Teaching by active learning: a laboratory experience on fundamentals of vibrations

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***Abstract*— This article shows the design and construction of a test bench for experiments on mechanical systems vibrations. The experimental setup is finalized to the development of a project work proposed to the students of the course "Mechanical System Vibrations", within the BSc faculty of Mechanical Engineering. The principal motivation that leads to the enrichment of the educational program of the aforementioned course is the willingness to involve the students in a valuable learning-by-doing activity, aligned to the recent paradigm of active learning. From a didactic perspective, such an approach improves the engagement of the students and promotes the discussion with their peers. In this framework, the curiosity towards the practical activity enhances the acquisition of knowledge. The project regards the theoretical modeling of a 4-storeys building exploiting a lumped-parameters approach, and the deployment of some laboratory sessions. Acceleration data gathered during laboratory activities are employed in analyses in both the time and the frequency domain, exploiting the procedure of "Experimental Modal Analysis". Conclusively, the project requires the students to design and construct two different types of Tuned Mass Dampers whose effectiveness will be directly tested on the building, subjected to a condition of harmonic motion imposed by an actuated sled. The final deliverable of this practical activity is a report that students should submit by the end of the course.**

Active learning, Experimental Modal Analysis, Learning-by-doing, Mechanical Engineering, Project work, Structural dynamics, Tuned Mass Damper, Vibrations

I. INTRODUCTION

The study and deep understanding of mechanical systems vibrations are hard tasks to accomplish for a vast portion of students. The major difficulties regard the link between the mathematics written in textbooks and the real behavior of mechanical systems. Students seeking to obtain their bachelor's degree in Mechanical Engineering at Politecnico di Milano are requested to face such difficulties in the context of the course "Meccanica delle vibrazioni" (Mechanical vibrations in the English translation), taught during the second semester of the third year of the BSc in Mechanical Engineering. To help students better understanding the effects of vibrations in the real world, for the academic year 2020/2021 it has been decided to include in the course program a project-based learning activity [1] [2]. Such an approach is nowadays getting more and more common among educators, deeming useful the exploitation of laboratory sessions to enhance the quality of the learning. This fact holds even truer within scientific faculties, including the course of Mechanical Engineering, as witnessed by many case studies described in literature ([3]-[7]). The project presented in this article involves the study of vibrations of a 75 cm height 4-storeys building. The structure, described in detail in section 3, is mounted on an actuated sled and provided with accelerometers, one per floor.

The work, whose final submission should be made of a report, is structured as follows. First, students participate in a practical session in which the structure moves under different initial conditions or it is excited employing a controlled and actuated sled, to then acquire acceleration signals through the transducers fixed to each floor. After having processed the raw data to extract the modal properties of the structure, it is performed a comparison with the results coming from the lumped-parameters model. Once concluded this first part, another practical session is organized: this time students have the opportunity to inspect the effect of a tuning mass damper both on the physical model and on the frequency response functions they should extract after such session.

Aim of this paper is to describe this didactic activity and, to the end, this paper is structured as follows. Section 2 provides information about the course and the objectives of the teaching activity. Section 3 gives a theoretical background on mechanical systems vibrations, also describing a simplified analytical model of the test bench. Section 4 is intended to describe the assignment of the project. The results obtained are discussed in Section 5, whereas some concluding remarks are reported in Section 6.

II. THE COURSE

The course "Meccanica delle Vibrazioni" is taken by students in their last year of the Bachelor's degree in Mechanical Engineering, within the School of Industrial Engineering of Politecnico di Milano. The course program encompasses most of the concepts linked to the vibrations of multi-degrees-of-freedom mechanical systems: starting from the simplest single-dof linear systems, students are taught how to extend the rationale to n-dof mechanisms, also including the method for obtaining the equations for small perturbation around equilibrium position of nonlinear systems. In its final part, the course introduces the concept of modal superposition approach, prior to the study of passive solutions (tuned mass damper) for vibrations control and attenuation. Being this the first time that students face such topics in their bachelor's, the course is deployed in a profoundly analytical way, explaining the concepts mainly from a theoretical point of view.

This approach to the course may be unfavorable in easing the students to understand the link between what they learn in terms of mathematics with the phenomena that affect mechanical systems in the real world. It seems a good idea to integrate the traditional teaching activity with a project work that allows students to work hands-on an experimental case study. The latter has been designed keeping in mind two objectives: on one hand, involving students as much as possible promoting practical sessions, on the other hand, trying to cover most of the theoretical concepts explained within the course.

Interestingly, the type of structure that has been realized for the project could turn useful also for the course of Mechanical Systems Dynamics, which is taken by Mechanical Engineering Master's degree students during their first year at Politecnico. Compared to "Meccanica delle Vibrazioni", the latter is centered on the study of distributed-parameter systems. Nonetheless, the experience with the test bench presented in this article still holds valid: indeed, it is sufficient to change the approach to the modeling of the system to treat it as a deformable body.

III. DESCRIPTION OF THE EXPERIMENT

A. Physical model

The experimental model aims at reproducing the dynamics of a multi-storey building whose foundations are subjected to a harmonic motion. The system is composed of five aluminum plates connected by steel laminas, modeling respectively the storeys and the pillars of the building (Figure 1,Figure 2). Each floor is a 200x200x20 mm plate while the laminas have a section of 0.5x50 mm. The whole building is mounted on a linear actuator that imposes the desired motion law, with a maximum stroke of 20 mm and a maximum frequency of 5 Hz. The motion of each floor is sensed with accelerometers.

This system can be completed by two tuned mass dampers (TMD), one devoted to the attenuation of the vibration of the lower frequency mode of the building and the other associated with the higher frequency modes. The first TMD is a pendulum-type (Figure 3a) with a mass of 0.4 kg oscillating with a radius of 0.4 m approximately. The mass can slide on the bar of the pendulum so that the system can be tuned on the exact frequency of the first mode of the building. The pendulum is pinned to the last floor of the building through a bearing.

The second TMD can be used for the higher frequencies of the system, up to the third. As shown in Figure 3b it is composed of a mass fixed on a vertical cantilever beam, clamped on the last floor of the building. Even in this case, the mass can slide over the beam to tune the frequency of the TMD.

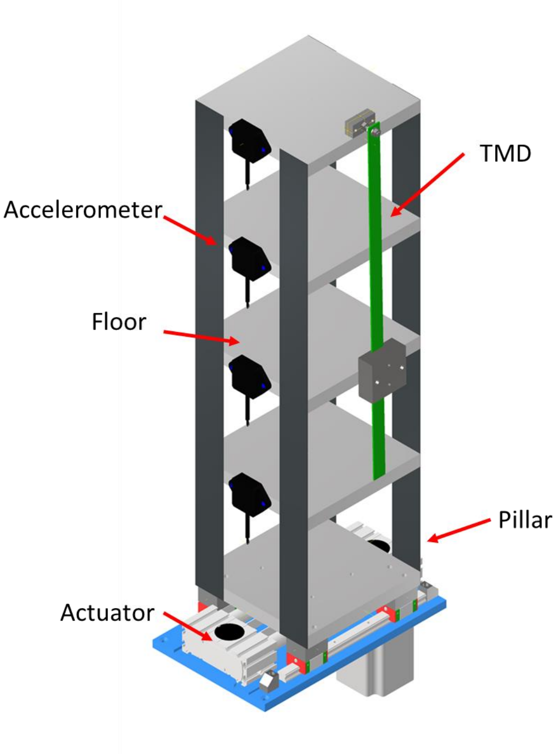
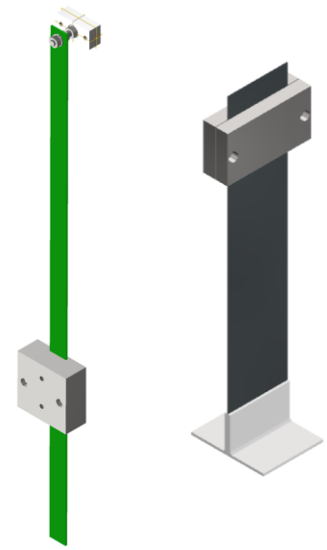


Figure 1 Physical model of the building

Figure 2 A photo of the test bench



a) b)

Figure 3 Pendulum-like (a) and beam-like (b) TMDs

B. *Mathematical model*

The assignment given to the students begins with the mathematical modeling of the building, performed utilizing Lagrangian equations [8]. As a first approximation, the system can be easily modeled through a lumped mass approach, given that the mass of each storey is much larger than the mass of laminas. Thus, the dynamic model consists of a series of masses connected by springs. The first mass, or the ground floor of the building, is subjected to an imposed motion. The value of the stiffness of the equivalent springs modeling the laminas can be derived by textbooks of structures mechanics such as [9], and it is the one of a clamped-clamped beam with one of the extremities subjected to a transversal displacement.

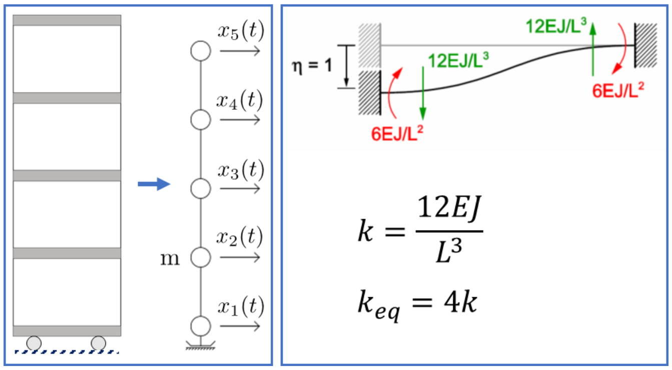


Figure 4 Lumped parameters modeling: main assumptions

Other approximations may be adopted to model the constraint between the storeys and the laminas. The principal alternative to the clamp-clamp is a pin-pin boundary with a certain amount of rotational stiffness, to be evaluated experimentally. However, the deformation that is observed in correspondence of the joint clearly shows that there is almost no relative rotation between the storey and the lamina, thus the constraint condition is well approximated by a clamp-clamp assumption. The total value of inter-storey stiffness k_{eq} is four times the one of a single beam. The system is modeled considering a planar motion using four degrees of freedom, which are the absolute horizontal displacements of the storeys. They can be grouped in a vector denoted as follows:

$$\overline{X} = [x_1\, x_2\, x_3\, x_4]^T \tag{1}$$

It is possible to express the equation of motion of the undamped system by means of a second-order ODE of the form:

$$M\ddot{\overline{X}} + K\overline{X} = \overline{F} \tag{2}$$

The mass matrix resulting from the modeling of the system is simply diagonal:

$$M = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{bmatrix} \tag{3}$$

while the stiffness matrix reflects the interaction of each floor with the superior and inferior ones:

$$K = \begin{bmatrix} 2k_{eq} & -k_{eq} & 0 & 0 \\ -k_{eq} & 2k_{eq} & -k_{eq} & 0 \\ 0 & -k_{eq} & 2k_{eq} & -k_{eq} \\ 0 & 0 & -k_{eq} & k_{eq} \end{bmatrix} \tag{4}$$

The vector of forcing terms depends on the first-floor displacement:

$$\overline{F} = \left[k_{eq}y(t)\, 0\, 0\, 0\right]^T \tag{5}$$

Given this dynamical model is requested to students to derive the undamped eigenfrequencies and eigenmodes of the system. By analyzing the homogeneous equation of the system:

$$[\mathrm{M}]\ddot{\overline{X}} + [K]\overline{X} = 0 \tag{6}$$

and imposing an exponential solution:

$$\overline{X} = \overline{X}_0 e^{\lambda t} \tag{7}$$

The following matrix equation is obtained:

$$\lambda^2[\mathrm{M}] + [\mathrm{K}] = 0 \tag{8}$$

By imposing that the solution is non-trivial and equating the determinant of the left-hand matrix to zero the characteristic equation of the system is got in the form:

$$\mathrm{a}\lambda^{2\mathrm{n}} + \mathrm{b}\lambda^{\mathrm{n}} + \mathrm{c} = 0 \tag{9}$$

The solutions of (9) are the eigenvalues of the dynamic system, whose imaginary part corresponds to the eigenfrequencies. By substituting one by one the eigenfrequencies in (8) the corresponding eigenmode is obtained. The following procedure can be easily performed in a numerical tool (for example Matlab by Mathworks [10]) by invoking the function "eig" on the matrix:

$$A = [M]^{-1}[K] \tag{10}$$

The estimation of the damping matrix, whose entries are difficult to evaluate by theory, is part of the assignment taken by students. One way suggested to them to estimate the damping ratio ξ of the system is to adopt the logarithmic decrement. Given a free-decay of one of the modes of the building the damping ratio can be estimated by evaluating the quantity [8]:

$$\delta_i = \frac{1}{n} ln \frac{q_i(t)}{q_i(t+nT)} \tag{11}$$

Where T is the period of oscillation of the time history and $q_i(t)$ is the value of any two successive peaks. The damping ratio of the i-th mode is related to δ_i through:

$$\xi_i = \frac{1}{\sqrt{1+\left(\frac{2\pi}{\delta_i}\right)^2}} \tag{12}$$

After evaluating the damping of the system also the Frequency Response Function of the system can be formulated analytically. Given the complete equation:

$$[\mathrm{M}]\underline{\ddot{X}} + [R]\underline{\dot{X}} + [K]\underline{X} = \underline{F} \tag{13}$$

By imposing a tentative solution in the form:

$$\underline{X} = \underline{X}_0\, e^{i\Omega t} \tag{14}$$

and making derivatives the following equation is obtained:

$$(-[\mathrm{M}]\Omega^2 + \mathrm{i}\Omega[\mathrm{R}] + [\mathrm{K}])\underline{\mathrm{X}}_0 = \underline{\mathrm{F}} \tag{15}$$

The left-hand matrix in brackets, after inversion, represents the matrix of the FRFs of the dynamic system:

$$H(\Omega) = (-[\mathrm{M}]\Omega^2 + \mathrm{i}\Omega[\mathrm{R}] + [\mathrm{K}])^{-1} \tag{16}$$

And

$$\underline{X}_0 = \mathrm{H}(\Omega)\underline{\mathrm{F}} \tag{17}$$

In the present case, by assuming $y = 1$ the response to unitary displacement of the first floor is obtained.

When a TMD is mounted on the building the system is augmented with an additional degree of freedom (i.e. θ, the pendulum rotation). In the pendulum-type TMD, the oscillating weight has mass M, at a distance L from the pin. The bar has mass m_a, length $2l_a$ and moment of inertia J_a. The vector $\overline{X}$ is now:

$$\underline{X} = [x_1\, x_2\, x_3\, x_4 \theta]^T \tag{18}$$

The mass matrix of the system becomes:

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & (m+m_a+M) & (m_a l_a + ML) \\ 0 & 0 & 0 & (m_a l_a + ML) & (m_a l_a^2 + ML^2 + J) \end{bmatrix} \tag{19}$$

And the stiffness matrix is added with the gravitational restoring of the pendulum:

$$K = \begin{bmatrix} 2k_{eq} & -k_{eq} & 0 & 0 & 0 \\ -k_{eq} & 2k_{eq} & -k_{eq} & 0 & 0 \\ 0 & -k_{eq} & 2k_{eq} & -k_{eq} & 0 \\ 0 & 0 & -k_{eq} & k_{eq} & 0 \\ 0 & 0 & 0 & 0 & (m_a g l_a + MgL) \end{bmatrix} \tag{20}$$

In the case of the beam TMD the addition of the damping system is equivalent to the addition of a fifth floor with mass equal to the oscillating mass and stiffness equal to the stiffness of a beam with clamped-free extremities:

$$\mathrm{k_{TMD}} = \frac{3\mathrm{EJ}}{\mathrm{L}^3}$$

IV. ASSIGNMENT

This section aims at describing the set of activities that students are requested to carry out for this project work.

Prior to accessing to the laboratory where model testing is performed, students must complete a short course on safety measures inside laboratories and are instructed on specific safety issues related to the experimental tests.

The preliminary phase consists of a visual inspection of the test bench. The objective is to make the students aware of how the structure is built, to let them be able to collect the physical parameters and the kind of constraints needed to set up a mathematical model. Students are requested to define a mathematical model of the building using a lumped parameter approach and to model it in a software for numerical analysis.

Then, the program foresees the organization of a first practical session. During this activity, students have the opportunity to directly interact with the model. Indeed, the process of data gathering involves the setup of the transducers, together with the acquisition system, and then the perturbation of the initial equilibrium condition so that the free decay phenomenon can be captured. Some acceleration time histories are given to students so that they can compare the experimental outcomes with the numerical model they have built, and they are incited to highlight pitfalls in the mathematical modeling or in the selection of parameters.

From a modal analysis of the building, students are requested to extract the modal parameters, with particular attention to the techniques adopted for the estimation of the damping coefficients. Discussion and practical demonstration both of modal analysis and modal parameters identification techniques are given to them.

The modal analysis is also a fundamental prerequisite for the last part of the project work: the design and test of a tuned mass damper, aimed at reducing the vibration of the model in correspondence with the natural frequencies. In this regard, students are asked to choose between two different kinds of TMDs – a pendulum and a clamped steel foil – and then design them to properly damp, one at a time, the effects of the different resonances. The design phase consists of the selection of the right radius of oscillation for the pendulum TMD or steel beam length for the cantilever TMD. Masses of different entities are prepared so that students can compare the effect of mass ratio on the functionality of TMD, first of all on their numerical model. Once accomplished the task, a conclusive practical session is organized to let the students test their TMDs on the various natural frequencies, with the structure excited by means of the actuated sled.

As already mentioned in the introduction, the final deliverable consists of a report summarising the activities carried out within the project work.

V. RESULTS

Let us now present some results of the project work. Unfortunately, the unpredicted outbreak of the pandemic caused by the spread of Covid-19 has forbidden the organization of the practical sessions, since students were not allowed to attend lessons and participate in experimental activities and laboratories.

Consequently, what is shown in this section does not include the testing of the tuned mass dampers designed by the students. However, as an example, the results of the application of a TMD designed by us are proposed. More specifically, it is presented a comparison between the lumped-parameters numerical model and the experimental results, to validate the assumptions underlying the modeling activity. Then, the effects of the TMD on the structure are shown in terms of attenuation of the vibrations associated with the first natural frequency of the system. Both the numerical simulation and the postprocessing of the experimental data are performed exploiting a commercial software.

In Figure 4a it is represented the time domain response to a free decay of the structure obtained perturbing the equilibrium position of the 4th floor. The signal has been registered by the accelerometer attached to the 4th floor of the building. From this figure, one may estimate the damping and the fundamental frequencies of the vibration mode. At this step of learning, students know the technique of logarithmic decrement, but more refined analyses might be done by analyzing the FRF (with the half-power damping method, to cite one [11]). As an example, Figure 5b represents the same signal shown in Figure 5a, but low-pass filtered with a cutoff frequency of 2 Hz, so to highlight only the first vibration mode. Applying the logarithmic decrement technique, a damping ratio of around 1% is obtained.

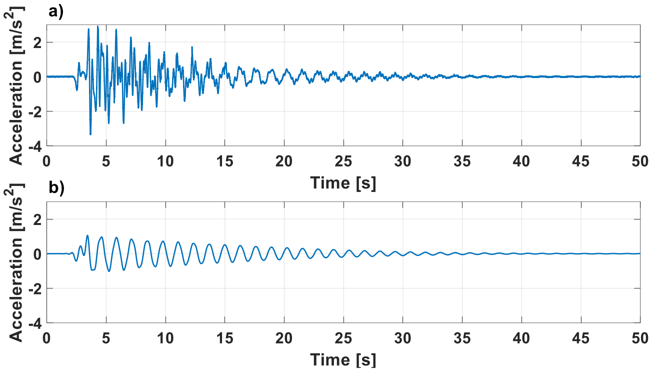


Figure 5 Free decays measured on the last floor of the building. (a) no filters. (b) 2 Hz low-pass filter applied to the signal

Subsequently, Figure 6presents some experimental outcomes in the frequency domain: what is shown is the Frequency Response Function considering each accelerometer, one per floor, in correspondence with an input force applied to the 4th floor. The input force has been applied to the system using an impact hammer.

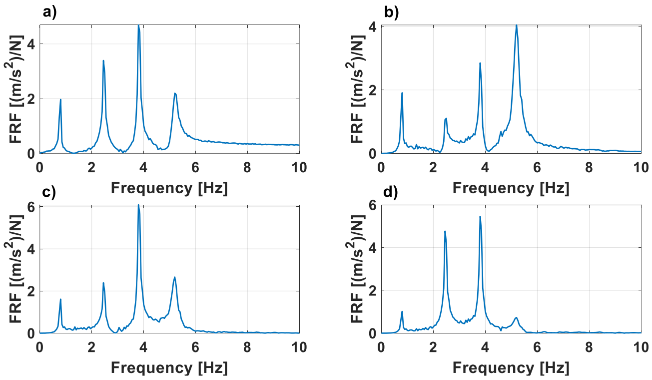


Figure 6 FRFs calculated from the signals measured by each accelerometer, one per floor, with input applied to the 4th floor. (a) 1st floor. (b) 2nd floor. (c) 3rd floor. (4) 4th floor.

Then, to highlight the comparison between experimental and numerical results, the natural frequencies inspected in the two cases are summarized in Table 1. What can be noticed is that the numerical model tends to overestimate the natural frequencies, suggesting that the clamp-clamp boundary assumption on the steel laminas leads to a more rigid structure compared to the actual one. Conclusively, exploiting the single-mode approximation, the results from the FRFs (module and phase), should also allow students to obtain information about the vibration modes.

Table 1 Natural frequencies from the numerical and the experimental model.

Mode	Numerical model [Hz]	Experimental model [Hz]
1	1.09	0.79
2	3.15	2.44
3	4.83	3.79
4	5.94	5.19

Lastly, it is discussed the effect of the TMD on the first mode shape. The tuning of the latter consists of determining the length of the pendulum to set its natural frequency equal to the building one that is intended to damp out. Referring to the experimental results, a 390 mm lamina has been chosen: recalling the properties of the pendulum, one should infer that a frequency of 0.79 Hz is obtained. To inspect the effectiveness of the TMD, two sets of tests with an impact hammer were performed, five hits each, with and without the TMD respectively. The input has been delivered to the 4th floor, while each storey has been instrumented with an accelerometer. Figure 7shows the difference in terms of the frequency response functions of the last floor in case a TMD is applied or not to the structure. As theoretically expected, the first natural frequency of the building per-se is fully damped out, whereas two new frequencies originate. In particular, the system with the TMD shows an anti-resonance at the same frequency where the original first mode was located.

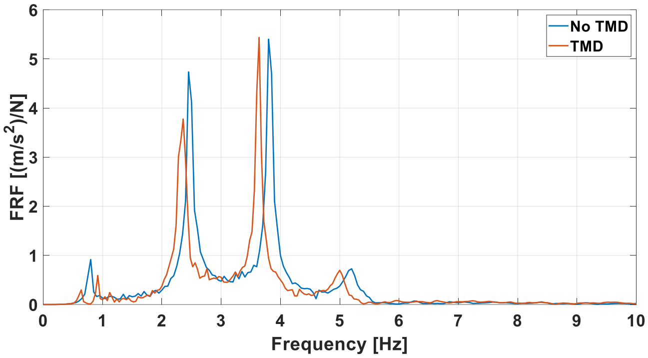


Figure 7 Comparison in terms of FRF of the last floor with or without TMD.

VI. CONCLUSIONS AND FURTHER DEVELOPMENTS

The present article describes the ideation and testing of a project work to propose to students attending the course of Mechanical Vibrations during their Bachelor degree in Mechanical Engineering at Politecnico di Milano. The educational objectives are:

- create engagement with the students exploiting a learning-by-doing approach. This in turns allows them to work hands-on on a real project, with all the difficulties that the in-field testing implies;
- learn how to apply the mathematical model of discrete mechanical systems to model a real structure;
- try to exploit the theoretical knowledge acquired during the course to extract meaningful information from the acquired data: time domain analysis and frequency domain analysis;
- design a self-made tuned mass damper to test on the structure and inspect its effectiveness;
- practice with the use of a commercial software both to build a numerical model to compare to experimental results and to analyze the experimental data as well.

The test bench has been successfully realized, then data were gathered and analyzed to evaluate whether they could be good enough to perform the study requested within the project work. The numerical model resulted to be slightly stiffer than the physical model, given the simplified modeling of constraint conditions. It is actually an important take-home message for students to discern what is the origin of differences between numerical and physical model, and what are the assumptions that have a stronger impact on the validity of results. The pendulum-like tuned mass damper was tested and proved to be properly effective, as theoretically expected.

Unfortunately, the Covid-19 pandemic did not let students actively participate in the active learning activities, thus the full implementation of the project has been forcibly postponed to the next academic year. Eventually, the following developments are related to the feedbacks that will be provided by students attending lab classes, with the aim to tailor the project as much as possible to enhance the engagement and educational effectiveness of the initiative.

The Authors declare that there is no conflict of interest.

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