Reachable domain analysis for analytical design of end-of-life disposal

Xiaodong Lu, Prof. Camilla Colombo
Politecnico di Milano
New Frontiers of Celestial Mechanics: theory and applications
• Background and motivations
• Modelling of the long-term evolution of s/c
• Phase space structure of orbital elements
• Post-mission disposal
• Conclusion and future work
Background and motivation

- Number of debris increases
- Earth’s orbit region becomes crowded

- Necessities of mitigation measures
  - Post-mission disposal, largest contribution
  - ...

- Trade-off of mitigation measures
  - Space environment improvements
  - Feasibility and cost

- Post-mission disposal
  - Technically feasible
  - Economical in the sense of fuel consumption

  • ESA’S Annual Space Environment Report, April 2022
Background and motivation

- Aim: end-of-life disposal design techniques leveraging perturbations to decrease the fuel consumption during the disposal
  - Follow the natural evolution of the s/c orbit
  - Enhance the natural perturbations by manoeuvres
    - Impulsive
    - Continuous
  - semi-analytical models of s/c dynamics
  - Hamiltonian formalism
    - Phase space structure of orbital elements
Modelling of the long-term evolution of s/c

Semi-analytical dynamics model based on averaging technique

- Lagrange planetary equations
  \[
  \begin{align*}
  \frac{da}{dt} &= \frac{2}{na} \frac{\partial R}{\partial M} \\
  \frac{dc}{dt} &= -\frac{b}{na^3c} \frac{\partial R}{\partial \omega} + \frac{b^2}{na^4} \frac{\partial R}{\partial M} \\
  \frac{di}{dt} &= -\frac{1}{nab\sin i} \frac{\partial R}{\partial \Omega} + \frac{b}{nab\sin i} \frac{\partial R}{\partial \omega} \\
  \frac{d\Omega}{dt} &= \frac{nab\sin i \frac{\partial \Omega}{\partial i}}{\cos i} \\
  \frac{d\omega}{dt} &= -\frac{\cos i}{nab\sin i} \frac{\partial \omega}{\partial i} + \frac{b}{nab\sin i} \frac{\partial R}{\partial \omega} \\
  \frac{dM}{dt} &= n - \frac{2}{na} \frac{\partial R}{\partial a} - \frac{nab^2}{na^4} \frac{\partial R}{\partial e} 
  \end{align*}
\]

- Filtering out the short period effects
  \[\bar{F}(\alpha) = \frac{1}{T} \int_{t_0}^{t_0+T} F(\alpha)dt = \frac{1}{2\pi} \int_0^{2\pi} F(\alpha)dM.\]

- Possible reference frame choices:
  - Equatorial
  - Lunar plane
  - Ecliptic

- Single average, over period of s/c
  - \(J_2\) perturbation
  \[\bar{R}_{J_2} = \frac{\mu J_2 R_0^2}{4a^3\eta^3} (2 - 3\sin^2 i).\]
  - Third-body attraction
  \[\bar{R}_{3b} = \sum_{l \geq 2} \frac{\mu_3}{r_3^l} \left( \frac{a}{r_3} \right)^l F_l(A, B, e).\]

- Double average, over period of the third body
  \[\bar{\bar{R}}_{3b} = \sum_{l \geq 2} \frac{\mu_3 a^l}{\eta_3^l} \sum_k (F_{l,k} \cos k\omega_3 + G_{l,k} \sin k\omega_3).

- Total disturbing potential
  \[\bar{R} = \bar{R}_{J_2} + \bar{R}_{\text{Moon}} + \bar{R}_{\text{Sun}}\]

- Single-averaged
  \[\bar{R} = \bar{R}_{J_2} + \bar{R}_{\text{Moon}} + \bar{R}_{\text{Sun}}\]

- Double-averaged
  \[\bar{R} = \bar{R}_{J_2} + \bar{R}_{\text{Moon}} + \bar{R}_{\text{Sun}}\]
Modelling of the long-term evolution of s/c

- Model for lunar and solar ephemerides

- Sun: circular Earth’s orbit

- Moon:
  - Vallado, 2013
  - Simplified model used in double-averaged dynamics
    - Constant semimajor, eccentricity, inclination referred to the ecliptic
    - Linear regression of the lunar node
    - Linear advance of the lunar perigee
Modelling of the long-term evolution of s/c

- Validation of the model
- INTEGRAL
  - [87736 km, 0.824, 53 deg, 102 deg, 302 deg, 83 deg]
  - From 13-Nov-2002, propagate for 50 years
  - Including J2, and lunisolar perturbations

- Gauss and Cartesian
  - Exact force model for third body attraction

- Lagrange, single-averaged and double-averaged
  - Truncated up to 4th order of the Legendre form
Phase space structure of orbital elements

- Double-averaged elements referred to Moon plane

- Constructing the phase space maps
  - Propagate the orbital elements using double averaged equations
  - Transfer to the orbital elements referred to the equatorial plane to those referred to the Moon plane
  - Construct the $e - \omega - i$ maps
Phase space structure of orbital elements

- $e - \omega - i$ map, double-averaged elements referred to Moon plane

- Lunar perturbation only
  - Layer structure for different $(e, i)$ pairs
  - Corresponding to Kozai parameter, $\Theta = (1 - e^2) \cos^2 i$
    - integral corresponding to the $z$-component of the angular momentum
  - Smaller $\Theta$, libration regime
    - Large variations for $e, \omega, i$
    - Libration centre at $\omega = 90$ deg
  - $\Theta$ increases
    - Libration centre move towards lower eccentricity and lower inclination
  - Higher $\Theta$, rotation regime
    - Almost no libration centre
    - Rotating apsidal line
  - Variation of $a, \omega$ only cause movement within the same layer of the phase portrait

- Kozai Y. “Secular perturbations of asteroids with high inclination and eccentricity”, 1962
Phase space structure of orbital elements

- $e - \omega - i$ map, double-averaged elements referred to the Moon plane

- Lunisolar perturbations
  - Slightly deviate from the case with only lunar perturbation
  - Libration regime
    - Similar to lunar case
    - Periodic
  - Rotation regime
    - Quasi-periodic
  - Due to small inclination of the lunar orbit with respect to the ecliptic, $i_{\text{Moon}} = 5.145$ deg

- $J_2$ and lunisolar perturbations
  - Deviate from the lunisolar case
Post-mission disposal

Possible re-entry solution aided by natural perturbations

- Post-mission disposal for s/c in HEO
  - INTEGRAL
  - Targeting a re-entry

Phase space structure provide possible solutions for re-entry.

- Give an initial impulsive manoeuvre
- \( \textit{kep} = \textit{kep}_0 + \Delta \textit{kep} \)
- \( \Delta \textit{kep} = \text{Gauss’ equations}(\textit{kep}_0, \Delta \textit{v}) \)
- \( \Delta \textit{v} = |\Delta \textit{v}| \begin{bmatrix} \cos \alpha \cos \beta \\ \sin \alpha \cos \beta \\ \sin \beta \end{bmatrix} \), in TNH frame
- Propagate for 50 years using double averaged dynamics
- Maximal eccentricity

Criterion for re-entry

- Perigee height \( h_p \leq 100 \text{ km} \)
- Critical eccentricity \( e_{\text{crit}} = 1 - \frac{R_\oplus + h_p}{a} \)
- \( e_{\text{max}} \geq e_{\text{crit}} \), possible to re-enter

- Colombo, C., End-of-life re-entry for highly elliptical orbits: the INTEGRAL mission, 2014
- Scala F., Analytical design of end-of-life disposal manoeuvre for Highly Elliptical Orbits under the influence of the third body’s attraction and planet’s oblateness”, 2018
Post-mission disposal

- Possible re-entry solution aided by natural perturbations
- Grid search for possible manoeuvre for INTEGRAL’s re-entry
- Initial Keplerian elements of INTEGRAL
  - [81120 km, 0.8977, 55.8956 deg, 109.4648 deg, 293.0959 deg, 0 deg]
  - Date = [2020 12 8]
- 1st search
  - $||\Delta v|| = 100\sim150$ m/s
  - $\alpha = 0\sim180$ deg, $\beta = 0\sim90$ deg
  - Possible solutions lay in the cone with axis T
  - Either increase or decrease the velocity along the tangential axis
- 2nd search
  - $||\Delta v|| = 110$ m/s
  - $\alpha = 0\sim30$ 150~180 deg, $\beta = 0\sim30$ deg
Post-mission disposal

- Possible re-entry solution aided by natural perturbations

- Summary of the results
  - successful re-entry manoeuvres lie in the cone of the tangential direction
  - most effective: T or -T direction

- In the phase space
  - Get the lowest perigee height
  - Increase the maximal eccentricity within 50 years
  - Move the evolution of elements to another layer of the phase portrait, corresponding to different values of the Kozai parameter
    - Change inclination, hard and consuming more fuel
    - Change eccentricity, feasible, along the tangential direction

- Possible re-entry solution aided by natural perturbations

  - Colombo, C., End-of-life re-entry for highly elliptical orbits: the INTEGRAL mission, 2014
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\[ \Delta v = 100 \text{ m/s}, \beta = 0 \text{ deg} \]
Conclusion

• Implemented the averaged dynamics for s/c under $J_2$ and lunisolar perturbations
• Constructed and analysed the phase space structure of Keplerian elements
  ‒ Especially $e - \omega - i$ maps
• Post-mission disposal analysis targeting an atmospheric re-entry
• Serve as preliminary work for future optimisations

Future work

• Optimisation of the enhancing manoeuvre for the post-mission disposal
• Analytical design of the post-mission disposal using the phase space structure
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