



On the Correct Implementation of the Walsh Series Loader

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Abstract—Quantum state preparation is a preliminary step needed by quantum algorithms that require a specific input state. Exact state preparation methods scale exponentially with the number of qubits, making them impractical on noisy intermediate-scale quantum (NISQ) devices. Approximate approaches offer therefore more flexible and scalable alternatives. In this paper, we discuss the Walsh Series Loader (WSL), a recently proposed approximate algorithm for state preparation. In particular, we observe that the paper does not describe how to implement the term of order zero of the operator WSL is based on. While this does not affect the theoretical validity of WSL, it poses obstacles for practitioners aiming to use WSL because, as we show in our experiments, an incorrect implementation leads to states with very poor fidelity. In this paper, we describe how to implement the full quantum circuit required by WSL, including the term of order zero, releasing our source code online, and show that the algorithm works correctly. Finally, we empirically show how, as was theoretically demonstrated in the original article, truncated WSL does not prepare well highly entangled states.

Index Terms—quantum computing, quantum algorithms, state preparation, amplitude encoding.

I. INTRODUCTION

State preparation is a key task to tackle in order to map classical data onto quantum systems and consists in loading a complex-valued vector x of dimension $N = 2^n$, with $n \in \mathbb{N}$, into the amplitudes of a quantum register $|x\rangle$ of n qubits. This encoding is called *amplitude encoding*. In particular, if $x = (x_0, x_1, \dots, x_{N-1})$, the state to prepare is given by:

$$|x\rangle = \frac{1}{\sqrt{\sum_{i=0}^{N-1} |x_i|^2}} \sum_{i=0}^{N-1} x_i |i\rangle \quad (1)$$

where the state $|i\rangle$ identifies the eigenstate given by the binary representation of the integer i . State preparation is a preliminary step required by many quantum algorithms relying on input data, for example in quantum machine learning [1]–[3], finance [4]–[6] and chemistry [7]–[9], without which those algorithms would not be usable in practice.

In general, preparing a quantum state exactly requires a number of quantum gates that grow exponentially as the number of qubits n increases. Some exceptions exist if the quantum state exhibits certain structures, for example Matrix Product States (MPSs), where exact methods that scale well are known [10]–[12]. In general, unless the state exhibits a structure for which an efficient state preparation method

exists, its preparation can be considerably expensive. This is a substantial obstacle for the future applicability of many quantum algorithms. Significant attention has been devoted to approximate strategies for state preparation, in particular with the current Noisy Intermediate-Scale Quantum (NISQ) devices, each of them relying on specific assumptions and exhibiting specific limitations. A large family of methods belong to the family of variational algorithms, where a parametric quantum circuit is optimized iteratively to prepare a certain quantum state [4], [13], [14]. While being very flexible and general, since in principle they allow to prepare any quantum state, they suffer from several limitations, such as a lack of control on the error of the approximation, as well as the presence of barren plateaus limiting the trainability the parametric circuits as the number of qubits increases. Other approximate methods exist to prepare specific states, such as MPSs [15], [16], states used in quantum chemistry [17], [18] or states represented by series of functions [19], [20].

The Walsh Series Loader (WSL), developed by Zylberman and Debbasch [20], is a *repeat-until-success* approximate method of state preparation. WSL is based on the expansion of a real-valued function $f : [0, 1) \rightarrow \mathbb{R}$ with a generalized Fourier series, called *Walsh series*.

The contributions of this paper are the following:

- **The implementation of the WSL algorithm is incomplete:** While the original paper describes how to implement the operators for the Walsh terms of order ≥ 1 , it does not mention that the term of order zero is a special case that requires a different implementation;
- **The missing description limits reproducibility:** An incomplete implementation of WSL in which the term of order zero is missing leads to very poor results compared to those reported in the original paper, highlighting its importance;
- **We provide a correct WSL circuit:** we identify how the missing Walsh term can be added into the quantum circuit that implements WSL and show empirically that the method works correctly. Furthermore, we show that WSL does not prepare well highly entangled states.¹

¹The correct implementation of the WSL algorithm that we describe in this paper is available at: <https://github.com/qcpolimi/WSL>.

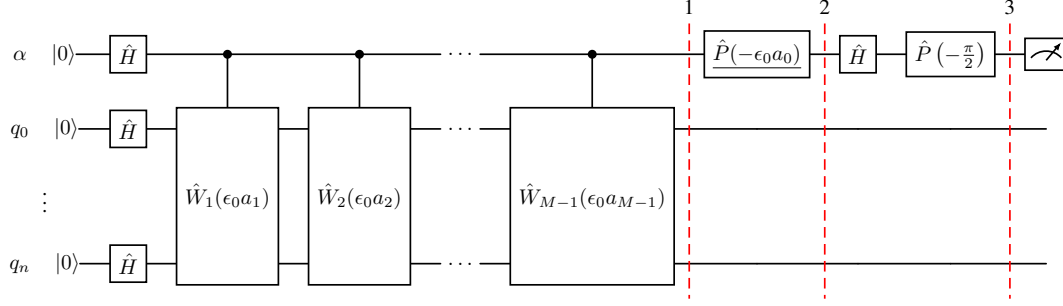


Fig. 1. Quantum circuit of the WSL algorithm. The least significant qubit is on the top of the circuit. \hat{H} identifies an Hadamard gate, the operator $\hat{W}_h(\epsilon_0 a_h)$ identifies a term of the Walsh series of f , while $\hat{P}(\beta)$ identifies a phase gate that introduces a phase β . Note that gate $\hat{P}(-\epsilon_0 a_0)$ is underlined to highlight we added it as part of our contribution.

II. WALSH SERIES LOADER

In this section, we provide a brief description of a correct implementation of the Walsh Series Loader (WSL).

A. Walsh Series, Walsh Functions and Walsh Transform

Given the discrete real function $f : D \rightarrow \mathbb{R}$ defined over $N = 2^n$ points,² with $D = \{0, 1, 2, \dots, N-1\}$, we can expand such discrete function as a *Walsh Series*, i.e., a finite sum of N *Walsh terms*. Each term is the product between a coefficient a_h , called *Walsh transform*, and a function $w_h(k)$ called *Walsh function*, where h represents their order. The Walsh Series is defined as:³

$$f(k) = \sum_{h=0}^{N-1} a_h w_h(k) \quad (2)$$

Generally, we keep all the N Walsh terms to represent correctly the function f , but notice that function f can be approximated by using $M < N$ Walsh terms, where M can be determined based on the accepted error $\epsilon_1 \in (0, 1)$. In particular, $M = 2^m$ with $m = \lceil \log_2 \left(\frac{1}{\epsilon_1} \right) \rceil$. Since f is discrete, M must be a power of two to ensure the points are contained in D .

The Walsh function $w_h(k)$ is defined as follows:⁴

$$w_h(k) = (-1)^{\sum_{j=0}^{n-1} h_j k_{n-j}} \quad (3)$$

where h_j and k_j represent the j -th least significant bit in the binary representation of h and k .

The Walsh transform a_h over M terms is defined as:

$$a_h = \frac{1}{M} \sum_{k=0}^{M-1} w_h(k) f(k) \quad (4)$$

²While f is defined over the interval $[0, 1)$, since it has to be mapped into a quantum state we assume it is discrete. The cardinality of the domain D must be a power of 2 and we use directly integer numbers to identify the values of f .

³Since f is a discrete function (2) identifies a full Walsh series; if f was continuous it would identify a truncated one.

⁴In the original paper [20], the definition is slightly different because k_j was defined based on a dyadic expansion. Here we simplify the notation using for both h and k a binary expansion.

B. Walsh Operators

The WSL algorithm is based on an operator \hat{U}_f proposed by Welch et al. [21] to represent Walsh terms, which is applied as a controlled unitary. In this section we discuss the non-controlled unitary:

$$\hat{U}_f = \prod_{h=0}^{M-1} e^{-ia_h \hat{w}_h} = \prod_{h=0}^{M-1} \hat{W}_h(a_h) \quad (5)$$

A Walsh function w_h can be implemented with a quantum operator \hat{w}_h , called *Walsh operator*. On n qubits, the Walsh operator \hat{w}_h is defined as [21]:

$$\hat{w}_h = \hat{Z}^{h_0} \otimes \hat{Z}^{h_1} \otimes \dots \otimes \hat{Z}^{h_{n-1}} \quad (6)$$

where \hat{Z} is the Z -Pauli operator and h_j is the j -th least significant bit of h , with:

$$\hat{Z}^{h_j} = \begin{cases} \hat{Z} & \text{if } h_j = 1 \\ \hat{I} & \text{if } h_j = 0 \end{cases} \quad (7)$$

We adopt the convention that the most significant qubit is the leftmost one. Notice that operator in (6) which is applied to the most significant qubit is elevated to the *least significant bit* of h . Of particular importance for our contribution is to highlight that the Walsh operator of order $h = 0$ corresponds to the identity.

The Walsh operator in (6) is equivalent to a $C\hat{N}OT$ staircase, constructed as follows. First, given h we define q_p as the most significant qubit on which \hat{w}_h applies a \hat{Z} gate. Then, we define operator \hat{A}_h as:

$$\hat{A}_h = \prod_{i=0}^{p-1} \left(C\hat{N}OT_p^i \right)^{h_i} \quad (8)$$

where $C\hat{N}OT_p^i$ is a unitary, defined on n qubits, which applies a $C\hat{N}OT$ gate, controlled by q_i with target q_p . Then, the Walsh operator \hat{w}_h can be written as:

$$\hat{w}_h = \hat{A}_h \left(\bigotimes_{i=0}^{p-1} \hat{I} \otimes \hat{Z} \otimes \bigotimes_{i=p+1}^n \hat{I} \right) \hat{A}_h^{-1} \quad (9)$$

Figure 2 shows an example of \hat{w}_h .

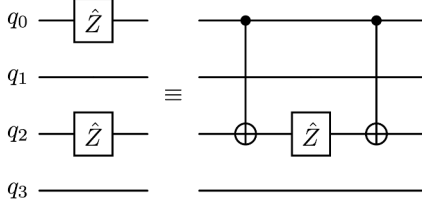


Fig. 2. Equivalence between a layer of \hat{Z} gates and the $C\hat{N}OT$ staircase for $h = 10$, hence with $h_0 = 0, h_1 = 1, h_2 = 0, h_3 = 1$.

Based on this it is straightforward to determine how to implement the operator $\hat{W}_h(a_h)$ as a quantum circuit for $h > 0$, see (10). Notice that gate \hat{Z} becomes unitary $e^{-ia_h\hat{Z}}$, corresponding to a $\hat{R}_z(2a_h)$ gate.

$$\hat{W}_h(a_h) = \hat{A}_h \left(\bigotimes_{i=0}^{p-1} \hat{I} \otimes \hat{R}_z(2a_h) \otimes \bigotimes_{i=p+1}^n \hat{I} \right) \hat{A}_h^{-1} \quad (10)$$

At this point, however, we encounter a crucial issue. The \hat{W}_0 operator produces a global phase which is not possible to implement with a $\hat{R}_z(\theta)$ gate. This means that implementing \hat{W}_0 requires an ad-hoc strategy.

$$\hat{W}_0(a_0) = e^{-ia_0\hat{w}_0} = e^{-ia_0 \bigotimes_{j=0}^{n-1} \hat{I}_j} = e^{-ia_0} \quad (11)$$

If the Walsh operator is used in a non-controlled fashion, the missing term of order zero is not an issue because it would only produce a global phase that can be discarded. Hence there is no need to include it.

On the other hand, the WSL algorithm applies the operator in a controlled way, which means that the phase it generates is *relative* on the ancilla qubit and therefore cannot be ignored. However, the description of the original WSL algorithm does not discuss this special case and does not describe how to include the term into the quantum circuit. In the following section we discuss how this issue can be addressed.

C. Description of the Correct Walsh Series Loader

The Walsh Series Loader performs state preparation by applying the Walsh operator \hat{U}_f on a register of n qubits, as a unitary controlled by an ancilla qubit α in superposition. The algorithm is a repeat-until-success method, where $|f\rangle$ has been prepared correctly only if the ancilla qubit is $|1\rangle$. The WSL algorithm introduces a coefficient $\epsilon_0 > 0$ multiplied to the Walsh transform a_h to control the error on the prepared state as well as the probability of success. As $\epsilon_0 \rightarrow 0$ the fidelity of the prepared state increases. Note that by choosing ϵ_0 so that the rotation angle induced by operators $\hat{W}_i(a_i\epsilon_0)$ is arbitrarily small, we can control the fidelity of the prepared state at the cost of a reduced probability of success. However, while this approach is theoretically sound, the limited resolution with which rotations can be applied on real quantum hardware will be a limiting factor on the fidelity WSL can achieve [22].

Given the discrete function f defined over N points, the state we aim to prepare is:

$$|f\rangle = \frac{1}{\sqrt{\sum_{k=0}^{N-1} f(k)^2}} \sum_{k=0}^{N-1} f(k) |k\rangle \quad (12)$$

The quantum circuit of the WSL algorithm is composed by a quantum register of n qubits where we wish to prepare state $|f\rangle$ and one ancilla qubit. The first step is to compute the N Walsh functions $w_h(k)$ and the Walsh transform a_h of f . For a truncated series of $M < N$ terms, we can apply (4).

The second step is to create the quantum circuit shown in Figure 1. Notice how every operator $\hat{W}_h(a_h\epsilon_0)$ is controlled by the ancilla qubit α . At the step labeled with 1 in Figure 1, the state is:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} (|s_0\rangle |0\rangle + |s_1\rangle |1\rangle) \quad (13)$$

where $|s_0\rangle$ is not relevant because the state is prepared correctly only if the ancilla is equal to $|1\rangle$, while $|s_1\rangle$ is:

$$|s_1\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \exp\left(-i\epsilon_0 \sum_{h=1}^{M-1} a_h w_h(k)\right) |k\rangle \quad (14)$$

where the innermost summation corresponds to the Walsh terms of order $h \geq 1$. Notice that the Walsh term of order $h = 0$ is missing because, as we discussed, it cannot be implemented in the same way as the other terms. Since the Walsh operators are applied as controlled unitaries by an ancilla, the global phase generated by $\hat{W}_0(a_0\epsilon_0)$ will end up as a relative phase on the ancilla and, as such, it cannot be ignored.

In order to address this issue and include $\hat{W}_0(a_0\epsilon_0)$ we must add a phase $e^{-ia_0\epsilon_0}$ to all states of the quantum register, which corresponds to a relative phase applied to state $|1\rangle$ of the ancilla qubit. We achieve this by adding a phase gate $\hat{P}(-\epsilon_0 a_0)$ on the ancilla qubit immediately after the last controlled Walsh operator, see Figure 1.

With our integration, this correct WSL circuit can work as intended. At the end of the circuit, for small enough ϵ_0 , state $|1\rangle$ of the ancilla qubit is entangled with a state of the quantum register asymptotic to $|f\rangle$. Since integrating this term only requires one gate, it has a negligible impact on the scaling of WSL.

III. EXPERIMENTAL ANALYSIS

In this section we aim to show that if the Walsh term $\hat{W}_0(a_0\epsilon_0)$ is not correctly included in the implementation of WSL, the fidelity of the prepared state is very poor. This highlights the importance of this apparently small component.

A. Experimental Setup

In order to assess the fidelity of the states prepared with WSL we use the same functions f as in the original paper. Notice that all functions are defined as continuous over the interval $[0, 1)$, then are discretized on $N = 2^n$ points, with $n = \{7, 8, 9, 10, 11, 12, 13\}$ qubits.

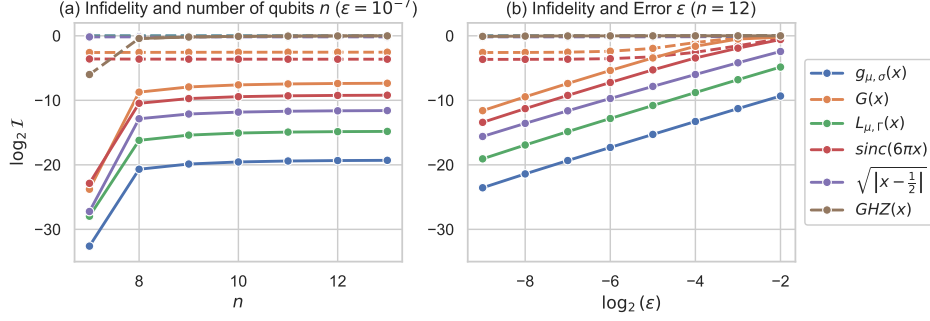


Fig. 3. (a) Relationship between the number of qubits and the logarithm of the infidelity of the prepared state, lower infidelities are better. As in the original paper, we set $\epsilon_0 = \epsilon_1 = 2^{-7}$. The number of Walsh terms used is therefore $M = 2^7$. Note that the logarithm of the infidelity for $GHZ(x)$ on 7 qubits is missing because it is $-\infty$. (b) Relationship between the logarithm of $\epsilon = \epsilon_0 = \epsilon_1$ and the logarithm of the infidelity of the prepared state, for a register of 12 qubits. Notice that the number of Walsh terms $M = \epsilon_1^{-1}$ is always lower than $N = 2^{12}$ and, in this case, the state $GHZ(x)$ is always prepared with a very low fidelity. For both plots, dashed lines represent states prepared with the incomplete WSL algorithm, while solid lines represent states prepared with the correct WSL described in this paper.

The functions are:

- Gaussian:

$$g_{\mu,\sigma}(x) = \frac{1}{\sigma} \exp\left(-\frac{1}{2\sigma^2}(x - \mu)^2\right) \quad (15)$$

with $\mu = 0.5$ and $\sigma = 1$;

- Bimodal Gaussian:

$$G(x) = s g_{\mu_1,\sigma_1}(x) + (1 - s) g_{\mu_2,\sigma_2}(x) \quad (16)$$

with $\mu_1 = 0.25$, $\mu_2 = 0.75$, $\sigma_1 = 0.3$, $\sigma_2 = 0.04$ and $s = 0.1$;

- Lorentzian:

$$L_{\mu,\Gamma}(x) = \frac{\Gamma}{\Gamma^2 + 4(x - \mu)^2} \quad (17)$$

with $\Gamma = 1$ and $\mu = 0.5$;

- Cardinal sine⁵:

$$\text{sinc}(x) = \frac{\sin(6\pi x)}{6\pi x} \quad (18)$$

- A non-differentiable function:

$$f(x) = \sqrt{\left|x - \frac{1}{2}\right|} \quad (19)$$

To provide a more complete picture of WSL, we also include a Greenberger-Horne-Zeilinger state on n qubits, which is maximally entangled and already discrete:

$$GHZ(x) = \begin{cases} 0 & \text{if } x \in (0, \frac{N-1}{N}) \\ \frac{1}{\sqrt{2}} & \text{otherwise} \end{cases}$$

As in the original paper, we set $\epsilon_0 = \epsilon_1 = 2^{-7}$, which correspond to $M = 2^7$ Walsh terms.

The prepared state $|\tilde{f}\rangle$ is evaluated based on its *infidelity* \mathcal{I} with respect to the target state $|f\rangle$, defined as:

$$\mathcal{I}(\tilde{f}, f) = 1 - |\langle \tilde{f} | f \rangle|^2$$

⁵Note that while the original paper states that the function has argument $6\pi x$, it reports a plot of a cardinal sine but with argument $19\pi x$.

B. Results

In this section we compare the results of an incomplete WSL implementation that is missing term $\hat{W}_0(a_0\epsilon_0)$, with a correct one that includes it as we described previously. As in the original paper, we report the logarithm of the infidelity of the prepared state which we show in relation to n and to the value of $\epsilon = \epsilon_0 = \epsilon_1$ (see Figure 3).

Plot (a) in Figure 3 shows the relationship between the logarithm of the infidelity of the prepared state and the number of qubits. We notice that, given a state to prepare $|f\rangle$, the incomplete WSL algorithm exhibits an infidelity that is orders of magnitude worse compared to that obtained with the correct WSL algorithm we describe. Furthermore, the results obtained using the correct WSL implementation are comparable to those shown in the original article highlighting how the original results are reproducible only by addressing this special case.

Plot (b) shows the relationship between the infidelity of the prepared state and ϵ , which is linear for the correct WSL algorithm. Again, the incomplete WSL algorithm does not allow to reproduce the original results, but the correct version does. Indeed, the incomplete WSL algorithm also numerically violates the theoretical upper bound $\mathcal{I}(\tilde{f}, f) \leq \epsilon$ that is proven in the original paper, which are instead consistent with the correct WSL. Lastly, Theorem 1 of the original WSL paper [20] showed that truncated WSL may not prepare well highly entangled states. Indeed, operator \hat{W}_h does not introduce entanglement. We can observe the effect empirically on the Greenberger-Horne-Zeilinger state which is prepared with reasonable fidelity only when $M = N$. In Plot (a), in fact, the infidelity related to the state $GHZ(x)$ is always 1, when $n \geq 8$, while the number of Walsh terms used is equal to $M = 2^{-7}$.

Figure 4 shows the infidelity of the state prepared by the complete WSL implementation by varying the number of truncated terms M for $n = 12$ qubits and therefore $N = 2^n = 4096$. Most states can be prepared with a very limited number of terms, validating the effectiveness this truncation can have,

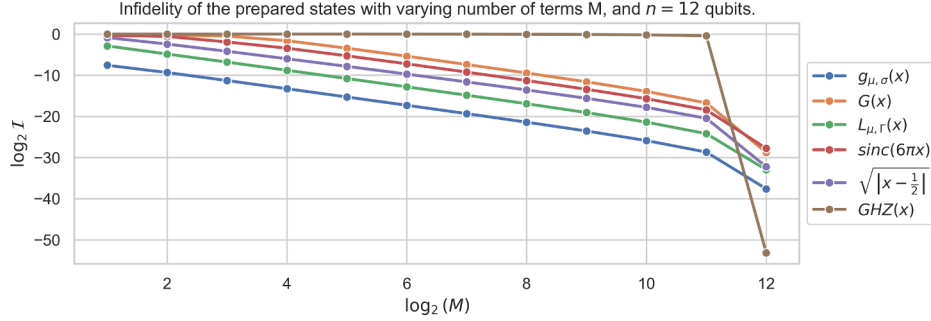


Fig. 4. Plot representing the value of the logarithm of the infidelity \mathcal{I} between the exact states and the states prepared by the WSL algorithm using M Walsh terms. Different states are represented by lines of different colors. The number of qubits of the states is $n = 12$. Notice that the state $GHZ(x)$ is prepared with very high infidelity for all the $M < 2^{12}$. In the case $M = 2^{12}$, the infidelity in preparing the state $GHZ(x)$ is 0, therefore its logarithm is not defined. We chose to set a very low value of infidelity, namely 10^{-16} , for visualization purposes.

especially for the Gaussian and Lorentzian, while the cardinal sine and Bimodal Gaussian are more difficult to prepare. As expected, the $GHZ(x)$ state can be prepared effectively only when $M = N$. Such evidence confirms that highly entangled states cannot be prepared with a truncated number of Walsh terms.

IV. CONCLUSIONS

In this paper, we have discussed how the implementation of the algorithm described in the paper “Efficient quantum state preparation with Walsh series” [20] does not mention that the Walsh term operator \hat{W}_0 is as special case that cannot be implemented like the others. Without this term the results of the algorithm are very poor and is not possible to reproduce the results reported in the original paper. We have shown how the missing term can be included by adding a phase gate on the ancilla qubit, and that this complete algorithm behaves as expected in preparing states with reasonable fidelity. Finally, we analyze empirically the approximation introduced by truncating the Walsh series we observe that, for highly entangled states, such truncation reduces drastically the fidelity.

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REFERENCES

- [1] J. Biamonte, P. Wittek, N. Pancotti, P. Rebentrost, N. Wiebe, and S. Lloyd, “Quantum machine learning,” *Nature*, vol. 549, no. 7671, pp. 195–202, Sep 2017. [Online]. Available: <https://doi.org/10.1038/nature23474>
- [2] C. Ciliberto, M. Herbster, A. D. Ialongo, M. Pontil, A. Rocchetto, S. Severini, and L. Wossnig, “Quantum machine learning: a classical perspective,” *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 474, no. 2209, p. 20170551, Jan 2018. [Online]. Available: <https://doi.org/10.1098/rspa.2017.0551>
- [3] K. Gui, A. M. Dalzell, A. Achille, M. Suchara, and F. T. Chong, “Spacetime-efficient low-depth quantum state preparation with applications,” *Quantum*, vol. 8, p. 1257, 2024. [Online]. Available: <https://quantum-journal.org/papers/q-2024-02-15-1257/>
- [4] K. Nakaji, S. Uno, Y. Suzuki, R. Raymond, T. Onodera, T. Tanaka, H. Tezuka, N. Mitsuda, and N. Yamamoto, “Approximate amplitude encoding in shallow parameterized quantum circuits and its application to financial market indicators,” *Phys. Rev. Res.*, vol. 4, p. 023136, May 2022. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevResearch.4.023136>
- [5] D. Herman, C. Googin, X. Liu, Y. Sun, A. Galda, I. Saffro, M. Pistoia, and Y. Alexeev, “Quantum computing for finance,” *Nature Reviews Physics*, vol. 5, no. 8, pp. 450–465, Aug 2023. [Online]. Available: <https://doi.org/10.1038/s42254-023-00603-1>
- [6] S. Wilkens and J. Moorhouse, “Quantum computing for financial risk measurement,” *Quantum Information Processing*, vol. 22, no. 1, p. 51, Jan 2023. [Online]. Available: <https://doi.org/10.1007/s11128-022-03777-2>
- [7] B. Bauer, S. Bravyi, M. Motta, and G. K.-L. Chan, “Quantum algorithms for quantum chemistry and quantum materials science,” *Chemical Reviews*, vol. 120, no. 22, pp. 12 685–12 717, Nov 2020. [Online]. Available: <https://doi.org/10.1021/acs.chemrev.9b00829>
- [8] Y. Cao, J. Romero, J. P. Olson, M. Degroote, P. D. Johnson, M. Kieferová, I. D. Kivlichan, T. Menke, B. Peropadre, N. P. D. Sawaya, S. Sim, L. Veis, and A. Aspuru-Guzik, “Quantum chemistry in the age of quantum computing,” *Chemical Reviews*, vol. 119, no. 19, pp. 10 856–10 915, Oct 2019. [Online]. Available: <https://doi.org/10.1021/acs.chemrev.8b00803>
- [9] S. McArdle, S. Endo, A. Aspuru-Guzik, S. C. Benjamin, and X. Yuan, “Quantum computational chemistry,” *Rev. Mod. Phys.*, vol. 92, p. 015003, Mar 2020. [Online]. Available: <https://link.aps.org/doi/10.1103/RevModPhys.92.015003>
- [10] K. C. Smith, E. Crane, N. Wiebe, and S. Girvin, “Deterministic constant-depth preparation of the aklt state on a quantum processor using fusion measurements,” *PRX Quantum*, vol. 4, p. 020315, Apr 2023. [Online]. Available: <https://link.aps.org/doi/10.1103/PRXQuantum.4.020315>
- [11] K. C. Smith, A. Khan, B. K. Clark, S. Girvin, and T.-C. Wei, “Constant-depth preparation of matrix product states with adaptive quantum circuits,” *PRX Quantum*, vol. 5, p. 030344, Sep 2024. [Online]. Available: <https://link.aps.org/doi/10.1103/PRXQuantum.5.030344>
- [12] Y. Zhang, S. Gopalakrishnan, and G. Styliaris, “Characterizing matrix-product states and projected entangled-pair states preparable via measurement and feedback,” *PRX Quantum*, vol. 5, p. 040304, Oct 2024. [Online]. Available: <https://link.aps.org/doi/10.1103/PRXQuantum.5.040304>
- [13] C. Zoufal, A. Lucchi, and S. Woerner, “Quantum generative adversarial networks for learning and loading random distributions,” *npj Quantum Information*, vol. 5, no. 1, p. 103, Nov 2019. [Online]. Available: <https://doi.org/10.1038/s41534-019-0223-2>
- [14] N. Mitsuda, T. Ichimura, K. Nakaji, Y. Suzuki, T. Tanaka, R. Raymond, H. Tezuka, T. Onodera, and N. Yamamoto, “Approximate complex amplitude encoding algorithm and its application to data classification problems,” *Phys. Rev. A*, vol. 109, p. 052423, May 2024. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevA.109.052423>
- [15] J. Iaconis, S. Johri, and E. Y. Zhu, “Quantum state preparation

- of normal distributions using matrix product states,” *npj Quantum Information*, vol. 10, no. 1, p. 15, Jan 2024. [Online]. Available: <https://doi.org/10.1038/s41534-024-00805-0>
- [16] M. Ben-Dov, D. Shnaiderov, A. Makmal, and E. G. Dalla Torre, “Approximate encoding of quantum states using shallow circuits,” *npj Quantum Information*, vol. 10, no. 1, p. 65, Jul 2024. [Online]. Available: <https://doi.org/10.1038/s41534-024-00858-1>
- [17] S. Fomichev, K. Hejazi, M. S. Zini, M. Kiser, J. Fraxanet, P. A. M. Casares, A. Delgado, J. Huh, A.-C. Voigt, J. E. Mueller, and J. M. Arrazola, “Initial state preparation for quantum chemistry on quantum computers,” *PRX Quantum*, vol. 5, p. 040339, Dec 2024. [Online]. Available: <https://link.aps.org/doi/10.1103/PRXQuantum.5.040339>
- [18] C. H. Chee, D. Leykam, A. M. Mak, and D. G. Angelakis, “Shallow quantum circuits for efficient preparation of Slater determinants and correlated states on a quantum computer,” *Phys. Rev. A*, vol. 108, p. 022416, Aug 2023. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevA.108.022416>
- [19] M. Moosa, T. W. Watts, Y. Chen, A. Sarma, and P. L. McMahon, “Linear-depth quantum circuits for loading Fourier approximations of arbitrary functions,” *Quantum Science and Technology*, vol. 9, no. 1, p. 015002, Oct. 2023. [Online]. Available: <http://dx.doi.org/10.1088/2058-9565/acfc62>
- [20] J. Zylberman and F. Debbasch, “Efficient quantum state preparation with Walsh series,” *Phys. Rev. A*, vol. 109, p. 042401, Apr 2024. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevA.109.042401>
- [21] J. Welch, D. Greenbaum, S. Mostame, and A. Aspuru-Guzik, “Efficient quantum circuits for diagonal unitaries without ancillas,” *New Journal of Physics*, vol. 16, no. 3, p. 033040, Mar 2014. [Online]. Available: <https://dx.doi.org/10.1088/1367-2630/16/3/033040>
- [22] T. Bialecki, T. Rybotycki, J. Tworzyno, and A. Bednorz, “Testing the accuracy of qubit rotations on a public quantum computer,” *Frontiers in Physics*, vol. 12, Mar. 2024. [Online]. Available: <http://dx.doi.org/10.3389/fphy.2024.1360080>