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Multi-objective optimal control of a simple stochastic climate-economy model

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Abstract: Integrated assessment modelling of climate change aims to provide quantitative solutions to inform international climate policy by employing models where socio-economic and climatic systems are integrated. Among these models, DICE (Dynamic Integrated Climate-Economy), is used to perform cost-benefit analysis that returns as output the optimal emission reduction pathway. The model makes some important assumptions: future socio-economic and climate system evolution is deterministic and economic damages of climate change are a quadratic function of the atmospheric temperature. In this study, propose a multi-objective stochastic optimal control problem formulation of the DICE model in order to account for stochastic disturbances and to align with physical targets posed by international agreements on climate change mitigation. The solutions are control policies that can handle stochastic disturbances outperforming the static inter-temporal optimization approach traditionally adopted. Moreover, such control policies are able to deal with multiple objectives making explicit the trade-offs between economic and environmental objectives.

Keywords: Climate change impact and adaptation measures; Environmental decision support systems.

1. INTRODUCTION

Integrated assessment models (IAMs) of climate change use both simulation and optimization to explore different policies aimed at reducing future climate change impacts on the coupled human-natural Earth system (Parson et al., 1997; Kelly and Kolstad, 1999). Such quantitative results are used to inform policy makers and the public debate about the stakes involved in the decision making process (Weyant, 2017). Optimization models traditionally formulate cost-benefit analysis as a static optimization problem under deterministic conditions and include uncertainty considerations only a posteriori via sensitivity analysis (Crost and Traeger, 2013).

Nonetheless, many uncertainties involved in both socio-economic and climatic system call for an adaptive and target based approach (Allen and Frame, 2007; Farmer et al., 2015). For this reason, the problem has already been framed under the optimal control perspective using dynamic programming (Bahn et al., 2008; Webster et al., 2012; Lontzek et al., 2015; Lemoine and Traeger, 2016) and model predictive control (Weller et al., 2015; Hafeez et al., 2016, 2017; Faulwasser et al., 2018). For a complete review of optimal control practice in the context of integrated assessment modelling of climate change see Kellett et al. (2019).

Yet, to our knowledge, stochastic disturbances have been considered in the problem either as output noise measurement (Weller et al., 2015), implicitly trusting the equations of the model, or as binary stochastic jumps which result in alternative model trajectories (Lontzek et al., 2015).

In addition to that, the $2°C$ threshold has been identified as crucial in order to avoid triggering cascading tipping points resulting in abrupt and catastrophic changes for the Earth system (IPCC, 2018; Steffen et al., 2018). With the Paris Agreement (UNFCC, 2015), many countries agreed upon reducing GHGs emissions to keep the average global temperature increase with respect to pre-industrial temperatures well below $2°C$. Nonetheless, many IAMs optimal policies still result in overshooting this temperature threshold (Nordhaus, 2017). This points out that economic analysis should be integrated with targets on the climate system. Multi-objective optimization has been used to handle the conflicting objectives in this context by Garner et al. (2016) but not yet applied together with optimal control techniques.

For the above reasons, we extend the well known DICE model formulating a multi-objective stochastic optimal control problem including a stochastic disturbance on the atmospheric temperature transition equation and introducing a second objective, i.e. the sum of atmospheric temperatures over the horizon, which is used as a proxy to minimize climate change.

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2. BACKGROUND: DICE MODEL DESCRIPTION

The DICE (Dynamic Integrated Climate Economy) model (Nordhaus, 1992, 2017) is an optimization model whose aim is to maximize economic welfare of the coupled climate-economy system over a predefined time horizon. It was used to estimate the social cost of carbon (SCC) (IAWG, 2013), value that should be adopted as a carbon tax to internalize the economic damages produced by CO_2 emissions, the only GHG explicitly modelled in DICE.

DICE is composed by three interconnected components: carbon cycle, temperature and economy. The dynamical system described in the model has six state variables: $\mathbf{M}_t$, i.e. the carbon masses of the three reservoirs used to model the carbon cycle, $\mathbf{T}_t$, the temperatures in the two boxes used to model temperature dynamics, and K_t, the capital stock in the economy.

In order to fully describe the coupled climate-economy system, the model relies also on external input variables whose trajectory is exogenously given. These are: total factor productivity A_t, population L_t, carbon intensity of the economy σ_t, backstop technology cost θ_t^1, forcing due to other GHGs F_t^{EX} and natural CO_2 emissions E_t^{land}.

The decision variables (or control variables) are two: the emission control μ_t (i.e. the fraction of reduced emissions with respect to a business-as-usual scenario) and the gross savings rate s_t (i.e. the fraction of world GDP that is reinvested in the economy). In its current version (DICE-2016R), the model adopts a 5-year time step and an horizon covering years from 2015 to 2515.

Using these variables, the dynamics of the model evolve according to the following equations:

$$\boldsymbol{x}_{t+1} = \boldsymbol{f}(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{w}_t) \qquad (1a)$$

$$\{\boldsymbol{w}_t\}_{t=0,...,H-1} \quad given \qquad (1b)$$

$$\boldsymbol{x}_0 \quad given \qquad (1c)$$

$$t = 0, ..., H-1 \qquad (1d)$$

$$\boldsymbol{x}_t = \begin{bmatrix} M_t^{AT} & M_t^{UP} & M_t^{LO} & T_t^A & T_t^O & K_t \end{bmatrix} \in \boldsymbol{X} \qquad (1e)$$

$$\boldsymbol{u}_t = [\mu_t \ \ s_t] \in \boldsymbol{U} \qquad (1f)$$

$$\boldsymbol{w}_t = \begin{bmatrix} A_t & L_t & \sigma_t & \theta_t^1 & F_t^{EX} & E_t^{land} \end{bmatrix} \in \boldsymbol{W} \qquad (1g)$$

where $\boldsymbol{x}_t$, $\boldsymbol{u}_t$ and $\boldsymbol{w}_t$ are respectively the state, control and exogenous input variables, $\boldsymbol{f}(\cdot)$ represents the state transition function, t is the time index, and H is the length of the horizon. $\boldsymbol{X}$, $\boldsymbol{U}$, $\boldsymbol{W}$ are the set of admissible values for state, control and exogenous input variables respectively.

In the following, we shortly describe the endogenous dynamics of the model. For a more detailed description of the model and of the exogenous variables trajectories, please refer to Nordhaus and Sztorc (2013).

2.1 Carbon cycle

The carbon cycle is described by a three-reservoir model. The three reservoirs correspond to atmosphere, upper ocean and biosphere, and deep ocean. The atmospheric carbon reservoir has an additional input, CO_2 emissions, which are described in the economy component.

The transition equation for the carbon component is the following:

$$\mathbf{M}_{t+1} = \mathbf{\Phi^M} * \mathbf{M}_t + [E_t \Delta t/3.666 \ \ 0 \ \ 0]^T \qquad (2a)$$

$$\mathbf{M}_t = \begin{bmatrix} M_t^{AT} & M_t^{UP} & M_t^{LO} \end{bmatrix}^T \qquad (2b)$$

$$\mathbf{\Phi^M} = \begin{bmatrix} 1-\phi_{12} & \phi_{12}\phi_1 & 0 \\ \phi_{12} & 1-\phi_{12}\phi_1-\phi_{23} & \phi_{23}\phi_2 \\ 0 & \phi_{23} & 1-\phi_{23}\phi_2 \end{bmatrix} \qquad (2c)$$

where $\mathbf{M}_t$ $[GtC]$ is the vector containing the three carbon mass reservoirs (M_t^{AT} atmospheric, M_t^{UP} upper ocean and biosphere, M_t^{LO} deep ocean), $\mathbf{\Phi^M}$ is the transition coefficients matrix, E_t $[GtCO_2]$ are CO_2 emissions, Δt $[y]$ is the time step and the factor 3.666 converts $[GtCO_2]$ into $[GtC]$.

2.2 Temperature

As for the temperature component, a two-box model describes atmospheric and ocean temperature dynamics that evolve according to the following equations:

$$\mathbf{T}_{t+1} = \mathbf{\Phi^T} * \mathbf{T}_t + [\xi_1 F_t \ \ 0]^T \qquad (3a)$$

$$\mathbf{T}_t = \begin{bmatrix} T_t^A & T_t^O \end{bmatrix}^T \qquad (3b)$$

$$\mathbf{\Phi^T} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \qquad (3c)$$

where $\mathbf{T}_t$ $[°C]$ is the vector containing atmospheric and ocean temperatures (T_t^A and T_t^O respectively), $\mathbf{\Phi^T}$ is the transition coefficients matrix. F_t $[W/m^2]$ is the radiative forcing and is computed as follows:

$$F_t = \eta \log_2 \frac{(1-\phi_{12})M_t^{AT} + \phi_{12}\phi_1 M_t^{UP} + E_t\Delta t/3.666}{M_{1750}^{AT}} + F_t^{EX} \qquad (3d)$$

where M_{1750}^{AT} $[GtC]$ is the mass of carbon in the atmosphere in year 1750 and F_t^{EX} $[W/m^2]$ is radiative forcing due to other greenhouse gases, exogenously given.

2.3 Economy

The economy is modelled according to the Solow's neo-classical growth theory:

$$Y_t = A_t (K_t)^\gamma (L_t)^{1-\gamma} \qquad (4a)$$

where Y_t $[trillion\ 2005\ USD]$ is the economic output or gross world product, A_t $[-]$ is the total factor productivity (exogenous variable), K_t $[trillion\ 2005\ USD]$ is the capital stock and L_t $[billions\ people]$ is the population (exogenous variable).

CO_2 emissions can be computed from economic output based on the following:

$$E_t = \sigma_t Y_t (1-\mu_t) + E_t^{land} \qquad (4b)$$

where total emissions E_t $[GtCO_2]$ are the sum of natural emissions E_t^{land} $[GtCO_2]$ and industrial emissions. The latter are computed based on carbon intensity σ_t $[GtCO_2/trillion\ 2005\ USD]$, which is an exogenous variable, and the emission control μ_t $[-]$, which is a control variable.

The emissions control comes at a cost which, together with the economic damages due to climate change, reduces the economic output as follows:

$$Y_t^{net} = Y_t[1 - a^1 T_t - a^2 (T_t)^2 - \theta_t^1 (\mu_t)^{\theta^2}] \qquad (4c)$$

where Y_t^{net} $[trillion\ 2005\ USD]$ is the net economic output, a^1 and a^2 are parameters of the quadratic function describing climate damages on the economy, θ_t^1 and θ^2 are parameters of the abatement cost function, exogenously given.

Last, the capital stock is the only endogenous state variable for the economic component of the model and it evolves according to the perpetual inventory equation:

$$K_{t+1} = (1-\delta^k)^{\Delta t} K_t + s_t Y_t^{net} \tag{4d}$$

where δ^k $[-]$is the depreciation rate of capital and s_t $[-]$ is the percentage of economic net output which is reinvested in the capital stock, i.e. the savings rate which is a control variable.

2.4 Original problem formulation

In the original DICE model the optimization problem is formulated as follows:

$$\max_{\{\mu_t, s_t\}_{t=0,\dots,H-1}} \quad J^e \tag{5a}$$

$$\text{s.t.} \quad Eq.\ 1 \tag{5b}$$

$$0 < \mu_t < 1 \tag{5c}$$

$$0 < s_t < 1 \tag{5d}$$

where the objective function to be maximized is the economic welfare, a function of the economic utility:

$$J^e = \sum_{t=0}^{H-1} \frac{U_t}{(1+\rho)^t} \tag{6a}$$

$$U_t = L_t \left[\frac{\left(\frac{(1-s_t)Y_t^{net}}{L_t}\right)^{1-\alpha} - 1}{1-a} - 1 \right] \tag{6b}$$

where U_t is the utility, α is the elasticity of the marginal utility of consumption, ρ is the pure rate of social time preference, which provides the welfare weights on the utilities of different generations. The solution of the problem defined in Eq. 5 yields the optimal sequence of emissions control and savings rate.

3. PROBLEM FORMULATION

Equation 5 defines a single-objective deterministic non linear programming problem which is traditionally solved using static inter-temporal optimization under the hypothesis of perfect foresight. These solutions are not adaptive and yield the optimum objective value only in the reference scenario considered during the optimization.

3.1 Introducing multiple objectives

As discussed in the introduction, in order to overcome the limitations posed by a single-objective problem formulation, we propose to introduce the following objective:

$$J^c = \sum_{t=0}^{h} T_t^A \tag{7}$$

Indeed, this objective aims at minimizing the integral of temperatures over the whole horizon, which is in line with the goal of staying well below $2°C$ aiming for $1.5°C$. By introducing a second objective we align economic objectives with agreements taken on the temperature goal and explore the trade-offs between the cost-benefit analysis and the climatic objective.

3.2 Introducing stochastic disturbances

Many uncertain parameters affect the model under examination, both on the socio-economic and on the climatic side (Butler et al., 2014; Gillingham et al., 2018). In the context of this work, we focus on the physical system only and introduce an additive stochastic disturbance $\varepsilon_{t+1}^{T^A}$ modeled as a white noise in the atmospheric temperature transition equation. In the following, we refer to atmospheric temperature as its increase with respect to pre-industrial temperature (IPCC, 2018).
First, we simulate the original DICE temperature model (both atmospheric and ocean components, with coefficients adapted to be used with an annual time step) using observed forcing obtained from the RCP database (Meinshausen et al., 2011) and compare atmospheric temperature with the average global temperature from the HadCRUT4 dataset (Morice et al., 2012) (see Fig. 1).

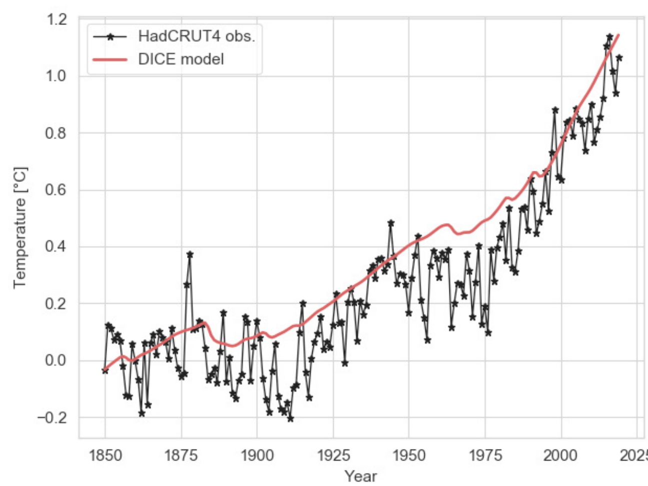

Fig. 1. Atmospheric temperature as simulated by DICE temperature model under historical radiative forcing and historical observations (HadCRUT4).

The model is able to capture the average behavior of the average global temperature but the residuals time series $\{\eta_t\}_{t=1850,\dots,2018}$, computed as:

$$\eta_t = T_t^{HadCRUT4} - T_t^A \tag{8a}$$

is correlated and its mean is significantly different from zero. Since a process noise has to be identified, we can see the time series of residuals $\{\eta_t\}_{t=1850,\dots,2018}$ as the realization of an AR(1) process:

$$\eta_{t+1} = \beta_{11}^y \eta_t + \varepsilon_{t+1}^{T^A} \tag{8b}$$

Therefore, differencing the residuals time series using the auto-regressive coefficient β_{11}^y, i.e. the annual time-step equivalent of β_{11} in Eq. 3, we obtain the time series $\{\varepsilon_t^{T^A}\}_{t=1851,\dots,2018}$, whose components are approximately uncorrelated and normally distributed as shown in Fig. 2.

The temperature model showed in Eq. 3 is modified accordingly:

$$\mathbf{T}_{t+1} = \mathbf{\Phi^T} * \mathbf{T}_t + [\xi_1 F_t \ \ 0]^T + \left[\varepsilon_{t+1}^{T^A} \ \ 0\right]^T \tag{9a}$$

$$\varepsilon_{t+1}^{T^A} \sim N(0, \sigma_{T^A}^2) \tag{9b}$$

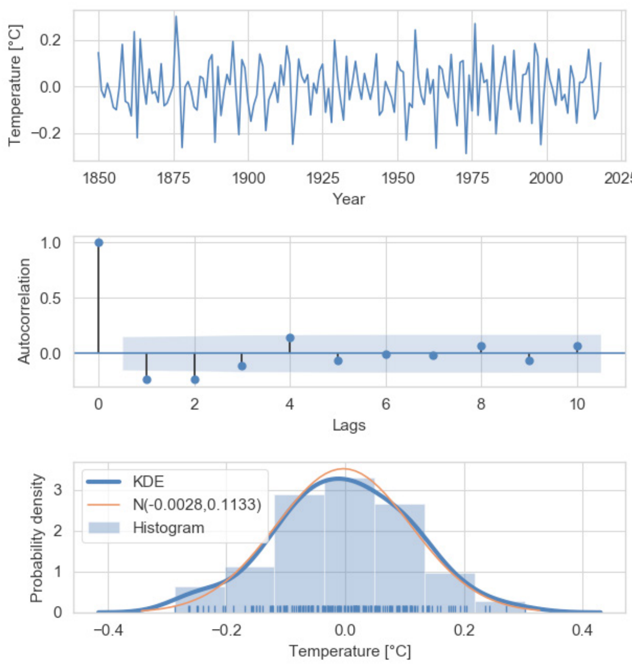

Fig. 2. Residuals of auto-regressive differentiation: time series (top panel); auto-correlation (middle panel); KDE probability density estimation, normal distribution inferred from data and histogram (bottom panel).

where the variance of the time series $\{\varepsilon_t^{T^A}\}_{t=1851,\dots,2018}$ is used as an estimate for $\sigma^2_{T^A}$, which is set equal to $0.01284\ [(^\circ C)^2]$. The derived stochastic temperature model is demonstrated over the different historical and future forcing derived from the RCPs in Fig. 3.

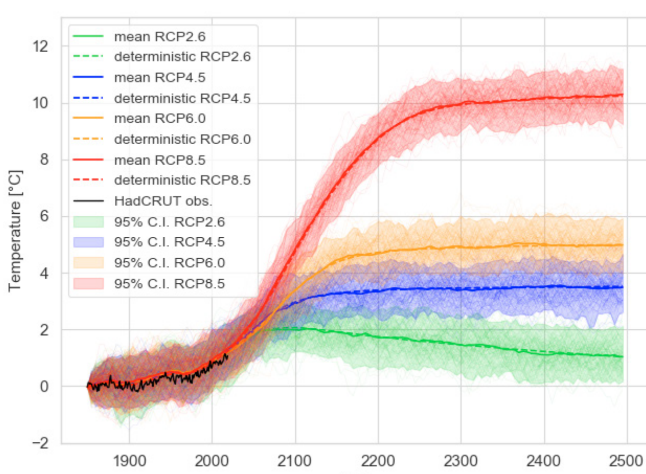

Fig. 3. Stochastic temperature model simulations under historical and future forcing of the four RCPs.

3.3 Multi-objective stochastic optimal control problem

The multi-objective stochastic optimal control problem can be formulated as follows:

$$
\begin{aligned}
&\min_{p} \; \underset{\{\boldsymbol{\varepsilon}_t\}_{t=1,\dots,H}}{E} \; |-J^e \quad J^c| && \text{(10a)}\\
s.t. \quad & Eq.\ 1 && \text{(10b)}\\
& \boldsymbol{u}_t = p(t, \boldsymbol{x}_t) && \text{(10c)}\\
& \boldsymbol{u}_t \in U(\boldsymbol{x}_t) && \text{(10d)}\\
& \boldsymbol{\varepsilon}_{t+1} \sim \phi_t(\cdot) && \text{(10e)}\\
& |\mu_{t+1} - \mu_t| < 0.2 && \text{(10f)}\\
& |s_{t+1} - s_t| < 0.1 && \text{(10g)}\\
& \mu_0 = 0.039 && \text{(10h)}\\
& s_0 = 0.258 && \text{(10i)}
\end{aligned}
$$

where the uncertainty is filtered using the expected value operator over the different objectives realizations, Eq. 1 has been updated to consider the stochastic temperature model in Eq. 9, $p(\cdot)$ is the control policy, $U(\boldsymbol{x}_t)$ is the set of admissible control variables, $\phi_t(\cdot)$ is the stochastic disturbance probability density function.

Last, additional constraints on the control variables are added introducing Eqqs. 10f - 10i for control regularization and initialization purposes.

3.4 Evolutionary Multi-Objective Direct Policy Search

In order to solve the problem defined in Eq. 10, we use the Evolutionary Multi Objective Direct Policy Search (EMODPS) method (Giuliani et al., 2016). EMODPS is a simulation-based-optimization algorithm that optimizes the parameters of a control policy defined as a non-linear approximating network using the self-adaptive evolutionary algorithm Borg MOEA (Hadka and Reed, 2013). Indeed, EMODPS was proved to handle successfully nonlinear multi-objective optimal control problems, as the one examined in this case, by overcoming the need for linear systems hypothesis and scaling well with respect to the introduction of multiple objectives (Giuliani et al., 2018). The EMODPS algorithm is coupled with the CDICE model (Garner et al., 2016) as implemented by Lamontagne et al. (2019) to replicate the original DICE model, that allows simulating open-loop sequences of control variables. The model has been modified in order to determine the control at each time step by using closed-loop control policies which map state observations into control variables.

4. RESULTS

The performance of the solutions obtained for the problem defined in Eq. 10 using control policies is shown in Fig. 4 and compared with the optimal solution of the problem defined in Eq. 5 under the same stochastic disturbance. Along the horizontal axis the value of the climatic objective can be read while on the vertical axis the economic utility is reported. Since our aim is to maximize economic utility and minimize the sum of atmospheric temperature over the horizon, the ideal solution would be in the top left corner. As an additional information, since it strongly correlates with the temperature objective, the average atmospheric temperature reached at year 2100 is reported for each solution using a color scale from green (low temperature) to red (high temperature).

First, it is easy to notice that the static optimization solution is dominated by all control policy solutions that achieve better economic and climatic performance. Indeed,

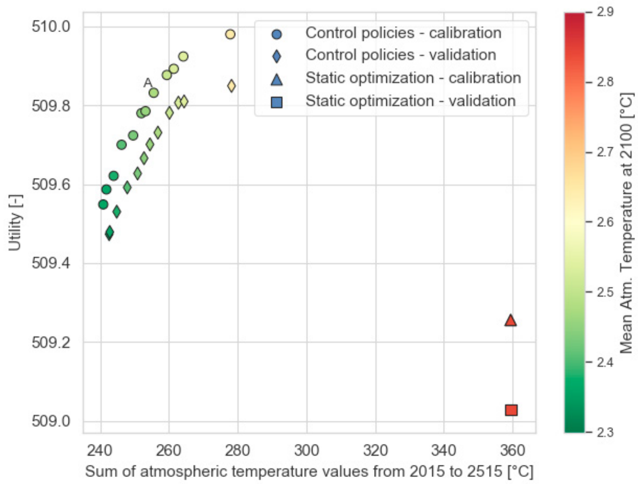

Fig. 4. Solutions of the multi-objective stochastic optimal control problem. The solution obtained via static optimization is reported too. Scores obtained in calibration and validation over 10000 simulations are shown. Additionally, the color bar shows the average atmospheric temperature at 2100.

the best control policy with respect to the economic objectives is able to improve the economic utility while committing to less warm temperature by $0.2°C$ at 2100 and in the future. This proves that under stochastic conditions static optimization is outperformed by closed-loop control policies that can better deal with stochastic disturbances (Bennett, 1996).

In addition to that, the Pareto front exhibits a strong trade-off between the objectives, especially when we want to achieve very low temperatures. This is in line with the expectation that a strong reduction of temperatures implies high abatement costs resulting in a substantial departure from a purely economic analysis.

Now we will focus on the trajectories of atmospheric temperature and control variables for the control policy solution marked with the letter A in Fig. 4. This solution can substantially improve the economic utility with respect to the static optimization solution while substantially improving the temperature objective reaching an average $2.5°C$ temperature increase with respect to pre-industrial levels.

For example, looking at this policy trajectories in Fig. 5, when temperatures are higher than expected, emission control increases in order to make the effort more effective in reducing temperature increase in the long term. On the other hand, if temperature does not increase as expected, there is more space to spread the abatement effort over time. This happens from 2040 on as until that time a strong increase in reduction is profitable under the cost-benefit perspective too and all the solutions propose the same level of emission control until 2035.

Additionally, the role played by the savings rate is also relevant, even tough this is usually not even optimized but supposed constant. When the temperature is lower than average, since climate damages are low and abatement effort increases slowly, more economic output is available to gain some extra consumption and the savings rate decreases accordingly. In the opposite case, when temperatures are higher than expected, the economy is more affected and emission control has to increase rapidly leading to higher costs. Nevertheless, investments (i.e. savings rate) have to increase in order to bring economic output and capital stock closer to their expected trajectories at the next step. This reduces consumption in the short term but ensures that at next step enough capital is available to eventually increase emissions control, hedge against future climate damages and guarantee future consumption.

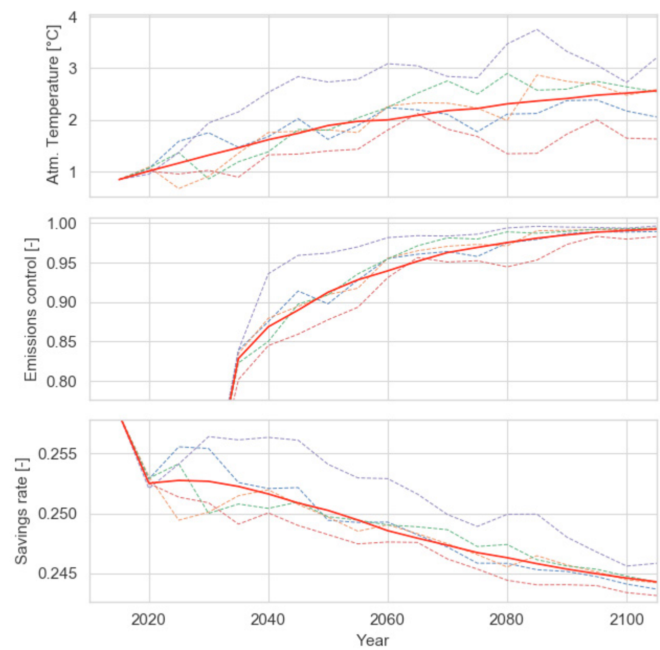

Fig. 5. Trajectories for the control policy solution attaining similar economic objective value of static optimization solution. Atmospheric temperature (top panel); emission control (middle panel); savings rate (bottom panel). Dashed lines display five different realizations of atmospheric temperature and corresponding control variables. The solid red line represents the average over 50 realizations.

5. CONCLUSION

In this work, we reformulated the DICE integrated assessment model as a multi-objective stochastic optimal control problem by: (i) adding stochastic disturbances on atmospheric temperature transition equation; (ii) introducing an objective on the physical climate system. The solution of the problem provides a set of Pareto-optimal control policies that outperform the standard static optimization approach and provide new perspective on the integration of standard cost-benefit analysis with environmental targets. Further research will focus on the introduction of other stochastic disturbances, that are especially needed in the economic component (Gillingham et al., 2018; Farmer et al., 2015). We also recognize that the constraints added on the control variables (Eqqs. 10f-10g) limit significantly the chance of reaching a $2°C$ target and we plan to further verify those assumptions too. Moreover, the employment of climatic models with a higher level of detail than the one considered in the original DICE model, might allow exploring solutions which are hidden and in line with the $2°C$ long term temperature goal.

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