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Optimizing the return window for online fashion retailers with closed-loop refurbishment

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Abstract

The strategic positioning of online fashion retailers is defined, in part, by how they handle the complex task of managing returns. Although customers demand lenient policies such as free and late returns, tight control of returned items is mandated by the high costs of re-transportation and product value erosion. We model closed-loop fashion supply chains in order to describe, analyze, and optimize the performance of both forward and backward networks, including a secondary market. The model is based on a queueing system that combines the effect of new products entering the network for the first time with returned products entering the network for the second or nth time. We derive a closed-form expression for optimal service rates at both the test and refurbishment facilities and for the optimal return window. We then analyze the economic effects—on supply chain profit—of return rate, multiple looping of the same item in the supply chain, returned items being sold in a secondary market, and controlled delays in the refurbishment cycle. We apply our model to data obtained from a German fashion online retailer and report the managerial insights derived from this approach. Our results indicate how a company can set its return window strategically so as to maximize her profit as well as how it can decide whether to refurbish merchandise or sell it in the secondary market. In addition, we describe how refurbishment activities can sometimes lead to greater benefits even though the secondary market is usually an attractive opportunity for product returns.

Keywords: return window, customer sojourn time, product aging, refurbishment, online fashion retailers, closed-loop supply chains, returns management, secondary markets

1 Introduction

Recent decades have seen increasing attention given to reverse logistics and closed-loop supply chains—in large part because of stricter environmental legislation that aims to blunt the effects of diminishing resources and growing population. Researchers have started to focus on the problems of efficiently and effectively designing supply chains in this new context, and then extracting value from them (Blackburn et al. 2004, Guide et al. 2006). Especially since the explosive growth of e-business, the closed-loop supply chain (CLSC) and in particular returns management have drawn considerable interest from both research and industry (Fraunhofer 2011, ibi research 2013). More and more e-tailers, i.e., retailers selling online, face the issue of returns. For an online business—as with a traditional brick-and-mortar store—the likelihood of sold items being returned is much greater because customers cannot see and try out the items before buying them (ibi research 2013). In order to attract customers, an online retailer must be sensitive to their needs; with respect to return policies, customers demand that purchased products be returnable both free of charge and as late as possible (PriceWaterhouseCooper 2013). Hence the online retailer encounters higher return rates and related costs, e.g., transportation, testing, refurbishment, and value erosion. The key challenge for e-tailers is therefore to find the optimal trade-off between market pressure, i.e., customers demanding lenient return policies and profit pressure, i.e., the high cost of returns management.

In our paper, we develop a model for closed-loop fashion supply chain and cross-check our analytic results with data from Zalando, Europe's largest fashion online retailer. A recent study conducted by A.T. Kearney (2013) describes the rapidly expanding nature of German online business generally and discusses in particular the explosive growth of German online retailer Zalando. Zalando's sales grew in five years faster than either Amazon or Walmart (*Financial Times* 2012).

Zalando' s fast growth and exceptional returns rate

Zalando was founded in Berlin in October 2008 by three former students of the WHU–Otto Beisheim School of Management (Germany). In 2014 the company went public; its present market capitalization is some €5.54 billion, and its stock price continues to climb. Much like the US online retailer Zappos.com, Zalando initially specialized in shoes for its own country's market but soon (within a few years) also sold clothes, accessories, and beauty products while expanding into 15 other European countries. Although Zalando's year-to-year revenue growth has been extraordinary (from €150 million in 2010 to more than €3.6 billion in 2016, with almost 70 million orders in 2016), the company has only recently reported a profit (Zalando 2017). Besides the high initial investments required of Zalando, its losses are due mainly to the high rate of returns (more than 50% in the German market and approx. 40% in the overall market). Considering that the estimated cost of a return is approx. €10 (industry benchmark; EHI Retail Institute 2015), we can easily get an idea of how costly it is for Zalando to manage product returns. Yet by 2012 Zalando was profitable in German-speaking countries and, in the other European countries broke even for the first time in 2014 (Zalando 2017). The authors were given a tour of the company by its Senior Vice President (SVP) Operations and visited the Zalando warehouse in Berlin, the logistics center in Erfurt (where also the refurbishment activities take place) and Mönchengladbach as well as the outlet store in Berlin-Kreuzberg. Zalando adopts a fairly liberal return policy, offering customers free returns for up to 30 days in most European markets (100 days in Germany only). In Figure 1 we represent Zalando's positioning in terms of return policy as compared to some other major online retailers. As a consequence of its liberal policy, Zalando attracts a large number of customers but must also face a substantial amount of returns (and related costs)—which often come back after a long customer sojourn time.

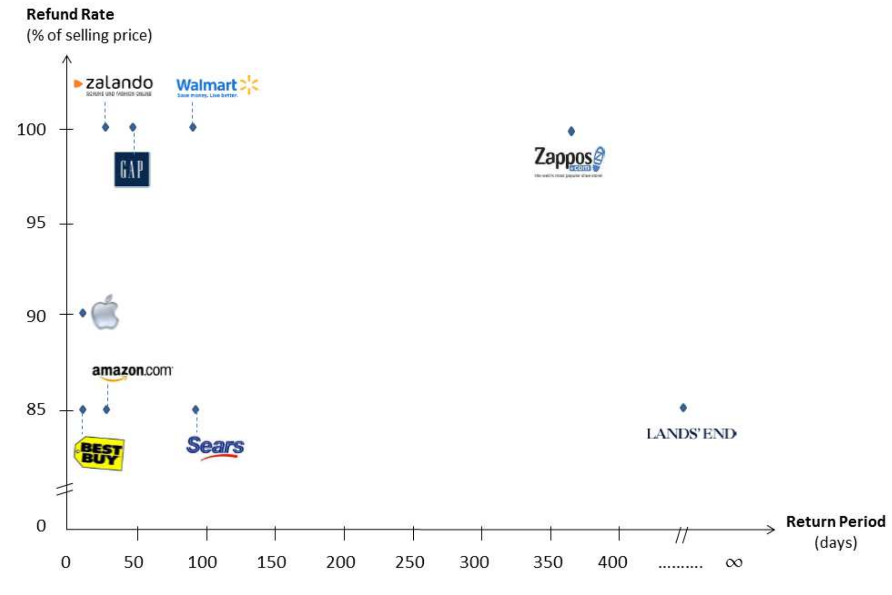

Figure 1: Return periods and refund rates[1] for some major companies

Delayed returns go hand in hand with product value erosion; especially for products with a short life cycle, e.g., fashion products. Longer delays through the supply chain translate into increased erosion of product value and hence losses for the company (see Blackburn et al. 2004).

Supply chain design for online retailers

Companies can design their supply chains in order to establish better coordination among the

[1]Some retailers might charge a different restocking fee depending on the product category or the condition of the returned item. Therefore, the reader should take this variability into account when looking at Figure 1.

activities involved with returns management and to improve supply chain performances. One option (Figure 2) is to sell returned items to third parties (via the retailer) that will take charge of

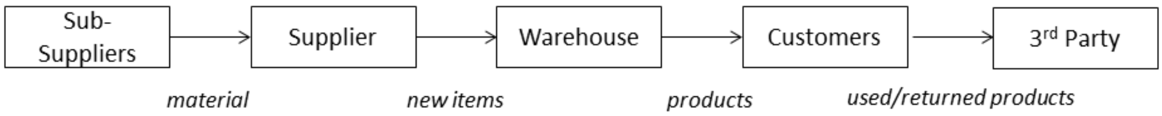

Figure 2: Open-loop supply chain—example design

any further action (recovery and/or reselling). This approach is the one taken by Amazon, which resells returned products to third parties that in turn deal with all returns management issues. Alternatively, companies could choose to manage themselves (either all or some of) the activities necessitated by returned items and plan their supply chain accordingly in order to integrate forward and backward flow. Figure 3 depicts the supply chain design for this second option. We can see

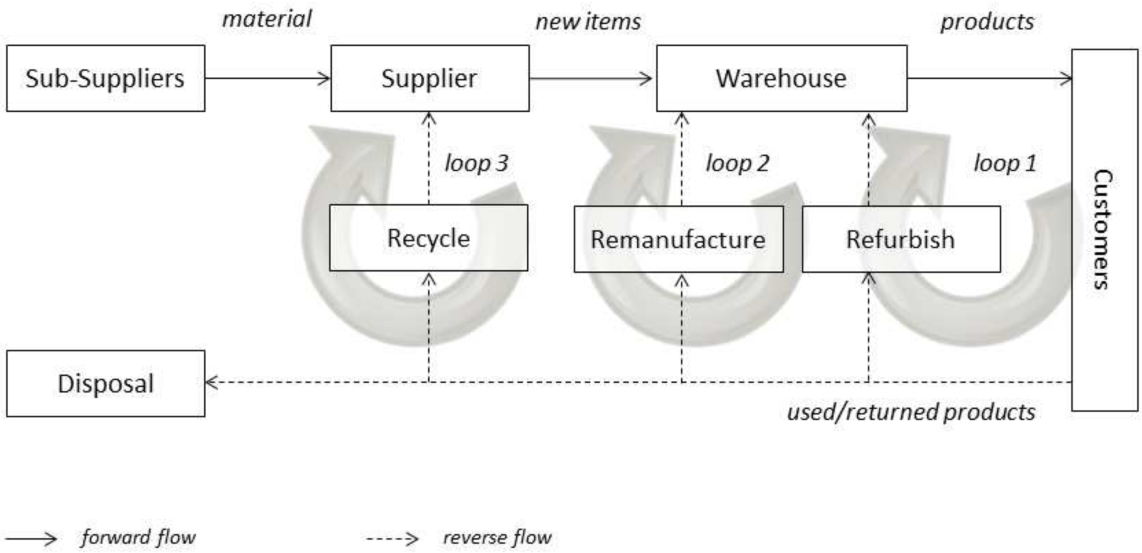

Figure 3: Closed-loop supply chain for fashion items

that, unlike the previous option, this design is not "linear" because the supply chain incorporates those activities (refurbishing, remanufacturing, and recycling) that "close the loop". Thus recovery activities initiate three sub loops within the closed-loop supply chain: returns can be re-introduced into the primary market after refurbishment (loop 1), they can be remanufactured and then re-introduced into the primary market (loop 2), or they can be recycled and re-introduced into the primary market in the form of new raw material (loop 3). In this paper, we follow the design described in Figure 3 and model the network as a closed-loop supply chain. We observe in practice that the same product can be sold and returned several times, so we model the scenario of a closed-loop supply chain in which the same item can be returned and re-introduced into the primary market multiple times. After a maximum number of iterations, the product leaves the supply chain via either sale in the secondary market or outright disposal. In this context, we develop a model for CLSCs that incorporates the effects of both product age[2] and operational delays through the chain and also due to customer sojourn time. Our paper aims to extend the existing literature that models CLSCs with a secondary market by allowing items to be re-introduced into the market several times and by analyzing the effect, on a company's overall profit, of allowed return window to send the item back, the resale of refurbished items in a secondary market, and controlled delays in the CLSC. In particular, we identify the conditions under which it is possible to influence the customer's delay as well as the maximum number of times companies should allow returns to re-enter the forward chain.

The work reported in this paper is motivated by the limited extent to which current literature reflects the real-life practice of online business. We refer in particular to the model presented by Guide et al. (2006); these authors emphasize the relevance of delays through the CLSC and propose strategies to reduce them and thereby increase competitive advantage. Guide et al. mainly consider problems related to the electronic/power tool industry. An online fashion business naturally has different characteristics and presents different challenges, which—together with the empirical evidence from Zalando—was an additional motivation for our study. Guide et al. (2006) suggest that retailers employ and propose a so-called preponement strategy (see Blackburn et al. 2004) in order to extract more value from returns. Yet in the fashion online industry there is no physical retailer to which items can be physically returned, which precludes the preponement solution. So in many cases, online retailers must implement different solutions in order to manage their returns. The main limitation of Guide et al.'s model is that it ignores the strategic function of the delay at the customer, which can have a significant effect on supply chain performance.

Another limitation of the literature is that it usually views a given item as being able to cycle through the forward chain only once. However, that scenario does not describe fashion retailers, where the same product can loop through the CLSC multiple times and thus lead to even greater erosion of product value. Our model therefore applies to the "multiple looping" scenario. We remark that our research is especially applicable to products (such as fashion retailing) with a short life cycle because such products, more than others, rapidly lose their value, and to products that require "low touch" refurbishment, i.e., polishing, steam ironing and repackaging (this justifies the repeated looping of returns that Zalando and other fashion retailers adopt).

[2]With the term "age" we refer to the average number of times the same item loops in the supply chain, i.e., how many times an item is returned and, after possible refurbishment, re-introduced into the primary market. We note that this corresponds to the minimum quality level defined by the retailer.

The principal aim of our research is to answer four questions: How can online retailers strategically set the return window? What is the impact—on supply chain performace—of the amount of time that items spend at the customers before being returned? Should online retailers engage in refurbishment activities? Does a secondary market pose a threat or represent an opportunity? In this paper we shall employ without distinction the terms "remanufacture" and "refurbish"; although they are actually two different processes, our interest is in representing the general activity of bringing a used product into an "as new" condition. In other words, our model can easily be applied to both cases.

The paper is structured as follows. In Section 2 we review the relevant literature. In Section 3 we present the analytical model and in Section 4 we conduct a sensitivity analysis on the key parameters. We conclude in Section 5.

2 Literature review

In this section, we describe and classify five of the most relevant streams in the literature of closed-loop supply chains: *product quality*, which in this context involves the processing of returns according to their residual quality; *return policies*, in particular the effect of charging customers a restocking fee; *the role of secondary markets*, or the interaction effects between primary and secondary market for an online retailer; *the time value of returns*, which addresses the economic value of time-sensitive products and *industry studies* on returns management.

Product quality

Souza et al. (2002) present a model in which returned items are grouped into different classes according to their residual quality and then, depending on this classification, are either remanufactured or sold "as is" at a lower price. In this way, companies sell to different market segments and provide different product quality at different prices. Modeling the various quality classes as a GI/G/1 queue network, Souza et al. determine the profit-maximizing proportion of returns to be remanufactured. Their model also introduces a service-level constraint, which is defined as the maximum throughput time that items can spend in the reverse chain. Galbreth and Blackburn (2006) present the problem of determining how many unsorted returned products a remanufacturer should buy from a third party and also the sorting policy for those items, where one purpose of sorting is to identify which returns should be remanufactured and which should be scrapped. Guide et al. (2008) define—as a function of the required remanufacturing time—the conditions under which it is preferable to remanufacture returns (and then sell them in the secondary market) rather than selling them "as is" (at a salvage value). In their model, remanufacturing costs are indeed increasing with remanufacturing time, and the price on the secondary market is time sensitive (i.e., the secondary-market price decreases with remanufacturing time). Thus an increase in remanufacturing delays will increase the benefit of selling the product "as is".

Ferguson et al. (2009) develop a production planning problem in order to find the optimal returns mix and amount of inventory to carry for future periods when different quality levels of returned items are available. Tao et al. (2012) present a model that determines the optimal order/remanufacturing policy: one that minimizes the total expected discounted costs over a finite horizon when different quality levels of returned items are available. Differently from the current literature, in our paper the main driver for returns classification is product age. So up to cycle count $N-1$, returns are re-introduced into the primary market after prior refurbishment whereas at cycle count N, they are sold in the secondary market or discarded.

Returns policies

A second stream of literature deals with returns policies and, in particular, with the role of restocking fees in returns management. We observe that customers are typically sensitive to returns policies and so a lenient one (in terms of restocking fee, time interval permitted for returns, store credit versus money back, additional documentation required for returning, etc.) should increase customer demand (see e.g. Ketzenberg and Zuidwijk 2009, PriceWaterhouseCooper 2013, Janakiraman et al. 2016). It would therefore seem reasonable to adopt more lenient return policies in order to attract more customer segments. However, recent studies (Shulman et al. 2010, *Bloomberg Businessweek* 2013, Ruiz-Benítez et al. 2014, EHI Retail Institute 2014) have shown that some customers tend to abuse return policies—for example, those who buy digital cameras or dresses for special occasions like weddings and then return them right after usage. In addition, a lenient return policy also tends to increase the return rate.

In order to reduce returns management costs, firms discourage customers from abusing return policies and reduce the returns rate by allocating some return costs to the buyer. For instance, the US retailer Target charges a restocking fee of 15% for certain portable electronics and does not cover the cost of shipping the returned item unless its deficiency was Target's fault (Shahar and Posner 2011). Several papers emphasize how introducing a restocking fee reduces retailer costs, reduces the quantity of returns, and discourages customers from behaving opportunistically (see Shulman et al. 2009, Su 2009, Shulman et al. 2011). However, some countries have enacted legislation that does not allow sellers to charge restocking fees. This restriction has long been characteristic of European countries, in which (until June 2014) online retailers could not charge a restocking fee for products worth more than €40. After the recent enactment of legislation allowing EU retailers to charge a restocking fee, the percentage of online retailers offering free returns declined sharply (in Germany, it dropped from about 66% to about 42%; EHI Retail Institute 2014).

There have been studies (e.g., Petersen and Kumar 2009, Bower and Maxham 2012) that demonstrate the importance of lenient policies. Bower and Maxham show how customers who receive a free return significantly increase their post-return buying level (from 158% to 457%); conversely customers who must pay a return fee significantly decrease their post-return buying level (minimum -75% all the way to -100%). A less drastic alternative to the introduction of a restocking fee is to increase the "hassle" costs for customers who wish to return products. A return process that is more complex (e.g., requiring customers to fill out a detailed form explaining why the item was returned) will discourage some customers from making a return. In this way, companies can still claim to have a free return policy and can benefit (as described before) yet simultaneously discourage product returns. Another measure that online retailers have begun to adopt is the exclusion of customers with high return rates; in some cases, this policy eliminated more than two thirds of the firm's customers (*Lebensmittel Zeitung* 2014). Altung (2012) analyzes a dynamic return policy, in which the return policy is a decision variable that can change every period and can affect demand, product returns and the cost of return. In their paper, Hsiao and Chen (2012) analyze customers who are heterogeneous both to the product evaluation and to their costs of returning an item. The manufacturer' s goal is to find the optimal pricing and return policy strategy, choosing whether to try to accommodate all customers or to focus only on one segment. Akcay et al. (2013) analyze the benefit for a retailer to restock returned products and resell them as open-box items at a discount. Ülkü et al. (2013) determine the optimal price and return period to be settled by a retailer and the impact of such variables on customer' s valuation function. The model also includes the possibility of fraudulent customers. Moreover, the results show that, in case products depreciate quickly, it may not be profitable to operate with a return policy. Altung and Aydinliyim (2016) focus on the return policy decision in presence of strategic consumers who defer their purchases until the clearance period, in order to buy at a discounted price.

In our model, we formulate customer demand as function of the returns policy; in particular, we include the effect of the return window in terms of the maximum number of days allowed for returning an item. We also investigate how the introduction of a restocking fee would affect our results.

Role of secondary markets

A third stream of related literature addresses the interaction between primary and secondary markets and the role played by returned goods in contracting between manufacturer and retailer. Petruzzi and Monahan (2003) identify when it is preferable for a retailer to terminate sales in the primary market and unload the remaining product in the secondary market. Some papers employ the notion of multiple secondary markets: Mitra (2007) proposes a model in which returns are sold in different secondary markets according to their residual quality; and Yin et al. (2010) distinguish different secondary markets according to the seller's identity (i.e., retailer versus peer to peer). Shulman and Coughlan (2007), Oraiopoulos et al. (2012), and Gümüs et al. (2013) focus on optimal contracting between manufacturer and retailer in the presence of product returns and highlight that even the manufacturer benefits from a secondary market. Gümüs et al. (2013) analyze in particular how used goods markets affect the incentive of manufacturers to offer the retailer a returns contract, and Oraiopoulos et al. (2012) the phenomena of cannibalization effects. Here we consider the presence of a secondary market that allows the retailer (but *not* the manufacturer) to reach a larger customer population. We ignore the role of returned products in contracting between manufacturer and retailer.

Time value of returns

The models proposed by Blackburn et al. (2004), Guide et al. (2005), and Guide et al. (2006) are innovative because they reveal that speed is of special relevance in the recovery of some product—such as "fast clockspeed" items (electronic, cosmetic, fashion-oriented)—because they rapidly lose their value. These authors concentrate on how to extract value from closed-loop supply chains while focusing on the potential value of time-sensitive returns. So far, few studies have considered the economic value of time in CLSCs. Blackburn et al. (2004) highlight the business relevance of time value for product returns and present alternative reverse supply chain designs that reflect the different needs of companies; they also propose some strategies for improving network responsiveness. Guide et al. (2005) focus on the returns of notebook and desktop personal computers. These authors estimate the value erosion of returns (as they await warehousing, testing, and remanufacturing) and develop an optimization model for maximizing the firm's overall profit and reducing lead times for its refurbishment activities (and thereby reducing returns-related value erosion).

Guide et al. (2006) consider the problem of designing the reverse supply chain so as to maximize net asset value recovered from the flow of returned products. In their paper, exponential price and variable cost decay functions are assumed for both new and remanufactured products. This means that, upon the occurrence of a delay (e.g., between products being returned and being evaluated and/or resold), the firm's overall profit declines. Minimizing the processing rate (and thus the waiting times) at each facility enables the firm to maximize its profit. Our paper extends the current literature on time value of returns in the following sense: in addition to modeling the delay at testing and refurbishment facilities, it incorporates product sojourn time with the customer—yet another source of delay.

Industry studies

Some recent empirical studies examine the extent and relevance of additional costs deriving from returns management activities. An Accenture (2011) report analyzes electronics returns in the US market and the cost to manufacturers and retailers of coping with returns management activities. That study also identifies a significant opportunity for the industry to cut costs and reduce the level of product returns. Fraunhofer (2011), based on surveys of different business sectors (including automotive, electronic, pharmaceutical, and fashion goods) in the German market, reveals that the processing time for returns is slow and that products often sojourn at customers for a long time. With regard to the fashion sector, the study shows that more than one quarter of the surveyed companies take goods back even two years after their delivery.

In "Recovering Lost Profit by Improving Reverse Logistics", UPS (2012, esp. pp. 5, 8) shows how product returns have a direct influence on the supply chain's bottom line and how recovery activities represent a challenge for companies that extends beyond their associated costs. According to the returns management study conducted by ibi research (2013) on more than 350 German online retailers, 70% of a typical return's total cost is due to inspection, repackaging, and restocking activities while only 30% is due to shipping. However, some 40% of online retailers are unable to estimate these cost breakdowns for their own company. These circumstances argue for the relevance of identifying and then appropriately managing return costs. Our model, which is validated with real-world data from Zalando, offers new insights into the online business market and especially with reference to fashion e-tailers.

3 Closed-loop fashion supply chain with multiple returns

We first present our system in a general setting, and then introduce our notation and assumptions to formulate the profit model for a fashion supply chain.

The retailer buys new items from the supplier, stores them in the warehouse and delivers them to customers. Some products can be returned by customers: product returns are processed and then they are either reintroduced into the primary market or sold into a secondary market/disposed of. Figure 4 represents the product flow in the general setting. Products flow from node to node as they are processed, according to a first-in-first-out basis. More in particular, returns arriving

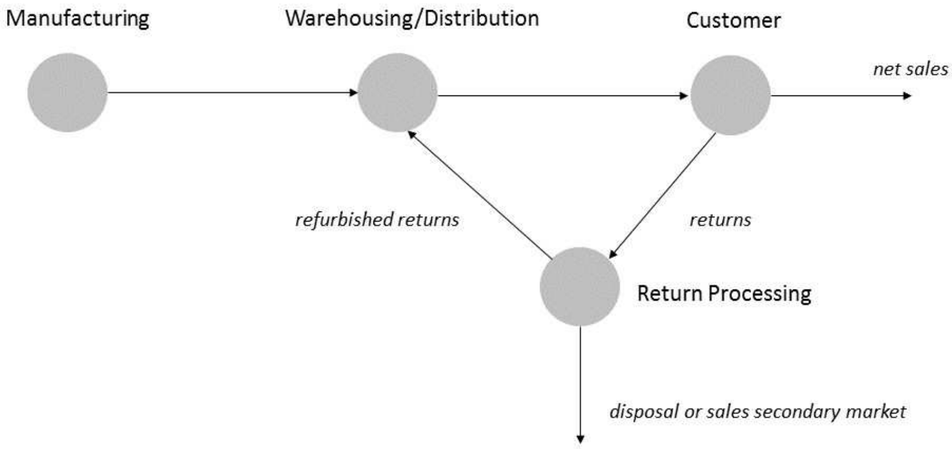

Figure 4: Product flows in the general setting

randomly at the processing center require a service, i.e., sorting and refurbishment. For reasons of tractability, following Guide et al. (2006), we model the delay at the processing facility with the expected flow time through an M/M/1 queue and therefore approximate the network flow time with the expected value. As we will better discuss later, our model applies to the steady-state phase of the return life cycle because of the high volume of products and the processing and resales of returns which happens during the steady-state phase.

3.1 Model formulation

In this section, we formulate the profit model for a fashion supply chain in steady state. In each period, the retailer buys x new products from the supplier at a unit price p_a and sells them in the primary market at a unit price $p[t]$. A fraction $1-\alpha$ of all units sold is returned (after a delay W_c spent at the customer). The retailer reimburses the customers (issuing a credit of $p[t]$), and returns are collected and shipped to the sorting and testing center. All the items that have been returned for less than N times are refurbished and then resupplied to the primary market, while all the items that have been returned for N times exit the primary market. Thus to meet demand in each period:

$$x\left(1+(1-\alpha)+...+(1-\alpha)^{N-1}\right)=D, \tag{1}$$

i.e., $x=\dfrac{\alpha}{1-(1-\alpha)^N}D$. In each period, the number of units that have been returned N times is $(1-\alpha)^N x$. Of these returned units, a fraction $\beta^{'}$ is sold to the secondary market (at a price of $p_2[t]$) and a fraction $1-\beta^{'}$ is disposed of (at a unit cost k_d). Thus the amounts sold to the secondary market and disposed of are, respectively,

$$\begin{aligned}\beta'(1-\alpha)^N x =&\beta'(1-\alpha)^N \frac{\alpha}{1-(1-\alpha)^N} D \quad \text{and} \\ (1-\beta')(1-\alpha)^N x =&(1-\beta')(1-\alpha)^N \frac{\alpha}{1-(1-\alpha)^N} D.\end{aligned} \tag{2}$$

We now define β_N and γ_N as:

$$\begin{aligned}\beta_N =&\beta'(1-\alpha)^N \frac{\alpha}{1-(1-\alpha)^N} \quad \text{and} \\ \gamma_N =&(1-\beta')(1-\alpha)^N \frac{\alpha}{1-(1-\alpha)^N}.\end{aligned} \tag{3}$$

Given the definition of x, β_N and γ_N, it follows:

$$\begin{aligned}(\alpha+\beta_N+\gamma_N)D &= \left(\alpha+\beta'(1-\alpha)^N \frac{\alpha}{1-(1-\alpha)^N} + (1-\beta')(1-\alpha)^N \frac{\alpha}{1-(1-\alpha)^N}\right) D \\ &= \left(\alpha+(1-\alpha)^N \frac{\alpha}{1-(1-\alpha)^N}\right) D = \frac{\alpha}{1-(1-\alpha)^N} D = x,\end{aligned} \tag{4}$$

i.e., $x = (\alpha+\beta_N+\gamma_N)D$.

In a similar way, it results $x\left((1-\alpha)+...+(1-\alpha)^{N-1}\right) = (1-\alpha-\beta_N-\gamma_N)D$.[3] We provide an overview of the notation used in Table 1 and represent the supply chain for our model in Figure 5. We choose to model multiple loops because of empirical evidence concerning online business, where most retailers face returns not only on a daily basis but also at a much higher rate (as compared with sales) than do traditional retailers. In particular, we observe that a given item can be returned and re-introduced into the forward supply chain several times after possible refurbishment activities. So notwithstanding the consequences of delays due to refurbishing operations and customer sojourn time, supply chain profit could be affected also by the age of returns. Our model tracks the aging of returns by counting how many times each item re-enters the loop. We observe that—especially for an online business—it is essential to have an efficient information technology system for tracking products and managing supply chain activities; hence also tracking product age does not impose any meaningful extra cost.

[3]Using the definitions of β_N and γ_N given by equation (3), it follows:

$$(1-\alpha-\beta_N-\gamma_N)D = \left(1-\alpha-\beta'(1-\alpha)^N \frac{\alpha}{1-(1-\alpha)^N} - (1-\beta')(1-\alpha)^N \frac{\alpha}{1-(1-\alpha)^N}\right) D = \frac{1-\alpha-(1-\alpha)^N}{1-(1-\alpha)^N} D.$$

From the definition of x, recalling that $\sum_{k=1}^{N-1}(1-\alpha)^k = \frac{1-(1-\alpha)^N}{\alpha} - 1$, it results $x\left((1-\alpha)+...+(1-\alpha)^{N-1}\right) = (1-\alpha-\beta_N-\gamma_N)D$.

Table 1: Notation

	Definition
Decision variable	
μ_i	Service rate at facility i
$W_{c_{\max}}$	Return window (defined by the return policy)
N	Maximum number of times the same item can be returned before being either disposed or sold in the secondary market
Parameter	
i,j	Subscripts for nodes: s, supplier; w, online retailer warehouse; c, customers; f, sorting center; r, refurbishment/remanufacturing center; 2, secondary market; d, disposal
D	Product demand in the primary market
α	Net sales rate in the primary market
β_N	Proportion of items sold in the secondary market
γ_N	Proportion of items disposed of (destroyed)
λ_{ij}	Product flow between node i and node j
k_d	Unit cost for disposal
c_{ij}	Unit transportation cost between node i and node j
$p[t]$	Unit selling price in the primary market at time t
$p[0]$	Unit selling price in the primary market at time 0
$p_2[t]$	Selling price function in the secondary market at time t
p_a	Purchasing price of one new product from the supplier
p_s	Salvage price
g_i	Cost function at facility i
h	Unit holding cost
W_i	Delay between the beginning and the end of processing at node i
W_{loop}	Delay through the loop
τ_{ij}	Transportation time between node i and node j
δ_2	Discount rate of price in the secondary market
ε	Elasticity of secondary market demand to secondary market price
W_c	Sojourn time at the customer
e_1	Sensitivity of the primary market demand to $W_{c_{\max}}$
e_2	Sensitivity of the customer sojourn time to $W_{c_{\max}}$
$a > 0$	Continuous-time price decay in the primary market
$b > 0$	Continuous-time discount rate in the selling season
$n = 1, \ldots, N$	Number of times the same item can be returned
π	Steady-state profit
Π	Steady-state discounted profit

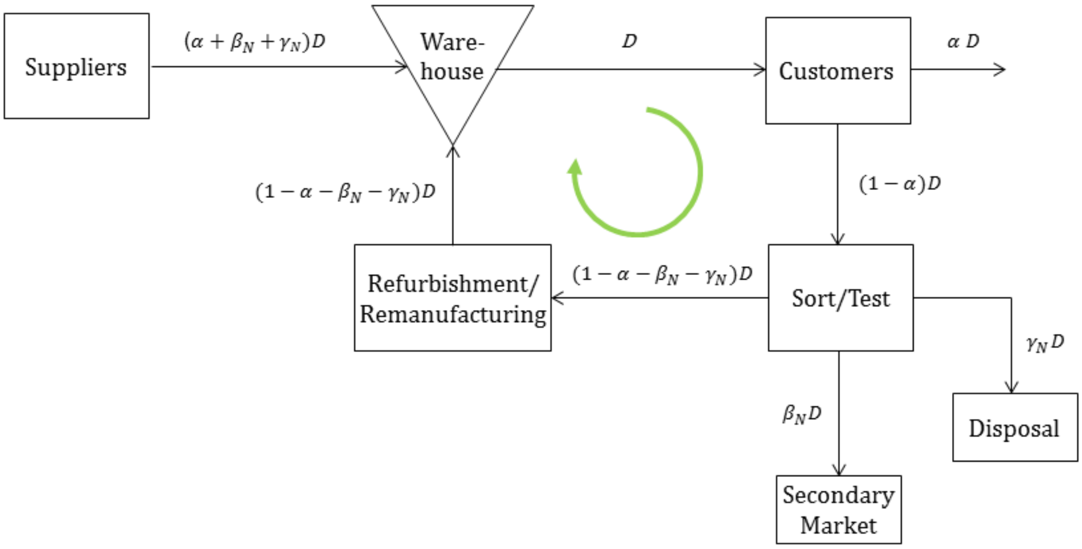

Figure 5: Stylized closed-loop supply chain model for an online fashion retailer in steady state

3.2 Life cycle of returns

Figure 6 plots the three phases of the returns life cycle, where we have adopted the following notation (refer to Table 1 for subscripts): $T_1 = \tau_{wc} + W_c$; $T_2 = T_1 + \tau_{cf}$; $T_3 = T_2 + W_f$; $T_4 = T_3 + \tau_{fr}$; $T_5 = T_4 + W_r$; $T_6 = T_5 + \tau_{rw}$; $T_7 = T_6 + (N-1)(\tau_{wc} + W_c + W_{\text{loop}}) = N(\tau_{wc} + W_c + W_{\text{loop}})$; $T_8 = T + \tau_{wc} + W_c$ and $W_{\text{loop}} = \tau_{cf} + W_f + \tau_{fr} + W_r + \tau_{rw}$. We need to distinguish between T_1 through T_8 so that we can fully model the ramp-up phase of the CLSC. At the start of the selling season (phase-in), companies face no returns because products, after being sold, sojourn for a certain time with the customers. Fulfilling demand therefore requires that companies buy 100% new materials from the suppliers (0–T_1). Once returns commence, they are collected, inspected, refurbished, and re-introduced into the primary market (T_1–T_6) but with a delay due to inspection, refurbishment, and transportation activities. At this point, companies buy a lower quantity of new products from the suppliers because the difference can be made up by returned products. After cycling for further $N-1$ times (T_6–T_7), returned items are either disposed of or sold in a secondary market (T_7–T). Companies stop selling products when the season ends, yet customer delays entail some returns thereafter (T–T_8); these quantities are collected and salvaged without any additional processing. See Section 3.3 for more details.

We want to stress the importance of customer sojourn time on the returns life cycle. Figure 6 shows that if customers keep products for a longer time before returning them (i.e., if W_c increases), then the beginning of the steady-state phase is delayed and so its length is reduced (the end of the steady-state phase is fixed because it coincides with the end of the selling season), decreasing the

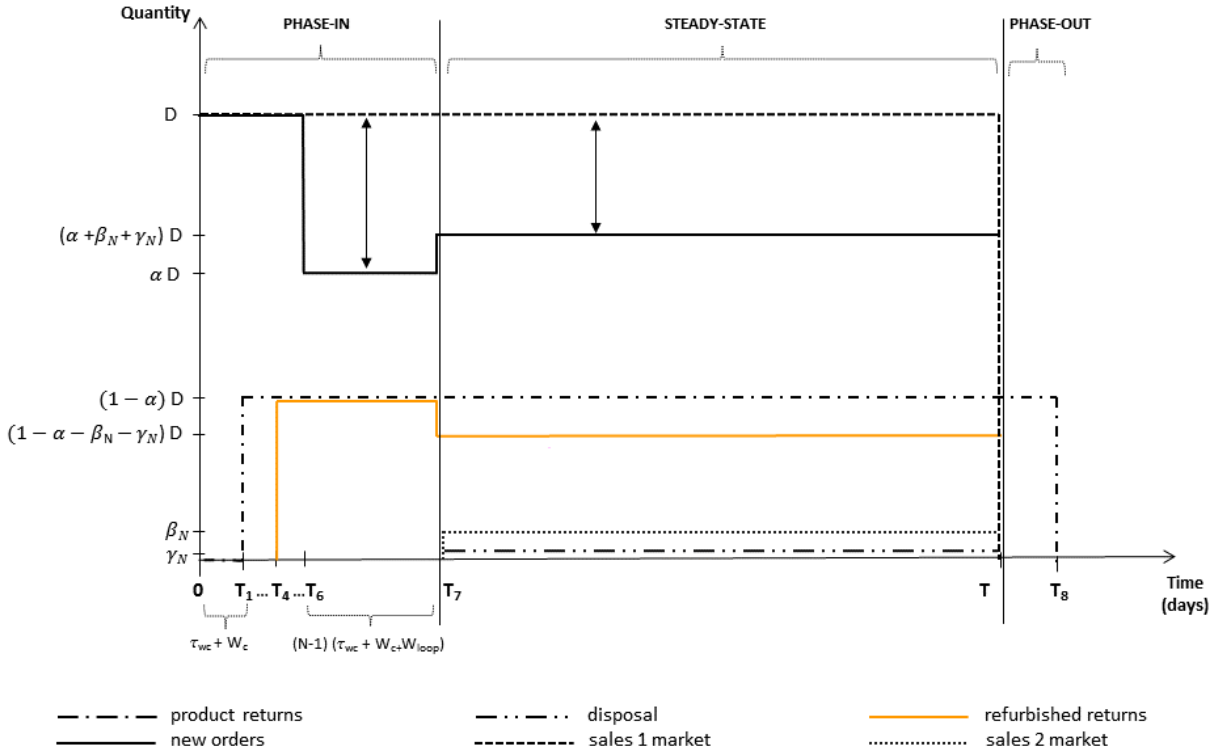

Figure 6: Life cycle of returns

significant savings in purchasing new items from the suppliers.

3.3 Assumptions and the profit function

Given the time-sensitive value of products such as fashion goods, we want our model to capture product value declines caused by delays in the supply chain. Similarly to Guide et al. (2006), we introduce an exponential decay price function: we define this price function in the primary market at time t as $p[t] = p[0]e^{-at}$, where the decay parameter a can be viewed as a measure of industry clockspeed. Before delineating the analytical model, we summarize the main assumptions as follows.

(A1) Customer demand in the primary market is independent of product returns.

(A2) Customer demand in the primary market is constant: $D = D_o + e_1 ln[W_{c_{\max}}]$ for D_o a nonnegative constant.

(A3) Product sojourn time at the customer is defined by: $W_c = W_o + e_2 ln[W_{c_{\max}}]$ for W_o a nonnegative constant.

(A4) Inverse demand function in the secondary market[4] is $p_2[t] = \delta_2 p[t] - \varepsilon\beta_N D$.

(A5) Supply lead time (i.e., from suppliers to retailer warehouse) is zero.

(A6) There are no capacity constraints at the sorting/testing and refurbishment centers.

(A7) Any product that has been returned N times will leave the primary market through secondary market sales or disposal.

(A8) Customers receive a full refund on returned items in the *primary* market.

(A9) A strict no-return policy is adopted in the *secondary* market.

(A10) Product waiting time at the retailer warehouse is zero (for both new and returned items).

Consistent with the empirical evidence, we assume that customer sojourn time is increasing with the number of days allowed by the return policy ($W_{c_{\max}}$): if customers are allowed to keep the item longer, then the resulting sojourn time should increase. However, as observed in practice (see also Figure 16) and confirmed by Zalando's SVP Operations, such effect becomes less significant for higher values of $W_{c_{\max}}$[5]. Therefore, we model such effect using a logarithmic curve. Similar considerations apply to customer demand in the primary market.

Under Assumptions (A1)–(A10), the time-t total expected supply chain profit during steady-state period (for the sake of completeness, we present in the Appendix A1 the supply chain profits relatively to the phase-in and phase-out) can be expressed as follows:

$$\begin{aligned}\pi[t] = {} & Dp[t] + \beta_N D(\delta_2 p[t] - \varepsilon\beta_N D) \\ & - (1-\alpha)Dp[t]e^{-b(\tau_{wc}+W_c+\tau_{cf})} - (\alpha+\beta_N+\gamma_N)Dp_a \\ & - (1-\alpha)Dg_f - (1-\alpha-\beta_N-\gamma_N)Dg_r \\ & - \beta_N Dg_2 \\ & - (1-\alpha-\beta_N-\gamma_N)DhW_{\text{loop}} - \gamma_N Dh(\tau_{cf}+W_f) \\ & - \beta_N Dh(\tau_{cf}+W_f+\tau_{f2}+W_2) \\ & - \gamma_N Dk_d - \sum_{(i,j)} \lambda_{ij}c_{ij}.\end{aligned} \tag{5}$$

The first term in this expression represents revenue from the sale of products in the primary market

[4]Zalando secondary market is comprised of outlet stores located in Berlin, Cologne and Frankfurt: it is common knowledge that customers buying in an outlet store are aware that they will get products at a lower price because of lower quality, limited variety, size availability, end of season products, etc. For these reasons, Zalando secondary market cannot compete with the primary market and the company simply chooses to clear the inventory in the outlet store. The price in the secondary market is depepndent on the price in the primary market, discounted by the rate δ_2.

[5]It means that, for example, if the retailer offers a 30 days rather than a 15 days return policy, the effect on customer sojourn time will be much more significant than the case when the retailer offers a 100 days rather than a 85 days return policy.

and in the secondary market, and the second term is the money that the company fully refunds to customers after returns —however discounted by the delay between the sales and the refund—and the purchasing cost for new items. The third term of (5) represents the sorting/testing cost and the refurbishment cost; the fourth term captures the secondary market cost (of running an outlet store). The fifth term represents the holding cost of returns through the loop and the holding cost of products that will be disposed of; while the sixth term is the holding cost of returns sold in the secondary market. Finally, the last term represents the disposal cost and the transportation cost between each pair of nodes. To derive a closed-form expression for the optimal service rates (at the sorting and refurbishment facilities) and the optimal return window, we calculate the expected discounted profit Π in $[0,T]$: $\Pi = \int_0^T \pi[t]e^{-bt}\ dt$. We can then collect expressions for the transportation times between node i and node j (τ_{ij}) and for the expected delays at the inspection center (W_f), refurbishment center (W_r), secondary market (W_2), and customer (W_c) to rewrite the profit function for the steady-state period as:

$$\begin{aligned}
\Pi = &- e^{-b(W_c+\tau_{cf}+\tau_{wc})}\frac{Dp[0]}{a+b}(1-\alpha)(1-e^{-(a+b)T}) \\
&- (\tau_{cf}+W_f)\frac{Dh}{b}(1-\alpha)(1-e^{-bT}) \\
&- (\tau_{fr}+W_r+\tau_{rw})\frac{Dh}{b}(1-\alpha-\beta_N-\gamma_N)(1-e^{-bT}) \\
&- (\tau_{f2}+W_2)\frac{Dh\beta_N}{b}(1-e^{-bT}) \\
&- \frac{p_a(\alpha+\gamma_N)+g_f(1-\alpha)+g_r(1-\alpha-\beta_N-\gamma_N)}{b}(1-e^{-bT})D \\
&- \frac{D\gamma_N k_d + Dp_a\beta_N + Dg_2\beta_N + D^2\varepsilon\beta_N^2 + \sum_{(i,j)}\lambda_{ij}c_{ij}}{b}(1-e^{-bT}) \\
&+ \frac{1+\beta_N\delta_2}{a+b}(1-e^{-(a+b)T})Dp[0].
\end{aligned} \tag{6}$$

It is evident from this expression that the delay at the testing center W_f, the delay at the refurbishment center W_r, the delay at the secondary market W_2, and the transportation delays τ_{fr}, τ_{rw}, and τ_{f2} have a negative effect on CLSC profit. As a result, an increase in one (or more) of these delays will reduce that overall profit. The transportation delay τ_{cf} could have both a positive effect (the benefit from offering credit to customers) and a negative effect (transportation delay through the loop) on profit depending on the values assumed by the parameters a, b, h, and $p[0]$. However, delay at the customer W_c and transportation delay τ_{wc} have a positive effect on company profit because they represent gains from providing credit to customers (i.e., when paying back at time $t+\tau_{wc}+W_c+\tau_{cf}$, companies must refund the same price that customers paid at time t while accounting for the discount rate b). That being said, we show next that W_c can also have a negative overall effect on profit.

3.4 Linear testing and refurbishment cost functions

In order to optimize the supply chain profit, we must determine the optimal number of times an item can cycle through the CLSC, the optimal return window offered by retailers and the optimal level of responsiveness at the inspection center and refurbishment centers. For that purpose we proceed by first solving an optimization problem to derive a closed-form expression for the service rates (and thus for delays through the chain); we then use this optimization problem's output to find a closed-form expression for the optimal return window. Afterwards, we conduct a sensitivity analysis on the number of cycles (i.e., after how many loops should a typical item leave the supply chain). This approach is motivated in part by the intractability of simultaneously optimizing all the variables (optimal number of cycles, service rates and return window).

Recalling equation 3, the size of the secondary market β_N can be expressed solely as a function of α, N and β'. On the one hand, we observe that increasing the size of the secondary market automatically reduces the number N of cycles because a higher proportion of returns can be absorbed by the secondary market instead of being re-introduced into the primary market. On the other hand, increasing the number of cycles reduces the percentage $(1-\alpha)^N$ to a point where, in the extreme, no returned unit ever leaves the chain until it is sold in the primary market. Our model defines N as a finite integer number so that, after a limited number of loops, returns can leave the CLSC through sales in the secondary market or disposal.

At this point, we assume a linear cost function for sorting and testing and also for refurbishment costs; thus, respectively,

$$\begin{aligned} g_f[\mu_f] &= A_f\mu_f + B_f \quad \text{and} \\ g_r[\mu_r, W_c] &= A_r\mu_r + B_rW_c + C_r, \end{aligned} \tag{7}$$

for nonnegative constants A_f, B_f, A_r, B_r, C_r. Thus we have decided to model the refurbishment cost as a function of the service rate μ_r and also of the customer sojourn time W_c, since one may reasonably assume that, the longer the customer keeps the product, the more effort is required to make it “as new”. For notation convenience, we define the following discounted quantities:

$$\begin{aligned} \tilde{p}[0] &= \frac{1-e^{-(a+b)T}}{a+b}p[0]; \qquad \tilde{p_a} = \frac{1-e^{-bT}}{b}p_a \\ \tilde{g_f} &= \frac{1-e^{-bT}}{b}g_f; \qquad \tilde{g_r} = \frac{1-e^{-bT}}{b}g_r. \end{aligned} \tag{8}$$

In a similar way, we define the other "tilde" parameters. Furthermore, we introduce for tractability the following assumption: $e^{-b(\tau_{wc}+W_c+\tau_{cf})} \approx 1-(\tau_{wc}+W_c+\tau_{cf})$, guaranteeing a maximum approximation error in the sensitivity analysis of ca. 0.26%.

3.5 Optimal service rates

Computing the optimal service rates requires that we make some assumptions about the distribution of returns and about how best to model sorting and refurbishment activities. First of all, we observe that the returns probability follows a hypergeometric distribution that we can use to compute the *return probabilities* at every cycle—in other words, the likelihood that the same item (that has already been returned and re-introduced into the primary market once or more times), will be returned again. These return probabilities are given by $P(X = k) = \frac{\binom{K}{k}\binom{S-K}{l-k}}{\binom{S}{l}}$, where S is the population size (i.e., units sold in the primary market), K is the number of success states in the population (i.e., how many units that have already been returned n times are there among the S units sold), l is the number of draws (i.e., the number of units returned), and k is the number of successes (i.e., how many units that have already been returned n times are there among the l returned items). Since the mean of the distribution is simply lK/S, it follows that we can easily compute the required probabilities[6]. We then model the sorting/testing activity and the refurbishment activities as a single-class, single-server queueing station with Poisson arrivals and exponential service time. It would be more realistic to use a multi-server representation, but for our purposes, it is reasonable to consider each queueing station as part of a single macro entity. Because there are no priority constraints on returns processing, the service discipline is based on a policy of first-come, first-served.

Given these assumptions, we can express the delay at the two stations as the expected throughput time of an M/M/1 queue. In particular, we define the expected flow time at the sorting/testing facility and at the refurbishment facility as (respectively)

$$\begin{aligned} W_f &= \frac{1}{\mu_f - (1-\alpha)D} \quad \text{and} \\ W_r &= \frac{1}{\mu_r - (1-\alpha-\beta_N-\gamma_N)D}. \end{aligned} \tag{9}$$

These definitions enable us to derive the profit expression with respect to the service rates μ_f and μ_r. Obtaining the optimal solution now requires that we and apply the first-order conditions to

[6] As a simple example, consider, in steady state, $S = 100$ units sold in the primary market, $K = 10$ units (among the 100 sold in the primary market) that have already been returned twice (i.e., age = 2) and $l = 40$ units returned. The average number of items of age 2 that will be returned for the third time is given by $lK/S = 40 * 10 / 100 = 4$ units.

expression (6):

$$\frac{\partial \Pi}{\partial \mu_f} = (1-\alpha)D\left(\frac{\tilde{h}}{(\mu_f-(1-\alpha)D)^2} - \tilde{A}_f\right) = 0; \tag{10a}$$

$$\frac{\partial \Pi}{\partial \mu_r} = (1-\alpha-\beta_N-\gamma_N)D\left(\frac{\tilde{h}}{(\mu_r-(1-\alpha-\beta_N-\gamma_N)D)^2} - \tilde{A}_r\right) = 0. \tag{10b}$$

Since $\frac{\partial^2 \Pi}{\partial \mu_f \partial \mu_r} = \frac{\partial^2 \Pi}{\partial \mu_r \partial \mu_f} = 0$, sufficient conditions for the optimality of these first-order conditions are given by the following second-order conditions:

$$\frac{\partial^2 \Pi}{\partial \mu_f^2} = \frac{-2(1-\alpha)D\tilde{h}}{(\mu_f-(1-\alpha)D)^3}; \tag{11a}$$

$$\frac{\partial^2 \Pi}{\partial \mu_r^2} = \frac{-2(1-\alpha-\beta_N-\gamma_N)D\tilde{h}}{(\mu_r-(1-\alpha-\beta_N-\gamma_N)D)^3}. \tag{11b}$$

In both cases, the numerators are negative and (owing to network stability conditions) the denominators are positive. Hence the partial derivatives are strictly negative; it follows that the Hessian matrix is negative definite. We therefore conclude that the solution to (6) is the optimal service rate that we seek.

The optimal solutions for the capacities of the testing and refurbishment stations of the CLSC are as follows:

$$\begin{aligned} \mu_f^* &= (1-\alpha)D + \frac{\sqrt{\tilde{h}}}{\sqrt{\tilde{A}_f}}; \\ \mu_r^* &= (1-\alpha-\beta_N-\gamma_N)D + \frac{\sqrt{\tilde{h}}}{\sqrt{\tilde{A}_r}}. \end{aligned} \tag{12}$$

The optimal service rate μ_f^* is clearly influenced by the return rate, the sorting/testing cost, and the holding cost. We notice in particular that an increase in the holding cost will also increase the service rate: if holding the product becomes more expensive, companies will tend to speed up their processing time so that items can leave the reverse chain as quickly as possible. In contrast, an increase in the sorting/testing cost will reduce the optimal service rate because the processing of returns will then become more expensive. The optimal service rates also increases with product demand, and hence with the return window $W_{c_{max}}$. With regard to the optimal service rate at the refurbishment facility, we observe that—for reasons similar to the case of μ_f^*, it increases with the holding cost and with product demand, and decreases with the refurbishment cost. Furthermore, μ_r^* is an increasing function of N; that is, it increases with product age. The implication is that, if N increases, then quantity sold in the secondary market β_N decreases and so products loop more often before leaving the supply chain. Thus if the value of γ_N is held constant, then in each successive period there will be a higher quantity of products to refurbish (i.e., $1-\alpha-\beta_N-\gamma_N$

increases). As a consequence, companies must put more effort into refurbishment activities if they are to continue processing returns.

At this point, we substitute the optimal values for the service rates given by (12) into the profit function given by (6) and optimize with respect to the return window $W_{c_{max}}$. Our model is able to compute a closed-form solution for the optimal $W_{c_{max}}$, which is given in Appendix A2. Proposition 1 characterizes the optimal result (proof is provided in Appendix A2).

Proposition 1 *Given the $\alpha - \beta_N - \gamma_N$ model as described before, the optimal return window $W^*_{c_{max}}$ and the effect of customer sojourn time W_c (and initial customer sojourn time W_o) are defined as follows:*

$$\begin{array}{llll} \text{(i)} & W^*_{c_{max}} = \max_{W_{c_{max}}} \Pi[\mu^*_f, \mu^*_r] & \textit{and} \quad \dfrac{\partial \Pi}{\partial W_c} < 0,\ \dfrac{\partial \Pi}{\partial W_o} < 0 & \textit{when} \quad \tilde{p}[0] < K_1 \\ \text{(ii)} & W^*_{c_{max}} = \max_{W_{c_{max}}} \Pi[\mu^*_f, \mu^*_r] & \textit{and} \quad \dfrac{\partial \Pi}{\partial W_c} > 0,\ \dfrac{\partial \Pi}{\partial W_o} > 0 & \textit{when} \quad K_1 < \tilde{p}[0] < K_2 \\ \text{(iii)} & W^*_{c_{max}} \longrightarrow \infty & \textit{and} \quad \dfrac{\partial \Pi}{\partial W_c} > 0,\ \dfrac{\partial \Pi}{\partial W_o} > 0 & \textit{when} \quad \tilde{p}[0] > K_2 \end{array} \tag{13}$$

with $K_1 = \frac{\tilde{B}_r(1-\alpha-\beta_N-\gamma_N)}{b(1-\alpha)}$ *and* $K_2 = \frac{e_1(\tilde{A}_f(1-\alpha)^2+\tilde{A}_r(1-\alpha-\beta_N-\gamma_N)^2+\tilde{\varepsilon}\beta_N^2)+e_2(1-\alpha-\beta_N-\gamma_N)\tilde{B}_r}{e_2 b(1-\alpha)}$.

Cases (i) and (ii) identify an internal solution for the problem, i.e., there exists a finite value for the optimal return window. However, in case (i) the profit function is decreasing with the customer sojourn time, while in case (ii) is increasing. Case (iii) identifies an infinite solution, i.e., it results optimal to set the return window as large as possible (ideally infinite). In this case, the profit function is increasing with the customer sojourn time. If N=1, then, recalling expression (3), it results $1 - \alpha = \beta_N + \gamma_N$, i.e., the retailer will not undertake refurbishment activities and will sell all the returned items (net of disposal) in the secondary market. It follows that $\tilde{p}[0] > K_1$ and the profit is increasing with the customer sojourn time because of the benefit of credit to customers before returning. In the practical evidence, when the retailer undergoes refurbishment activities, we should focus only on case (i) because here the conditions on $\tilde{p}[0]$ define the scenario where the positive effect of $W_{c_{max}}$ and W_c—deriving from credit to customers—does not exceed the negative effect of $W_{c_{max}}$ and W_c—deriving from returns management costs—and the purpose of our study is not to speculate on customers. This means that there exists a finite optimal solution for the return window $W_{c_{max}}$ and that the profit function is decreasing with the customer sojourn time W_c. We also note that in some countries there might be regulations constraining the optimal solution, to which retailers should always pay attention (e.g., the 14 days minimum return window required by the current legislation in the European countries).

In the next section, we conduct a sensitivity analysis of the main parameters and develop some extensions of the base model.

4 Sensitivity analysis

In this section, we focus on the most relevant parameters of our model: customer sojourn time W_c, return window $W_{c_{\max}}$, maximum age of returns N, and return rate $1-\alpha$. Using data from Zalando as input, we conduct a sensitivity analysis of these parameters by considering their effect on the discounted profit. We posit a single-product scenario and consider a globally marketed product that is average in terms of cost/price and delays. In particular, we set the following baseline values for the European market: $D_o = 25$ units, $p[0] = €60$, $\delta_2 = 0.6$, $\varepsilon = 1$, $\alpha = 0.6$, $\gamma_N = 0.01$, $T = 180$ days, $a = 2.5 \times 10^{-3}$, $b = 1.4 \times 10^{-4}$, $W_o = 9$ days, $c_{ij} = €5$, $g_2 = €1.1$, $k_d = €1$, $p_a = €30$, $h = €1$, $\tau_{wc} = 3.5$ days, $\tau_{cf} = 1$ day, $\tau_{f2} = 1$ day, $\tau_{fr} = 0.1$ day, and $\tau_{rw} = 0.1$ day (the sorting/testing and refurbishment activities usually take place in the same location, so τ_{fr} and τ_{rw} are extremely short). From the nonnegativity of β_N (and its relation with α, γ_N, and N), it follows that $N < 5$. Throughout our analysis, we express the cost and profit values in euro, and the waiting times in days.

4.1 Effect of return window and customer sojourn time

Here we direct our attention to the return window and the sojourn time that an item spends with the customer before being returned; we also analyze the effect of such customer delay on the profit. The graphs in Figures 7 and 8 show the effect of the return window $W_{c_{max}}$ on Π for

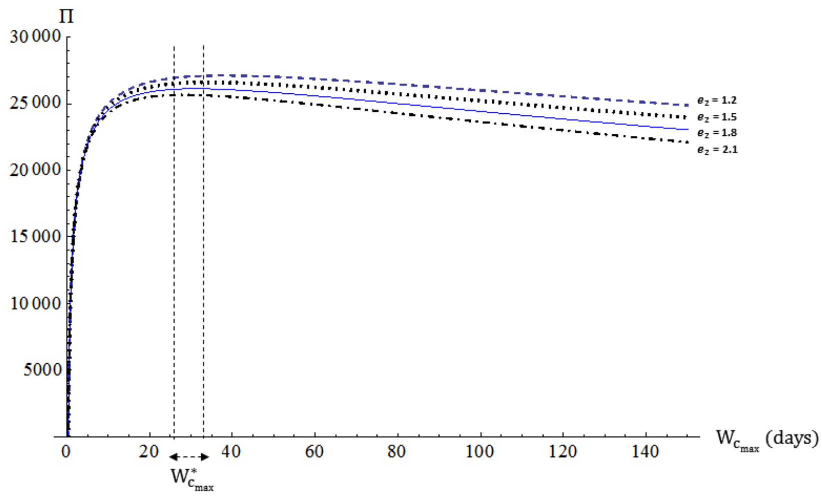

Figure 7: Optimal return window for the Zalando case (European market)

both the European and the German only market. We highlight the range of values for the optimal return window $W^*_{c_{max}}$ according to the different choice of the customer's sensitivity to $W_{c_{max}}$ (i.e., parameter e_2). Our results indicate that the analytical solution provided by Proposition 1 is quite

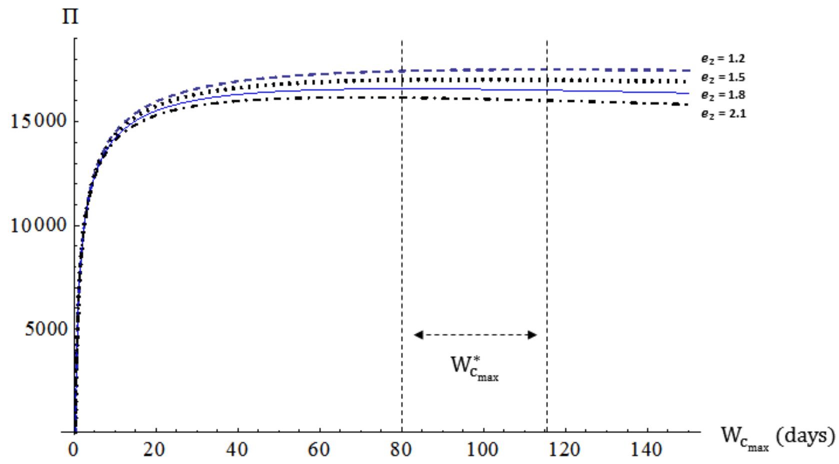

Figure 8: Optimal return window for the Zalando case (German market only)

close to Zalando's current return window (30 days in Europe and 100 days in Germany). Such result is even more evident for the European market (see Figure 7). Unfortunately, we could not have access to the data necessary for the estimation of the parameter e_2 (i.e., sensitivity of customer sojourn time to $W_{c_{max}}$); however, as confirmed by the Zalando's SVP Operations, the interval of values for e_2 that we represent in Figures 7 and 8 model quite precisely the empirical evidence.

The results of Proposition 1 show that there is still a margin for improvement when we reduce the initial sojourn time at the customer W_o. We also note that a reduction in W_o will decrease the optimal return window $W^*_{c_{max}}$ (i.e., $\partial W^*_{c_{max}} / \partial W_o < 0$, see Appendix A2). We shall now propose a solution to reduce customer delays in order to further increase the profit. The company cannot fully control customer delay W_o; however, it can "nudge" customers so that, when they decide to return an item, customers have an incentive to do so as early as possible. This is exactly the purpose of the incentive function g_c, which we now introduce. In order to decrease the delay at the customer W_o, companies must incentivize them and so will sustain an incentive cost. A decrease in W_o is indeed justified by an increase in g_c. Examples of incentives that companies can offer include payment advantages (e.g., the option of paying upon delivery or even within a grace period *after* the product is delivered) as well as gift vouchers and special discounts that reward returns occurring within a certain number of days.

Let $g_c[\Delta W_o] = A_c \Delta W_o + B_c$ $(A_c, B_c > 0)$ be the customer incentive cost function in order to decrease the customer delay by ΔW_o, and let G_c be the cumulative discounted incentive cost.

We evaluate the trade-off between the profit benefit due to lower W_o and the cost of making that reduction. Figure 9 illustrates the trade-off between a five-day reduction in customer delay W_o (and the resulting increase in profit, $\Delta\Pi$) and the associated cost G_c (represented by the coefficients A_c and B_c). For each value of the incentive function *below* the line $\Delta\Pi = G_c$ it will always be preferable

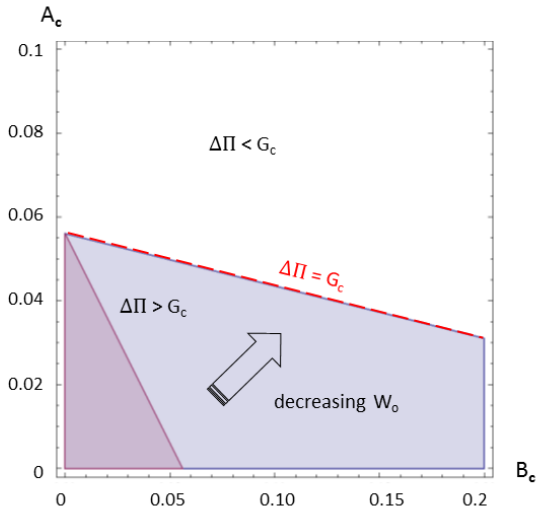

Figure 9: Trade-off between increase in profit and customer incentive cost

to invest in customer incentives because the increase in profit exceeds the required costs. For values *above* the line, it will never be preferable to invest in customer incentives because the costs of doing so are greater than the benefits. All values *on* the line $\Delta\Pi = G_c$ satisfy the indifference condition.

When applied to Zalando, these results imply a fairly low cost limit for investing in customer incentives. If it seeks to reduce customer delay by five days, for example, Zalando can invest no more than ca. €0.3 per item. Yet if W_o is reduced further then the frontier line $\Delta\Pi = G_c$ shifts upward, which shows that the company could invest more in customer incentives because the additional reduction in W_o would yield a greater profit benefit. It is thus profitable for Zalando to invest in customer incentives only when their cost is relatively low and/or the profit benefit is relatively high, as confirmed by the trends graphed in Figure 9. It is worth noting that such incentives could lead to opportunistic behavior on the part of customers pursuing further discounts. Zalando addresses this problem by way of measures that include keeping track of "bad" customers; however, this issue is not within the scope of our paper.

4.2 Effect of product aging and return rate

In this section, we discuss the role that product aging plays in supply chain profit and seek the best choice of N. In other words, we ask: After how many loops and under what conditions is it more profitable for companies to sell returns in the secondary market or dispose of them? In the case of Zalando, the discounted profit is, up to $N = 3$, an increasing function of N. For $N = 4$ the profit function decreases (see Figure 10)[7]. The trends plotted in this figure indicate that it is optimal for the company to refurbish and re-introduce returns into the primary market $N = 3$ times, rather than selling them into the secondary market directly and buying new ones from the suppliers. This solution indicates that, although some online retailers (e.g., Amazon) set $N = 1$, that solution may prove to be less than optimal. The results naturally depend also on the company's cost—price

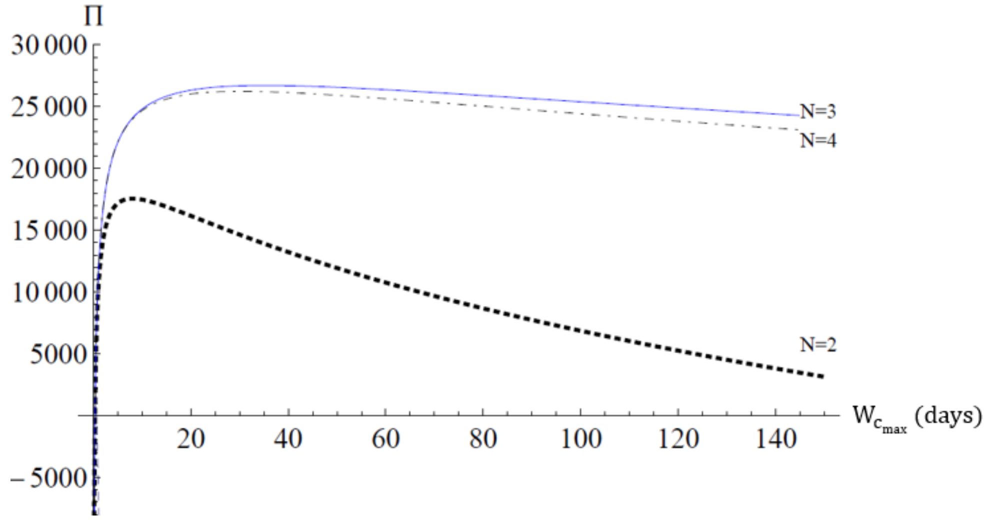

Figure 10: Profit trend as function of the number of cycles N (for N=1 the profit is very negative and decreasing)

functions. For this reason, we analyze the choice of N in terms of the company's refurbishment costs, price for new materials and secondary market's profitability (because secondary-market costs were observed not to affect our findings, they are excluded from our analysis). For the two extreme cases the result we obtain is not surprising: when buying new product from suppliers is far more expensive than refurbishing used items and secondary market is not very attractive, it makes more sense for the company to loop returns several times (i.e., set N as high as possible) so that it will be refurbishing a greater quantity of items; conversely, if refurbishing is more expensive than resupplying and secondary market quite attractive, then the firm should hasten the exit of returns from the CLSC (i.e., set N as low as possible).

However, our findings also reveal that it is not a trivial task to define the optimal N for values of purchasing price, refurbishment costs and secondary market's profitability *between* the two extreme cases just described. Figure 11 offers an example of when a different choice for these parameter

[7] We observe that the step between two "consecutive" profit functions (corresponding to two consecutive values of N) is decreasing because the effect of N on the profit is defined by a power function of a quantity smaller than one (which is function of the return rate), i.e., its effect decreases with the increase of N.

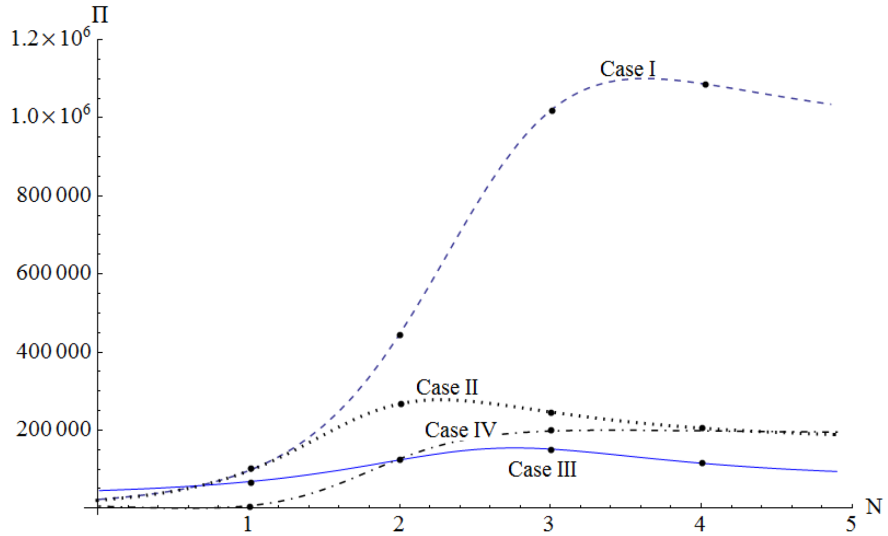

Figure 11: The optimal N will vary under different sets of parameters (Case I: $A_r =$ 0.001, $p_a =$ €20, $\delta_2 = 0.9$; Case II: $A_r = 0.1$, $p_a =$ €20, $\delta_2 = 0.9$; Case III: $A_r = 0.001$, $p_a =$ €40, $\delta_2 = 0.2$; Case IV: $A_r = 0.1$, $p_a =$ €20, $\delta_2 = 0.2$)

values can lead to different choices of N: the optimal choice is $N = 4$ in Case I, whereas it is $N = 2$ in Case II. What we observe in practice is that, in the market for electronic products, it is seldom preferable to loop returned items several times because the testing, refurbishing, and remanufacturing costs are extremely high. However, this is not the case for the fashion supply chain, where returns require only a modest extent of refurbishment and where purchasing new products is rather expensive. These considerations hold even more strongly in the case of online business, so modeling fashion online supply chains definitely requires that we identify the optimal number of loops.

Return rate represents one of the biggest issue for online retailer. We now address the effect of return rate on the profit and discuss its effects on the major parameters of our model. Since a different return rate $1 - \alpha$ might also affect the optimal return age N, from the nonnegativity of β_N (and its relations with α, γ_N and N), we consider in Figures 12 and 13 only the feasible combination of values for $1 - \alpha$ and N. In Figure 12 we represent the optimal return window $W^*_{c_{max}}$ as function of the return rate and return age. The optimal return window is rather large for lower values of the return rate, and becomes sharply very short (almost zero) for higher values of the return rate. This sharp decrease is more evident for lower values of N. Figure 13 shows the effect of return rate on the profit: initially the profit is sharply decreasing with the return rate; however, after reaching a minimum value (correspondent to the sharp decrease of the optimal return window, see Figure 12), it slowly starts to increase again. We can conclude that, when the return rate significantly increases, then, despite the benefit deriving from the purchasing cost savings, it

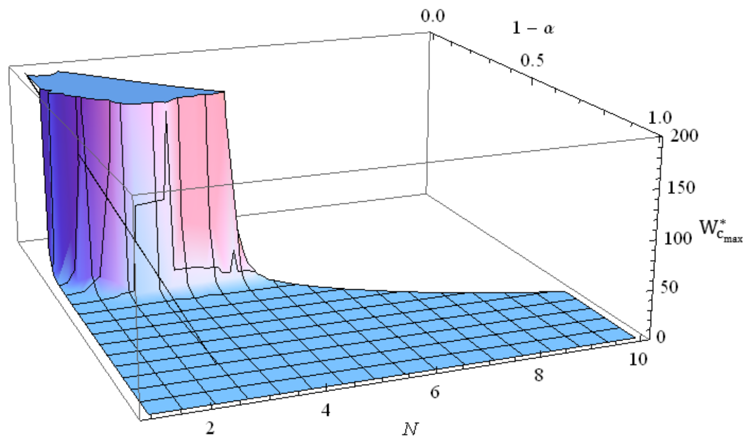

Figure 12: Optimal return window (in days) as function of the return rate

becomes prohibitive for the retailers to manage so many returned items. Therefore, in such case, it results optimal for the retailers to set a very narrow return window, which would also decrease product demand (according to Assumption (A2)) and consequently the number of returned items to process. Furthermore, the shorter return window will also decrease product sojourn time at the

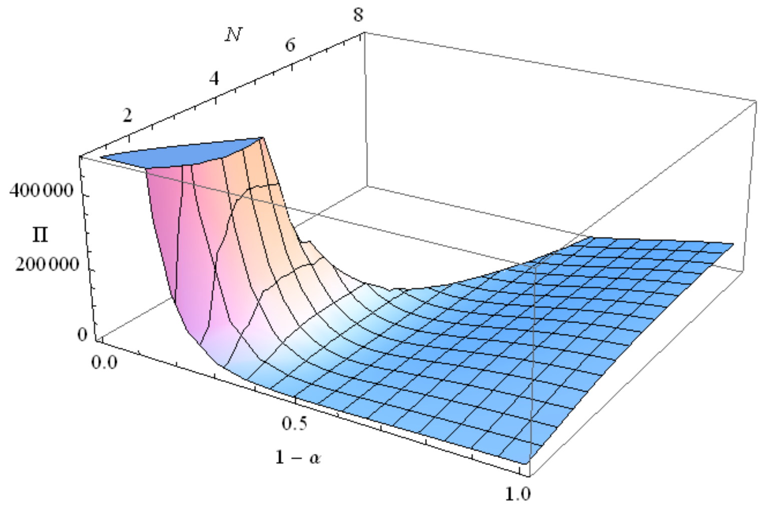

Figure 13: Sensitivity to the return rate and the product age

customers (according to Assumption (A3)), leading to a profit increase (see Proposition 1) that can partially compensate the returns management costs.

4.3 Effect of restocking fee

So far we have analyzed the scenario of retailers offering free returns. At such point, how would our results be affected if retailers charge customers for returning an item? Consistent with the literature (see for example Wood 2001, Anderson et al. 2009, Shulman et al. 2011), we assume that the introduction of a restocking fee f decreases product demand, i.e., $D \Rightarrow\ D_o + e_1 ln[W_{c_{\max}}] - e_3 f$

(where e_3 represents the sensitivity of demand to the restocking fee) and it also decreases the return rate, i.e., $1-\alpha \Rightarrow 1-\alpha(1+e_4 f)$ (where e_4 represents the sensitivity of the return rate to the restocking fee). Furthermore, when customers return an item, the retailer will now pay back $(1-f)p[t]e^{-b(\tau_{wc}+W_c+\tau_{cf})}$. We consider a range of values for $e_3 \in [70;350]$ and $e_4 \in [0.5;4]$. Following the same procedure as before, we find the optimal return window $W^*_{c_{\max}}$, which, this time, is affected also by the value of the restocking fee. Figure 14 shows that the optimal return window $W^*_{c_{\max}}$ is always increasing with the restocking fee f, and such increase is more significant for higher values of the return age N. Figure 15(a) shows that also the profit is increasing with the restocking

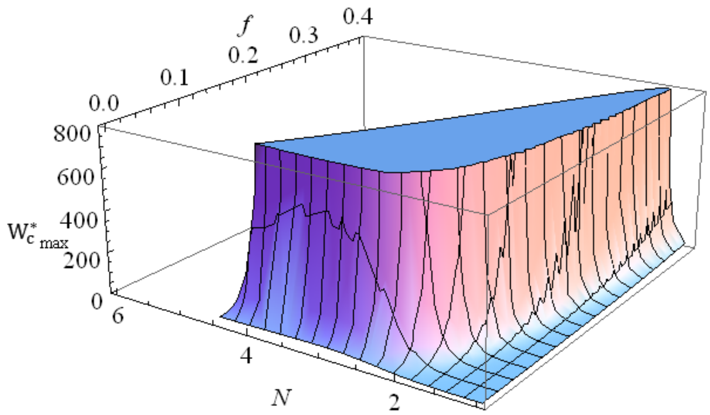

Figure 14: Optimal return window (in days) as function of the restocking fee

fee (such increase is more significant for higher values of the return age N). However, in case of very high return rate (Figure 15(b)), then the profit starts to decrease for increasing values of the restocking fee, up to a minimum value. After that, it starts to increase again (as observed before, such effect is more significant for higher values of the return age N). The decreasing trend derives from the high returns management costs associated with the very high return rate; the benefit of introducing a higher restocking fee will gradually compensate it and let the profit function increase again. We also observed that our results are barely affected by changes in the parameters e_3 and e_4.

Our analysis does not investigate the consequences of a restocking fee on the post-return buying level, which, as already discussed in Section 2, can be very significant. These considerations often lead retailers to keep offering a full refund policy, despite the attractive opportunity of introducing a restocking fee.

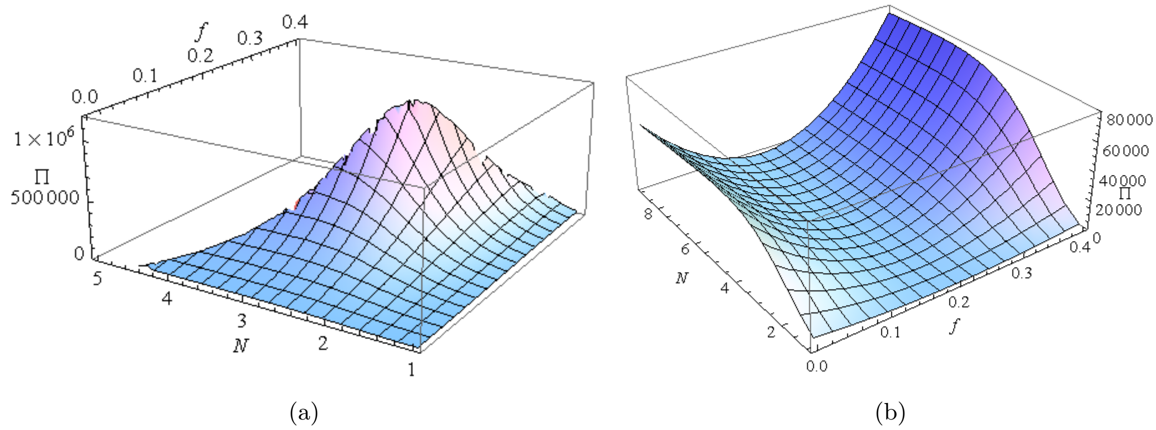

Figure 15: Profit as function of the restocking fee for (a) $1-\alpha$ = 0.4, e_4 = 1.5 and (b) $1-\alpha$ = 0.8 and e_4 = 1.5

5 Conclusions and managerial implications

In this paper, we modeled a closed-loop fashion supply chain while focusing on returns sojourn time at customers, return window, and product aging. We address our findings mostly to the online business sector because it is one of the most affected by multiple and late returns. We derived a closed-form expression for optimal service rates at the sorting and refurbishment facilities and the optimal return window, and then conducted a sensitivity analysis on the major parameters.

We show how a company can *strategically* set its return window as to maximize her profit and we assessed the effect of delays (especially customer delay) on CLSC profit and demonstrated how inducing variations in customer delay could improve the company's profit. We then explained how product aging affects profit and the choice of refurbishment versus resale in secondary markets. The management issue involves deciding when to forgo refurbishment and instead either discard merchandise (via sale in a secondary market) or scrap it. Our results indicate that this choice is far from trivial and is driven by the trade-off between refurbishment costs, the purchasing cost of new materials and the secondary market profitability. So even though a secondary market is often an attractive option for product returns, there are some cases (such as that of Zalando) in which refurbishment activities yield greater benefits.

In summary, management should always consider the multiple dimensions of the returns problem because it reflects the complex reality of online business. When setting the maximum number of times that product returns can loop, companies should consider the main drivers of this calculation—namely, refurbishment costs, the purchasing price of new items from suppliers and the secondary market profitability. By considering these factors, online retailers should be able to set parameters appropriately and to undertake actions that will increase their competitive advantage.

Although it may seem attractive for retailers to enlarge the return window in order to increase product demand, this will not always result in a profit benefit: besides the negative effect associated with customer sojourn time, when facing high return rates, it results more profitable for companies to keep a low return window (despite the decrease in product demand) in order to contain the otherwise unbearable returns management costs.

We observe that most German online start-ups set a return window of 100 days, which is the same as Zalando's return policy in Germany. Despite Zalando's continuing to offer a 100-day return policy in the German market (to match its competitors), the firm maintains high performance by speeding up its supply chain in two ways: on the company side, through fast delivery to customers, fast refurbishment, and fast resale (achieving these objectives requires Zalando to maintain a refurbishment center in every national market); on the customer side, through customer reminders within two weeks of product delivery. Hence customers who intend to return an item are incentivized—by means of a quick refund—to do so promptly. Zalando confirms that this rapid refund strategy is extremely effective (so that other major online retailers like Amazon have already started to implement it; Bloomberg Businessweek 2013); as shown in Figure 16, about 72% of returns do occur within 15 days of delivery. The combination of timely returns from

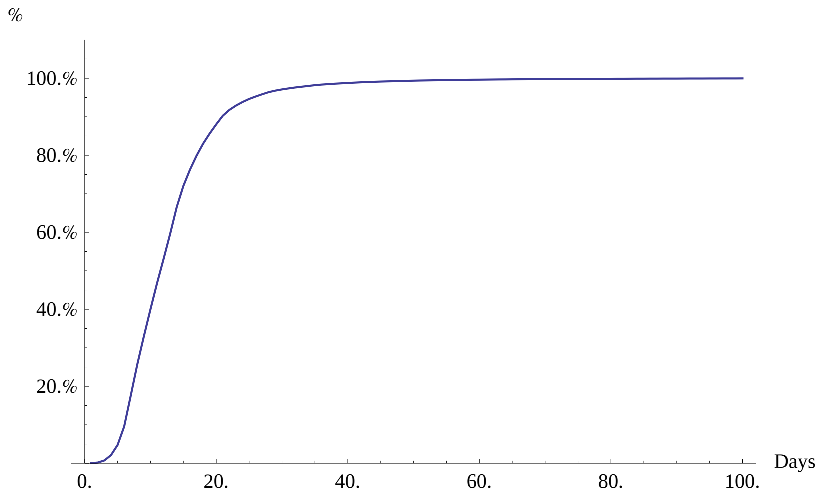

Figure 16: Percentage of items returned to Zalando as function of the number of days after delivery (*Source:* Zalando company data)

customers and quick internal processing allows Zalando to re-introduce more than two thirds of returned items into the primary market within three or four days (one or two days if we consider the German market). Therefore, most returned items can pass through the supply chain loop twice in the period of a month.

Fast fashion supply chains are shortening their lead times even further, and the number of planned seasons has increased significantly in response to consumer demand for "newness" (Barnes and Lea-Greenwood 2006, Bhardwaj and Fairhurst 2010). Even if there are 12 selling seasons every year, Zalando can maintain a 100-day (resp., 30-day) return policy in Germany (resp., in other European countries)—and thereby achieve a competitive advantage—because it can minimize lead times and maximize how often an item loops through the supply chain (here, twice in each season). Unlike e-tailers that follow stricter measures (e.g., blacklisting customers with excessive return behavior; *Lebensmittel Zeitung* 2014), Zalando has adopted a strategy based on communication and involvement with customers combined with incentives for them. In various test markets, e.g., Amsterdam, customers can also make use of an app and request pick up of their return by a messenger within one hour. That strategy makes the e-tailer, on the one hand, more attractive than brick-and-mortar stores by offering lenient return policies and, on the other hand, better able to manage its cost competitiveness by minimizing late returns. We believe that the shift to fast fashion will clearly show the Zalando supply chain to be an industry benchmark.

References

Accenture (2011). A returning problem: Reducing the quantity and cost of product returns in consumer electronics.

Akcay, Y., T. Boyaci, and D. Zhang (2013). Selling with money-back guarantees: The impact on prices, quantities and retail protability. *Production and Operations Management 22*(4), 777–791.

Altung, M. S. (2012). Optimal dynamic return management of xed inventories. *Revenue Pricing Management 11*(6), 569–595.

Altung, M. S. and T. Aydinliyim (2016). Counteracting strategic purchase deferrals: The impact of online retailers' return policy decisions. *Manufacturing & Service Operations Management 18*(3), 376–392.

Anderson, E., T. Hansen, and D. Simester (2009). The option value of returns: Theory and empirical evidence. *Marketing Science 28*(3), 405–423.

A.T. Kearney (2013). Online retail is front and center in the quest for growth.

Barnes, L. and G. Lea-Greenwood (2006). Fast fashioning the supply chain: shaping the research agenda. *Journal of Fashion Marketing and Management: An International Journal 10*(3), 259–271.

Bhardwaj, V. and A. Fairhurst (2010). Fast fashion: response to changes in the fashion industry. *The International Review of Retail, Distribution and Consumer Research 20*(1), 165–173.

Blackburn, J., G. Souza, and L. Van Wassenhove (2004). Reverse supply chains for commercial returns. *California Management Review* (46), 6–22.

Bloomberg Businessweek (09/30/2013). Don't even think about returning this dress.

Bower, B. and G. Maxham (2012). Return shipping policies of online retailers: Normative assumptions and the long-term consequences of fee and free returns. *Journal of Marketing 76*, 110–124.

EHI Retail Institute (2014). Versand-und-retouren-management im e-commerce 2014: Anforderungen, trends und strategien der onlinehändler.

EHI Retail Institute (2015). Versand-und-retouren-management im e-commerce 2015: Anforderungen, trends und strategien der onlinehandler.

Ferguson, M., D. Guide, E. Koka, and G. Souza (2009). The value of quality grading in remanufacturing. *Production and Operations Management 18*, 300–314.

Financial Times (12/11/2012). Amazon and law of the jungle.

Fraunhofer (2011). The processing of returns at logistics centers.

Galbreth, M. and J. Blackburn (2006). Optimal acquisition and sorting policies for remanufacturing. *Production and Operations Management 15*, 384–392.

Guide, D., E. Gunes, G. Souza, and L. Van Wassenhove (2008). The evolution of closed-loop supply chain research. *Operations Management review 1*, 6–14.

Guide, D., L. Muyldermans, and L. Van Wassenhove (2005). Hewlett-packard company unlocks the value potential from time-sensitive returns. *Interfaces 35*, 281.

Guide, D., G. Souza, L. Van Wassenhove, and J. Blackburn (2006). Time value of commercial product returns. *Management Science 58*(2), 1200–1214.

Gümüs, M., S. Ray, and S. Yin (2013). Returns policies between channel partners for durable products. *Marketing Science 32*, 622–643.

Hsiao, L. and Y.-J. Chen (2012). Returns policy and quality risk in e-business. *Production and Operations Management 21*(3), 489–503.

ibi research (2013). Retourenmanagement im online handel.

Janakiraman, N. H., A. Syrdal, and R. Freling (2016). The effect of return policy leniency on consumer purchase and return decisions: A meta-analytic review. *Journal of Retailing 92*(2), 226–235.

Ketzenberg, M. and R. A. Zuidwijk (2009). Optimal pricing, ordering, and return policies for consumer goods. *Production and Operations Management 18*, 344–360.

Lebensmittel Zeitung (04/07/2014). Douglas baut crosschannel aus.

Mitra, S. (2007). Revenue management for remanufactured products. *The International Journal of Management Science 35*, 553–562.

Oraiopoulos, N., M. Ferguson, and L. Toktay (2012). Relicensing as a secondary market strategy. *Management Science 58*, 1022–1037.

Petersen, J. and V. Kumar (2009). Are product returns a necessary evil? antecedents and consequences. *Journal of Marketing 73*, 35–51.

Petruzzi, N. and G. Monahan (2003). Managing fashion goods inventories: dynamic recourse for retailers with outlet stores. *IEEE Transactios of Engineering Management. 35*, 1033–1047.

PriceWaterhouseCooper (2013). Rückgabegebühr: Weniger retouren, weniger kunden?

Ruiz-Benítez, R., M. Ketzenberg, and E. Van der Laan (2014). Managing consumer returns in high clockspeed industries. *Omega 43*, 54–63.

Shahar, O. B. and E. A. Posner (2011). The right to withdraw in contract law. *The Journal of Legal Studies 40*, 115–148.

Shulman, J. and A. Coughlan (2007). Used goods, not used bads: Profitable secondary market sales for a durable goods channel. *Quantitative Marketing and Economics 5*, 191–210.

Shulman, J., A. Coughlan, and R. Savaskan (2009). Optimal restocking fees and information provision in an integrated demand-supply model of product returns. *Manufacturing & Service Operations Management 11*, 577–594.

Shulman, J., A. Coughlan, and R. Savaskan (2010). Optimal reverse channel structure for consumer product returns.

Management Science 29, 1071–1085.

Shulman, J., A. Coughlan, and R. Savaskan (2011). Managing consumer returns in a competitive environment. *Management Science 57*, 347–362.

Souza, G., M. Ketzenberg, and D. Guide (2002). Capacitated remanufacturing with service level constraints. *Production and Operations Management 11*, 231–248.

Su, X. (2009). Consumer returns policies and supply chain performance. *Manufacturing & Service Operations Management 11*, 595–612.

Tao, Z., S. Zhou, and C. Tang (2012). Managing a remanufacturing system with random yield: Properties, observations, and heuristics. *Production and Operations Management 21*, 797–813.

Ülkü, M. A., C. Dailey, and H. Yayla-Küllü (2013). Serving fraudulent consumers? the impact of return policies on retailer's profitability. *5*(4), 296–309.

UPS (2012). Recovering lost profits by improving reverse logistics.

Wood, S. L. (2001). Remote purchase environments: The influence of return policy leniency on two-stage decision. *Journal of Marketing Research 38*(2), 157–169.

Yin, S., S. Ray, H. Gurnani, and A. Animesh (2010). Durable products with multiple used goods markets: products upgrade and retail pricing implications. *Marketing Science* (29), 540–560.

Zalando (2017). https://www.zalando.de/.

Appendix

A1 Phase-in and phase-out profit

The phase-in profit function (π_{in}) is:

$$\pi_{\text{in}}[t] = \begin{cases} Dp[t] - Dp_a - Dc_{wc} = \pi_{01}, & t \in [0, T_1]; \\ \pi_{01} - (1-\alpha)Dc_{cf} - \dfrac{(1-\alpha)DhW_{\text{loop}}}{2} = \pi_{12}, & t \in [T_1, T_2]; \\ \pi_{12} - (1-\alpha)Dp[t]e^{-b(\tau_{wc}+W_c+\tau_{cf})} & \\ \qquad -(1-\alpha)Dg_f = \pi_{23}, & t \in [T_2, T_3]; \\ \pi_{23} - (1-\alpha)Dc_{fr} = \pi_{34}, & t \in [T_3, T_4]; \\ \pi_{34} - (1-\alpha)Dg_r = \pi_{45}, & t \in [T_4, T_5]; \\ \pi_{45} - (1-\alpha)Dc_{rw} = \pi_{56}, & t \in [T_5, T_6]; \\ \pi_{56} + (1-\alpha)Dp_a - \dfrac{(1-\alpha)DhW_{\text{loop}}}{2}, & t \in [T_6, T_7]. \end{cases} \tag{A1}$$

Until time T_1, the absence of any returns means that the company buys from its suppliers the entire amount of items necessary to fulfill demand. Starting from time T_1, after a sojourn time spent with the customers (W_c), some items are returned. However, the company continues to buy 100% of product inventory from its suppliers. The reason is that the company must wait to re-integrate the returned items because of several delays: the returns collection delay (up to time T_2), a sorting delay (up to time T_3), transportation between sorting and refurbishment facilities (up to time T_4), refurbishment delay (up to time T_5), and transportation from refurbishment center to the retailer warehouse (up to time T_6). Starting from time T_6, returned items can be re-introduced into the forward supply chain and the company can buy less product from suppliers because refurbished product can make up the difference. Only after N loops do some of the returned items leave the supply chain—to be either sold in the secondary market or disposed of. During the phase-in, returns are gradually entering the loop until time T_6; we therefore evaluate the holding costs of returns based on the average sojourn time in the loop.

The online retailer sells no products after the selling season; yet because of delays with customers and throughout the return loop, the company must continue to handle returns (and their associated refund and transportation costs), which are salvaged without any additional processing. The phase-out profit function (π_{out}) is:

$$\pi_{\text{out}}[t] = (1-\alpha)Dp_s - (1-\alpha)Dp[t]e^{-b\left(\tau_{wc}+W_c+\tau_{cf}\right)} - (1-\alpha)Dc_{cf}, \qquad t \in [T, T_8]. \tag{A2}$$

We note that the phase-in period is relatively short (as compared with the six-months selling season) because delay through the loop is minimal (for Zalando it is 1.44 days when both German and other European markets are considered together) and the maximum number of cycles N is single digit. Similar considerations apply to the phase-out period and so, with regard to computing the optimal solution, we refer to the CLSC model during the steady-state period only (see also Guide et al. 2006).

A2 Proof of Proposition 1

Obtaining the optimal solution now requires that we apply the first-order conditions to expression (6) after substituting the optimal values for the service rates:

$$\begin{aligned}
\frac{\partial \Pi}{\partial W_{c_{max}}} = \frac{1}{W_{\text{c}_{\max}}} \Big(& -\left(D_o + \ln\left[W_{\text{c}_{\max}}\right] e_1\right) \Big((-1+\alpha)^2 \tilde{A}_f e_1 - b\tilde{p}[0]e_2 + b\alpha\tilde{p}[0]e_2 + \tilde{B}_r e_2 - \alpha \tilde{B}_r e_2 \\
& - \tilde{B}_r e_2 \beta_N + \tilde{\varepsilon} e_1 \beta_N^2 - \tilde{B}_r e_2 \gamma_N + \tilde{A}_r e_1 \left(-1+\alpha+\beta_N+\gamma_N\right)^2 \Big) \\
& + e_1 \left(\tilde{p}[0] + (-1+\alpha) \left(\sqrt{\tilde{h}} \sqrt{\tilde{A}_f} + \tilde{B}_f - (-1+\alpha)\tilde{A}_f \left(D_o + \ln\left[W_{\text{c}_{\max}}\right] e_1\right) \right) - \tilde{g}_2 \beta_N - \tilde{k}_d \gamma_N - \tilde{p}_a \left(\alpha + \beta_N + \gamma_N\right) \right. \\
& + \left(-1+\alpha+\beta_N+\gamma_N\right) \left(\sqrt{\tilde{h}} \sqrt{\tilde{A}_r} + \tilde{C}_r + \tilde{B}_r \left(\ln\left[W_{\text{c}_{\max}}\right] e_2 + W_o\right) - \tilde{A}_r \left(D_o + \ln\left[W_{\text{c}_{\max}}\right] e_1\right) \left(-1+\alpha+\beta_N+\gamma_N\right) \right) \\
& + \beta_N \left(-\tilde{\varepsilon} \left(D_o + \ln\left[W_{\text{c}_{\max}}\right] e_1\right) \beta_N + \tilde{p}[0]\delta_2 \right) + (-2+\alpha) \sum_{(ij)} \lambda_{ij} \tilde{c_{ij}} - \sqrt{\tilde{h}} \gamma_N \left(\sqrt{\tilde{A}_f} + \sqrt{\tilde{h}} \tau_{\text{cf}} \right) \\
& - \sqrt{\tilde{h}} \beta_N \left(\sqrt{\tilde{A}_f} + \sqrt{\tilde{h}} \left(W_2 + \tau_{\text{cf}} + \tau_{\text{f2}}\right) \right) + \sqrt{\tilde{h}} \left(-1+\alpha+\beta_N+\gamma_N\right) \left(\sqrt{\tilde{A}_f} + \sqrt{\tilde{A}_r} + \sqrt{\tilde{h}} \left(\tau_{\text{cf}} + \tau_{\text{fr}} + \tau_{\text{rw}}\right) \right) \\
& \left. + (-1+\alpha)\tilde{p}[0] \left(1 - b\left(\ln\left[W_{\text{c}_{\max}}\right] e_2 + W_o + \tau_{\text{cf}} + \tau_{\text{wc}}\right)\right) \right) \Big) = 0
\end{aligned} \tag{A3}$$

Sufficient conditions for the optimality of these first-order conditions are given by the following second-order conditions:

$$
\begin{aligned}
\frac{\partial^2\Pi}{\partial W_{c_{max}}^2} = \frac{1}{W_{c_{\max}}^2} \Big(&-2e_1 \Big((-1+\alpha)^2\tilde{A}_f e_1 - b\tilde{p}[0]e_2 + b\alpha\tilde{p}[0]e_2 + \tilde{B}_r e_2 \\
&-\alpha\tilde{B}_r e_2 - \tilde{B}_r e_2\beta_N + \tilde{\varepsilon}e_1\beta_N^2 - \tilde{B}_r e_2\gamma_N + \tilde{A}_r e_1 \left(-1+\alpha+\beta_N+\gamma_N\right)^2\Big) \\
&+ \left(D_o + \ln\left[W_{c_{\max}}\right] e_1\right) \Big((-1+\alpha)^2\tilde{A}_f e_1 - b\tilde{p}[0]e_2 + b\alpha\tilde{p}[0]e_2 + \tilde{B}_r e_2 \\
&-\alpha\tilde{B}_r e_2 - \tilde{B}_r e_2\beta_N + \tilde{\varepsilon}e_1\beta_N^2 - \tilde{B}_r e_2\gamma_N + \tilde{A}_r e_1 \left(-1+\alpha+\beta_N+\gamma_N\right)^2\Big) \\
&- e_1 \Big(\tilde{p}[0] + (-1+\alpha) \Big(\sqrt{\tilde{h}}\sqrt{\tilde{A}_f} + \tilde{B}_f - (-1+\alpha)\tilde{A}_f \left(D_o + \ln\left[W_{c_{\max}}\right] e_1\right)\Big) \\
&- \tilde{g_2}\beta_N - \tilde{k}_d\gamma_N - \tilde{p}_a\left(\alpha+\beta_N+\gamma_N\right) \\
&+ \left(-1+\alpha+\beta_N+\gamma_N\right) \Big(\sqrt{\tilde{h}}\sqrt{\tilde{A}_r} + \tilde{C}_r + \tilde{B}_r \left(\ln\left[W_{c_{\max}}\right] e_2 + W_o\right) \\
&-\tilde{A}_r \left(D_o + \ln\left[W_{c_{\max}}\right] e_1\right)\left(-1+\alpha+\beta_N+\gamma_N\right)\Big) \\
&+ \beta_N \left(-\tilde{\varepsilon}\left(D_o + \ln\left[W_{c_{\max}}\right] e_1\right)\beta_N + \tilde{p}[0]\delta_2\right) \\
&+ (-2+\alpha)\sum_{(ij)}\lambda_{ij}\tilde{c_{ij}} - \sqrt{\tilde{h}}\gamma_N \left(\sqrt{\tilde{A}_f} + \sqrt{\tilde{h}}\tau_{\mathrm{cf}}\right) - \sqrt{\tilde{h}}\beta_N \left(\sqrt{\tilde{A}_f} + \sqrt{\tilde{h}}\left(W_2 + \tau_{\mathrm{cf}} + \tau_{\mathrm{f2}}\right)\right) \\
&+ \sqrt{\tilde{h}}\left(-1+\alpha+\beta_N+\gamma_N\right)\left(\sqrt{\tilde{A}_f} + \sqrt{\tilde{A}_r} + \sqrt{\tilde{h}}\left(\tau_{\mathrm{cf}}+\tau_{\mathrm{fr}}+\tau_{\mathrm{rw}}\right)\right) \\
&+(-1+\alpha)\tilde{p}[0]\left(1 - b\left(\ln\left[W_{c_{\max}}\right] e_2 + W_o + \tau_{\mathrm{cf}} + \tau_{\mathrm{wc}}\right)\right)\Big)\Big)
\end{aligned}
\tag{A4}
$$

Expression (A4), evaluated in the stationary point, is negative when

$$
\tilde{p}[0] < \frac{e_1(\tilde{A}_f(-1+\alpha)^2 + \tilde{A}_r(-1+\alpha+\beta_N+\gamma_N^2+\tilde{\varepsilon}\beta_N^2) + e_2(1-\alpha-\beta_N-\gamma_N)\tilde{B}_r}{be_2(1-\alpha)}
\tag{A5}
$$

The optimal solution for the return window is therefore the solution to (A3):

$$
\begin{aligned}
W_{c_{max}}^* = Exp[1 \Big/ \Big(\Big(2e_1 &\left((-1+\alpha)^2\tilde{A}_f e_1 - b\tilde{p}[0]e_2 + b\alpha\tilde{p}[0]e_2 + \tilde{B}_r e_2 - \alpha\tilde{B}_r e_2 - \tilde{B}_r e_2\beta_N + \tilde{\varepsilon}e_1\beta_N^2 - \tilde{B}_r e_2\gamma_N + \tilde{A}_r e_1\left(-1+\alpha+\beta_N+\gamma_N\right)^2\right)\Big)\Big) \\
&\Big(\Big(D_o e_2 \left(-b(-1+\alpha)\tilde{p}[0] + \tilde{B}_r\left(-1+\alpha+\beta_N+\gamma_N\right)\right) + \\
&+ e_1 \Big(\alpha\tilde{p}[0] + 2\sqrt{\tilde{h}}(-1+\alpha)\sqrt{\tilde{A}_f} - \tilde{B}_f + \alpha\tilde{B}_f - \tilde{C}_r + \alpha\tilde{C}_r - 2(-1+\alpha)^2\tilde{A}_f D_o - \alpha\tilde{p}_a + b\tilde{p}[0]W_o \\
&- b\alpha\tilde{p}[0]W_o - \tilde{B}_r W_o + \alpha\tilde{B}_r W_o - \tilde{g_2}\beta_N + \tilde{C}_r\beta_N - \tilde{p}_a\beta_N - \tilde{h}W_2\beta_N + \tilde{B}_r W_o\beta_N - 2\tilde{\varepsilon}D_o\beta_N^2 \\
&+ \tilde{C}_r\gamma_N - \tilde{k}_d\gamma_N - \tilde{p}_a\gamma_N + \tilde{B}_r W_o\gamma_N + 2\sqrt{\tilde{h}}\sqrt{\tilde{A}_r}\left(-1+\alpha+\beta_N+\gamma_N\right) - 2\tilde{A}_r D_o\left(-1+\alpha+\beta_N+\gamma_N\right)^2 \\
&+ \tilde{p}[0]\beta_N\delta_2 - 2\sum_{(ij)}\lambda_{ij}\tilde{c_{ij}} + \alpha\sum_{(ij)}\lambda_{ij}\tilde{c_{ij}} - \tilde{h}\tau_{\mathrm{cf}} + \tilde{h}\alpha\tau_{\mathrm{cf}} + b\tilde{p}[0]\tau_{\mathrm{cf}} - b\alpha\tilde{p}[0]\tau_{\mathrm{cf}} - \tilde{h}\beta_N\tau_{\mathrm{f2}} - \tilde{h}\tau_{\mathrm{fr}} \\
&+\tilde{h}\alpha\tau_{\mathrm{fr}} + \tilde{h}\beta_N\tau_{\mathrm{fr}} + \tilde{h}\gamma_N\tau_{\mathrm{fr}} - \tilde{h}\tau_{\mathrm{rw}} + \tilde{h}\alpha\tau_{\mathrm{rw}} + \tilde{h}\beta_N\tau_{\mathrm{rw}} + \tilde{h}\gamma_N\tau_{\mathrm{rw}} - b(-1+\alpha)\tilde{p}[0]\tau_{\mathrm{wc}}\Big)\Big)\Big)]
\end{aligned}
\tag{A6}
$$

We can conclude that, when condition (A5) holds, then expression (A6) maximizes the profit function. When condition (A5) is not satisfied, then expression (A6) minimizes the profit function.

If we now calculate the first derivative for the profit function provided in (6) as function of the customer sojourn time W_c and initial customer sojourn time W_o, it results

$$
\frac{\partial\Pi}{\partial W_c} = \frac{\partial\Pi}{\partial W_o} = \left(-b(-1+\alpha)\tilde{p}[0] + \tilde{B}_r\left(-1+\alpha+\beta_N+\gamma_N\right)\right)
\tag{A7}
$$

which is negative when

$$
\tilde{p}[0] < \frac{\tilde{B}_r\left(1-\alpha-\beta_N-\gamma_N\right)}{b(1-\alpha)}.
\tag{A8}
$$

Therefore, the profit is a decreasing function of W_c and W_o when condition (A8) holds.

Furthermore, we observe that the optimal return window is decreasing with the initial customer sojourn time

W_o, i.e.,

$$\frac{\partial W^*_{c_{max}}}{\partial W_o} = W^*_{c_{max}} \frac{-b(-1+\alpha)\tilde{p}[0]+\tilde{B}_r(-1+\alpha+\gamma_N+\beta_N)}{2\left(e_1(-1+\alpha)^2\tilde{A}_f-e_2(-1+\alpha+\gamma_N)\tilde{B}_r+be_2(-1+\alpha)\tilde{p}[0]-e_2\tilde{B}_r\beta_N+\tilde{\varepsilon}e_1\beta_N^2+e_1\tilde{A}_r(-1+\alpha+\gamma_N+\beta_N)^2\right)} \tag{A9}$$

which is negative when condition (A8) holds.