Frequency-Variant Phase-Shifters in Bandstop Extracted-Pole Cascade Synthesis

Matteo Oldoni
Dip. di Elettronica, DEIB
Politecnico di Milano
Milan, Italy
matteo.oldoni@polimi.it

Stefano Tamiazzo
Research & Development
Andrew Corporation, Amphenol
Agrate Brianza, Italy

Giuseppe Macchiarella
Dip. di Elettronica, DEIB
Politecnico di Milano
Milan, Italy

Gian Guido Gentili
Dip. di Elettronica, DEIB
Politecnico di Milano
Milan, Italy

***Abstract*—Microwave bandstop filters several finite transmission zeros are commonly synthesized as a sequence of frequency-invariant phase shifters followed by hung resonators. The physical realization however often involves transmission lines, whose phase delay introduces a linear variation with frequency and hence limit the design effectiveness to narrow band and heavy subsequent optimization. Leveraging a revisited version of classical cascade synthesis, the formulation presented here allows to include the known frequency variation of such components at synthesis time. The resulting compound technique yields a synthesized circuit much closer to its physical implementation up to relative bandwidth 13%.**

***Index Terms*—Cascade Synthesis, Fully-Canonical Response, Stopband Response, Section Extraction.**

I. INTRODUCTION

Most synthesis techniques for microwave filters today rely on the implementation of rational functions, implementable exactly by means of lumped components, such as LC resonators, frequency-invariant susceptances and frequency-invariant couplings and, in specific conditions, frequency-variant couplings.

Modern filters, however, involve high-order transfer functions and requirements spanning relatively far from the central frequency, and the lumped-component synthesis thus may exhibit a poor correspondence once actual frequency-variant distributed components are introduced in practice. This is especially apparent in bandstop filters with several finite transmission zeros, implemented by a common line tapped with hung resonators as in fig. 1, where the electrical length of the phase-shifter segments varies linearly around the central frequency and thus strongly affects the actual response.

This shortcoming is shared by both coupling matrix and section extraction techniques, even in their extensions for synthesis directly in the bandpass domain. But while the former does not impose any *a priori* constraint on the topology, the latter can address more easily the considered tapped-line structure. The present work in fact focuses on section extraction, leveraging a recent method [1] conceived to mitigate numerical sensitivity but which also uniquely opens up the possibility of taking into account the frequency-variant nature of phase shifters in extracted-pole stopband filters. The technique relies on closed-form formulas, containing the proposed corrections as first-order approximation. The method is illustrated in section II with examples in section III.

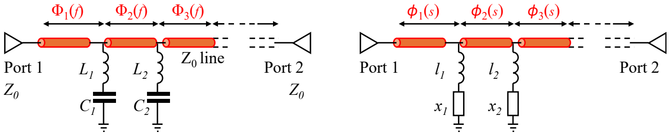

Fig. 1. Left: stopband filter circuit, with red elements representing frequency-variant phase shifters. Right: equivalent normalized highpass circuit.

II. SYNTHESIS WITH FREQUENCY-VARIANT PHASES

A. Approach

Synthesis of extracted pole filters with several finite transmission zeros (TZs), occurs via specific techniques [2]–[4] or via cascade synthesis. Numerical sensitivity issues of the latter have been addressed in several ways, including extended-precision arithmetic for polynomials and suitable transformations [5], [6], whereas [1] proposes a revisited version of cascade synthesis which no longer handles polynomials. [1] observes that the components at each extraction are determined from the *evaluation* of rational functions (specifically the input susceptance and its first derivative) at the N transmission zeros; only these values are passed along and transformed after each extraction, progressively discarding the TZs.

The method has recently been improved explicitely for bandstop responses [7], yielding a cascade of frequency-invariant phase shifters and hung resonators.But when phase shifters are implemented by transmission lines, the actual frequency-variant phase strongly distorts the response, thus requiring more extensive optimizations of the physical structure. However, since the considered synthesis process relies only on *evaluations*, and their recomputation after each extraction, a frequency-variant phase shifter such as a transmission line section can be embedded in the recomputation, so that subsequent extractions are properly compensated. The formulas are given in section II-B while the termination of the synthesis is described in section II-C, following the algorithm in fig. 2.

B. Inclusion of frequency-variant phase shifters

For simplicity, only finite real-frequency TZs ($j\Omega_i$ with i from 1 to N) in the normalized domain s are considered here, in the form of fig. 1 (right). The generic extraction step k fully extracts a section implementing the TZ in $j\Omega_k$, by means of a

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Given: from $S_{11}$ $\{\Omega_i\}$, $\{b_i^{(1)}\}$, $\{b_i^{(1)\prime}\}$, $i$=1:$N$
Iterate for k=1 to N:
    Compute $\phi_k$, $l_k$, $x_k$ (eq. 2 and 3);
    Convert $\phi_k$ to $\phi_{0,k}$ (eq. 2);
    Compute the remainders:
        $\{b_i^{(1)}\}$, $\{b_i^{(1)\prime}\}$ for $i>k$ (eq. 4, 5, 6)
Compute the terminal section
Implement the circuit:
    Tx lines with el. length $\phi_{0,k}$ at the
      central freq., linear var. as eq. 2.
    Shunt branches as series resonators

Terminal Section (general form):
    Given: (from $S_{22}$) $\hat{b}_N$, $\hat{b}'_N$, $\hat{b}_{N-1}$ , $l_N$ , $\phi_{0,N}$, $\Omega_N$, $\Omega_{N-1}$
    (Optional: extract a unitary-imped. phase $\varphi_2$)
    Compute J-inverter on port 2 ($J_T$)
    Compute the NRN susceptance $B_{L2}$
    Extract a unitary inverter $J_L$=1
    Compute susceptance $B_{L1}$ in parallel to $l_N$, $x_N$
    Transform (as in [9] eqs. 5, 6, 8):
        scale the $B_{L2}$ node so that $J_T$ becomes 1;
        compute an equiv. phase shifter for $B_{L1}$-$J_L$-$B_{L2}$
          (potentially $Z_E$≠1 and stray susceptances).
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Fig. 2. Proposed algorithm for the synthesis by section extraction. Details of the terminal sections given in section II-C

phase shifter ϕ_k and a hung resonator represented by a shunt arm with an inductance l_k in series with a frequency-invariant reactance x_k. The extraction step k (from 1 to N) needs:

- $\{\Omega_k; \ldots; \Omega_N\} \in \mathbf{R}$: the normalized TZs to be extracted;
- $\left\{b_k^{(k)}; \ldots; b_N^{(k)}\right\}, \in \mathbf{R}$: the input susceptance b, remaining after the previous $k-1$ extractions, evaluated in the remaining TZs
- $\left\{b_k^{(k)\prime}; \ldots; b_N^{(k)\prime}\right\}, \in j\mathbf{R}$: the derivative of the input susceptance b with respect to s, remaining after the previous $k-1$ extractions, evaluated in the remaining TZs.

Based on these, formulas in [7] return the required phase shift at Ω_k and the hung resonator l_k, ϕ_k; they also return the remaining susceptance and its first derivative at the remaining transmission zers, ready for the next extraction step $k+1$.

The critical modification proposed by this work stems from the phase shifter ϕ_k which will actually be implemented by a transmission line with phase delay $\Phi_k(f) = \phi_k f/f_k = \phi_{0,k} f/f_0$, with f_0 denoting the central frequency of the stopband and $B_n = B_W/f_0$ its normalized band, so that $\phi_{0,k}$ is the phase delay at central frequency f_0. Thus, in the normalized lowpass domain it behaves as:

$$\Omega \approx \frac{f-f_0}{B_W/2} \to \phi_k(s) = \phi_{0,k}\left(\frac{sB_n}{2j}+1\right);\ \phi_k'(s) = \frac{\phi_{0,k}B_n}{2j}. \quad (1)$$

Since the value of ϕ_k is determined solely to have a short circuit at $j\Omega_k$ after the phase shifter, the original formula for ϕ_k stands, now converted to the corresponding value at f_0:

$$\begin{aligned}\phi_k &= \arctan\left(b_k^{(k)}\right) - \tfrac{\pi}{2} + N_k\pi;\ \text{for any integer } N_k \\ \phi_{0,k} &= f_0\phi_k/f_k = \phi_k/(1+0.5\cdot B_n\Omega_k)\end{aligned} \quad (2)$$

The resonator must then match the derivative of the input susceptance at $j\Omega_k$ (after the frequency-variant shifter). The formula for l_k thus needs to be rederived following [7]:

$$\begin{aligned} l_k &= \tfrac{jb_k^{(k)\prime}}{1+b_k^{(k)2}} - \phi_{0,k}\tfrac{B_n}{2} \\ x_k &= -\Omega_k l_k \end{aligned} \ \text{giving}\ \begin{aligned} B^{(k)}(s) &= \tfrac{-j}{l_k s + jx_k} \\ B^{(k)\prime}(s) &= \tfrac{jL}{(l_k s + jx_k)^2} \end{aligned} \quad (3)$$

The last operation is to compute the remainders and evaluate them at the remaining TZs. The remaining susceptance $b^{(Rk)}$ after extracting a unitary-impedance phase shifter $\phi_k(s)$ and a shunt susceptance $B^{(k)}(s)$ from a susceptance $b^{(k)}(s)$ is:

$$b^{(Rk)}(s) = \tfrac{b^{(k)}(s) - \tan\phi_k(s)}{1+b^{(k)}(s)\tan\phi(s)} - B^{(k)}(s) \quad (4)$$

$$b^{(Rk)\prime}(s) = \frac{\left(1+\tan^2\phi_k(s)\right)\left(b^{(k)\prime}(s) - (1+b^{(k)\,2}(s))\phi'(s)\right)}{\left(1+b^{(k)}\tan\phi_k(s)\right)^2} - B^{(k)\prime}(s)$$

The evaluation at $j\Omega_i$ with $i > k$ finally consists of:

$$b_i^{(k+1)} = b^{(Rk)}(s)\Big|_{s=j\Omega_i};\quad b_i^{(k+1)\prime} = b^{(Rk)\prime}(s)\Big|_{s=j\Omega_i} \quad (5)$$

For instance, the following is obtained for $N_k = 0$:

$$b_i^{(k+1)} = \tfrac{\tan p_i^{(k)} + b_i^{(k)}}{1-b_i^{(k)}\tan p_i^{(k)}} + \tfrac{1}{l_k(\Omega_i - \Omega_k)};\ p_i^{(k)} = \tfrac{B_n\Omega_i+2}{B_n\Omega_k+2}\,\mathrm{atan}\left(\tfrac{1}{b_k^{(k)}}\right) \quad (6)$$

Setting $B_n = 0$ brings back the original equations of [7]. The procedure then iterates incrementing k and repeating extractions until all TZs have been implemented.

C. Terminal Section

To conclude the synthesis once all TZs have been extracted, as visible in fig. 2 and fig. 3, a frequency-invariant terminal section B_{L1}, J_L, B_{L2}, J_T is determined from S_{22} as done in [7], which often entirely collapses into an irrelevant unitary-impedance phase shifter (ϕ_E, $Z_E = 1$, $B_{E1} = B_{E2} = 0$).

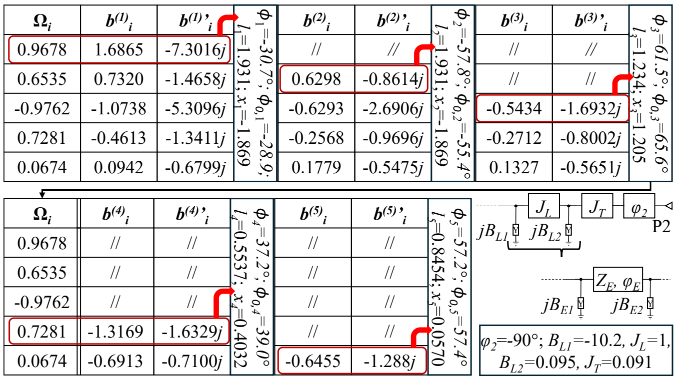
But the proposed modification with frequency-variant phase shifters forces a non-rational behavior and the analytically-correct terminal section would be very complicated. In practice however, the simple frequency-invariant terminal network described by [7] is still acceptable, as shown in section III. Compared to original cascade synthesis, the proposed method follows the ideal response better in the stopband and its neighborhood,while it may still present differences further away from the central frequency.

Furthermore, an additional unitary-impedance frequency-variant phase shifter ϕ_2 can be preliminarily removed from port 2 before synthesizing the terminal section: this modifies the susceptances to be presented at port 2 and hence affects the circuit values in the terminal section. This single degree of freedom (not available with the conventional frequency-invariant synthesis) can be used to adjust the terminal network until a more convenient one is obtained or until the response far from the stopband is deemed acceptable.

III. EXAMPLES

A 5-rd order example of a C-band bandstop filter for radar-interference applications is synthesized here based on the following specifications: Stopband IL $\geq$ 30 dB in $[5.15; 5.85]$ GHz, Passband RL $\leq$ 15 dB in $[4; 5.09] \bigcup [5.95; 7]$ GHz. The normalized passband is $B_n = 13\%$ around the central frequency $f_0 = 5.49$ GHz. The scattering polynomials are created in the lowpass normalized domain choosing 4 zeros: $-1.2j, -1.6j, 1.3j, 2j$ and applying a 5-th order lossless generalized Chebychev response. The reflection and transmission polynomials are then swapped to obtain a highpass response, which will be synthesized according to fig. 1 (right), with the five actual TZs $j\Omega_i$ shown in the leftmost column of fig. 3.

The corresponding input susceptance at port 1 and its first derivative with respect to s are evaluated at the TZs as



Ω_i	$b^{(1)}_i$	$b^{(1)\prime}_i$	$b^{(2)}_i$	$b^{(2)\prime}_i$	$b^{(3)}_i$	$b^{(3)\prime}_i$
0.9678	1.6865	-7.3016j	//	//	//	//
0.6535	0.7320	-1.4658j	0.6298	-0.8614j	//	//
-0.9762	-1.0738	-5.3096j	-0.6293	-2.6906j	-0.5434	-1.6932j
0.7281	-0.4613	-1.3411j	-0.2568	-0.9696j	-0.2712	-0.8002j
0.0674	0.0942	-0.6799j	0.1779	-0.5475j	0.1327	-0.5651j

Ω_i	$b^{(4)}_i$	$b^{(4)\prime}_i$	$b^{(5)}_i$	$b^{(5)\prime}_i$
0.9678	//	//	//	//
0.6535	//	//	//	//
-0.9762	//	//	//	//
0.7281	-1.3169	-1.6329j	//	//
0.0674	-0.6913	-0.7100j	-0.6455	-1.288j

Fig. 3. Table: TZs and input susceptances as the synthesis progresses. Bottom right: terminal section and its equivalence.

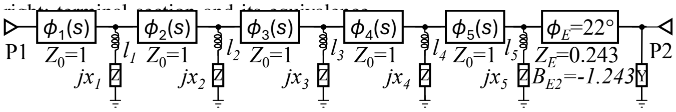


Fig. 4. Normalized highpass circuit from the synthesis example in fig. 3

described in [7], yielding $b_i^{(1)}$, $b_i^{(1)\prime}$. Then eqs. (2) and (3) yield ϕ_1, l_1 and x_1 which extract the TZ at $j\Omega_1$, using $N_k = 0$ for simplicity. Finally, the remainders are evaluated at the *remaining* TZs, via eqs. (4) to (6), giving $b_i^{(2)}$ and $b_i^{(2)\prime}$.

Then the procedure iterates through all TZs $k = 2, \ldots 5$.

For the terminal section in this case an inconsequential $90\,\mathrm{deg}$ phase shifter from port 2 is extracted, then computing the terminal stage according to [7]. The terminal section is transformed into a non-unitary impedance phase shifter and a shunt susceptance. Other terminal sections can be devised by changing the choice of the phase shifter on port 2, which slightly alters the rest of the terminal components to bring other solutions more or less acceptable and also affecting the response. The final circuit is shown in fig. 4, where the phase shifters ϕ_i denote transmission lines with electrical length varying linearly with frequency around that value.

The response in fig. 5 shows that the filter is not perfectly matched at infinity, due to the terminal section. However, both above and below the stopband, the return loss remains rather good and closely imitates the ideal response especially in the vicinity of the stopband, while $|S_{11}|$ deteriorates beyond $7\,\mathrm{GHz}$. The response achievable by a circuit synthesized by the traditional method is also shown, after it is implemented by physical transmission lines. Comparison demonstrates that the traditional method is less accurate due to the relatively high normalized bandwidth, and its response is in fact poor already close to the stopband. Moreover, the attenuation in the stopband is no longer equiripple. The response of the circuit synthesized with the proposed technique, instead, maintains the equiripple attenuation much better and also the closest reflection zeros are preserved as in the ideal response.

IV. CONCLUSIONS

This work proposes modified formulas for section extraction, in order to synthesize bandstop extracted-pole filters

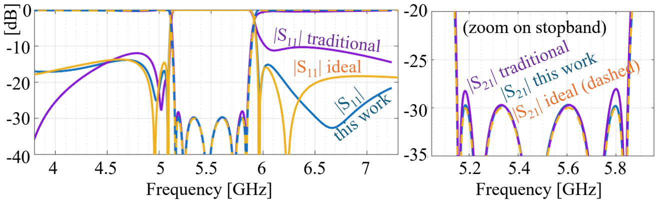


Fig. 5. Stopband response of the desired ideal model, compared to the traditional method which neglects frequency-variant phase shifters and hence undergoes significant deviations, and the result of the technique proposed in this work taking into account the frequency variation of phase shifters.

including frequency-variant phase shifters, as typical transmission line segments. The formulation does not require to handle polynomials, although the result in this case is an approximation. One main drawback emerges as a consequence: the frequency variation imposed by the phase shifters affects resonators too, and potentially unfeasible values might appear (as they would by optimizing the circuit). In such occurrences, the designer may alter the phase shifter (adding/removing 180°) and/or to change the TZ sequence of extraction.

The result anyway provides a better initial circuit than the exact method, benefitting subsequent optimizations. Tests have shown reasonable results up to a relative bandwidth of about 15% and with good out-of-band response up to 50%. One degree of freedom available in the terminal section can also be exploited to select the most palatable terminal components.

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REFERENCES

[1] M. Oldoni, S. Tamiazzo, G. Macchiarella, G. G. Gentili, and S. Caicedo, “Improved section extraction technique for fully-canonical filters.” Paris, France: IEEE, 2024, pp. 176–179.

[2] P. Zhao, “Matrix synthesis for filters with cascaded extracted-pole sections,” *IEEE Microwave and Wireless Components Letters*, vol. 32, pp. 1383–1386, 2022.

[3] S. Caicedo M., M. Oldoni, S. Moscato, G. Macchiarella, M. D’Amico, and G. G. Gentili, “Accurate synthesis of extracted-pole filters by topology transformations,” *IEEE Microwave and Wireless Components Letters*, vol. 31, no. 1, pp. 13–16, Jan 2021.

[4] S. Caicedo Mejillones, M. Oldoni, S. Moscato, G. Macchiarella, M. D’Amico, G. G. Gentili, and G. Biscevic, “Unified analytical synthesis of cascaded n-tuplets filters including nonresonant nodes,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 69, no. 7, pp. 3275–3286, July 2021.

[5] Y. Yang, M. Yu, and Q. Wu, “Advanced synthesis technique for unified extracted pole filters,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 64, pp. 4463–4472, 2016.

[6] G. Macchiarella and S. Tamiazzo, “An application-oriented design procedure for cascaded-block extracted-pole filters,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 69, pp. 647–658, 2021.

[7] M. Oldoni, S. Tamiazzo, G. Macchiarella, and G. G. Gentili, “Accurate cascade synthesis of high-order stopband filters,” in *55th European Microwave Conference (EuMC)*. Utrecht, The Netherlands: IEEE, 2025.