

A. DI GERLANDO - I. VISTOLI

SPECIAL TRANSFORMERS FOR THE MEASUREMENT
OF HARMONIC NOISE OF D.C. CURRENTS

SPEEDAM - SYMPOSIUM ON POWER ELECTRONICS,
ELECTRICAL DRIVES, ADVANCED ELECTRICAL MOTORS

POSITANO, 19-21 MAGGIO 1992



Symposium on
**Power Electronics, Electrical Drives,
Advanced Electrical Motors**

PROCEEDINGS

Positano (Italy), May 19-21, 1992

edited by



Special transformers for the measurement of harmonic noise of DC currents

Di Gerlando, A.; Vistoli, I.
Politecnico di Milano, Italy

ABSTRACT: some design problems regarding a special current transformer are presented and discussed: the transformer must be suited for the measurement of the harmonic pollution of D.C. currents generated or absorbed by static converters; for example, situations of this kind occur in the stationary and in the on board systems of the D.C. traction plants.

1. INTRODUCTION.

In several D.C. plants, connected with A.C. plants by static converters, an important problem is the limitation and the measurement of the D.C. side harmonic pollution generated by the converters. On the one hand, in order to limit this pollution, especially devoted power filters are frequently employed; on the other hand, sometimes the verification of the levels of the harmonic content of the currents on the D.C. side requires the measurement of alternative components considerably smaller than the direct component, in a frequency range within a few Hz to some kHz.

A typical situation is that of the D.C. traction lines: for example, the limits of the harmonic noise for the on board conversion units (such as the auxiliary inverters) impose that the maximum allowed amplitude of the alternative components be of the order of 10^{-3} to 10^{-4} times the direct component, from a few Hz up to 10 kHz (fig.1).

Usually the precision class of the current transducers is given in terms of the maximum value of the measured current: considering that the amplitude of the harmonic components is low compared with the average value, the accuracy with which the current harmonic content can be estimated is greatly limited.

On the other hand, the presence of the direct component at the output terminals of the transducer, if not suitably filtered, implies several problems, in particular for the harmonic analysers: in fact, also their precision and resolution are related to the maximum instantaneous value of

the signal to be analysed, then the instrument errors are affected by the direct component too.

The use of a current transformer (C.T.), suitably designed, as an interface device between the power circuit and the measurement system, involves certain advantages: the galvanic insulation (for safety reasons and for the de-coupling of the acquisition channels of the instruments) and the filtering of the direct component (obtained without high-pass filters added).

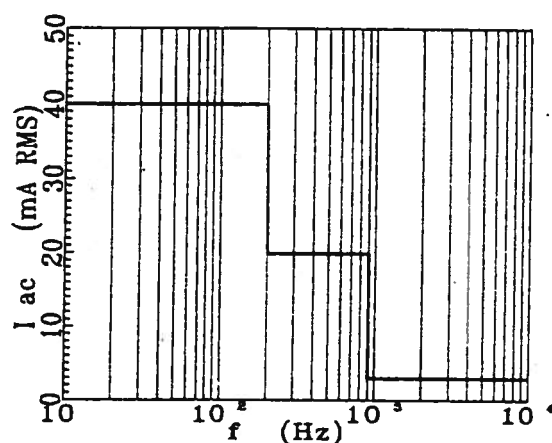


Fig.1 - Allowed limit threshold values of the harmonic pollution generated by auxiliary static converters for on board traction plants: line voltage: 3000 V d.c.; converter rating: 45 kVA.

The design problems are numerous: on one hand the high direct current requires the adoption of gapped cores, in order to prevent the magnetic saturation; on the other hand, the existence of a gap implies high values of the magnetizing components of the harmonic currents.

Leaving the winding and core data out of consideration, the highest errors occur at low frequencies, where the primary current is mostly magnetizing, and at high frequencies, where the leakage secondary reactance grows and the capacitive phenomena occur.

2. BEHAVIOUR OF MAGNETIC MATERIALS WITH D.C. AND A.C. FIELDS SUPERPOSED.

The presence of a great direct component in the current to be analysed causes a magnetic behaviour quite different from that in A.C. operation: as known, the A.C. operation of a magnetic material is described by the normal magnetization curve, or the curve $\mu = \mu(B)$, where μ is called normal permeability, or, simply, permeability.

On the contrary, the experience shows that, given an alternating magnetic strength ΔH and a biasing constant magnetic strength (H_{dc}) superposed, the alternative flux density ΔB is not μ times ΔH , but sensibly lower: in this case there are asymmetrical hysteresis cycles having a slope defined by another permeability coefficient, called incremental permeability (μ_{Δ}); μ_{Δ} decreases with the increase of H_{dc} (fig.2.a), and it increases with the increase of ΔH (fig.2.b).

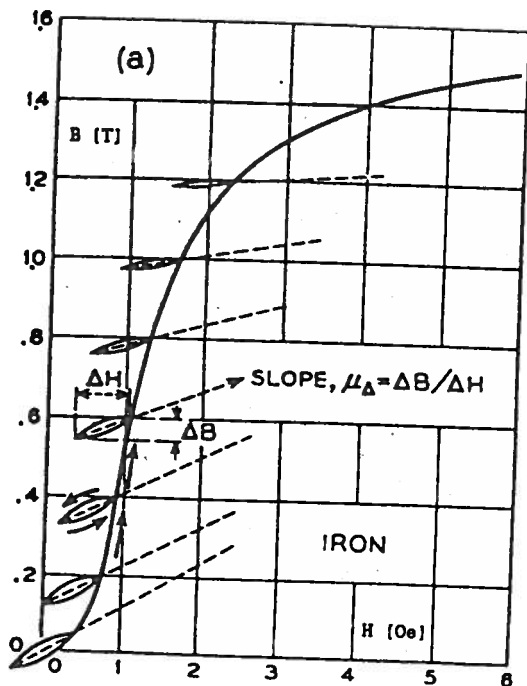


Fig.2.a.- Minor hysteresis loops in iron²: μ_{Δ} (slope of the loops) as a function of the biasing field strength (H_{dc}), being constant the alternating magnetic strength (ΔH).

$$1 \text{ Oe} = 10^3 / (4 \cdot \pi) \text{ A/m}$$

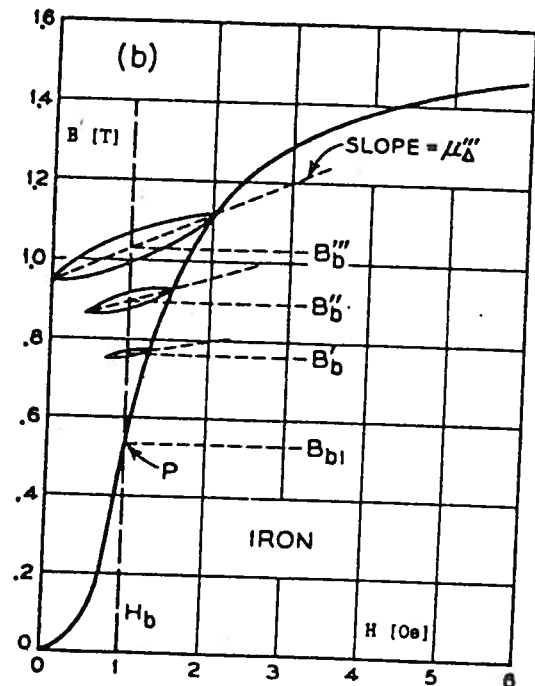


Fig.2.b.- Minor hysteresis loops in iron²: μ_{Δ} (slope of the loops) as a function of the alternating magnetic strength (ΔH), being constant the biasing field strength (H_{dc}).

It is defined reversible permeability (μ_{rev}) the following quantity:

$$\mu_{rev} = \lim_{\Delta H \rightarrow 0} \left(\frac{\Delta B}{\Delta H} \right)_{H_{dc} = \text{const}} = \lim_{\Delta H \rightarrow 0} \mu_{\Delta} \quad (1)$$

Considering the low amplitude of the alternative components that occur in the present case, in the following we will refer to the reversible permeability rather than to the incremental permeability.

It is defined initial permeability the quantity:

$$\mu_i = \lim_{\Delta H \rightarrow 0} \left(\frac{\Delta B}{\Delta H} \right)_{H_{dc} = 0} = \lim_{\Delta H \rightarrow 0} \mu \quad (2)$$

On the basis of some theoretical-experimental studies² we can assume that the reversible permeability obeys the following law:

$$\mu_{rev} \approx \mu_i \cdot \left(1 - \frac{B_{int}}{B_{sat}} \right)^k \quad (3)$$

where:

- B_{sat} is the material saturation flux density;
- B_{int} is the intrinsic biasing flux density, that can be assumed equal to:

$$B_{int} = B_{dc} - \mu_0 \cdot H_{dc} \approx B_{dc} \cdot (1 - \mu_0 / \mu(B_{dc}));$$

considering that usually B_{dc} is quite lower than saturation value B_{sat} , we have $\mu(B_{dc}) \gg \mu_0$, from which: $B_{int} \approx B_{dc}$;

- k is an exponent depending on the kind of the magnetic material: its value is included in the range 0.75-2.

From eq. (3) we can deduce that the most suited magnetic material has high values of initial permeability μ_i and saturation flux density B_{sat} . During the design, the adoption of a material with high values of B_{sat} and μ_i allow to choose a high biasing flux density B_{dc} (so that it is not necessary to adopt an air-gap too much wide), without reducing too much the reversible permeability.

The examination of the characteristics of the materials commercially available suggests that materials having the best values of these parameters are not silicon-iron alloys, but Mumetal alloys (fig.3), that have:

$$\mu_i \approx 25 \cdot 10^3 \mu_0; B_{sat} \approx 0.7 \text{ T}; k \approx 0.75.$$

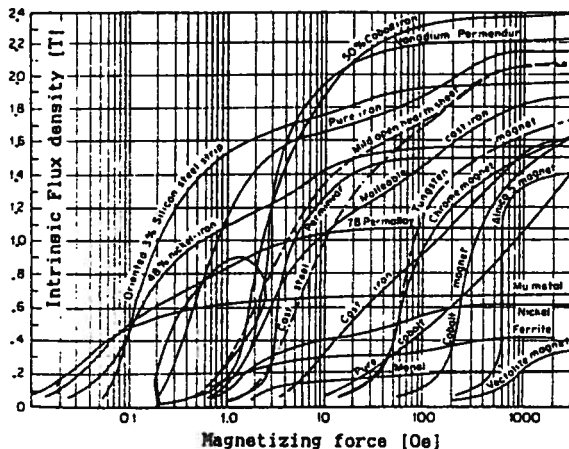


Fig.3 - Typical normal magnetization curves in a wide range of commercial magnetic materials.

3. THE D.C. POLARIZED C.T. IN THE OPERATION AT LOW FREQUENCY.

The high value of the direct current component in the C.T. primary winding imposes the adoption of a gapped core, in order to limit the saturation level of the material.

This structural characteristic is typical also for the current transformers designed for the measurement of transient and fault currents: about these devices, a wide literature and Standard references can be found.

In the D.C. polarized C.T. the comparative levels of the harmonic components with respect to the D.C. component implies some peculiar characteristics: because of this, the study of the design and of the model of this kind of device deserves a special analysis.

During operation at low and medium frequencies (when the effects of the parasitic capacitances are negligible) the parameters of the "T" equivalent circuit of a C.T. are, as known, the following (referred to the same number of turns):

- primary series impedance: $\bar{Z}_1 = R_1 + j \cdot \omega \cdot L_1$;
- secondary series impedance: $\bar{Z}_2 = R_2 + j \cdot \omega \cdot L_2$;
- parallel admittance: $\bar{Y}_0 = G_0 - j/(\omega \cdot L_0)$.

Called R_m the value of the measurement resistance, for each harmonic component the current ratio I_2/I_1 (RMS values) equals:

$$\frac{I_2}{I_1} = \frac{N_1/N_2}{\sqrt{\left[1 + G_0 \cdot (R_2 + R_m) + \frac{L_2}{L_0}\right]^2 + \left[G_0 \cdot \omega \cdot L_2 - \frac{R_2 + R_m}{\omega \cdot L_0}\right]^2}} \quad (4)$$

It is evident that the error increases as the frequency decreases: then, the maximum acceptable error at a minimum reference frequency must be indicated among the design specifications; in this situation, eq. (4) can be rewritten as follows:

$$\frac{I_2}{I_1} \approx \frac{N_1/N_2}{\sqrt{1 + \left[\frac{R_2 + R_m}{\omega \cdot L_0}\right]^2}} \quad (5)$$

The use of eq. (5) instead of eq. (4) corresponds to neglecting the effects of the leakage flux linked with the secondary winding compared with the main flux (in terms of parameters this means to assume $L_2 \ll L_0$), and to assuming negligible the magnetic material losses compared with the magnetizing reactive power (considering the presence of the air-gap, that absorbs the prevalent part of the reactive power, the parallel branch can be practically considered purely reactive: $G_0 \ll 1/(\omega \cdot L_0)$).

Then eq. (5) is an approximate expression: nevertheless, its employment (partially not in compliance with some classical theories³) is acceptable for design calculations, referring to the low frequency range and considering the peculiarities of the considered device.

In general, for reasons due to construction needs, one can assume that the core is always provided with two air-gaps, having the same geometrical thickness δ , disposed magnetically in series; moreover, the core has a geometrical length l_n and a net magnetic core section A_n .

Defined an equivalent magnetic air-gap:

$$\delta_{eq} = \delta + (l_n/2)/(\mu_{rev}/\mu_0) \quad (6)$$

(where μ_{rev} is the reversible permeability, evaluated by eq. (3)), the inductance L_0 , referred to the turns number N_2 , can be expressed as follows:

$$L_0 = N_2^2 \cdot \mu_0 \cdot A_n / (2 \cdot \delta_{eq}) \quad (7)$$

By substituting eq. (7) into eq. (5) one obtains:

$$\frac{I_2}{I_{1a}} \approx \frac{1}{\sqrt{1 + \beta^2}} \quad (8)$$

where $I_{1a} = I_1 \cdot N_1/N_2$ is the primary current referred to the secondary and β equals:

$$\beta = \frac{2 \cdot \delta_{eq} \cdot (R_2 + R_m)}{\omega \cdot N_2^2 \cdot \mu_0 \cdot A_n} \quad (9)$$

ANTONINO DI GERLANDO, IVO VISTOLI
SPECIAL TRANSFORMERS FOR THE MEASUREMENT OF HARMONIC NOISE OF D.C. CURRENTS

The quantity β can be defined as "error function". Besides the current ratio, it is of interest to give the approximate expressions of other functional quantities:

- phase error γ :

$$\gamma = \angle \bar{I}_2 - \angle \bar{I}_1 \approx \arctg(\beta) ; \quad (10)$$

- e.m.f. per turn E_1 :

$$E_1 = \frac{\omega \cdot \Phi}{\sqrt{2}} \approx N_1 \cdot I_1 \cdot \frac{R_2 + R_m}{N_2^2} \cdot \frac{1}{\sqrt{1 + \beta^2}} ; \quad (11)$$

- magnetizing current:

$$I_\mu \approx I_1 \cdot \frac{\beta}{\sqrt{1 + \beta^2}} ; \quad (12)$$

- voltage V_m at the terminals of the measurement resistance R_m :

$$V_m \approx \frac{N_1}{N_2} \cdot \frac{R_m \cdot I_1}{\sqrt{1 + \beta^2}} . \quad (13)$$

When β increases, the magnetizing current and the ratio and phase errors increase too, while the e.m.f. per turn and the voltage at the measurement resistance terminals decrease: then, the more β increases, the more the C.T. performances are getting worse.

With regard to the influence of the various parameters one can say:

- air-gap δ : the presence of the air-gap is necessary in order to avoid an excessive reduction of the reversible permeability μ_{rev} , caused by the direct component I_{dc} of the primary current. As a matter of fact, the biasing flux density B_{dc} can be practically assumed as follows:
- $$B_{dc} \approx \mu_0 \cdot N_1 \cdot I_{dc} / (2 \cdot \delta) ; \quad (14)$$
- on the other hand, a too high value of the air-gap increases β , worsening the performances;
- total secondary resistance: a high value of such a resistance ($R_2 + R_m$) causes greater C.T. errors; on the other hand, high values of the voltage V_m can be obtained by high values of R_m ;
 - frequency: as the frequency increases, there is a decrease of the errors, at least in the low and medium frequency range ($f < 1$ kHz, when the effects of the parasitic capacitances are negligible);
 - number of turns N_2 of the secondary winding: the increase of N_2 implies a remarkable reduction of the errors, thanks to its quadratic influence; on the other hand, when N_2 increases, the voltage V_m decreases proportionally to the inverse of N_2 ;
 - section A_n of the magnetic core: a high value of A_n reduces the errors, but it requires a heavier, bulkier and more expensive design;
 - middle geometric length ℓ_n of the magnetic core: it is implicit in the expression (6) of the equivalent air-gap δ_{eq} : if the material is characterized by high values of μ_{rev} , ℓ_n affects

very weakly the errors: in this case the magnetic length of the core equals a few percent of the air-gap length. The definition of the geometric length ℓ_n depends on the area of the window necessary for the windings and on technological needs regarding the construction of the core.

One can observe that, from the qualitative point of view, some of these parameters have an influence on the design and on the operation of the C.T. that is similar to all the other kinds of C.T.; nevertheless, in this case this influence shows particular quantitative characteristics, due to the special-operation conditions.

4. GENERAL DIMENSIONING CRITERIA FOR THE DESIGN OF THE D.C. POLARIZED C.T..

The design begins from the specification of the ratio error, that can be defined as follows:

$$\epsilon = 1 - \frac{I_2}{I_{2a}} . \quad (15)$$

In particular, the error ϵ to be considered is that ($\tilde{\epsilon}$) corresponding to a certain minimum reference angular frequency ($\tilde{\omega}$); by manipulating eq. (8) and (15), one obtains the corresponding value of the error function $\tilde{\beta}$:

$$\tilde{\beta} = \frac{\sqrt{2 \cdot \tilde{\epsilon} - \tilde{\epsilon}^2}}{1 - \tilde{\epsilon}} . \quad (16)$$

With regard to the lamination thickness, the need of a linear response of the transducer in a frequency range up to some kHz requires the adoption of a maximum value of thickness equal 0.3 mm, in order to limit the dissipative and demagnetizing effects of the eddy currents.

With regard to the core design, the possible solutions could be numerous: the toroidal shape would be the best one (negligible leakage fluxes and immunity to disturbances); on the other hand, for constructive reasons it is better to adopt a "double C" shaped core. Besides, the two configurations are not so different each other (the "double C" shaped core is a toroidal core slightly squashed). For this reason, in the following a toroidal core, equivalent to the "double C" shaped one, will be considered: the equivalence means equal value of section and equal middle length. In this way, it is easy to obtain simple expressions of the quantities of interest.

A toroidal magnetic core having a rectangular section is characterized by the internal core diameter (D_n) and by core widths in the radial direction (a_n) and in the axial direction (b_n). Let be $K_L = b_n/a_n$ the width ratio, $K_n = a_n/D_n$ the core coefficient and K_{st} the stacking factor; the section of the core A_n equals:

$$\lambda_n = K_{al} \cdot K_L \cdot K_n^2 \cdot D_n^2 \quad (17)$$

and the middle geometric length ℓ_n of the magnetic core equals:

$$\ell_n = \pi \cdot (1 + K_n) \cdot D_n \quad (18)$$

Eq. (17) and (18) show that the core shape is defined by the internal diameter D_n and by the adimensional factors K_L and K_n .

By referring the magnetic semilength of the core to the value of the air-gap δ by means of the coefficient λ_n , called core magnetic factor:

$$\lambda_n = [(\ell_n/2)/(\mu_{rev}/\mu_0)]/\delta, \quad (19)$$

from the comparison with eq. (6) one can write:

$$\delta_{eq} = (1 + \lambda_n) \cdot \delta \quad (20)$$

From eq. (19) and (20) one can observe that the parameter λ_n has the same meaning of the saturation ratio, usually employed for the study of the most kind of gapped magnetic circuits.

On the other hand, by substituting eq. (18) in eq. (19) one obtains an expression of λ_n , useful for the design:

$$\lambda_n = \frac{\pi \cdot (1 + K_n)}{2 \cdot (\mu_{rev}/\mu_0)} \cdot \frac{D_n}{\delta} \quad (21)$$

In order to limit the influence that the amplitudes of the direct and alternative components have on the current ratio, the core magnetic factor λ_n must be suitably lower than unity (for example: $\lambda_n \approx 0.1$).

It has been already observed that the error function $\tilde{\beta}$ decreases when the number of turns N_z increases, following a quadratic law: then, it is convenient to increase N_z as much as possible. On the other hand, eq. (9) contains the resistance R_z/N_z^2 : this resistance, indicated with R_z^* , equals:

$$R_z^* = \frac{R_z}{N_z^2} = \rho \cdot \frac{\ell_{mz} \cdot N_z}{A_{cz}} \cdot \frac{1}{N_z^2} = \rho \cdot \frac{\ell_{mz}}{A_{rz}} \quad (22)$$

where ℓ_{mz} is the length of the middle turn of the secondary winding, A_{cz} is the conductor section, A_{rz} is the total copper section of this winding: by keeping constant A_{rz} during the variation of N_z , nor the current density of the secondary winding neither its gross utilization area varies.

Considering the variation of the design parameters, in particular the resistance R_m , a need to be observed is to obtain a suited value of the voltage V_m at the terminals of the measurement resistance: eq. (13) shows that, neglecting the effect of β , this voltage does not vary considerably if the ratio R_m/N_z (indicated with r_m) is maintained constant:

$$V_m \approx r_m \cdot \frac{N_z \cdot I_s}{\sqrt{1 + \tilde{\beta}^2}} \quad (23)$$

By manipulating eq. (9), the error function $\tilde{\beta}$ changes as follows:

$$\tilde{\beta} = \frac{2 \cdot \delta_{eq}}{\tilde{\omega} \cdot \mu_0 \cdot A_n} \cdot \left[\frac{R_z^*}{N_z} + \frac{r_m}{N_z} \right] \quad (24)$$

Eq. (24) points out the important fact that $\tilde{\beta}$ decreases when the number of turns N_z increases (with r_m constant): then the C.T. errors decrease too, being unvaried the amplitude of the measurement signal V_m .

From eq. (24), considering that $R_z^* \ll r_m/N_z$, and by the eq. (14), (16), (17), (21), (23), one obtains an expression of the product $N_z \cdot D_n^2$ particularly interesting:

$$N_z \cdot D_n^2 = \frac{1 + \lambda_n}{\sqrt{2 \cdot \tilde{\omega}^2 - \tilde{\omega}^2}} \cdot \frac{1}{\tilde{\omega} \cdot K_{al} \cdot K_L \cdot K_n^2} \cdot \frac{V_m \cdot I_{dc}}{B_{dc} \cdot I_s} \quad (25)$$

As a matter of fact, this expression includes the main functional and constructive quantities, that affect the product $N_z \cdot D_n^2$:

- the core magnetic factor λ_n has a weak influence;
- the choice of the reference angular frequency $\tilde{\omega}$ and of the corresponding current ratio error $\tilde{\omega}$ is very important: this choice can make the design very heavy, in case of too severe specifications;
- the characteristics of the magnetic material are included in the value of the core magnetic factor λ_n (inversely proportional to μ_{rev}) and in the flux density B_{dc} (the higher is B_{sat} , the higher can be B_{dc});
- the product $N_z \cdot D_n^2$ increases as the signal V_m increases;
- the product $N_z \cdot D_n^2$ increases as the direct component I_{dc} increases, while it decreases as the alternative component I_s increases.

The quantities not explicitly expressed in eq. (25) are the air-gap and the primary number of turns. With regard to the air-gap, from eq. (18) and (19) one can obtain:

$$\delta = \frac{\pi}{2} \cdot \frac{1 + K_n}{\mu_{rev}/\mu_0} \cdot \frac{D_n}{\lambda_n} \quad (26)$$

then the air-gap increases with the internal core diameter D_n and with the core coefficient K_n ; the higher is the relative reversible permeability, the smaller is the air-gap: in practise, above all for construction reasons, it is better to fix the ratio δ/D_n at the beginning of the design.

With regard to the primary number of turns, by solving eq. (14) one obtains:

$$N_z \approx \frac{2}{\mu_0} \cdot B_{dc} \cdot \frac{\delta}{I_{dc}} \quad (27)$$

Finally, the observation of eq. (25), (26), (27) suggests the possibility to employ the same core for measurements of harmonic noise of cur-

ANTONINO DI GERLANDO, IVO VISTOLI
SPECIAL TRANSFORMERS FOR THE MEASUREMENT OF HARMONIC NOISE OF D.C. CURRENTS

rents having quite different average components I_{dc} : this corresponds to the use of the same transformer for different nominal values.

As a matter of fact, eq. (25) contains the ratio I_{dc}/I_1 : then, if the harmonic content is constant in per unit when referred to the I_{dc} component, the product $N_2 \cdot D_n^2$ does not vary as I_{dc} increases; from eq. (26) one can deduce that the air-gap does not change too. Then, according to eq. (27), the number of turns N_2 must decrease as I_{dc} increases, with a law of inverse proportionality. In conclusion, within certain limits and provided that $N_2 \geq 1$, the same core with the same number of turns N_2 can be employed, also for performing measurements regarding fluctuating direct currents having amplitude different from the value considered for the design of the current transformer.

5. CONCLUSIONS.

In this paper it has been presented and discussed a new type of current transformer, suited to perform the measurement of small amplitude harmonic noise associated to high values of direct current components, absorbed or generated by static converters, for example, the converters used in D.C. traction lines.

The design criteria of the transformer are quite different from those adopted for A.C. current measurements, either for the choice of the magnetic material or for the dimensioning of the core and windings.

The studies continue, in order to improve the design and the high frequency model of the C.T., also by means of experimental tests using prototypes.

REFERENCES

- [1] "Capitolato tecnico di fornitura dei convertitori statici della potenza di 45 kVA o superiore per la ricarica delle batterie e l'alimentazione delle utenze di bordo delle carrozze FS munite di impianto di condizionamento." TV 9.3/CSA/99.5/27.5, maggio 1982.
- [2] R. Bozorth: "Ferromagnetism"; D. Van Nostrand, New York, 1951.
- [3] B. Hague: "Instrument Transformers"; Pitman & Sons, London, 1936.

LIST OF SYMBOLS

A_{c2} conductor section of the secondary winding
 a_n radial width of the core section
 A_n net magnetic core section
 A_{r2} total section of the secondary winding

B_{dc} D.C. biasing flux density
 B_{int} intrinsic D.C. biasing flux density
 B_{sat} magnetic material saturation flux density
 b_n axial length of the core section
 β error function (generic value, see eq. (9))
 $\tilde{\beta}$ error function (design value, see eq. (16))
 D_n internal diameter of the toroidal core
 δ geometrical air-gap
 δ_{eq} equivalent magnetic air-gap (see eq. (6))
 E_t e.m.f. per turn
 ε current ratio error (generic value)
 $\tilde{\varepsilon}$ current ratio error (design value)
 Φ magnetic flux in the core
 G_o parallel conductance of the equiv. circuit
 γ phase error
 H_{dc} biasing D.C. magnetic strength
 I_{dc} D.C. component of the primary current
 I_μ magnetizing current
 I_1 A.C. component (single harmonic) of the primary current (RMS)
 I_{1s} A.C. primary current, referred to secondary
 I_2 secondary current (single harmonic)
 K_L ratio of the core widths ($K_L = b_n/a_n$)
 K_n core coefficient ($K_n = a_n/D_n$)
 K_{st} lamination stacking factor
 l_{m2} length of the middle secondary turn
 l_n geometric length of the magnetic core
 L_o parallel inductance of the equiv. circuit
 L_1 primary series induct. of the equiv. circuit
 L_2 secondary series induct. of the eq. circuit
 λ_n core magnetic factor (see eq.s. (19) e (20))
 μ normal permeability
 μ_i initial permeability
 μ_Δ incremental permeability
 μ_o vacuum permeability
 μ_{rev} reversible permeability
 N_1 number of turns of the primary winding
 N_2 number of turns of the secondary winding
 r_m specific measurement resist. ($r_m = R_m/N_2$)
 R_m measurement resistance
 R_1 primary series resist. of the equiv. circuit
 R_2 secondary series resistance of the C.T. equivalent circuit
 R_2^* specific secondary resistance, referred to 1 secondary global equiv. turn (see eq. (22))
 ρ copper resistivity
 V_m measurement voltage at the terminals of R_m
 ω angular frequency of the A.C. quantities (single harmonic)
 $\tilde{\omega}$ angular frequency corresponding to the error $\tilde{\varepsilon}$ (design value)
 \bar{Y}_o parallel admittance of the equiv. circuit
 \bar{Z}_1 primary series impedance of the "T" shaped C.T. equivalent circuit
 \bar{Z}_2 secondary series impedance of the "T" shaped C.T. equivalent circuit.