

# Managing Input Parameter Uncertainty in Digital Twins

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**Abstract:** Discrete event simulation models can be used in digital twins to support the design and decision-making process of manufacturing systems. In many industrial contexts, the collection of real-time data from the system is a costly task and the performance predicted by digital twins may be affected by input uncertainty, due to the scarcity of data used to input simulation parameters, thus leading stakeholders to biased decision-making. Literature approaches treat this problem mainly from a theoretical point of view and are applied on very simple systems that do not adequately represent real factories.

The aim of this paper is to explore the advantages and drawbacks of an input uncertainty simulation technique, namely the metamodel-assisted bootstrapping procedure. This technique is applied and extended to evaluate the production rate of a lab-scale manufacturing system. We show it is possible, despite the scarcity of data, to build a reliable confidence interval on the production rate and to identify those parameters whose effect on the performance is most relevant. Moreover, the marginal contribution of each input parameter to the performance can be quantitatively assessed, thus enabling stakeholders to identify which parameters to focus on in data collection activity.

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**Keywords:** Simulation under input uncertainty; Metamodel-assisted bootstrapping; Digital twins; Manufacturing systems' optimization

## 1. INTRODUCTION

The performance evaluation of manufacturing systems has been subject of high interest over the last decades. Taking advantage of the increasing computational power of computers, a digital representation of a manufacturing system can now be twinned with its physical counterpart to provide real time support in the decision making for production control. Indeed, discrete event simulation models are proposed to be used as *digital twins* of production systems and to provide prediction and what-if analysis functionalities during the system operation (Lugaresi and Matta, 2018). An overall review of industrial applications of digital twins is provided by Liu et al. (2021).

In this context, prediction accuracy of simulation is increasing a major role as the simulation applications get closer and closer to the physical system. Two types of error are related with performance estimation of simulated systems: an *intrinsic error* due to the stochastic nature of simulation and the finite length of simulation runs, and an *extrinsic error* due to wrong estimation of the parameters used as input to the simulator (Barton et al., 2014). Input simulation parameters define the input distributions and they can be estimated in the light of the collected *on-field* data. While, on one hand, experts deal with intrinsic uncertainty defining confidence intervals on the performance instead of point estimates, on the other hand, the only way to reduce extrinsic uncertainty is the collection of a larger amount of empirical data. Therefore, a fundamental aspect to be taken into account for the correct modelling of a manufacturing system is

the possibility of collecting data to estimate parameters (such as the reliability of machines or the cycle time of workstations, etc). However, it is sometimes impossible, or very costly, to collect sufficient data from which to properly estimate the values of input parameters. This problem is emphasized in digital twin applications, where the simulation is coupled in real time with a system that may change in its behaviour and design and parameters estimation must be repeated at every major change of the real system. In such cases, therefore, extrinsic uncertainty may lead to misjudgements of the system's performance. Ignoring the scarcity of data, and the resulting extrinsic uncertainty, may lead companies to misjudgements of the physical system's performance. This implies wrong control choices and optimization procedures.

This paper studies the input uncertainty problem in manufacturing systems applications. Existing literature treats the input uncertainty problem from a theoretical point of view and demonstrates the analytical asymptotic validity of the proposed formulations. However, the existing procedures are tested only on very simple queuing systems (such as single server  $M/M/1/q$  queuing systems) to prove their reliability and robustness whereas more complex manufacturing applications are not fully explored yet. In this work, the *metamodel-assisted bootstrapping* is applied (Barton et al. (2014)) and extended in its analysis for manufacturing systems. Starting from the few data that can be collected in real-time from the physical system, we describe how extrinsic uncertainty of input parameters can be rigorously quantified and the extent to which it

propagates into performance estimation uncertainty. Using the proposed procedure, reliable performance estimates and design insights can be provided to the stakeholders.

This content is organized as follows. In section 2, a brief literature analysis on the input uncertainty is reported. Section 3 describes the proposed procedure to deal with the extrinsic error of simulation models for manufacturing systems. In section 4, a case of study is presented to show the applicability of the procedure to a manufacturing line. Section 5 concludes this work by drawing some considerations useful for further developments.

## 2. LITERATURE REVIEW

Traditional simulation approaches for the performance evaluation of manufacturing systems rely on the hypothesis of known input parameter values. Jahangirian et al. (2010) and Negahban and Smith (2014) provide a comprehensive review of multiple discrete event simulation approaches with a focus on manufacturing systems' design.

However, simulation output depends on the input distributions used to drive the model. When these distributions are fitted using finite samples of real-world data, errors in the input distributions may arise and these errors inevitably propagate into the output estimation. Yet this error is rarely considered and assessed in simulation output analysis. In recent years, several methodologies have been developed in order to keep into account input parameter uncertainty when estimating the performance of a manufacturing systems. Three main techniques are described in literature to adapt traditional simulation approaches to the case in which lack of input data is experienced and the value of input parameters values has to be inferred: Delta method, Bayesian approach and Bootstrap resampling. Barton (2012) presents a discussion of input uncertainty issues and the developed methodological approaches to characterize the impact on simulation output arising from input parameters errors. A description of Delta method, Bootstrap resampling and Bayesian approach is provided in his tutorial.

The Delta method (Cheng and Holloand (1997) and (2004)) makes it possible to evaluate separately the contribution to performance estimation error of *intrinsic stochastic uncertainty* and *extrinsic parameter uncertainty*. The overall estimation error is quantified as the sum of the two contributions. This method exploits the asymptotic properties of parameters' maximum likelihood estimators and the Taylor series approximation of the expected system's response. Delta method is used by Corlu and Biller (2013) to provide a subset selection procedure, which is a Ranking & Selection optimization process. Their objective is to provide a decision rule which is able to identify a subset of system designs whose performance is within a user-specified distance from the performance of the true best system. Song and Nelson (2019) use the Delta method for their optimization procedure which aims at the identification of the real-world optimal from a finite amount of system designs being compared. The main concerns about Delta method are the asymptotic properties of maximum likelihood estimators (that are not always verified in case the amount of available data is not sufficient), the Taylor series approximation on which it is based (which requires

the computationally costly calculation of gradients) and, finally, the fact that the extrinsic error inflates as the number of input parameters increases, thus making the technique ineffective for complex systems involving many uncertain input parameters.

In Bayesian approach, a simulation experiment is run at each of many repeated samplings from the posterior distributions of input parameters to evaluate the impact of input uncertainty on performance estimation. Bayesian approach is applied by Zouaoui and Wilson (2003) and (2004), thanks to the implementation of a Simulation Replication Algorithm, with the aim of specifying a confidence interval on the performance that keeps rigorously into account input uncertainty. Xie et al. (2014) introduce a Bayesian framework to measure the overall simulation error arising from the lack of collectable input data. Corlu and Biller (2015) develop a subset selection procedure which identifies a subset of system designs including the best with a probability higher than an user-specified threshold. The decision rule guiding the subset selection is implemented exploiting the Bayesian approach and the already mentioned simulation replication algorithm (Zouaoui and Wilson (2004)). The main drawbacks of Bayesian approach are the computation of the posterior distributions (which could be a quite complex task) and the high simulation time required to evaluate the performance of the system at each one of the combinations of input parameters sampled from the posterior distributions.

Finally Bootstrapping is presented by Cheng and Holloand (1997) and (2004) as a method to estimate the uncertainty of input parameters, indeed the sampling distributions of input parameters are identified thanks to bootstrap resampling of data. How the variability of input parameter values propagates into the output is then assessed by means of simulation experiments run at multiple samplings from the sampling distributions of input parameters. This technique is known as *direct bootstrapping*. Also in this case, as for the Bayesian approach, the simulation effort required to evaluate the performance of the system in the multiple combinations of resampled input parameters is considerable. On the other hand, the main advantage of bootstrapping is the fact that it does not require the strict assumptions on input parameters of the Delta method, nor the complex computations required to build the posterior distributions characterizing the Bayesian approach.

To overcome the problem of the long required simulation time, Barton et al. (2014) describe a *metamodel-assisted* procedure. The metamodel allows the computation of the output performance as an analytical smooth function of input parameters, thus overcoming the limitations of traditional bayesian and bootstrapping approaches which require many simulation experiments. Furthermore, metamodel-assisted procedures allow for the saving of a considerable amount of time by computing analytically the value of the expected performance in the multiple combinations of input parameters sampled from the posterior or sampling distributions.

The literature cited in this section addresses from a theoretical point of view the problem of evaluating the performance of systems subject to input uncertainty. The proposed techniques are applied to very simple systems,

typically single-server queueing systems. This work aims, instead, to apply metamodel-assisted bootstrapping on more complex systems with the aim to identify a confidence interval for the performance of interest, and to extend it so that it can be used to guide the process of decision making of the experts thanks to the identification of the achievable performance targets and of the parameters whose effect on the performance is more relevant.

### 3. THE PROCEDURE

Metamodel-assisted bootstrapping is applied and extended in this section so to evaluate the performance of a manufacturing line in the light of few available real-time data. The ease of construction of sampling distributions of uncertain parameters, the few required assumptions on the input distributions and the asymptotic properties of the metamodel led to the choice of metamodel-assisted bootstrapping for this study. The procedure is broken down in two stages: generation of input scenarios and performance evaluation, which are described in sections 3.1 and 3.2 respectively. When few data are available, the value of the input parameters is uncertain, and therefore, in the first step, a multitude of input parameters' scenarios that can occur is generated via bootstrapping. Finally, in the second step, exploiting a metamodel, the performance of the system is evaluated in each generated scenario so to keep input uncertainty into account when providing performance estimates. A brief introduction to the notation is provided in this section.

#### 3.1 Input Uncertainty Quantification

The aim of this first step is to quantify the uncertainty in the value of parameters. Quantifying means identifying for each parameter, on the basis of the available information (real-time data), the range within which its exact value lies.

Bootstrap resampling is used to obtain the sampling distribution of each of the parameters. In this way, if  $M$  sets of empirical data are observed, not only the best estimates of the parameters  $\hat{\theta}$  are computed, but also a measure of uncertainty is defined.  $P$  uncertain parameters  $\theta = \{\theta_1, \theta_2, \dots, \theta_P\}$  for  $M$  input distributions have to be estimated in the light of  $M$  sets of input data.

The input uncertainty quantification procedure is identified by the following steps:

- (1) *Data collection*: observation of  $M$  sets of  $n_m$  real world variates for the  $M$  distributions.

$$\mathbf{X}_m = (x_m^1, x_m^2, \dots, x_m^{n_m}) \quad (1)$$

$m=1,2,\dots,M$

- (2) *Identification of estimators*: computation of the maximum likelihood estimators of the parameters:  $\hat{\theta}^1 = \{\hat{\theta}_1^1, \hat{\theta}_2^1, \dots, \hat{\theta}_P^1\}$ . The maximum likelihood estimators of the parameters are the values maximizing the log-likelihood function (Cheng and Holloand (1997))
- (3) *Bootstrapping*: resampling of the set of empirical data to obtain

$$\mathbf{X}_m^b = (x_m^{1,b}, x_m^{2,b}, \dots, x_m^{n_m,b}) \quad m = 1, 2, \dots, M \quad (2)$$

with  $b = 2, 3, \dots, B$ . The  $B$  resamplings of the original data provide experts with  $B$  input scenarios in which all the  $P$  input parameters are simultaneously varying.

- (4) *Sampling distributions*: estimation of the maximum likelihood estimators  $\hat{\theta}^b = \{\hat{\theta}_1^b, \hat{\theta}_2^b, \dots, \hat{\theta}_P^b\}$  with  $b = 2, 3, \dots, B$  to obtain:  $\{\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_P\}$ , where  $\hat{\theta}_p = \{\hat{\theta}_p^1, \hat{\theta}_p^2, \dots, \hat{\theta}_p^B\}$  identifies the sampling distributions of the  $p$ -th parameter.

Parameter	Scenario 1	Scenario 2	...	Scenario B
$\theta_1$	$\hat{\theta}_1^1$	$\hat{\theta}_1^2$	...	$\hat{\theta}_1^B$
$\theta_2$	$\hat{\theta}_2^1$	$\hat{\theta}_2^2$	...	$\hat{\theta}_2^B$
...	...	...	...	...
$\theta^P$	$\hat{\theta}_P^1$	$\hat{\theta}_P^2$	...	$\hat{\theta}_P^B$
<b>Metamodel output</b>	$\hat{\eta}^1$	$\hat{\eta}^2$	...	$\hat{\eta}^B$

Table 1. The  $B$  bootstrapped input scenarios.

The  $B$  bootstrap-resampled combinations of input parameter values  $\{\hat{\theta}^1, \hat{\theta}^2, \dots, \hat{\theta}^B\}$  can be directly used as input scenarios (where  $\hat{\theta}^b = \{\hat{\theta}_1^b, \hat{\theta}_2^b, \dots, \hat{\theta}_P^b\}$  is the  $b$ -th input scenario).

#### 3.2 Output performance evaluation

Bootstrap resampling defines the range and the probability of input parameter values allowing experts to keep into consideration that, because of the lack of data, multiple different input scenarios may occur. This step of the procedure involves evaluating the performance of the system in each of the  $B$  input scenarios generated in the input uncertainty quantification step (section 3.1). As already highlighted in section 2, running  $B$  simulation experiments is computationally too expensive. For this reason it is often necessary to resort to a *metamodel*. How to fit the metamodel  $\hat{\eta}$  approximating the response function  $\eta$  is out of the scope of this work (see Barton et al. (2014) and Xie et al. (2014) in which the metamodel is applied in a bootstrapping and a Bayesian framework respectively).

The output performance evaluation procedure is described by the following steps:

- (1) *Simulation experiments*:  $K$  design points, characterized by a different combination of input parameters' values, are generated (Barton et al. (2014)) and the expected performance of the system  $\bar{\eta}$  is computed via simulation in each design point  $\{\bar{\eta}_1, \bar{\eta}_2, \dots, \bar{\eta}_K\}$ .
- (2) *Metamodel construction*: based on the simulation results at the design points, the metamodel  $\hat{\eta}$  approximating the system's response function is fitted.
- (3) *Output performance evaluation*: once that the metamodel  $\hat{\eta}(\theta)$  has been built, it is possible to evaluate the expected performance of the system in the  $B$  generated input scenarios of Table 1 and to finally compute a quantile confidence interval on the performance of the system which keeps into account parameter uncertainty:

$$[\beta_l, \beta_u] = [\hat{\eta}_{[B\alpha/2]}, \hat{\eta}_{[B(1-\alpha/2)}]] \quad (3)$$

Parameter	Scenario 1	Scenario 2	...	Scenario B
$\theta_1$	$\hat{\theta}_1^1$	$\hat{\theta}_1^2$	...	$\hat{\theta}_1^B$
$\theta_2$	$\hat{\theta}_2^1$	$\hat{\theta}_2^2$	...	$\hat{\theta}_2^B$
...	...	...	...	...
$\theta_p$	$\hat{\theta}_p$	$\hat{\theta}_p$	...	$\hat{\theta}_p$
...	...	...	...	...
$\theta_P$	$\hat{\theta}_P$	$\hat{\theta}_P$	...	$\hat{\theta}_P^B$
<b>Metamodel output</b>	$\hat{\eta}^1$	$\hat{\eta}^2$	...	$\hat{\eta}^B$

Table 2. Performance of the system in  $B$  input scenarios in which the value of parameter  $\theta^p$  is fixed to  $\hat{\theta}^p$ .

It is also possible to provide a measure of the achievable performance target  $\eta_p$ :

$$\eta_p = \hat{\eta}_{\lfloor B(1-p) \rfloor} \quad (4)$$

where  $\hat{\eta}_{\lfloor B(1-p) \rfloor}$  is the  $(1-p)$  quantile of the  $B$  performance scenarios. Equation (4) simply reveals which minimum performance threshold is reached in a given percentage of input scenarios  $p\%$ .

The confidence interval in equation (3) informs about the expected performance that the system is able to achieve in  $(1 - \alpha)\%$  of the generated input scenarios. Although this confidence interval can sometimes be wide (the lower the amount of data, the wider the interval), it is consistently able to capture the true performance of the physical system, whereas a traditional confidence interval, assuming the correctness of the input parameter estimators, might not.

Moreover, equation (4) can be used, depending on the needs of the stakeholders, both to identify the probability of reaching a given performance target, and also, vice versa, the performance achievable with a given probability  $p$ .

It is worth pointing out that, in most of the industrial realities, not all the input parameters are equally important for the sake of the output performance. For a specific input parameter  $\theta^p$ , it is also possible to identify the parameter value  $\hat{\theta}^p$  that allows to reach a given performance target. To do so, it is sufficient to generate  $B$  input scenarios (from data bootstrap resampling), to set parameter  $\theta_p^b = \hat{\theta}_p$  for  $b = 1, 2, \dots, B$ , and to compute the value of the metamodel in all the scenarios as shown in Table 2. Thus it is possible to evaluate the achievable performance when parameter  $\theta_p$  is fixed to  $\hat{\theta}_p$  by applying formulas (3) and (4). Therefore, in the last step of the *output performance evaluation*, the parameters whose marginal contribution to the performance is greater and those parameters whose contribution to the performance is negligible can be identified. Specifically, it is possible to determine which value  $\hat{\theta}_p$  needs to be reached on a specific input parameter  $\theta_p$  so to achieve a given confidence interval on the performance. These aspects will be discussed further in section 4.

#### 4. LAB-SCALE SYSTEM CASE OF STUDY

In this section, it is shown how the procedure is applied on a lab-scale manufacturing line installed at the Mechanical Engineering department of Politecnico di Milano. This case study is more complex and closer to real applications with respect to the very simple cases presented in the cited literature. The analyzed system consists of six worksta-

Input distribution	Distribution family	parameters
M1 Processing Time	Normal	$(\mu_1, \sigma_1)$
M2 Processing Time	LogNormal	$(\mu_2, \sigma_2)$
M3 MTTR	LogNormal	$(\mu_3, \sigma_3)$
M4 MTTR	LogNormal	$(\mu_4, \sigma_4)$
M5 Processing Time	LogNormal	$(\mu_5, \sigma_5)$

Table 3. Input distributions, they are all 2-parameters distributions.

Maximum likelihood estimators	M1	M2	M3	M4	M5
Mean $\hat{\mu}_p$ [s]	7.1886	8.8465	16.4534	18.4269	6.7923
Std.Dev. $\hat{\sigma}_p$ [s]	0.8600	3.9453	0.5841	0.7956	2.7198
True parameters' values	M1	M2	M3	M4	M5
Mean $\mu_p$ [s]	7.1655	8.4932	16.6836	18.3586	6.7319
Std.Dev. $\sigma_p$ [s]	0.8366	3.9038	0.6038	0.8260	2.4318

Table 4. Maximum likelihood estimators of the 10 input parameters computed in the light of the  $n_m = 30$  observed empirical data and actually known true parameters' values.

tions and a quality-check station (Lugaresi et al. (2021)). The goal is to evaluate the throughput of the line and to obtain information on achievable performance targets and about parameters on which to focus to improve it. This case of study was selected since the true performance of the workstations is actually known. After generating a limited availability of data by running the physical system, it was possible to apply the procedure. The notation in this section is coherent with the one defined in section 3.

The uncertain input distributions are those defined by Table 3, therefore  $M = 5$  and  $P = 10$ . Ten input parameters have to be identified, two parameters for each distribution, mean  $\mu$  and standard deviation  $\sigma$  are selected.

Samples of 30 pieces of data were collected from the physical system for each input distribution  $n_1 = n_2 = \dots = n_5 = 30$ . Bootstrap resampling procedure described in section 3.1 was applied:

- (1) observation of the 5 sets of 30 real world variates for the 5 input distributions:  $\mathbf{X}_m = (x_m^1, x_m^2, \dots, x_m^{30})$   $m=1, 2, \dots, 5$
- (2) computation of the estimators of parameters:  $\hat{\theta}^1 = \{\hat{\mu}_1^1, \hat{\sigma}_1^1, \dots, \hat{\mu}_5^1, \hat{\sigma}_5^1\}$ , results are shown in Table 4.
- (3)  $b$ -th bootstrap resampling of the set of empirical data to obtain  $\mathbf{X}_m^b = (x_m^{1,b}, x_m^{2,b}, \dots, x_m^{30,b})$   $m = 1, 2, \dots, 5$
- (4) estimation of the  $b$ -th maximum likelihood estimators  $\hat{\theta}^b = \{\hat{\mu}_1^b, \hat{\sigma}_1^b, \dots, \hat{\mu}_5^b, \hat{\sigma}_5^b\}$  with  $b = 2, 3, \dots, B$  to obtain:  $\{\hat{\boldsymbol{\mu}}_1, \hat{\boldsymbol{\sigma}}_1, \dots, \hat{\boldsymbol{\mu}}_5, \hat{\boldsymbol{\sigma}}_5\}$  where  $\hat{\boldsymbol{\mu}}_1 = \{\hat{\mu}_1^1, \hat{\mu}_1^2, \dots, \hat{\mu}_1^B\}$  is the vector of the  $B = 1000$  bootstrap resampled values of parameter  $\mu_1$ , namely the sampling distribution.

Histograms in Figure 1, representing the sampling distributions of the parameters, make it possible to appreciate that the parameters' estimators calculated from the  $n_m$  collected empirical data (Table 4) do not necessarily correspond to the actual value of the parameters. In other words, the estimators computed in Table 4 can be seen as a sample taken from the sampling distributions and

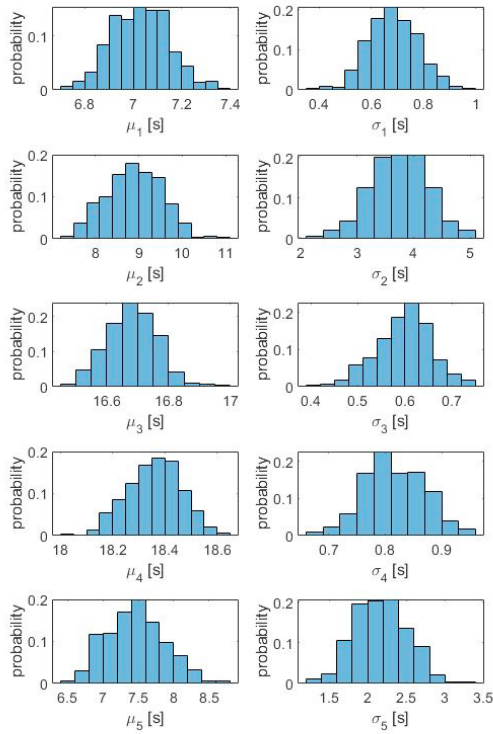


Fig. 1. Histogram of the  $B=1000$  bootstrap resamplings of the 10 input parameters in case  $n_m = 30$ .

assuming the correctness of those estimators could lead to misjudgements.

Second step of the procedure is performance evaluation. The  $B = 1000$  bootstrap resampled combinations of input parameters represented by vector  $\{\hat{\mu}_1, \hat{\sigma}_1, \dots, \hat{\mu}_5, \hat{\sigma}_5\}$  are treated as  $B$  input scenarios. It is important to stress once again that, if input uncertainty was neglected, a single combination of input parameters (therefore a single scenario) would have been computed and used in the simulator under the assumption of its correctness, underestimating the uncertainty of the output performance.

Once that the  $B$  input scenarios have been generated, the performance of the system has to be evaluated in each of them by means of the metamodel  $\hat{\eta}$  to build a quantile confidence interval using formula (3). This updated confidence interval is able to catch the actual performance of the system (which can be estimated since the true value of the parameters is actually known, as already mentioned earlier in this section). Whereas assuming the correctness of the estimators computed in the light of the few collected data (Table 4) would have lead to a biased estimation of the confidence interval. These results are reported in Figure 2. The performance target achievable with probability 80%, according to equation (4), is calculated:

$$Performance\ target = 256.80\ parts/h$$

We finally extended metamodel-assisted bootstrapping so that it also enables experts to identify the parameters whose marginal contribution to the performance is greater and those parameters whose contribution to the

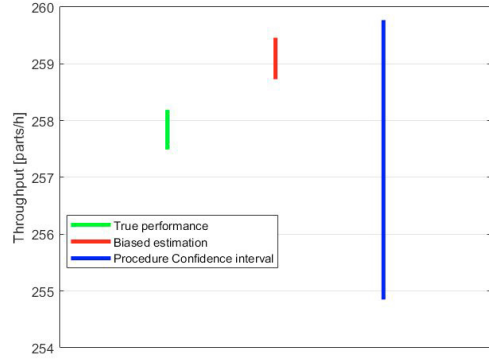


Fig. 2. Comparison of the confidence intervals computed using the true parameters' values, the biased parameters' values (Table 4) and the one obtained downstream of the procedure.

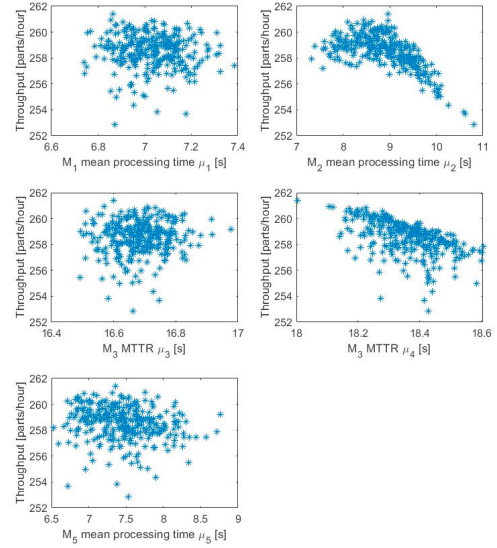


Fig. 3. Scatter-plot of the throughput in the  $B = 1000$  scenarios with respect to 5 uncertain input parameters.

performance is negligible. This can be done by plotting on a chart the metamodel-computed performance in the  $B = 1000$  input scenarios with respect to each one of the input parameters to detect the most relevant ones. In fact, if one takes a close look at the graphs in Figure 3, it can be seen that, as  $\mu_2$  and  $\mu_4$  increase, there is a decreasing trend in throughput which is not noticeable as the other parameters increase. This suggests that the mean processing time of station 2 and the mean time to repair of station 4 have to be taken into serious consideration for correct evaluation and optimization of line performance.

Moreover, once that the most critical parameters have been identified, experts might be interested in understanding the value of the critical parameters needed to reach a given performance. Focusing on the MTTR of machine 4, for example, we asked ourselves what performance could be achieved at increasing values of parameter  $\mu_4$ .  $B = 1000$  input scenarios are generated in which all the parameters

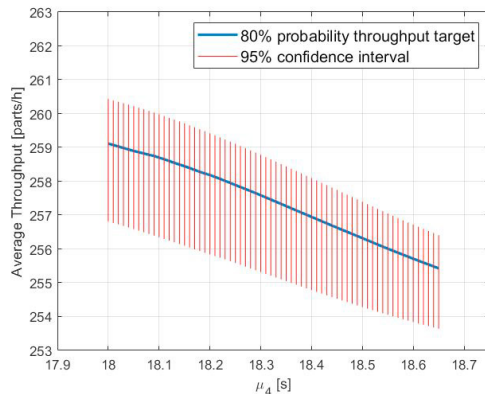


Fig. 4. Achievable performance at increasing values of  $\bar{\mu}_4$

change coherently with their sampling distributions except for the MTTR of station 4 which is fixed at  $\bar{\mu}_4$ . This procedure is then iterated for multiple values of  $\bar{\mu}_4$  (within the interval described by the sampling distribution of the parameter, as in Figure 1) and formulas (3) and (4) are applied so to obtain the chart of Figure 4.

## 5. CONCLUSION

This work shows how it is possible to evaluate the performance of a system using a simulation model as a digital twin with few data available for input modeling. The strength of this approach lies in the fact that it allows the construction of robust confidence intervals that take into account the scarcity of data. Not only the procedure allows the identification of unbiased confidence intervals on the performance, it also provides a measure of achievable performance targets and, finally, it allows the identification of those parameters whose effect on the output performance of interest is more relevant and of those whose impact is negligible.

The main drawback of this procedure lies in the effort required to fit the metamodel, which could be a quite computationally expensive task. The computational burden of the procedure is reasonable when applied to systems such as the lab-scale manufacturing line (which are however more complex than the case studies typically analysed in the literature). Moreover, the more the uncertain input parameters, the longer the time required to fit the metamodel. Therefore this procedure, as it is, is still time-demanding and computationally expensive when dealing with complex systems with many uncertain input parameters. Reducing the dimensionality of the problem by identifying negligible parameters is surely a useful step in leaning the procedure. Future developments will focus on coupling the approach with simulation-optimization techniques such as ranking & selection procedures. A limitation of this work is that only discrete event simulation models are considered in the digital twin whereas more detailed models of the equipment (e.g., stress models, geometric models, etc) are not taken into account. However, the input uncertainty procedure could be extended also to other kind of simulation models.

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