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ADELAIDE, AUSTRALIA
14-16 SETTEMBRE 1993

icema

International
Conference on
Electrical
Machines
in Australia

Adelaide
Australia

14-16 September 1993

PROCEEDINGS

VOLUME 3

14-18 September 1993
Adelaide
Australia

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Summary

The active magnetic bearings constitute a category of actuators rapidly developing, with wide perspectives of employment: their main advantage is the possibility to perform an active control of the rotor operating conditions, especially the damping of vibrations.
In the present paper, the design problems of the magnetic bearing are studied, starting from the mechanical specifications: in particular, the shape of the magnetic circuit and the winding structures are analysed, and the effects of these choices upon the characteristics of the supply and control systems are discussed. A numerical example illustrates the design algorithm.

1. Introduction

The active magnetic bearings are actuators with increasing perspectives of employment in the field of the rotating machines [5], [6]; their advantages are numerous: elimination of the lubrication systems, absence of contact and, above all, the active damping of vibrations, even in difficult conditions, such as operation at (or crossing of) one or more critical speeds.

The development of the magnetic bearing is related to the following research lines:

- mechanical modelling refinement, specifically oriented to the use of these bearings;
- advance of new control techniques, of adaptive type, even in non-linear ranges;
- optimisation of the magnetic bearing electromagnetic design.

The present paper is devoted to examine some aspects of the bearing design, by analysing its magnetic structure, the operating equations, the types of windings, and discussing the effects that these choices involve as regards the supply and the control systems.

2. Structure and operation of the magnetic bearings

The types of magnetic bearing studied till now differ each other for the number of magnets mainly; as regards the shape, frequently the pole shoe is larger than the pole body [3], [4]: this limits the force that can be developed, because of the magnetic saturation of the core.

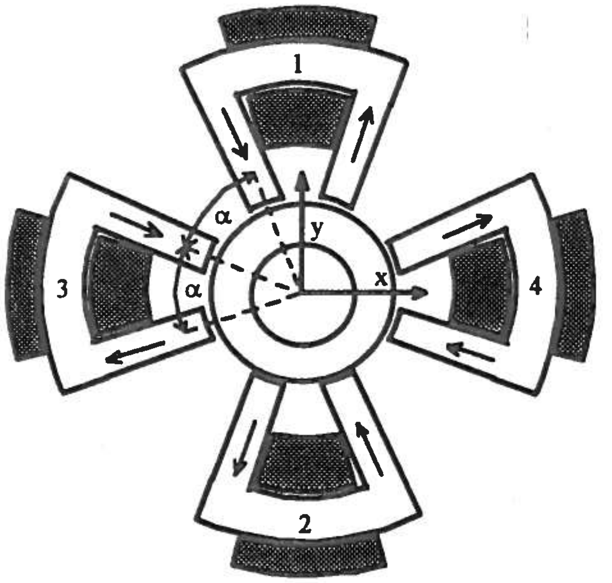
The analysis of this problem led to consider the structure shown in figs. 1 and 2: there are 4 U shaped laminated magnets, 2 by 2 placed along the x and y axes; the rotating part of the bearing consists of a cylinder (laminated too, in order to limit eddy currents), keyed on the rotor shaft.
If the device of fig.1 acts as a supporting bearing, the magnets 1 and 2 must develop an average vertical force dependent on the rotor weight; vice versa, in case of employment just as a damper, this resulting average force is zero, as for the magnets 3 and 4.
Let be K_e the polar arc portion along which the shoe is disposed; the air-gap area A_g equals:

$$A_g = K_e \cdot \alpha \cdot r \cdot \ell = \beta \cdot r \cdot \ell \approx w \cdot \ell \quad , \qquad (1)$$

with $\alpha=\pi/4$ the angle between poles, r the stator internal radius and ℓ the magnet axial length.

Let consider the vertical magnets 1 and 2 (the analysis for the magnets 3 and 4 is similar, except for the absence of the weight W); called b(t) the instantaneous air-gap flux density and $\Im_y(t)$ the total force (sum of W and of the force $f_y(t)$, having zero average value), one can write:

Research developed with the financial support of MURST 40% 1992.



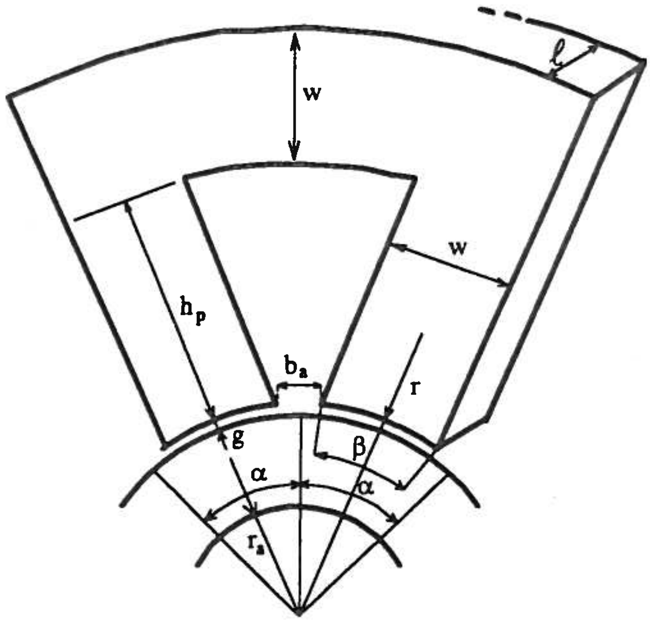


Fig.1: Schematic structure of a magnetic bearing with 4 magnets: the arrows indicate the senses of the total biasing m.m.f.s.

Fig.2: Main dimensions of the magnetic core with constant section of an electromagnet for a magnetic bearing.

$$\Im_y(t) = W + f_y(t) = K_B \cdot \left(b_1^2 - b_2^2\right) \ , \tag{2}$$

where:

$$K_B = (\alpha/\mu_0)\cdot\cos(\alpha/2)\cdot K_e \cdot (\ell/r)\cdot r^2 \ . \tag{3}$$

By adopting flux density values lower than the saturation limit, the following equations result:

$$b_1(t) = \mu_0 \cdot \frac{M_1 + m_y(t)}{g_n + \Delta(t)} \quad , \quad b_2(t) = \mu_0 \cdot \frac{M_2 - m_y(t)}{g_n - \Delta(t)} \quad , \tag{4}$$

where: M_1 and M_2 are the total biasing m.m.f.s; m_y is the regulation m.m.f. (for the vibration damping and the stability); g_n is the rated air-gap; Δ is the air-gap variation (along the y axis). Called M_{by} the basic biasing m.m.f., let define:

$$M_1 = M_{by} + M_w \quad ; \quad M_2 = M_{by} \ , \tag{5}$$

where M_w is the portion of the m.m.f. of the magnet 1 necessary to support the weight W:

$$W = K_F \cdot \left[\left(M_{by} + M_w\right)^2 - M_{by}^2\right] \ , \tag{6}$$

with

$$K_F = K_B \cdot \left(\frac{\mu_0}{2\cdot g_n}\right)^2 \ . \tag{7}$$

By manipulating the previous equations, the next expression for $f_y(t)$ can be obtained:

$$f_y(t) = K_F \cdot M_{by}^2 \cdot \left(M_w^* \cdot \left(M_w^* + 2\right)\cdot\left(\frac{1}{(1+\delta)^2} - 1\right) + 2\cdot\frac{M_w^*}{(1+\delta)^2}\cdot m + 4\cdot\frac{1 - m\cdot\delta}{\left(1-\delta^2\right)^2}\cdot(m-\delta)\right) \ , \tag{8}$$

where the following p.u. quantities are defined:

$$M_w^* = \frac{M_w}{M_{by}} \quad ; \quad m = \frac{m_y}{M_{by}} \quad ; \quad \delta = \frac{\Delta}{g_n} \ . \tag{9}$$

Eq. (8) suggests some remarks:

- the 1st and 2nd addend in the brackets exist only if the pair of magnets is supporting ($M_w^* \neq 0$);
- the non linearity of eq.(8) is basically due to the air-gap variation: the more δ is small (or, being equal δ, the more m is small), the more eq.(8) approaches linearity.

The linearization of eq.(8), by means of the insertion of (9), leads to the following equation:

$$f_y(t) \approx K_{my} \cdot m_y - K_{\Delta y} \cdot \Delta \ , \tag{10}$$

with $$K_{my} = 2 \cdot K_F \cdot (M_w + 2 \cdot M_{by}) \quad ; \quad K_{\Delta y} = 2 \cdot \frac{K_F}{g_n} \cdot \left[(M_{by} + M_w)^2 + M_{by}^2 \right] . \tag{11}$$

In a lot of described prototypes [3], [4], [5], [6] the magnet is equipped with just one coil: this choice causes some constraints concerning both the winding data and the supply features. As regards the last ones, frequently transconductance amplifiers are used, which are inherently characterized by a high level of losses, and then by a limit power rating too.

An interesting alternative is represented by the construction of two distinct coils: one coil produces the constant m.m.f., while the other one generates the regulation m.m.f.. In such a way, it is also possible to do some special connections: the biasing coils of the 4 magnets can be connected in series to constitute a biasing winding; also the 2 regulation coils of each axis can be connected in series to form an axis regulation winding. In conclusion, instead of adopting 4 independent transconductance amplifiers as in the previous case, just one d.c. supply and 2 regulation amplifiers must be used. The features of the regulation amplifiers are very different from those of the previously described linear amplifiers: the amplifiers of the regulation windings are interested by currents having practically zero average value and then they are characterized by a lower rating. Moreover, these amplifiers can be switching type devices, for example with PWM control: the inductive essence of the electromagnets and adequate values of the switching frequencies (up to 20 kHz) allow to limit the current ripple.

Finally, called L_n the nominal inductance of one regulation coil, the total axis inductance equals:

$$L_{axis} = L_1 + L_2 = \frac{L_n}{1+\delta} + \frac{L_n}{1-\delta} = 2 \cdot L_n \cdot \frac{1}{1-\delta^2} \ . \tag{12}$$

Eq.(12) shows that, while the coil inductance is subjected to the same p.u. air-gap variation δ, the axis inductance is almost constant, with advantages for the supply and the control devices.

3. Design elements of a magnetic bearing

The design of a magnetic bearing depends closely on the characteristics of the rotor-shaft system to be supported: for simplicity, in the following the shaft will be supposed rigid.

Let call m_r the rotor mass supported by each bearing, and K_{eq}, C_{eq} the stiffness and damping equivalent bearing coefficients respectively; if the external vibration force is due to an unbalance eccentricity ε, the equation of the rotor-bearing system at the speed ω equals [1], [2]:

$$f_{ext} - f_{bear} = m_r \cdot \frac{d^2\Delta}{dt^2} \quad \Rightarrow \quad \varepsilon \cdot m_r \cdot \omega^2 \cdot \sin(\omega \cdot t) - K_{eq} \cdot \Delta - C_{eq} \cdot \frac{d\Delta}{dt} = m_r \cdot \frac{d^2\Delta}{dt^2} \ . \tag{13}$$

From eq.(13) one obtains the condition of maximum vibration Δ_M (for the 1st critical speed ω_{cr}):

$$\Delta_M = \varepsilon \cdot \frac{\sqrt{K_{eq} \cdot m_r}}{C_{eq}} \quad ; \quad \omega_{cr} = \sqrt{\frac{K_{eq}}{m_r}} \ . \tag{14}$$

Eq.(14) determines the electromagnetic and control characteristics of the bearing.

As regards the control system, the simpler strategy employs a proportional-derivative (PD) control; therefore let adopt the following feedback law (y axis PD control):

$$m_y = K_{py} \cdot \Delta + K_{dy} \cdot \frac{d\Delta}{dt} \ . \tag{15}$$

By inserting (15) into (10), from the comparison with f_{bear} of (13) one can deduce that:

$$K_{eq} = K_{my} \cdot K_{py} - K_{\Delta y} \quad ; \quad C_{eq} = K_{my} \cdot K_{dy} \ . \tag{16}$$

Let consider a shaft having a diameter D_a=26 mm, on which is keyed a rotor mass m_{tot}=60 kg (m_r=30 kg), whose center of gravity is displaced by ε=30 μm from the geometric center; the two bearings must behave in such a way to set the 1st critical bending speed at N_{cr}=3000 t/m, and to limit the vibration to the value Δ_M=50 μm. From (14) one obtains the values of the bearing stiffness and damping equivalent parameters:

$$K_{eq} = m_r \cdot \omega_{cr}^2 = 5.26 \cdot 10^6 \quad [N/m] \;;\quad C_{eq} = m_r \cdot \omega_{cr} \cdot \varepsilon / \Delta_M = 7.54 \cdot 10^3 \quad [N \cdot s/m] \;. \quad (17)$$

For safety reasons and also for linearity purposes (employment of (10) instead of (8)), it is advisable that the nominal air-gap be adequately higher than Δ_M:

$$g_n = 20 \cdot \Delta_M = 1 \cdot 10^{-3} \; [m] \;. \quad (18)$$

Subsequently, K_B can be evaluated by eq.(2), where $\Im_y(t)$=W must be set; moreover, the total biasing flux densities must be chosen far from saturation, and compatible with the linearity hypotheses included in (10): one can assume that the supporting force with both the magnets biased equals a certain fraction (for example 2/3) of the force with the magnet 1 biased only:

$$K_B \cdot \left(B_1^2 - B_2^2\right) = \frac{2}{3} \cdot K_B \cdot B_1^2 \;\Rightarrow\; B_2 = B_1/\sqrt{3} \;. \quad (19)$$

Chosen the following values: B_1=0.8 T and B_2=B_{by}=0.45 T, one obtains:

$$K_B = \frac{W}{B_1^2 - B_2^2} = 673 \quad \left[N/T^2\right] \;. \quad (20)$$

Afterwards, r can be evaluated by means of (3), in which K_e=0.85 and (ℓ/r)=0.4 are set:

$$r = \sqrt{\frac{K_B}{(K_e/\mu_o) \cdot (\ell/r) \cdot \alpha \cdot \cos(\alpha/2)}} = 58.5 \cdot 10^{-3} \quad [m] \;, \quad (21)$$

from which ℓ =23.4 mm. There is a limit lower value of the radius r, that avoids the saturation of the rotor magnetic disc: by imposing that the radial dimension of this disc be not lower than the width w of the stator magnetic core, one obtains the following condition:

$$r \geq (r_a + g_n)/(1 - \alpha \cdot K_e) = 42.1 \cdot 10^{-3} \quad [m] \;. \quad (22)$$

On the basis of the hypothesis of magnetic linearity, the following m.m.f. values can be obtained:

$$M_{by} = \frac{B_{by} \cdot 2 \cdot g_n}{\mu_o} = 716 \quad [A] \;;\qquad M_w = \left(\frac{B_1}{B_{by}} - 1\right) \cdot M_{by} = 557 \quad [A] \;. \quad (23)$$

By means of (7) and (11) the corresponding bearing constants of the y axis can be evaluated:

$$K_F = 0.266 \cdot 10^{-3} \quad \left[N/A^2\right] \;;\quad K_{my} = 1.06 \quad [N/A] \;;\quad K_{\Delta y} = 1.13 \cdot 10^6 \quad [N/m] \;. \quad (24)$$

Finally, from (16) one can obtain the constants of the ideal PD controller of the y axis:

$$K_{py} = \left(K_{eq} + K_{\Delta y}\right)/K_{my} = 6.05 \cdot 10^6 \quad [A/m] \;;\quad K_{dy} = C_{eq}/K_{my} = 7.14 \cdot 10^3 \quad [A \cdot s/m] \;. \quad (25)$$

Considering that the external force f_{ext} acts in the same manner in every radial direction, the dynamic behaviour of the magnets of x axis and y axis must be identical: this condition can be defined by imposing, by means of the 1st eq. of (11), the following relation:

$$K_{mx} = K_{my} \;\Rightarrow\; M_{bx} = M_{by} + M_w/2 = 995 \; [A] \;. \quad (26)$$

The next values of the x axis bearing constants can be calculated:

$$K_{mx} = 4 \cdot K_F \cdot M_{bx} = 1.06 \quad [N/A] \;;\quad K_{\Delta x} = 4 \cdot K_F \cdot M_{bx}^2/g_n = 1.05 \cdot 10^6 \quad [N/m] \;. \quad (27)$$

Finally, again from (16) applied to the x axis, the x axis PD control parameters can be obtained:

$$K_{px} = \left(K_{eq} + K_{\Delta x}\right)/K_{my} = 5.98 \cdot 10^6 \quad [A/m] \;;\quad K_{dx} = C_{eq}/K_{mx} = 7.14 \cdot 10^3 \quad [A \cdot s/m] \;. \quad (28)$$

The comparison of (28) with (25) shows that, by imposing the 2nd of (26), the PD coefficients of the ideal controllers are the same or quite similar each other; actually, the real feedback system includes the transducer for the measurement of the air-gap, the signal amplifier, the power supply and the electromagnet, all with their dynamics: eq.(15) is just an ideal synthesis of them.

By applying (15) and (10) in sinusoidal operation, the phasors m.m.f. and developed force can be obtained at the critical speed; for the y axis one obtains (maximum values):

$$\overline{m}_y = \left(K_{py} + j \cdot \omega_{cr} \cdot K_{dy}\right) \cdot \Delta_M = 338\angle 26°[A] \; ; \; \bar{f}_y = K_{my} \cdot \overline{m}_y - K_{\Delta y} \cdot \Delta_M = 307\angle 31°[N] \; . \quad (29)$$

Similar calculations for the x axis give:

$$\overline{m}_x = 334\angle 27° \; [A] \; ; \; \bar{f}_x = 307\angle 31° \; [N] \; . \quad (30)$$

One can observe that, in spite of the difference (quite small) between the axis m.m.f.s, the forces are exactly the same, confirming the desired isotropic behaviour of the magnetic bearing.
It is interesting also to evaluate the linearity error connected to the employment of (10) instead of (8): for the y axis, considering the instant in which the air-gap variation is maximum:

$$M_w^* = 0.778 \; ; \; \delta_y = 0.05 \; ; \; m = m_y / M_{by} = 0.423 \quad (31)$$

eq.(8) and eq.(29) give:

$$f_{y_{exact}} = 253.6 \; [N] \; ; \; f_{y_{linear}} = 307 \cdot \cos(31°) = 263.2 \; [N] \; . \quad (32)$$

Therefore, the linearized model of the force leads, for m_y/M_{by}=0.423 and in the instant in which $\Delta=\Delta_M$, to an overestimate of 3.8 %, error not negligible, but anyway acceptable.

To conclude the bearing design, it is necessary to define its magnetic circuit and coils.
Four equal magnets must be constructed; the magnet 1 is that with the highest flux density:

$$B_{1max} = B_1 \cdot \left(1 + m_y / \left(M_w + M_{by}\right)\right) = 1.01 \; [T] \; ; \quad (33)$$

once more, this value is under the saturation limit sufficiently.
The width of the core (w) and the air-gap opening (b_a) equal respectively:

$$w = K_e \cdot \alpha \cdot r = 39.1 \cdot 10^{-3} \; [m] \; ; \; b_a = (1 - K_e) \cdot \alpha \cdot r = 6.9 \cdot 10^{-3} \; [m] \; . \quad (34)$$

For the calculation of the total copper section of the magnet 1, the following m.m.f. must be used:

$$M_{1tot} = M_w + M_{by} + m_y / \sqrt{2} = 1512 \; [A] \; . \quad (35)$$

By adopting a current density S=3 [A/mm^2] and a coil packing factor K_{cu}=0.36, the total area A_{w1} necessary for the insertion of the coils equals:

$$A_{w1} = M_{1tot} / (S \cdot K_{cu}) = 1400 \; \left[mm^2\right] \; . \quad (36)$$

By expressing A_{w1} as a function of the dimensions of fig.2, the height h_p of the pole body equals:

$$h_p = (r - w/\alpha) \cdot \left(\sqrt{1 + 2 \cdot \alpha \cdot A_{w1} / (\alpha \cdot r - w)^2} - 1 \right) = 51.6 \cdot 10^{-3} \; [m] \; . \quad (37)$$

The knowledge of h_p allows to estimate the saturation level, as a ratio between the magnetic voltage drop in the ferromagnetic core (U_{fe}, lamination Terni 1550) and in the 2 air-gaps (U_{2g}):

$$U_{fe} / U_{2g} = \mu_o \cdot H_{fe}(B_{1max}) \cdot ((2 + \alpha) \cdot h_p + 2 \cdot \alpha \cdot r) / (B_{1max} \cdot 2 \cdot g_n) = 0.018 \; . \quad (38)$$

It is a quite small value, that allows to consider the bearing magnet as magnetically linear.

The electrical time constant of the axis regulation winding is given by the following expression:

$$\tau = K_e \cdot \alpha \cdot \frac{\mu_o}{2 \cdot \rho} \cdot \frac{r}{g_n} \cdot \frac{\ell}{\ell_m} \cdot A_{cu\text{-}reg} = 8.71 \cdot 10^{-3} \; [s] \; , \quad (39)$$

where $\ell_m \approx 4 \cdot (\ell + w)$ is the average turn length, $A_{cu\text{-}reg} = m_y/(S \cdot \sqrt{2})$ is the total copper section of the regulation coil and ρ is the copper resistivity: τ is proportional to the dimensions (r) and to the copper section ($A_{cu\text{-}reg}$), while it is inversely proportional to the air-gap (g_n).

The apparent power absorbed by the y axis regulation winding for $\omega = \omega_{cr}$ equals:

$$A_{reg\text{-}y} = V \cdot I = \sqrt{1 + (\omega_{cr} \cdot \tau)^2} \cdot \frac{\rho \cdot \ell_m}{A_{cu\text{-}reg}} \cdot m_y^2 = 21.9 \; [V \cdot A] \; . \quad (40)$$

By adopting a wire with a diameter d_{co}=0.355 mm, the y axis regulation winding is characterized by the following coil turn number and winding terminal quantities (RMS values):

$$N_{y\text{-}reg} = A_{cu\text{-}reg} / A_{co} = 804 \; ; \; I = m_y / \left(\sqrt{2} \cdot N_{y\text{-}reg}\right) = 0.297 \; [A] \; ; V = A_{reg\text{-}y} / I = 73.8 \; [V] \; , \quad (41)$$

where $A_{co} = (\pi/4) \cdot (d_{co})^2$ is the regulation winding wire section.

As regards the y axis biasing coil, by adopting a wire with a diameter $d_{co\text{-}b}$=0.75 mm, the coils of the magnets 1 and 2 include N_1=961 and N_2=540 turns respectively, with a current I_b=1.33 A. For the x axis, from the values M_{bx} and I_b it follows a N° of turns $N_3=N_4$=751 for the biasing coils (equal to the average value between N_1 and N_2), while it is assumed $N_{x\text{-reg}}=N_{y\text{-reg}}$=804. Finally, the d.c. power and voltage of the biasing winding equal:

$$P_b = \rho \cdot (\ell_m / A_{co\text{-}b}) \cdot \left(\sum_{k=1}^{4} N_k \right) \cdot I_b^2 = 62.6\ [W]\ ;\ \ V_b = P_b / I_b = 47.2\ [V]\ , \tag{42}$$

where $A_{co\text{-}b}=(\pi/4)\cdot(d_{co\text{-}b})^2$ is the biasing winding wire section.

4. Conclusions

In the present paper the problem of the design of active magnetic bearings, for supporting the rotors of rotating machines and damping their vibrations, has been studied.

The geometric features of these devices have been examined, showing that the configuration with four magnets with constant section, 90° oriented, allows better performances, being equal the dimensions at the air-gap. The expression of the force has been obtained and linearized, showing its dependence on the air-gap and on the various m.m.f. contributions. The advantages in constructing two distinct coils for each magnet (one for biasing, the other for regulation) have been analysed: this simplifies the supply devices and reduces their rating. Finally, an example of magnetic bearing design algorithm has been described.

The studies will go on, by pursuing the following research lines:

- modelling of the various vibration modes and definition of the corresponding specifications regarding the characteristics and the positioning of bearings and intermediate dampers;
- analysis of the operation of bearing and control systems in case of a slight saturation level of the magnetic circuit;
- study of design and operation characteristics of the regulation feeding amplifiers.

Acknowledgements

The author greatly thanks Prof. S. Crepaz and Prof. I. Vistoli for their useful advice during the drafting of the paper.

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