# Life-Cycle Structural Reliability of Concrete Bridges Considering Spatial Variability of Corrosion and Model Updating

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ABSTRACT: This paper presents a computational approach to life-cycle structural reliability assessment of concrete bridges under chloride-induced corrosion considering spatial variability of damage and model updating based on the results of diagnostic activities to gather information on material properties and exposure scenario. The proposed approach implements random fields to account for the effects of spatial variability of corrosion and Bayesian inference for model updating. The main steps of the reliability assessment procedure are presented and discussed with emphasis on the application to an existing prestressed concrete box-girder railway bridge.

### 1. INTRODUCTION

Structural reliability of bridges can be very high if they are properly designed. However, the effects of aging and deterioration, combined with low durability of materials and/or lack of maintenance and repair activities, can drastically reduce the structural reliability in time (Biondini and Frangopol 2016).

Safety assessment and reliability analysis are of essence to plan maintenance and repair interventions of damaged bridges. In this type of evaluations, a realistic representation of the damage processes and a proper validation and calibration of the adopted probabilistic models are of key importance. As an example, in case of chloride-induced corrosion in concrete structures when chlorides, which penetrate through the concrete cover, reach a critical content at the steel reinforcing surfaces, the corrosion starts with possible localized effects. This process is spatially distributed over the volume of structural components due to multiple factors, including exposure to environmental conditions, variability of concrete cover and concrete diffusivity, and other geometrical and mechanical characteristics related to structural detailing and concrete cracking. Ignoring the effects of spatial variability of corrosion, considering also that localized damage of steel reinforcement may govern structural failures. might be an oversimplification in reliability assessment and can lead to underestimate the probability of failure (Darmawan and Stewart 2007, Stewart and Al-Harthy 2008).

In addition to the inherent randomness associated with damage processes, in timedependent reliability analysis a significant part of epistemic uncertainties may also arise due to lack of information about random variables (Zhang and Mahadevan 2000). Bayesian approaches are often used to quantify uncertainty associated with lack of data and incorporate new information, for example gathered from inspection and diagnostic activities, for model updating (Beck & Yuen 2004).

Life-cycle structural reliability assessment of bridges under chloride-induced concrete corrosion is investigated in this paper by considering spatial variability of corrosion damage and model updating. Spatial variability associated with the bride exposure is taken into account based on random fields. The results of diagnostic activities carried out to gather information on material properties and exposure conditions are incorporated in the assessment process using Bayesian inference. The main steps of the life-cycle reliability analysis procedure are presented and discussed using an application to an existing prestressed concrete (PC) box-girder railway bridge.

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## 2. PC BOX-GIRDER BRIDGE

The case-study refers to a multi-span PC boxgirder railway bridge simply supported over each span. The total length of the bridge is 297 m and includes twelve spans with length of 22.20 m. Figure 1 shows two views of the bridge.



Figure 1: PC box-girder railway bridge.

preliminary visual inspection and geometric survey, combined with the available technical documentation, allowed to describe the current as-built configuration for an accurate structural modeling of the bridge. Figure 2 shows the PC box-girder cross-sections at mid-span and supports, with geometry (Figure 2a) and layout of prestressing strands (Figure 2b) and reinforcing bars (Figure 2c). The prestressing system provides the main steel reinforcement and includes 160 pre-tensioned strands with effective diameter of 13.35 mm and estimated prestressing stress at mid-span  $\sigma_p$ =1018 MPa (net of instantaneous and long-term prestressing losses, as reported in the technical documentation). Ordinary design reinforcement includes 148 longitudinal steel bars.



Figure 2: PC box-girder deck cross-sections at midspan and end supports: (a) geometry and layout of (b) prestressing strands, and (c) reinforcing bars.

#### 3. MODELING OF MATERIALS

Concrete compression strength,  $f_c$ , and yielding strength of both reinforcing steel,  $f_{sy}$ , and prestressing steel,  $f_{py}$ , are modeled as lognormal random variables with probability density functions (PDFs), mean values, and coefficients of variation (CoV) listed in Table 1. Mean values have been estimated based on available technical documentation.

For concrete, the stress-strain law is defined by the Saenz's model in compression, with elastic modulus  $E_c = 9500 f_c^{1/3}$  and ultimate strain  $\varepsilon_{cu} = -0.35\%$ , and elastic-perfectly plastic model in tension with tensile strength  $f_{ct} = 0.25 f_c^{2/3}$  and ultimate strain  $\varepsilon_{ctu} = 2f_{ct}/E_c$ .

Random variable	Distribution type	Mean µ	CoV	
Concrete strength, $f_c$	Lognormal	44.63 MPa	0.15	
Steel yielding strength, <i>f</i> <sub>sy</sub> (Bars)	Lognormal	430 MPa	0.15	
Steel yielding strength, $f_{py}$ (Strands)	Lognormal	1471 MPa	0.10	

*Table 1: Material strengths: PDFs and descriptors (mean and CoV).* 

A bilinear hardening model is assumed for both reinforcing steel bars and prestressing steel strands, with ultimate strength  $f_{su}=1.25 f_{sy}$  and  $f_{pu}=1.20 f_{py}$ , and ultimate strain  $\varepsilon_{su}=8\%$  and  $\varepsilon_{pu}=5\%$ , respectively.

#### 4. STRUCTURAL MODEL

The structural model of the PC bridge is based on displacement-based PC beam finite elements (Malerba 1998). The model consists of 24 beam elements each one including ten sampling crosssections. The flexural capacity of the statically determinate simply-supported bridge deck can be investigated at cross-sectional level based on equilibrium. The cross-section is subdivided in 18 quadrilateral isoparametric subdomains, as shown in Figure 3. Numerical integration over the crosssection is based on a Gauss-Lobatto integration sampling points rule with  $4 \times 4$ for each subdomain. The discretization and sampling points of the structural model have been selected to achieve a suitable trade-off between accuracy and computational cost.

# 4.1. Load model

The load model includes the dead load due to the structural self-weight,  $G_1$ , the non-structural dead load (ballast),  $G_2$ , and the live load associated with an ETR500 train with axle loads, Q, as shown in Figure 4. The magnitudes of dead and live loads are modeled as lognormal random variables with mean values and CoV listed in Table 2.



Figure 3: Structural model of the bridge: (a) beam finite elements; (b) isoparametric sub-domains and prestressing strands layout.



Figure 4: Dead and live loads (ETR500 train).

*Table 2: Dead and live loads: PDFs and descriptors (mean and CoV).* 

Random variable	Distribution type	Mean $\mu$	CoV		
Dead load (self-weight), $G_1$	Lognormal	215 kN/m	0.10		
Non-structural dead load (ballast), G <sub>2</sub>	Lognormal	180 kN/m	0.20		
Live axle loads (train), Q	Lognormal	300 kN	0.15		

#### 5. DIFFUSION AND CORROSION

The bridge is considered exposed to chlorideinduced corrosion. The environmental exposure scenario is associated to a chloride attack from the bottom side of the bridge deck with surface concentration  $C_0$ .

The chloride concentration C is described at cross-sectional level using the 2D Fick's diffusion equation, which is solved numerically using cellular automata (Biondini et al. 2004, 2006). For the purpose of diffusion simulation, the deck cross-section is discretized in square cells with size  $\Delta x=20$  mm. Under service conditions, because of prestressing, cracking is expected to be limited and with negligible impact on concrete diffusivity  $D_c$  and related diffusion rate. Corrosion initiation is associated with the attainment of a critical concentration Ccr. A severe corrosion scenario is considered to exacerbate damage effects. with corrosion а rate V<sub>corr</sub>=1.16*i*<sub>corr</sub> based on the Faraday's law, where *i*corr is the corrosion density. Localized corrosion is considered based on a pitting factor R. Concrete diffusivity,  $D_c$ , surface concentration,  $C_0$ , critical concentration, Ccr, corrosion density, icorr, and pitting factor, R, are modeled as random variables with PDFs and descriptors listed in Table 3.

Table 3: Diffusion and damage variables: PDFs and descriptors (mean and CoV).

Random variable	Distribution type	Mean µ	CoV
Concrete diffusivity, D <sub>c</sub>	Lognormal	10 <sup>-11</sup> m <sup>2</sup> /s	0.30
Surface concentration, $C_0$	Lognormal	3.0 wt.%/c	0.30
Critical concentration, $C_{cr}$	Beta	0.6 wt.%/c	0.15
Corrosion density, <i>i</i> corr	Lognormal	16 mA/m <sup>2</sup>	0.25
Pitting factor, <i>R</i>	Normal Truncated [4;8]	6	0.20

As an example of diffusion process, Figure 5 shows the maps of chloride concentration  $C/C_0$  after 10 and 50 years of lifetime at mid-span cross-section as resulting from a single Monte Carlo realization.



Figure 5: Maps of chloride concentration  $C/C_0$  after (a) 10 years and (b) 50 years of lifetime at mid-span cross-section with location of the prestressing strands.

## 6. SPATIAL VARIABILITY OF DAMAGE

The effects of the spatial distribution of damage are modeled based on the discretization of the corresponding random field into sets of spatial correlated random variables (Vu and Stewart 2005). The structure is discretized into elements and a random variable is adopted to represent the random field over each element. For instance, in the midpoint method, the random field within an element is represented by the value at the centroid of that element and the value is assumed to be constant over the element (Der Kiureghian and Ke 1988, Hajializadeh et al. 2016). Moreover, the random variables of the elements are statistically correlated based on correlation characteristics of the corresponding random field (Vanmarcke 2010). Triangular, exponential and Gaussian correlation functions are generally adopted. A correlation length is assumed to represents, in a physical sense, the approximate length over which correlation persists in the random field. Several methods have been proposed for simulating random fields (Bocchini and Deodatis 2008, Shields et al. 2011, Srivaranun et al. 2022).

Among other available options, the Covariance Matrix Decomposition (CMD) is adopted in this paper due to its stability and robust computation. A Gaussian spatial random field  $\mathbf{s}$ , associated to the *n* components of the discretization, is simulated for the random variable *X* as follows (Srivaranun et al. 2022):

$$\boldsymbol{s} = \mu_X \boldsymbol{T} + \sigma_X \boldsymbol{L} \boldsymbol{u} \tag{1}$$

where  $\mu_X$  and  $\sigma_X$  are the mean and standard deviation of X, **T** is a vector of unitary components, **u** is a standard Gaussian vector, and **L** is a lower triangular matrix obtained from the correlation matrix **R**=**LL**<sup>T</sup> by Cholesky decomposition. The correlation coefficients of the matrix **R** are computed based on a correlation function (Srivaranun et al. 2022).

The effects of spatial variability of corrosion are considered with random fields associated to chloride surface concentration  $C_0$ , which in turn influences the random variables associated with corrosion initiation and propagation. The random fields describe the variability of chloride content at global level in each sampling section of the beam finite element model, as well as at local level at the bottom side of the cross-section in each exposed boundary cell of the grid of the cellular automata. In particular, two lognormal random fields are used with CMD and the random variables associated to the spatially distributed chloride content, i.e., within the two random fields, are statistically correlated based on a 2D exponential function (Srivaranun et al. 2022) of the distance between pairs of sampling crosssections (at global level) and cells (at local level).

Figure 6a shows the prestressing strands layout for half bridge. An example of the effects of the spatial distribution of corrosion of the prestressing strands is illustrated in Figure 6b and Figure 6c, respectively in terms of ten realizations of surface chloride concentration (Figure 6b) and percentage of steel mass loss  $\delta_p$  after 30 years of lifetime for three prestressing strands (Figure 6c) along the bridge longitudinal axis (sampling sections).



Figure 6: Spatial distribution model: (a) prestressing strands layout (half bridge deck); (b) ten simulations of surface concentration; (c) percentage of steel mass loss after 30 years for three prestressing strands.

#### 7. STRUCTURAL ANALYSIS

The effects of corrosion on the time evolution of the flexural behavior of the bridge deck is shown in Figure 7 in terms of mean bending moment vs curvature diagrams of the cross-section at mid-span over a lifetime of 50 years with time step  $\Delta t$ =10 years.

The results of the time-variant structural analysis show that the bending capacity of the bridge deck significantly decreases over the lifetime, particularly with respect to bending strength capacities at cracking and ultimate state and bending curvature ductility.

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Figure 7: Time-variant bending moment vs curvature diagrams of the bridge deck cross-section at mid-span over a lifetime of 50 years ( $\Delta t$ =10 years).

# 8. MODEL UPDATING

During the service life of the bridge, experimental activities have been conducted to investigate the bridge damage state and the residual capacity of the bridge. After 30 years of lifetime nondestructive tests (e.g., sclerometer and ultrasonic tests) and laboratory tests on both concrete cores and steel samples, have been performed. As an example, the amount of chloride content has been evaluated on concrete samples located along the external bridge surface. At mid-span cross-section a nominal chloride content  $C_0=1.70$  wt.%/c has been measured. This result is associated to a likelihood function modeled as a lognormal distribution  $LN[\mu=1.70; \sigma=0.30]$ . Figure 8 shows the prior, likelihood, and posterior PDFs of chloride surface content  $C_0$ . Bayesian statistical inference is used to combine prior information and new data gathered from diagnostic activities carried out at t=30 years for compressive concrete strength  $f_c$ , reinforcing steel yielding strength  $f_{sy}$ , and chloride surface content  $C_0$ .

#### 9. STRUCTURAL RELIABILITY

The lifetime structural reliability of the bridge is assessed with respect to a bending failure limit state under the loading condition illustrated in Figure 4, where the axle loads are applied to maximize the bending moment demand.



*Figure 8: Prior, likelihood, and posterior PDFs of chloride surface content*  $C_0$ *.* 

Subset simulation is used to assess reliability, with  $N=10^4$  samples for each subset and a target probability  $p_0=0.10$ . Figure 9 shows the time-variant reliability index profile without and with Bayesian updating at time t=30 years.



Figure 9: Time-variant reliability index on a 50-year lifetime ( $\Delta t$ =10 years) without (dashed line) and with (continuous line) Bayesian updating based on the results of diagnostics carried out at t=30 years.

The results show the impact of spatially distributed corrosion damage on the life-cycle structural reliability of the bridge and highlight the importance of exploiting the information from diagnostic activity for a more accurate prediction of the residual lifetime.

#### **10. CONCLUSIONS**

An approach to life-cycle structural reliability assessment of concrete bridges under corrosion has been proposed to account for both the spatial variability of damage and model updating based on the results of diagnostic activities to gather information on material properties and exposure scenario. The methodology has been applied to an existing PC box-girder railway bridge under corrosion. Random fields have been adopted to account the effects of spatial variability of damage due to chloride-induced corrosion. Bayesian inference has been adopted to account for the results of diagnostic activities adopted to gather information on material properties and exposure scenario. The results of the application allowed to quantify the impact of spatially distributed corrosion damage and to emphasize the role of diagnostic activity and related model updating in the time-variant structural reliability assessment and residual lifetime prediction of concrete bridges exposed to an aggressive environment. Further research is needed to investigate the role of several factors associated with structural behavior and uncertainty modeling in concrete bridges under corrosion, including multiple failure mechanisms, such as bond and shear failures, variability in space and time and residual levels of prestressing, multiple serviceability and ultimate limit states, internal stress-redistribution and redundancy effects in statically indeterminate systems under moving loads, among others.

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