Differential Algebra Based Model Predictive Control for Spacecraft Rendezvous in Cislunar Space

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Keywords: Spacecraft Rendezvous, Model Predictive Control, Differential Algebra, Cislunar Space.

Abstract NASA's Artemis program aims to establish a sustained human presence in cislunar space, using the Lunar Gateway (LG) as a central staging point for future missions. The LG will operate in a Near Rectilinear Halo Orbit whose highly nonlinear dynamics challenge traditional Guidance and Control algorithms, typically based on linearized models. This work presents a Model Predictive Control (MPC) based on Differential Algebra, designed to accurately represent highly nonlinear dynamics. The proposed controller is evaluated in a rendezvous scenario, and its performance is compared to a classical linear MPC. The results confirm that this approach provides substantial improvements in guidance accuracy while incurring only a moderate increase in computational time, which remains acceptable for real-time applications; in some cases it even reduces the overall maneuver cost.

Introduction

Through NASA's Artemis program, the future of space exploration is increasingly focused on returning to the Moon, with the aim of establishing a sustained human presence in cislunar space. One of the key components of the program will be the Lunar Gateway (LG), a space station intended to serve as a staging point for missions to the lunar surface. Among the orbits considered for the LG, a Near Rectilinear Halo Orbit (NRHO) was selected [1], introducing challenges for traditional Guidance and Control algorithms used in spacecraft rendezvous. Indeed, NRHOs are characterized by highly nonlinear dynamics, especially near the periselene [2], while classical algorithms are based on linearized models.

Model Predictive Control (MPC) is one of the most promising controller, given its optimality and capability of handling multiple constraints; it relies on solving an Optimal Control Problem (OCP) within a finite prediction horizon, making the model accuracy a critical factor in the performance and robustness of the controller. In the context of NRHOs, several formulations have been proposed, such as Robust MPC [3] or Nonlinear MPC [4], with the common objective of providing an algorithm capable of addressing the nonlinearities; although these approaches have proven effective, some result in increased fuel consumption, while others suffer from higher computational costs. Differential Algebra (DA), already applied in uncertainty propagation [5] and OCPs [6], offers an efficient way to capture higher-order system dynamics while keeping the computational cost contained. It enables the propagation of the flow by constructing a Taylor series that maps the final state as a function of the initial conditions; once computed, the high-order polynomial allows a straightforward evaluation, avoiding the need to solve the full system of Ordinary Differential Equations.

A Differential Algebra MPC (DAMPC) scheme is proposed with the aim of achieving a more accurate representation of the system dynamics while maintaining a relatively low computational cost. The polynomials representing the flow of the dynamics are computed online before every optimization, without requiring any offline intervention. The proposed controller is compared with a Linear MPC (LMPC) in a rendezvous scenario with a target placed on an NRHO, considering several initial conditions at both aposelene and periselene.

Differential Algebra MPC

The basic structure of the controller is outlined here. A quadratic cost function is employed, ensuring convexity while promoting smooth closed-loop transients. To guarantee operational safety, the position of the chaser is constrained within a predefined approach cone centered on the docking port axis, with a semi-aperture angle of 10°; the cone is approximated by four planar surfaces, allowing this path constraint to be expressed as a set of linear inequalities. In addition, upper and lower bounds are imposed on the control inputs, and soft docking terminal conditions are enforced to ensure a safe and gentle contact.

DA is employed to enforce the system dynamics. Specifically, let $\mathcal{M}_{\boldsymbol{x}_{k+1}}(\delta \boldsymbol{x}_k)$ denote the polynomial map expressing the high-order expansion of the flow, obtained by propagating the free dynamics from the time instant t_k to the successive one. Together with the discrete control input matrix $\boldsymbol{B}_k$, the state vector evolution can be propagated as in Eq. 1. Moreover, DA allows the analytical computation of derivatives of the polynomial maps with minimal computational effort, which is particularly valuable since classical optimization algorithms require the gradient and Hessian of the constraints to efficiently solve the OCP.

$$\boldsymbol{x}_{k+1} = \mathcal{M}_{\boldsymbol{x}_{k+1}}(\delta \boldsymbol{x}_k) + \boldsymbol{B}_k \, \boldsymbol{u}_k \tag{1}$$

To generate the polynomial maps, it is sufficient to propagate the free dynamics within the DA framework, where the initial condition serves as the expansion point. In principle, any propagation scheme can be adapted to operate with DA, enabling a straightforward implementation. Since the polynomials are local approximations, the resulting maps are valid only within a neighborhood of the expansion point; their accuracy therefore depends primarily on the displacement $\delta \boldsymbol{x}_k$, which measures the distance between the evaluation and the expansion point. The selection of the latter is therefore a critical aspect and should be carefully addressed. For an MPC, the optimal solution of the previous OCP is typically close to that of the successive one, and is therefore used as initial guess to ensure a rapid convergence. Accordingly, the expansion points can be extracted from this initial guess and propagated forward in time to obtain the polynomial maps, thereby limiting the displacement $\delta \boldsymbol{x}_k$ as the predicted states are expected to remain close to the expansion points. If N states are considered within the prediction horizon, an equal number of polynomial maps is required for the propagation. However, it is not necessary to regenerate all N maps at every optimization step. Once the complete set has been computed, it can be simply updated: the outdated first map is discarded, and only the last one is regenerated using the final term of the initial guess as the new expansion point. This strategy greatly reduces the computational cost, as only a single new polynomial map must be computed for each OCP. Mathematically, let $\boldsymbol{x}^{guess}$ be the chosen expansion point, and denote the free dynamics compactly as $f(\boldsymbol{x}, t)$; the following initial value problem must be solved within the DA framework:

$$\begin{cases} \dot{\boldsymbol{x}} = f(\boldsymbol{x}, t) \\ \boldsymbol{x}_0 = \boldsymbol{x}^{guess} \end{cases} \Rightarrow \boldsymbol{x}_{k+1} = \int_{t_k}^{t_{k+1}} f(\boldsymbol{x}, \tau)\, d\tau = \mathcal{M}_{\boldsymbol{x}_{k+1}}(\delta \boldsymbol{x}_k) \tag{2}$$

The final step concerns the selection of the expansion order. Higher orders generally provide greater accuracy, but they also require computing a larger number of coefficients to generate the maps, which significantly increases the computational cost. Therefore, a trade-off must be made.

Simulations and results

The DACEyPy library [7] is used for the DA framework. The dynamical models developed in [2] have been implemented. The Circular Restricted Three Body Problem is adopted and is assumed to represent the exact dynamics. The authors derived a set of nonlinear equations of relative motion, which are here employed by the DAMPC to generate the polynomial maps. A third-order expansion was chosen as the optimal compromise: at periselene, for sufficiently short propagation times, it achieves nearly the same accuracy as the fourth-order expansion, while offering a significant improvement over the second-order case. In addition the authors developed a linearized model which is exploited by the LMPC. Both controllers are initialized with the same set of parameters. The prediction horizon length is set to 30 steps, while the control horizon covers 15 steps. The gains vary depending on the initial conditions and have been manually tuned through a trial and error approach to ensure successful docking.

In the first set of simulations, the sampling time is fixed at 4 s. Different initial conditions are tested and are grouped into three categories based on the initial distance along V-bar: *short range* refers to cases where the chaser starts from 200 m, *medium range* from 2,000 m, and *long range* from 10,000 m; those initial conditions are used for simulations at both the aposelene and periselene. The results highlight the effectiveness of the proposed controller: DAMPC achieves prediction errors that are 2 to 3 orders of magnitude lower than those of the LMPC, clearly demonstrating its superior accuracy. This is illustrated in Fig. 1, which shows the average ratio of the LMPC prediction errors to those of the DAMPC across all simulations at the aposelene (similar results were observed at the periselene). Even if DAMPC is more accurate, the two controllers tend to yield the same results in terms of trajectory and control inputs, resulting in a similar level of optimality. On average, the percentage difference in Δv is on the order of 0.001%, demonstrating that there is no significant deviation between them for this metric in the present simulation setup. This result is likely due to the short sampling time: although the linear model is less accurate, it remains sufficiently precise under these conditions to produce an equally optimal control sequence.

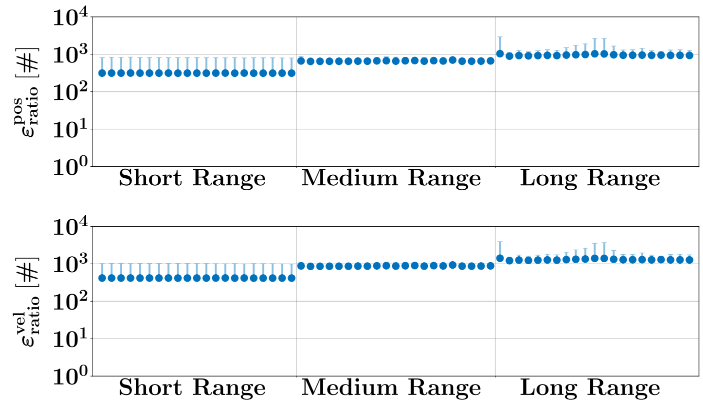

Figure 1: *Average position (top) and velocity (bottom) error ratios between LMPC and DAMPC across all simulations at the aposelene.*

A second set of simulations is conducted with a variable sampling time. The same initial conditions used previously are employed, however, only the *long range* scenario is considered. At the beginning the sampling time is set to 400 s, yielding a prediction horizon of over 3 hours. As the simulation progresses and the distance decreases, more frequent adjustments are required to maintain the level of accuracy necessary for completing the rendezvous, and therefore the sampling time is reduced. When the distance along V-bar decreases to $2,000$ m it is set to 40 s, and for the final 200 m it is further reduced to 4 s. The longer prediction horizon allows for more effective optimizations compared to the previous case, resulting in a lower overall Δv. However, this also amplifies the prediction errors, particularly when using linearized dynamics. If the model is not sufficiently accurate, the predicted trajectories may diverge significantly from reality, and the control inputs optimized on this basis could produce suboptimal or unexpected behavior, creating the necessity for corrections in subsequent iterations. Conversely, when the model is highly accurate, as in the case of the DAMPC, the predicted trajectory remains closer to the real one, making the optimization more effective and reducing the necessity for such corrections. This is verified in these simulations, with the DAMPC consistently requiring slightly lower Δv. At the aposelene, the maneuver cost obtained with the DAMPC is on average 0.49% lower than that of the LMPC, while at the periselene this difference increases to 1.15% as nonlinearities are more pronounced, thereby accentuating the difference between the two controllers.

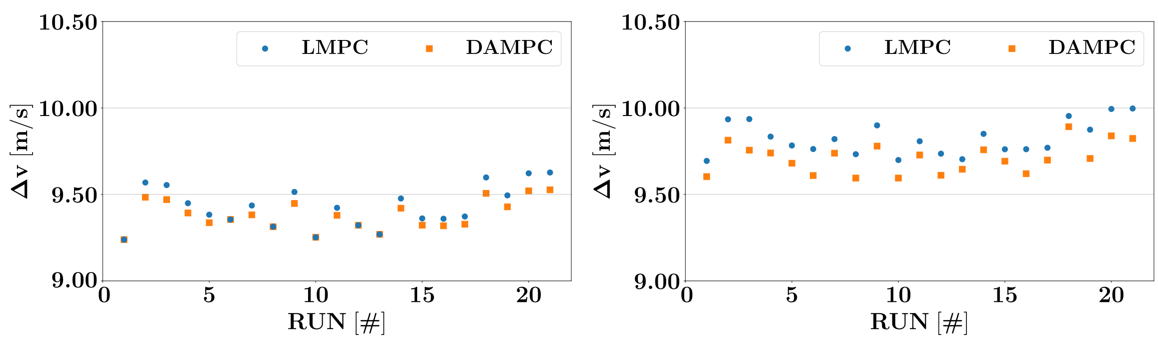

Figure 2: *Maneuver cost comparison at aposelene (left) and periselene (right).*

On an Intel i5-8300H processor, the DAMPC requires on average 57.6 ms per optimization, of which 22.7 ms are spent computing the polynomial maps and their derivatives. In comparison, the LMPC requires only 16.1 ms, making it about 3.5 times faster. As expected DAMPC is slower, but its computational cost remains manageable and suitable for a potential real-time implementation.

Conclusion

This work demonstrates how DA can be effectively embedded within an MPC framework to provide an accurate representation of the system dynamics in highly nonlinear environments. Although the proposed DAMPC requires higher computational resources, it can significantly reduce the prediction errors compared to the LMPC, leading, in some cases, to a reduction in maneuver cost. Nevertheless, some critical limitations remain in the current implementation. More advanced methodologies could be employed to better incorporate the control inputs, and improved strategies could be developed for the selection of the expansion points. Furthermore, the approach could be extended by integrating this formulation within a robust MPC scheme, such as Tube-based Robust MPC.

References

[1] R. Whitley, R. Martinez, Options for staging orbits in cislunar space, 2016 IEEE Aerospace Conference (2016) 1–9, https://doi.org/10.1109/AERO.2016.7500635

[2] G. Franzini, M. Innocenti, Relative Motion Dynamics in the Restricted Three-Body Problem, Journal of Spacecraft and Rockets (2019) 56:1322–37, https://doi.org/10.2514/1.A34390

[3] M. Mammarella, E. Capello, G. Guglieri, Robust Model Predictive Control for Automated Rendezvous Maneuvers in Near-Earth and Moon Proximity, 2018 AIAA SPACE and Astronautics Forum and Exposition (2018), https://doi.org/10.2514/6.2018-5343

[4] G. Bucchioni, F. Alfino, M. Pagone, C. Novara, A Minimum-Propellant Pontryagin-Based Nonlinear MPC for Spacecraft Rendezvous in Lunar Orbit, 62nd IEEE Conference on Decision and Control (2023) 8745–50, https://doi.org/10.1109/CDC49753.2023.10383572

[5] M. Valli, R. Armellin, P. Di Lizia, MR. Lavagna, Nonlinear Mapping of Uncertainties in Celestial Mechanics, Journal of Guidance, Control, and Dynamics (2013) 36:48–63, https://doi.org/10.2514/1.58068

[6] P. Di Lizia, R. Armellin, A. Morselli, F. Bernelli-Zazzera, High order optimal feedback control of space trajectories with bounded control, Acta Astronautica (2014) 94:383–94, https://doi.org/10.1016/j.actaastro.2013.02.011

[7] M. Maestrini, S. Carcano, Polynomial guidance laws for robust fuel optimal stationkeeping of geostationary satellites, Acta Astronautica, Volume 236, 2025, Pages 73-187, ISSN 0094-5765, https://doi.org/10.1016/j.actaastro.2025.06.047.