The essential work of fracture in relation to J-integral

Marta Rink, Luca Andena, Claudia Marano
Dipartimento di Chimica, Materiali e Ingegneria Chimica “G. Natta”, Politecnico di Milano, Italy

Abstract

The validity of the essential work of fracture (EWF) method and its relation to J-integral were questioned by applying it to three polymeric materials with different characteristics, in particular with respect to their yield behaviour. The importance of having an almost fully yielded ligament before crack onset was considered. Only when this condition is met, an essential work parameter can be identified which is equivalent to the J-integral at crack initiation, as determined by the Begley-Landes multispecimen method. No correlation was found between the slopes of relevant EWF and J-R curves.

Keywords

Polymers, Thin films, Fracture mechanics, J-integral, Essential work of fracture

1. Introduction

The essential work of fracture (EWF) method has become more and more widespread during the last decades as a way of characterizing fracture toughness of thin polymeric films and several reviews have been published [1-2].

The EWF method considers the overall energy (W_f) necessary to fracture a notched specimen as made of two components: an essential one (W_e), to create new surfaces in the so-called *fracture process zone*, and the non-essential work (W_p) dissipated for the plastic deformation of the surrounding area, the *process zone*. Accordingly, the specific work, w_f, can be written as the sum of two terms:

$$w_f = w_e + \beta w_p L \qquad eq. 1$$

w_e and $\beta w_p L$ are the essential and non-essential work respectively, β being a shape factor and L the length of the uncracked specimen width (ligament length). By performing a series of experiments on notched specimens with different ligament length L, an overall specific work w_f vs. L curve can be obtained in which w_e and $\beta w_p L$ are the intercept and slope of a linear interpolation of the data. The main limitations for the applicability of this method are as follows:

- *plane stress conditions must prevail*: this limits the minimum acceptable ligament length (which should be larger than about 5 times the specimen thickness);
- *no edge effects*: this condition limits the minimum notch length (i.e the maximum ligament length);
- *the uncracked cross-section should be fully yielded before crack onset*: this last requirement ensures that the fracture mechanism is the same irrespective of ligament length (with self-similar load vs. displacement curves for different L) and W_p is proportional to L^2, w_f thus being a linear function of L.

On this basis a testing protocol has been developed [3], even if it is not yet a standard both because of some experimental uncertainties [4] and some more conceptual arguments.

Indeed, the EWF concept itself has often been questioned as some researchers suggest that this approach is equivalent to a J-integral based one [5]5-11]. In most of these cases the w_f vs. L diagram is related to the J-resistance curve (J-R curve).

The determination of J is based on its definition as given by Rice [12]:

$$J = -\frac{1}{B}\left(\frac{dU}{da}\right)_{\delta} \qquad eq.\,2$$

in which U is the input mechanical energy up to a given displacement, δ; B is the specimen thickness and a the crack length. On assuming a linear J-R curve:

$$J = J_C + \frac{dJ}{da}\Delta a \qquad eq.\,3$$

in which J_C is the value at fracture initiation. By merging eq.2 and eq.3 and integrating the resulting expression, one can obtain the following relationship:

$$\frac{U}{BL} = J_C + \frac{1}{4}\frac{dJ}{da}L \qquad eq.\,4$$

Since the first term is nothing but the specific work w_f, a simple comparison between eq.1 and eq.4 suggests that the slope of the J-R curve should be equal to $4\beta w_p$ and J_c should coincide with w_e. An agreement between w_e and J_c has often been reported in the literature [5-9] while the correspondence between the slopes of the curves does not seem generally to hold; only in [6] is a reasonable agreement found. There still seems to be much to understand and clarify as the very meaning of the parameters differs for the two methods: w_e is the specific energy required to create new surfaces but it is not associated with a value for fracture initiation as J_c is; βw_p is an energy density necessary to plastically deform the surroundings of the fracture zone while the slope of the J-resistance curve is the incremental energy (dJ/da) needed to extend the crack during fracture propagation.

In this work three different polymeric films were considered and EWF and J-integral analysis were performed in order to verify whether the two methods really give the same information or not. The three materials studied were a polypropylene copolymer (EPCOP), a polyester copolymer (PETG) and polyetherimide (PEI). They were selected because their behaviour with respect to EWF requirements is very different: PETG gives rise to a fully yielded ligament before crack initiation while PEI initiates a crack with negligible yielding around the crack tip; EPCOP exhibits an intermediate behaviour.

2. Experimental details

2.1. Materials

Three commercially available films having significantly different yield thresholds were considered:

- EPCOP: Ethylene-propylene block copolymer by *BASF* (*Germany*) with *B* = 0.1 mm thickness;

- PETG: amorphous polyethyleneterephtalate by *NUDEC* (*Spain*) with *B* = 0.6 mm thickness;
- PEI: polyetherimide by *General Electric* (*Netherlands*) with *B* = 0.05 mm thickness.

Figure 1 shows the different stress-strain behaviour exhibited by the three materials.

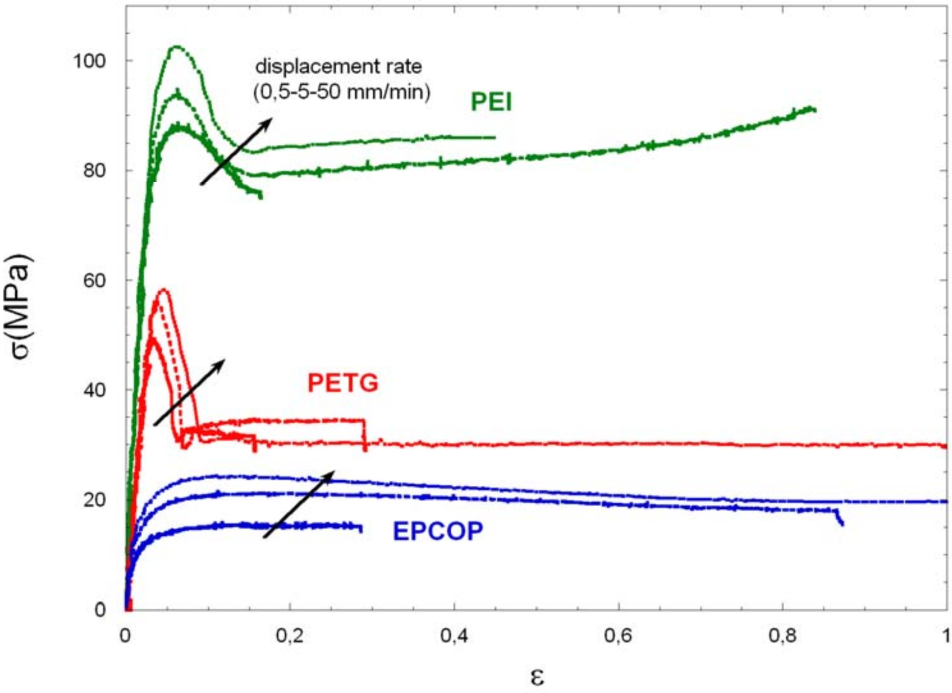

Figure 1. Tensile stress-strain curves for the three materials studied. The increase of yield stress with increasing displacement rate can be observed.

2.2. Test specimens

Double edge notched specimens in tension (DENT) were adopted, 35 mm wide and 85 mm long. Specimens were gripped so that the effective testing size was 35x35 mm. Notches were introduced into EPCOP and PEI specimens by hand-sliding a razor blade with a tip curvature radius below 5μm; for the thicker PETG specimens, machine razor sliding with a blade having a tip radius of about 13μm was performed. At least 11 specimens for each material, having ligament length between 5 and 16 mm, were tested. Ligament length and specimen thickness were accurately measured within ±1μm.

2.3. Testing

Tests were performed at 23°C with an electromechanical dynamometer at a constant displacement rate of 10 mm/min for EPCOP and PETG and 2mm/min for PEI. The load, P, was measured by means of a 500 N load cell.

Displacements on the specimens were measured by means of a video-extensometer. Local displacements were considered in order to obtain the energy dissipated only in the process and fracture zones, thus neglecting any viscoelastic dissipation in the rest of the specimen [13]. Pairs of markers, at distances $3/8L$, $5/8L$ and L, were placed on opposite ends of the specimens, symmetrically to the notch plane, vertically along the centreline, as shown in Figure 2. The closest pair still lying outside the plastic zone (whose size varies for the different materials and conditions) was chosen for measuring the displacement in each case.

Tests were video recorded using a 10 MPixel IDS UI-1495LE camera in order to observe yield and fracture phenomenology and perform crack length measurements which were used for evaluating J.

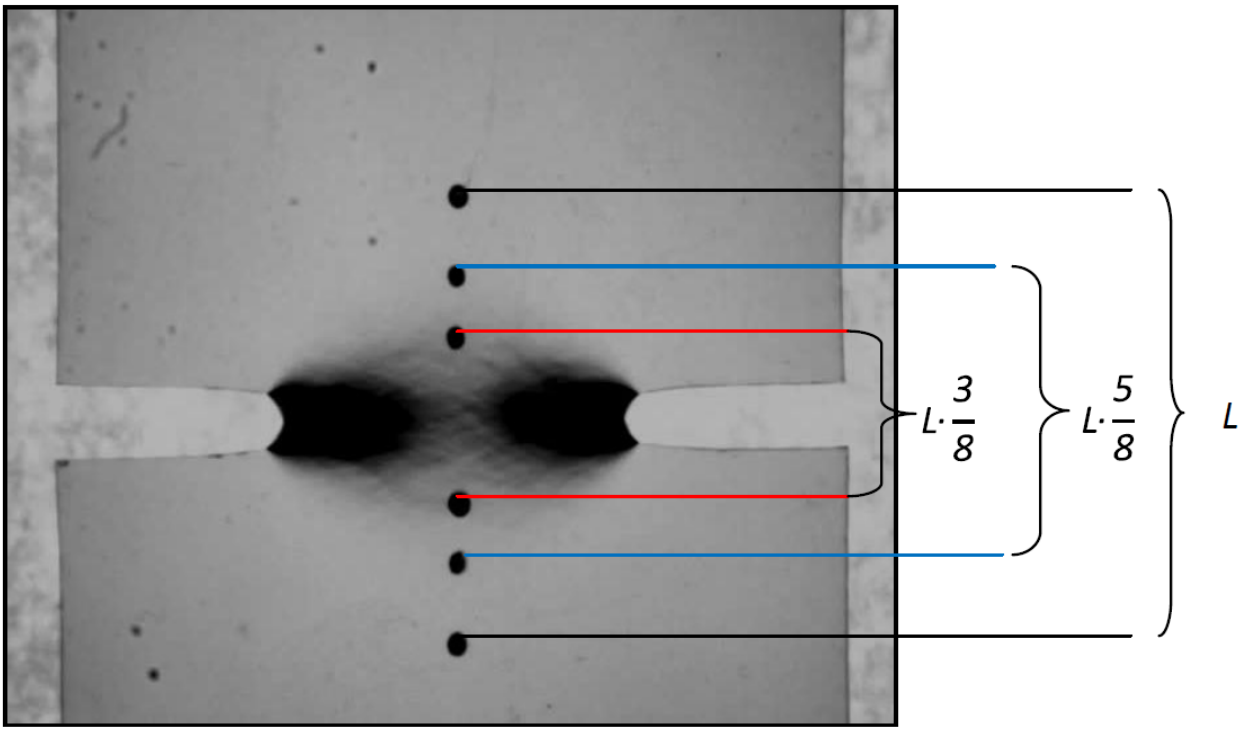

Figure 2. Relative position of the marker pairs used to measure local displacements immediately outside the fracture process zone, with L being the initial ligament length.

3. Results

In this section results and analysis of the fracture experiments are presented. The phenomenology is shown first, followed by the application of the EWF method and finally by the determination of J-R curves.

3.1. Fracture phenomenology

Figure 3a reports the load vs. displacement curves for the three materials. The points corresponding to crack initiation and to a fully yielded ligament are marked on the graphs; both events were detected visually by analysing the recorded videos. Key frames are displayed in figure 4 to present the yield and fracture phenomenology for each material. For PETG crack initiation clearly occurs after the ligament has fully yielded while for the PEI the opposite is observed. For the EPCOP the two points are quite close and the ligament becomes completely yielded shortly after crack initiation.

3.2. Essential work of fracture

From the area under the load-displacement curves in figure 3a the specific input energy, $w_f = W_f/BL$, was plotted as a function of the initial ligament length, L. A linear fit of the curves was used to identify values of w_e and βw_p, according to eq.1. Validity of data points was checked by verifying that load vs. displacement curves for the different ligament lengths were self-similar and that the stress and statistical criteria reported in [3] were met. Results are reported in figure 3b.

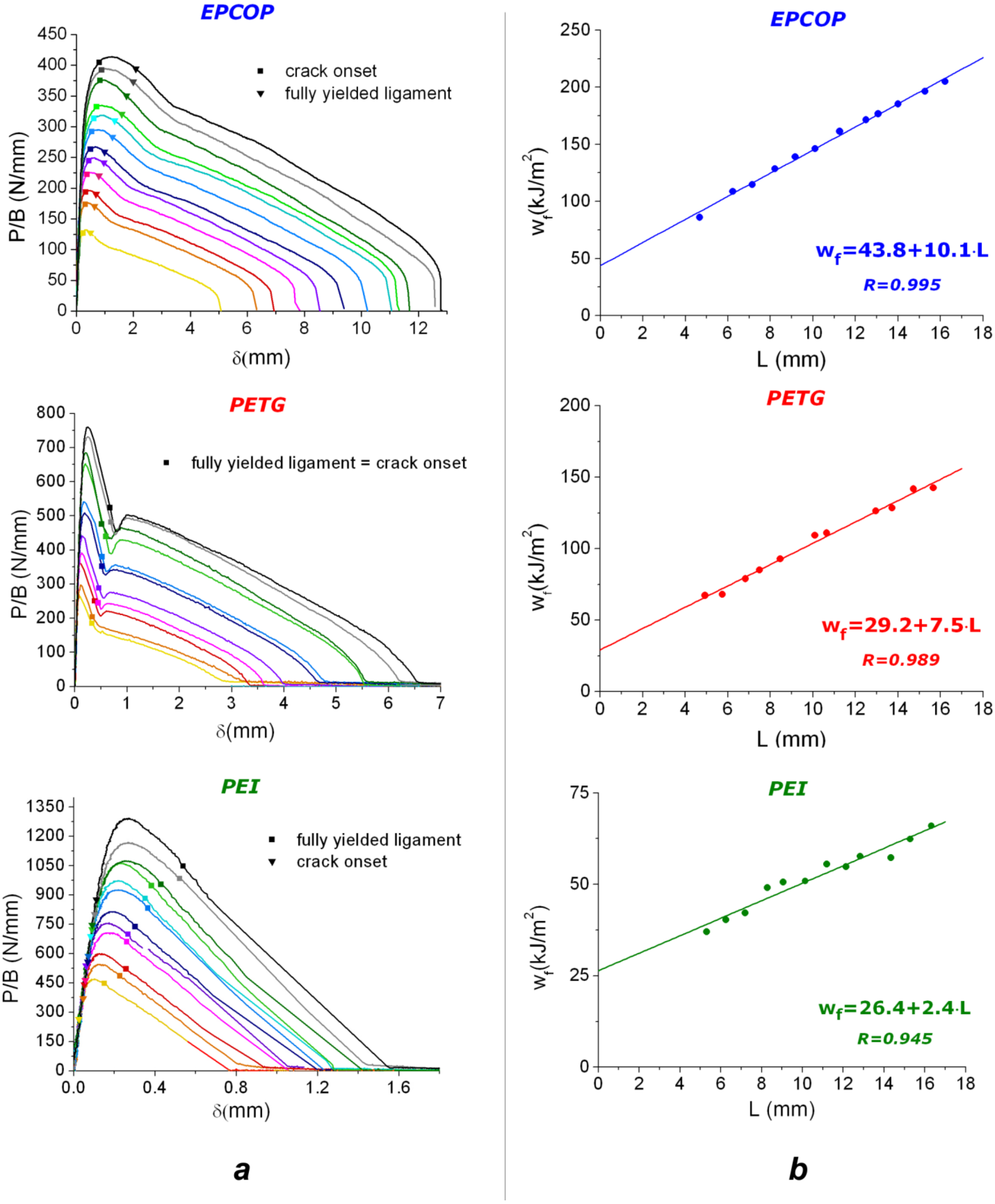

Figure 3. a) Specific load vs. displacement curves for the three tested materials; b) linear regression curves for the determination of w_e *and* w_p

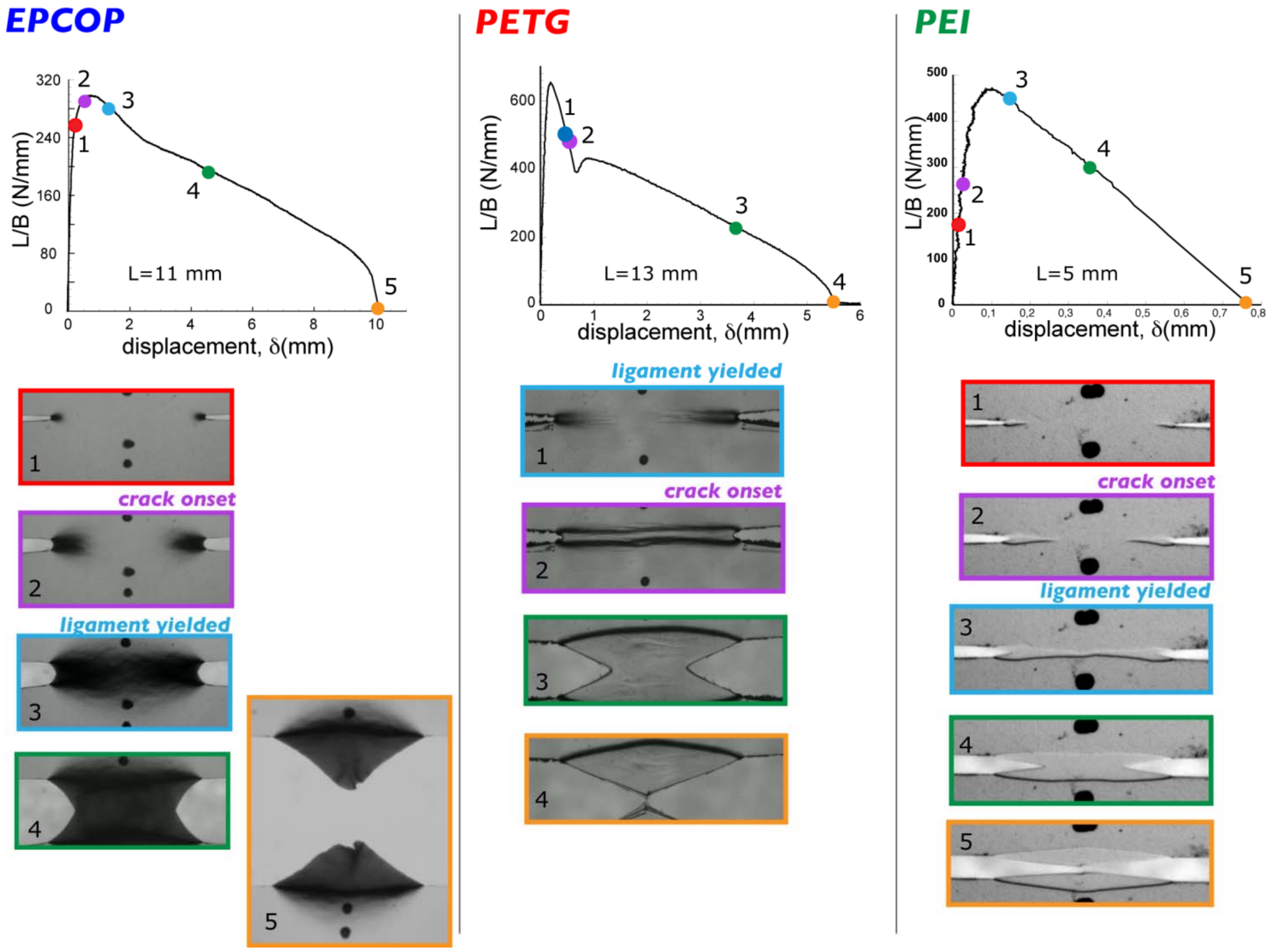

Figure 4. Fracture phenomenology of the three materials. Pictures were taken at different times during the experiments, as indicated by the numbers on the relevant load-displacement curves. This analysis allows to discriminate whether the ligament is (almost) completely yielded before crack initiation.

3.3. J-integral

In order to determine J the Begley-Landes [14] multi-specimen method can be followed:

1. specimens having different constant crack length are loaded up to fracture
2. diagrams of the input energy U as a function of crack length a are obtained by integrating each relevant load-displacement curve up to various displacements δ
3. the value of J as a function of displacement δ is calculated from the slopes of these curves

In this work load-displacement curves at constant crack length were obtained up to large displacements by adopting the procedure proposed by Hodgkinson and Williams [15]. This method entails the construction of "virtual" constant crack length curves from a set of experimental load-displacement ones. Since crack length is known at any given displacement from the video measurements, "virtual" constant crack-length vs. displacement curves could be obtained simply by interpolation of the different curves reported in figure 3a. Figure 5a shows these "virtual" constant crack length curves for the three materials: from their subsequent integration, the energy vs. (constant) crack length curves for varying displacements (shown in figure 5b) were then obtained.

It may be observed from figure 5b that in the case of EPCOP and PETG the energy, U, is a linear function of the crack length a for almost all displacements. This can be expected if the crack propagates from the beginning through a fully yielded ligament. In this case the material ahead of the crack tip is always in the same situation and the amount of energy required to advance the crack by a given amount is constant; therefore the total energy scales linearly with the ligament length. Conversely, in the case of PEI the extent of yielding surrounding the crack tip increases significantly during the early stages of crack propagation and the relation becomes linear only for larger crack lengths.

Differentiation of these curves with respect to a (according to eq.2) gives the values of J as a function of the displacement, δ; they are reported in figure 6a. For EPCOP and PETG, as expected, a single J vs. δ curve is obtained irrespective of crack length, although a significant degree of scatter is noticeable for PETG. This is due the presence of a peak in the load vs. δ curves, which is not considered in the interpolated iso-*a* curves; however, there is no apparent trend with the crack length a. Instead a trend is present for PEI and the different curves only merge for crack lengths of 12 mm or more: this value corresponds to the threshold above which crack propagation occurs through a fully yielded ligament.

The J vs. δ curves are almost linear over most of the displacement range. This is expected from the theory if one calculates J as proportional to the work done at the crack tip by a constant yield stress σ_y on the displacement δ, as reported in [6].

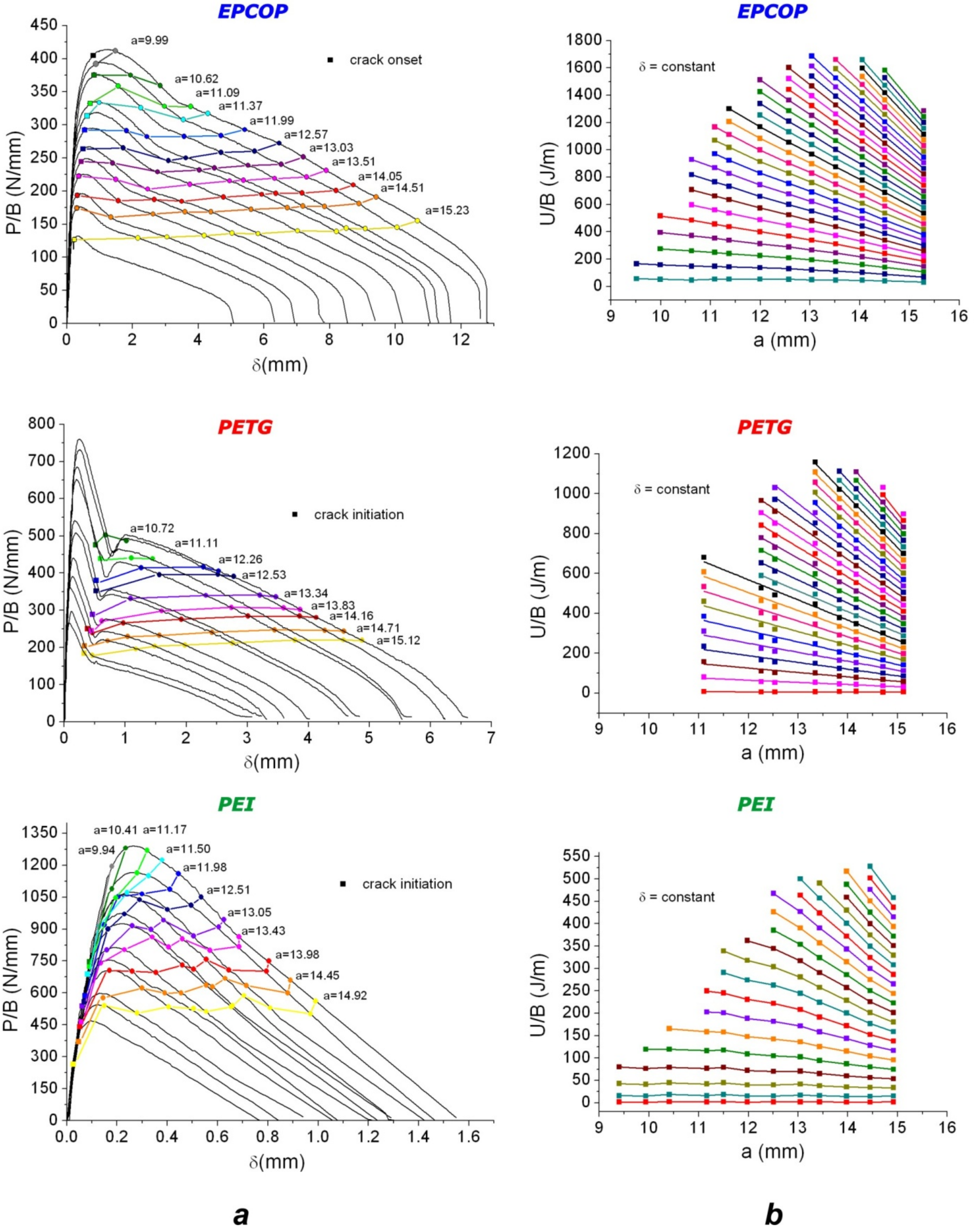

Figure 5. a) Load vs. displacement curves for the three tested materials; “virtual” constant crack length curves are highlighted. b) Energy vs. crack length curves obtained by integration of iso-*a* curves in figure 5a.

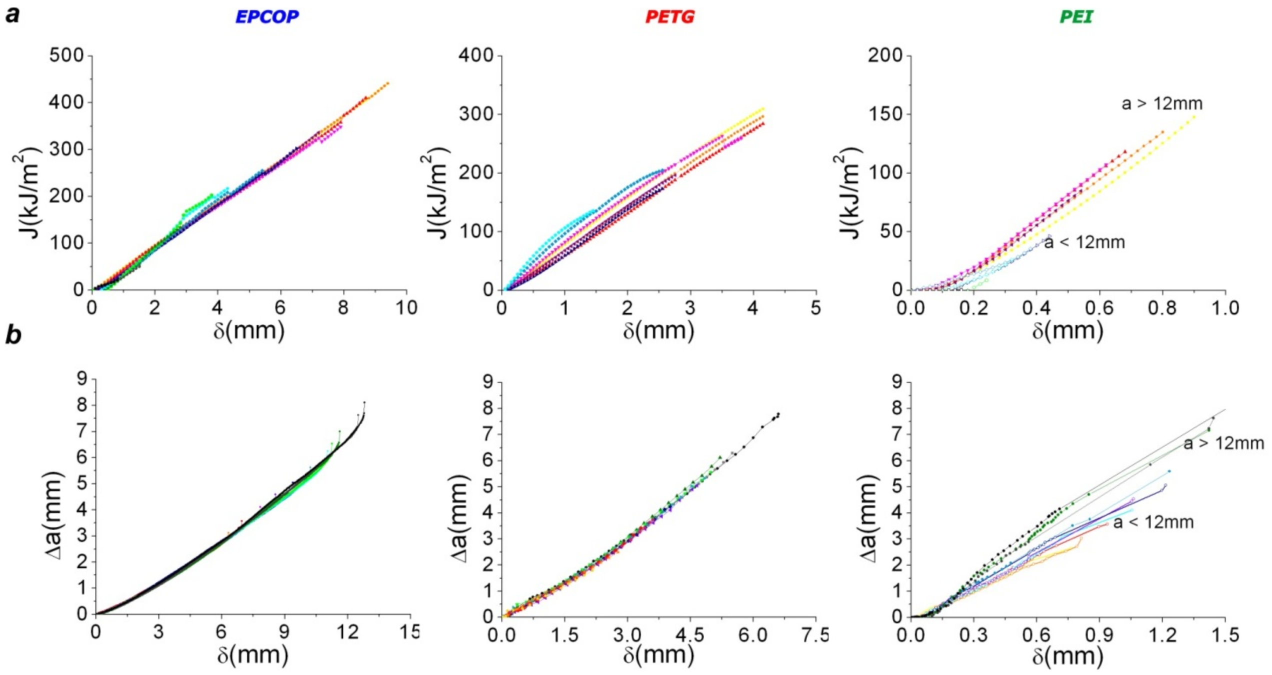

Figure 6. a) *J* vs. displacement curves and b) crack advancement vs. displacement curves for the three tested materials

The displacement vs. crack extension curves obtained from the videos are reported in figure 6b. Again, for EPCOP and PETG a single curve is obtained, irrespective of the initial ligament length (i.e. initial crack length), while for PEI this is only verified for crack lengths greater than 12mm.

The two set of curves in figure 6 were used together to determine the J-R curves for the three materials. Results are shown in figure 7, in which curves relative to different initial crack lengths have been merged together. For PEI, in view of the results previously discussed, only data for $a > 12$ mm was considered.

The curves in figure 7 show a linear trend after a certain crack extension (about 1mm) and a linear fitting was performed over this zone. The slope dJ/da and the intercept at $\Delta a = 0$, which can be thought of as a J value for fracture initiation (J_c), were determined and they are also reported in figure 7. While the slopes do not vary significantly, values for J_c show substantial differences among the three materials: PEI displays the lowest value while EPCOP is the toughest one.

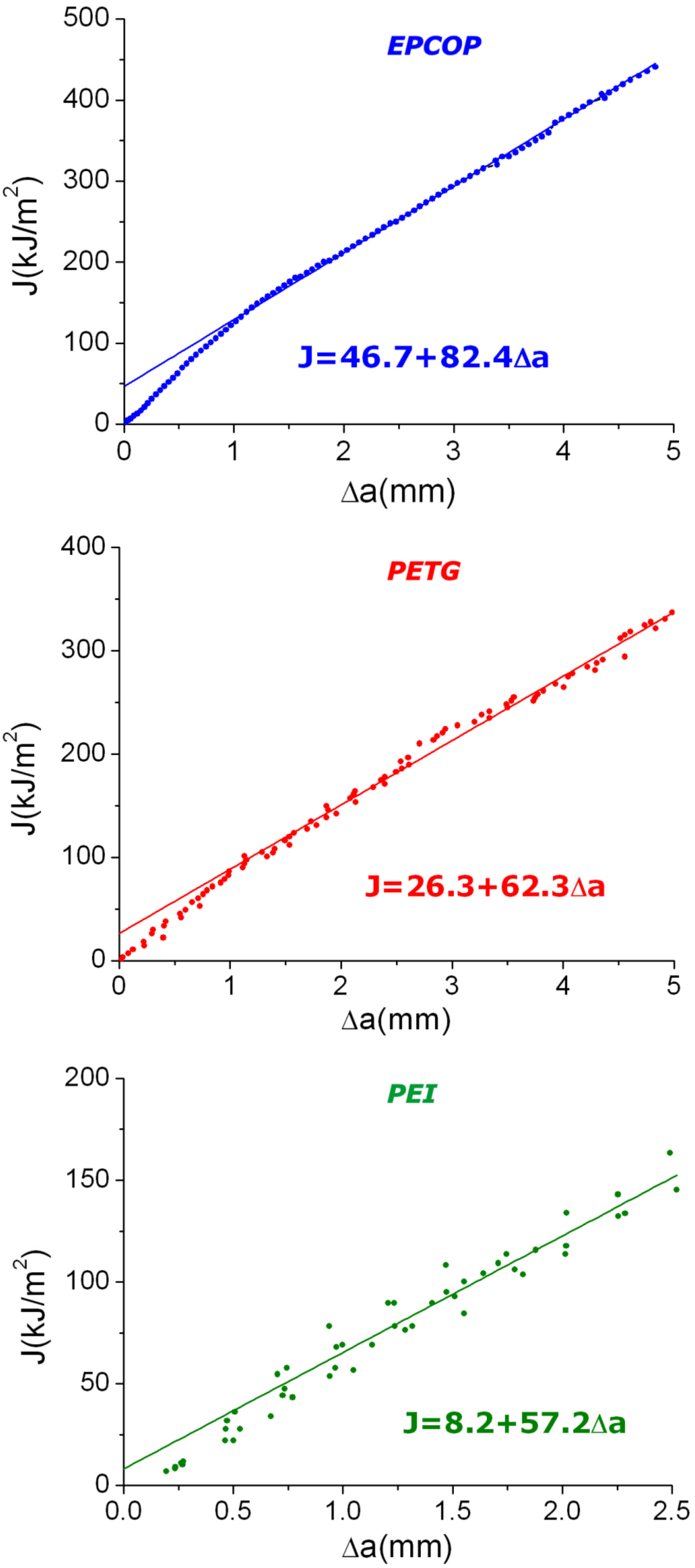

Figure 7. J-R resistance curves for the three materials (note the different scale used for the three materials)

As mentioned in the introduction, some theories predict that w_e and J_c and also $4\beta w_p$ and dJ/da should be equal. Such a comparison is set up in figure 8 for the three materials object of this study. For EPCOP and PETG w_e and J_c indeed turn out to be very similar but for PEI the value of w_e clearly exceeds that of J_c.

With regard to the slope of the J-resistance curve, dJ/da is much higher than $4\beta w_p$ for the three materials considered.

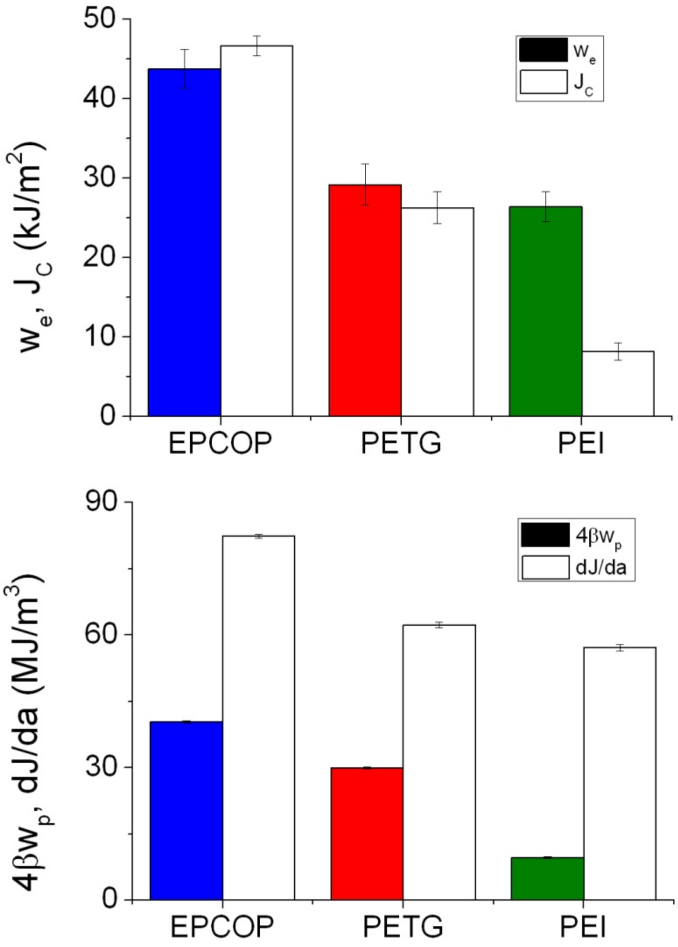

Figure 8. Comparison between characteristic parameters measured from EWF and J-R curves for the three materials

4. Discussion

The essential work of fracture w_e represents the fraction of the overall specific energy required to create a new fracture surface. The method inherently assumes a constant value for the whole fracture process, performing an average between energy required for initiation and propagation of the crack. The remaining fraction of the overall specific energy is related to yielding in the volume surrounding the crack and thus depends linearly on the ligament length, L. By getting rid of this linearly dependent term, the EWF method allows the identification of the essential contribution, w_e, without distinction between initiation and propagation. If the initiation contribution can be assumed as very small or similar to the propagation energy, w_e can be effectively considered a (constant) propagation value. Regarding the parameter J_c, it represents the energy per unit area necessary to initiate a crack and it is obtained by extrapolation to zero crack advancement of the J-R curve, which describes the energy per unit area necessary for the advancement of a propagating crack. In the present work J_c is determined from extrapolation of data obtained for cracks propagating across a fully yielded ligament; it thus represents the energy required to initiate a crack in an already yielded material. As the crack advances, fracture energy increases together with the plastic zone size, mainly because of geometrical constraint effects [16-17].

For the three materials examined, the application of the essential work of fracture method was apparently straightforward, since self-similar load vs. displacement curves were obtained and a straight line could be fitted to the w_f vs. L curves. Nevertheless, from the results obtained it seems that only when the requirement of having a fully yielded ligament before crack onset is met (or almost met, as for EPCOP), the EWF approach leads to identification of a parameter, w_e, which is equivalent to the J-integral value at fracture initiation, J_c. These findings show the importance of one of the method's main requirements not only to achieve self-similarity of crack propagation with all ligaments but also with regard to the meaning of the extrapolated essential work value.

In the case of PEI the assumption of a constant w_e value during crack propagation cannot be justified: crack advancement occurs partly across a fully yielded ligament, and partly not, with a varying proportion of the two regimes for small and large ligament values. This fact introduces an additional dependence of the overall specific energy on L which comes alongside the one related to yielding; the ability of the EWF method to correctly isolate an essential work of fracture value by a simple extrapolation is consequently thwarted, although this may not be immediately evident since the dependence of w_f on L remains approximately linear.

Even when the ligament is (almost) fully yielded before crack onset, as in the case of PETG and EPCOP, the w_e - J_c equivalence itself is not so obvious since the two quantities are obtained using very different methods and their immediate physical meaning is, at least apparently, different. In particular, w_e represents a fracture energy calculated as an average of the total energy required to initiate and propagate fracture along all ligaments while J_c, albeit extrapolated from propagation data, is a quantity related to fracture initiation, as mentioned earlier. However, in the case of a linearly increasing R-curve, the surplus energy required for propagation (for each ligament) is proportional to crack advancement – hence to ligament length: this additional contribution is then removed by the extrapolation perfomed in the EWF method and this can explain why w_e and J_c coincide.

It was not unexpected that a similar correlation could not be found between the slopes of the w_f vs. L and the *J-R* curve. Although formally the two quantities can be related as shown in the introduction (equation 1 and equation 4), the physical meaning of the two slopes clearly differs: dJ/da represents the increment of energy needed to advance fracture as the crack grows and is thus related to fracture phenomena occurring within the fracture process zone while βw_p is the average energy density dissipated in the (larger) plastically deformed process zone surrounding the whole ligament.

5. Conclusions

In this research three materials with a very different yielding behaviour were considered. The requirement of an entirely yielded ligament before crack onset was verified fully for PETG, almost for EPCOP and not at all for PEI.

The experimental data obtained on these materials allowed to critically review the postulated relationship between the essential work of fracture, w_e, and the J-integral value at fracture initiation, J_c, on one hand, and the plastic work contribution to EWF, βw_p , and the slope of the J-R curve, dJ/da. In fact a good correlation was found between w_e and J_c, provided the requirement of having a fully yielded ligament before crack onset is met: in this case the two quantities indeed represent the same fracture energy. It is important to remark that apparently the EWF method could be applied in a straightforward way even for a material (PEI) which did not satisfy the requirements; however, in this case the physical meaning of the w_e parameter itself becomes questionable.

As for the relationship between the βw_p and dJ/da, no evidence of any correlation was found and this is motivated by the very different physical meaning of these two parameters.

Acknowledgements

The authors wish to thank Sara Fazzini who performed most of the experimental work during her master's thesis and Oscar Bressan for helping with experiments and specimen preparation.

References

[1] Bárány T, Czigány T, Karger-Kocsis J. Application of the essential work of fracture (EWF) concept for polymers, related blends and composites: A review. Prog. Polym. Sci. 2010; 35;1257-1287.

[2] Martinez AB, Gamez-Perez J, Sanchez-Soto M, Velasco JI, Santana OO, Maspoch M Ll. The Essential Work of Fracture (EWF) method – Analyzing the Post-Yielding Fracture Mechanics of polymers. Eng. Fail. Anal. 2009; 16; 2604-2617.

[3] Clutton EQ. ESIS TC4 experience with the essential work of fracture method. In: Williams JG, Pavan A, editors. Fracture of Polymers, Composites and Adhesives, Netherlands: Elsevier; 2000, p. 187-99.

[4] Williams JG, Rink M. The standardisation of the EWF test. Eng Fract Mech 2007;74:1009–17.

[5] Mai YW, Powell P. Essential work of fracture and J-integral measurements for ductile polymers. J Polym Sci PartB Polym Phys 1991;29:785–93.

[6] Paton CA, Hashemi S. Plane-stress essential work of ductile fracture for polycarbonate. J Mater Sci 1992;27:2279–90.

[7] Hashemi S, Yuan Z. Fracture of poly(ether-ether ketone) films. Plast Rubber Compos Process Appl 1994;21:151-61.

[8] Wu JS, Mai YW. The essential fracture work concept for toughness measurement of ductile polymers. Polym Eng Sci 1996;36:2275–88.

[9] Arkhireyeva A, Hashemi S. Fracture behaviour of polyethylenenaphthalate (PEN). Polymer 2002;43:289–300.

[10] Martínez AB, Segovia A, Gamez-Perez J, Maspoch ML. Essential work of fracture analysis of the tearing of a ductile polymer film. Eng Fract Mech 2010;77:2654–61.

[11] Pardoen T, Marchal Y, Delannay F. Essential work of fracture compared to fracture mechanics-towards a thickness independent plane stress toughness. Eng Fract Mech 2002;69:617-31.

[12] Rice JR. A path independent integral and the approximate analysis of strain concentration by notches and cracks. J Appl Mech 1968;35:379-86.

[13] Gamez-Perez J, Santana O, Martinez AB, Maspoch ML. Use of extensometers on essential work of fracture (EWF) tests. Polym Test 2008;27:491–7.

[14] Landes JD, Begley JA. The J Integral as a Fracture Criterion. In: Corten HT, editor. Fracture Toughness – Proceedings of the 1971 national symposium on fracture mechanics – Part II, Philadelphia: America Society for Testing and Materials; 1972, p. 24-39.

[15] Hodgkinson JM, Williams JG. J and G_c analysis of the tearing of a high ductile polymer. J Mater Sci 1981;16:50-6.

[16] Xia L, Fong Shih C, Hutchinson JW. A computational approach to ductile crack growth under large scale yielding conditions. J Mech Phys Solids 1995;43:389-413.

[17] Wang J, Wang GZ, Xuan FZ, Tu ST. Derivation of constraint-dependent *J-R* curves based on modified *T*-stress parameter and GTN model for a low-alloy steel. Int J Fract 2013;183:155-68.