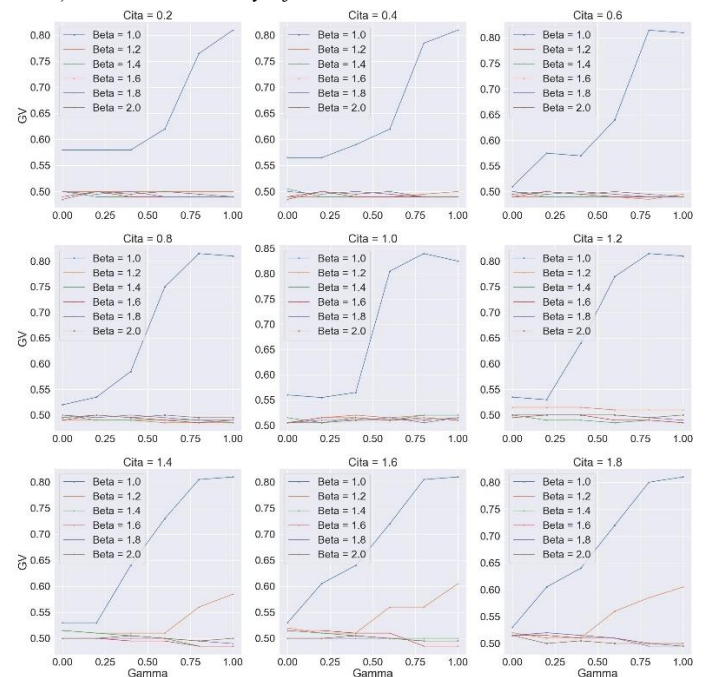


Fig. 7. The vulnerability of the IRPS.

Fig.7 gives the variation of the global vulnerability of the IRPS. When $\theta \leq 1.0$ and $\beta = 1.0$, the global vulnerability increases with the increase of γ . When $\theta > 1.4$, the influence of the capacity tolerance parameter γ on global vulnerability is increasing. It demonstrates that there is a cross-impact relationship between parameters θ and γ .

V. CONCLUSION

To assess the vulnerability of interdependent infrastructures, the functional IRPS is constructed based on the cascading failure model. Considering the distance between junction nodes with traffic lights and power stations, links between RN and PN are used to describe functional interdependence. A no-liner load-capacity model is proposed to characterize the ability of nodes to handle flows. What's more, an intelligent distribution rule is proposed to allocate the load of failed components. The proposed model is used to measure the vulnerability of parts of the RN and PN in Xi'an, China. Results show that: (1) The redistribution rule is an important tool to decrease the vulnerability of the RN, while for the PN capacity tolerance degree is more influential. (2) Parameters for the global vulnerability, there is a cross-impact between the capacity tolerance parameter, which should be studied in the future.

ACKNOWLEDGMENT

The author would like to thank all the collaborators and reviewers. This research was supported by the National Natural Science Foundation of China (Nos. 72171195 and 72002161) and the China Scholarship Council.

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