

Numerical Simulation of the Transition from Regular to Mach Reflection of Oblique Shocks

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Abstract. We use a rectangular computational domain to study numerically the transition from regular to Mach reflection of oblique shock waves in steady. The SU2 solver is used to solve the 2D Euler equation, where the desired shock conditions are obtained by imposing suitable boundary conditions. Results for the Mach stem height compare fairly well with those obtained on the conventional wedge geometry. Accurate Mach stem height requires a domain large enough to resolve the sonic throat.

Introduction

Oblique shock reflections are either regular reflections (RR) or irregular reflections (IR), further classified as Mach (MR), von Neumann (vNR), Guderley (GR) and Vasilev (VR) reflections [1]. Possible criteria for the transition between regular and Mach reflections are the detachment condition [2], the sonic condition [3], and the von Neumann condition [4]. RR can only exist below the detachment condition and MR only above the von Neumann condition. Both RR and MR are stable within the dual-solution domain. Identifying the correct transition point remains a challenge, but a length-scale criterion suggests that the sonic and the von Neumann conditions should be preferred, depending on the initial state of the flow [1].

Experiments conducted by Hornung and Robinson recorded transition at the von Neumann condition, supporting the length-scale principle [5]. Hysteresis effects, first discussed by Hornung et al. [3], were reproduced experimentally by Chpoun et al. [6]. Hysteresis was also observed in 2D numerical simulations, with the transition from RR to MR occurring at detachment, while the transition from MR to RR occurs closer to the von Neumann condition [7]. The discrepancy between numerical simulations and experiments lies in the strong three-dimensionality of the process. Small perturbations may also trigger the transition [8].

The geometry of the domain influences the transition and the shape of the reflection [9]. This study employs numerical simulations of oblique shock wave reflections to examine the transition in steady flows across different domain geometries and to define the optimal approach for obtaining consistent and reliable results.

Methodology

Euler 2D simulations were performed using SU2, an open-source CFD suite combined with the in-house adaptive mesh refinement tool Darwin. Unstructured triangular grids were generated using the in-house mesh software uhMesh.

The domain (Fig. 1) is rectangular: pre-shock inlet conditions are imposed on the left boundary, the bottom boundary is the symmetry plane where reflection occurs, while the right boundary is the outlet. The top boundary is divided into two segments: the shock is generated by imposing the post-shock state computed using the Rankine-Hugoniot jump conditions on the foremost segment (or gap); the aftermost segment is a wall.



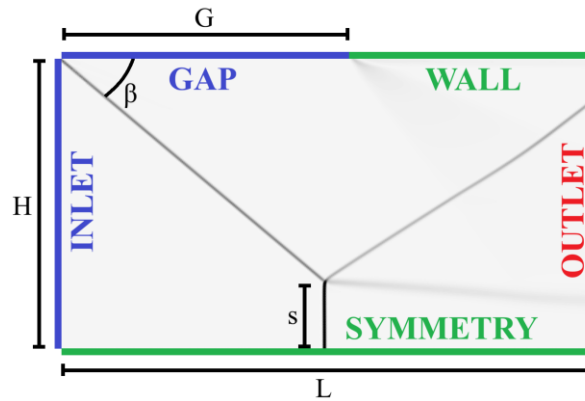


Figure 1: Numerical domain.

Table 1: Grid convergence.

Cell size h/H	Mach stem height s/H	Percentage variation
0.0100	0.223	—
0.0070	0.231	3.6
0.0050	0.238	3.0
0.0035	0.242	1.7
0.0025	0.244	0.8

The starting grid size is $h/H = 0.05$ as a balance between the advantage of better catching the transition at earlier steps of the iterations and the computational time. The mesh undergoes six levels of refinement. Mesh refinement is only used to assess the transition, but not when measuring the Mach stem height. In this case, a cell size of $h/H = 0.005$ is selected after performing a grid convergence study with parameters $M_0 = 2.9$, $\beta = 40$ deg, $G/H = 1$, reported in Table 1. Mach numbers range from 2 to 6, and incident shock angles from 31 degrees to 43 degrees.

Results

The simulation domain can be related to a corresponding configuration (Fig. 2) in which the shock is generated by the presence of a wedge by purely geometrical considerations, for any initial condition. In Fig. 3, a comparison of the normalized Mach stem heights in the two setups shows the expected agreement.

MR are not self-similar, thus requiring to define a length scale for a stable solution [8]. An expansion fan forms at the leading edge of the top wall and impinges on the slip line originating at the triple point of the MR. The region below the slip line behaves as a converging-diverging duct, selecting the required length scale [5]. The interaction of the expansion fan with the slip line can be controlled by changing the gap-to-height ratio, which is equivalent to changing the relative size of the wedge in the conventional configuration. As also observed by Bai [10], the Mach stem height is found to change linearly with the gap-to-height ratio as long as the fan does not interact with the incident shock, but uniquely with the reflected shock. The domain must be large enough to solve the sonic throat below the slip line to consistently predict the MR geometry (Fig. 4).

Conclusions

This numerical study investigates the transition of oblique shock reflections using a rectangular computational domain as an alternative geometry of generating the shock by imposing suitable boundary conditions. A constant number of elements between the symmetry plane and the wall is key to consistent, comparable results. This approach allows the use of fewer elements compared

to domains including wedges while preserving the overall number of cells when solving a wider range of initial conditions.

Reproducing accurate and reliable simulations can support theoretical findings, while also helping to predict the shape of shock reflections.

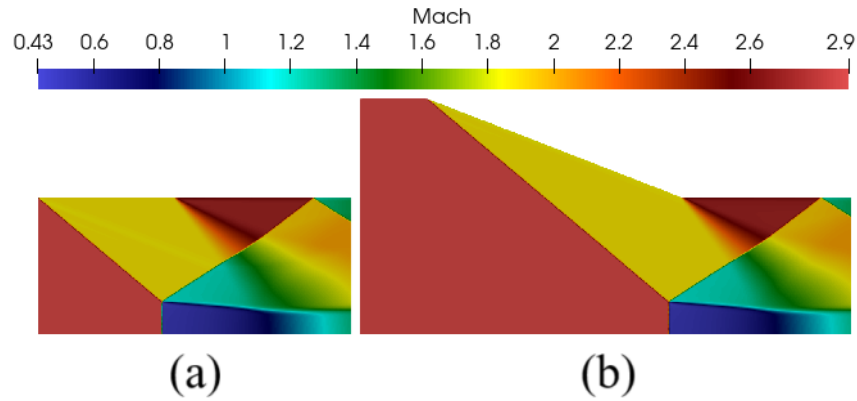


Figure 2: Rectangular domain (a) and corresponding domain with wedge (b) for $M_0 = 2.9$, $\beta = 40$ deg, $G/H = 1$.

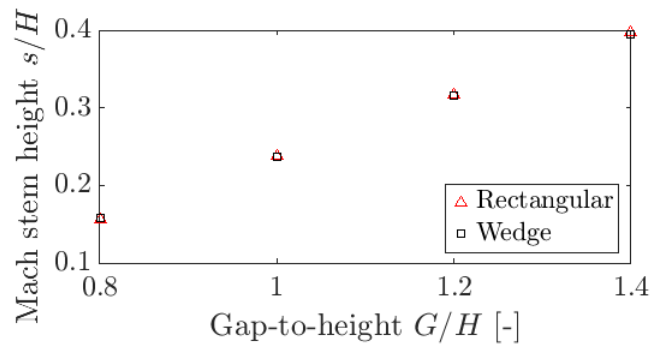


Figure 3: Comparison between the Mach stem height resolved in the rectangular domain and the domain with wedge ($M_0 = 2.9$, $\beta = 40$ deg).

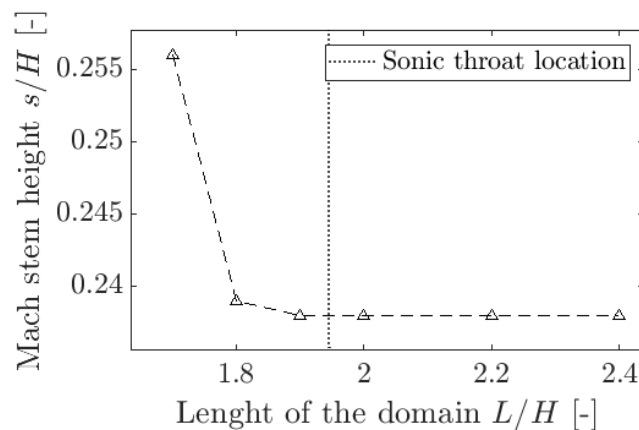


Figure 4: Mach stem height compared to the length of the domain. The height becomes constant only when the sonic throat is included inside the domain ($M_0 = 2.9$, $\beta = 40$ deg, $G/H = 1$).

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