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DESIGN OF A FLEXIBLE OPTIMAL TRAJECTORY DEFINITION TOOL FOR A MULTI-PAYLOAD MULTI-ORBIT INJECTION MISSION

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This paper proposes a two-step trajectory optimization tool for the multi-rendezvous problem, applied to the multi-payload multi-orbit capabilities of an upper stage. First, a bi-objective bi-level algorithm optimizes the sequence of orbit visitation and computes a first order impulsive solution to the transfers. Then, a multi-phase multi-shooting optimization algorithm uses the previous output as an initial guess to obtain a more accurate finite-thrust optimal trajectory. The result is a flexible tool which allows to obtain the optimal trajectories for a great range of multi-orbit injection problems, enabling sensitivity analyses and multi-disciplinary design of the upper stage for the mission definition.

INTRODUCTION

The irruption of new players in the space sector, and in particular private companies, has led into a dramatic increase in the demand for space access. This is partly due to the development of smaller and cheaper satellites, which has democratized the space medium in favor of small-to-medium institutions. However, it has also been pushed by the growing interest of constellation missions, with satellite numbers from the order of tens to thousands, such as OneWeb* or StarLink. While this situation can be understood as a great feat in technology development towards the use of space, current trends in demand of launching show a tendency beyond the growth capabilities of the logistics related to space transport systems. In fact, the number of planned small satellite launches is expected to increase this decade at a rate up to four times that of the previous one,[1] with an exponential evolution in the case of nano-satellites.[2] This situation has put in the spotlight the challenging position that the space access infrastructure must face not only in terms of logistic arrangements to accommodate all launches, but also in ecological terms due to the corresponding increase in material and fuel usage. Therefore, there is an obvious need for innovative and efficient space transport strategies.

There are no simple solutions in the upcoming space access bottleneck, although the most recurrent one is to deliver several satellites at once with a single vehicle, reducing the number of launches. Historically, this strategy has already been envisioned in the form of piggyback trips, in which secondary payloads are launched with a primary one, and delivered around the operational orbit of the latter. Achievable orbits are limited to the neighborhood of the primary one, transferring

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the maneuvering responsibilities (and cost) to the secondary payloads to reach their desired orbital position. This practice might discourage new projects, especially coming from small institutions or the the private sector.

The new space transport strategies should then focus on the capability of injection of the different payloads into their operational orbits, keeping the cost of maneuvering within the delivery vehicle. Such a concept has been already tested through the usage of kick-stages, which act as an intermediate stage moving from the primary orbit to the secondary ones. Among these, one can find the SL-OMV by MOOG or SHERPA by Andrews Space, both of which have flown with several payloads. In Europe, the ASTRIS kick-stage aims to enable this direct injection of satellites in geostationary orbits [*]. Multi-payload multi-orbit delivery vehicles present themselves to be strategic to the sustainable planning and performance of future launches.

While promising from the conceptual point of view, especially in terms of the increased flexibility and reduced overall cost, this type of mission requires the definition of a series of orbital maneuvers among consecutive target orbits which is not easy to determine. Establishing a multi-orbit visitation trajectory requires defining both the order of orbits to be followed as well as transfers among these.[3] This multi-rendezvous problem is normally cataloged as a dynamic variant of the Traveling Salesman Problem (TSP), which is a classic problem in the domain of optimization.[4] Solving it is crucial to determine the trajectory that must be followed by the spacecraft to deliver all the necessary satellites into their respective orbits. The application of the TSP in in space can be found in other types of missions, in particular those devoted to Active Debris Removal (ADR) and On-Orbit Servicing (OOS). Most of the literature is in fact related to these mission types, as it is observed in the following analysis of current solution strategies. Generally, these studies show a trend of separating the visitation and the transfer problems, and solving them separately in an effort to reduce the complexity and reach the optimal trajectory. The connection among these two problems, however, must be kept, which is conventionally done by attributing the cost of a set of transfers related to a specific orbital order, after fixing the latter. This information is then fed-back to contribute in the search for the optimal order of visitation, maintaining the coupling. This way, the problem is divided into two (the visitation sequence and the transfers definition) for whose solution several methods have been proposed.

The comparative of the sequential problem to the typical TSP has led to the proposal of typical solution methods. Among these, the most classic one is the use of extensive search algorithms such as the one used in Chen et al.[5] or Daneshjou et al.[6] The approach is to obtain the cost of the transfers to all possible orbit sequences and compare them to reach a global optimum. However, this strategy becomes unfeasible for larger number of orbits, as the number of possible sequence combinations grows in a factorial fashion. As a solution to this issue, other authors have used pseudo-extensive methods, such as tree-search algorithms in which the tours are built city by city by ”branching” limiting or ”pruning” their growth if a certain threshold in the overall cost is reached. The most popular of these methods is the Branch-and-Bound.[7–10] However, other strategies such as the Series Method are worth mentioning.[11] While reducing the problem of the search-space size, the branching limit must be carefully selected if it is desired to keep the feasibility of computational times while being sufficiently broad to reach as many different tours as possible. In fact, the gradual building method provokes a situation in which tours with lower initial cost are favored, although their global cost could be more expensive. To overcome both the computing effort and the limitation in possible

[*] `https://www.rst-rostock.com/news/news-detail/arianegroup-wins-esa-astris-\protect\@normalcr\relaxcompetition.html`

sequences it has been proposed to use heuristic algorithms that allow the evaluation of full paths at lower computational times, trading them for sub-optimal results. Typical strategies include Genetic Algorithms[7,12] , Simulated Annealing[13,14] , and Ant Colony Optimization.[15–17] Special interest has been shown in the latter algorithm due to its natural resemblance of the ant tour-building to the combinatorial tree-shape structure.

As stated before, the multi-rendezvous entails also the sub-problem of defining the transfers among the desired orbits. This part of the problem shows a much wider spectrum of approaches in literature, ranging from the use of diverse complex optimization algorithms, to its complete overlook. In fact, in some cases, the transfer legs are not even considered in the optimization process.[4] However, most of the studies do consider them in different manners. A typical simplified way is to generate a data storage of costs attributed to the individual transfers (generally in terms of ΔV) which are pre-computed and assumed constant. The solver can then quickly access the matrix and attribute them to a certain sequence. These costs can be assumed to be time-independent and equal to a Hohmann-transfer ΔV, in order to make it as simple and closer to the baseline TSP as possible.[9,18] However, to make it more realistic, most studies which use the pre-computed cost approach include the time-dependence by generating a time grid and computing all possible transfers between two orbits at all possible discrete times.[8,10,14] These cost arrays allow for fast correlation between sequence and cost, trading it for the need of large data storage. To compensate this issue, Bang et al.[19] proposes a preliminary local optimization of each one of the individual transfers, such that only local optima are kept as possible transfers for the global optimization while the rest of possible maneuvers are filtered out.

These types of approaches do not ensure to provide a sufficiently good solution for the optimization problem, as their results highly dependent on the design variables (such as the size of the step of the time grid). For this reason, the transfers themselves are also desired to be optimized. These are generally modeled as impulsive maneuvers, being Lambert targeting the most prominent approach in literature. The optimization is usually performed by means of heuristic algorithms, given the enormous size of the search space. Typical strategies include Genetic Algorithms[12] or Evolutionary Algorithms.[7,13] The usage of these requires the discretization of time, as for the data-storage approaches, limiting the range of possible transfers that can be computed. This can be overcome by using algorithms which function in the continuous domain, such as the Particle Swarm Optimization (PSO).[5,6,20] It must be noted, however, that all the presented strategies optimize the transfers in order according to the given sequence, one by one, instead of all at once. Thus, later transfers are influenced by the decisions taken to optimize previous ones, limiting the overall search-space. This approach could affect negatively the full sequence generation as solutions might stagnate in sub-optimal tours.

While the full multi-rendezvous problem has been extensively studied, the main focus has always remained within the generation of the optimal tour, using the transfers as a means for this end. Transfers are greatly simplified to impulsive maneuvers (or simplified analytical low-thrust legs).[7] Very rarely the transfer problem is considered as a continuous thrust problem to be solved by means of Nonlinear Optimal Control (NOC) to get the thrust control vector for the full trajectory. In fact, the only solutions found in this regard are provided for low-thrust simplified trajectories.[21,22] This issue is also pointed out in Cerf,[8] although left out as a future work. In the case of the upper stage, the possibility of high thrust strategies should be envisaged which would allow not only to study the correctness of approximating the finite thrust maneuvers by an impulsive one, but also to evaluate the effect of other physical aspects such as disturbances, engine dynamics or thrust direction. These could potentially affect the results coming from the multi-rendezvous solution given by the simplified model, as they could impact the feasibility of some transfers as well as the transfer accuracy with respect to ideal cases.

The current paper focuses on the solution of the time and fuel mass constrained multi-payload multi-orbit delivery mission, with the aim of visiting all orbits and minimizing both propellant consumed and time of flight. For this purpose, an optimization tool has created that works in two steps: a first step dealing with the simplified impulsive problem; and a second step which uses the solution of the first as an initial guess for the full NOC problem. The objective of this study is to close the gap in the generation of trajectories for the multi-rendezvous problem in the case of high thrust which include the full dynamics and allows the study of different parameters in a more realistic fashion. For this purpose, first the mission as well as the mathematical formulation will be described. Then, it will be explained the approach followed for developing the tool. Following, two different cases will be studied with the tool: one to verify it, and one to study its behavior with a more complicated mission scenario. Finally, a summary of the paper and its main conclusions will be presented.

PROBLEM STATEMENT

As stated before, the multi-payload multi-orbit is a complex optimization problem which intertwines in its trajectory definition the decision on the sequence to be followed and the transfers between consecutive orbits. In this section, the complete mission and mathematical formulation are presented as to first explain the requirements and constraints, and then present the full problem to be solved.

Mission Definition

To develop the tool, it is first necessary to establish a baseline mission. Consider an upper stage or kick stage which is used to inject a set of N payloads into N differentiated orbits. The desired orbital parameters, as well as the physical properties of the payloads are known beforehand, and specified towards the design of the trajectory. The motion of the vehicle is considered to start after the release of the first satellite, meaning that the initial launch leg is not taken into account in the overall cost of the mission. The selection of this first orbit, however, is still a decision variable for the optimization tool. It is assumed that the overall cost to reach any of these orbital regions from the ground will be similar, and carried out by other elements of the launcher. The course finishes once the vehicle delivers the last satellite and performs a final transfer towards a pre-defined disposal orbit, as to comply with the space debris mitigation guidelines.[23] The selection of this disposal orbit is arbitrary, as its definition or optimization are both outside the scope of this study. Thus, the overall mission is summarized as:

1. The spacecraft, after deploying the first satellite, performs a two-burn transfer towards the next orbit, where the second payload is released.

2. This sequence is repeated $N-1$ times, until all satellites are released. Then, it performs an N^{th} maneuver towards the disposal orbit. At this point the mission is considered to be finished.

3. As the activity, generally, does not require the satellite to be at any specific position within the final orbit, all points are equally valid and considered for the transfer final position.

4. Considering a hypothetical industrial point of view (in which customers require they payloads to be operational as soon and as cheaply as possible) both the fuel consumption and mission time are to be minimized.

5. Considering the classic upper stage propulsion system type, the engine is assumed to be high-thrust, and approximated as impulsive shots if necessary.

It is important to note that in the case under consideration, it is the fuel mass that is used as a measure cost of the transfers, and not the associated ΔV as it is typically done in literature. This is related to the fact that the overall mass is expected to evolve in a discrete manner, as payloads are being released. In addition, this discrete drop in total mass is not equal in all orbits, as every satellite to be injected might have a different weight. Therefore, the correlation between ΔV and fuel mass consumed is not direct, and highly depends on the satellites' masses that have been delivered or are yet to be injected. Thus, for a correct evaluation of the cost, it is the propellant mass that needs to be evaluated.

Mathematical Formulation

The multi-rendezvous mission above defined falls into the category of TSP-related problems in optimization.[4] In this type of problems, a pre-defined set of nodes must be visited sequentially, only once, while minimizing the cost to move among them. The problem is defined as a graph problem $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, in which $\mathcal{V} = \{1, ..., N\}$ is the set of nodes to be visited, and $\mathcal{A} = \{(i, j) : i, j \in \mathcal{V}, i \neq j\}$ is the set of arcs among the nodes, each of these corresponding to a certain cost. Within the graph problem, the set of satellites to be injected is defined as $\mathcal{S} = \{1, ..., N\}$, with the set of orbits equal to this set plus the final disposal orbit $\mathcal{V} = \{1, 2, ..., N, N+1\} = \mathcal{S} \cup \{N+1\}$. However, the current mission optimization problem has two main differences with respect to the classic TSP. First, the route is not closed, as the initial and final orbits are not the same. This affects the sequence generation, as the final disposal orbit must be included in the tour and different strategies might be looked for by the optimizer which might differ from a closed loop scenario. Second, and more importantly, the arcs are highly time-dependent due to the non-linearity of the orbital dynamics involved in the vehicle's motion. There is an infinite number of possible arcs between two orbits depending on vehicle's initial and final positions and its duration, making the search space infinite. This is summarized in Figure 1, where the different possible connections between the nodes are schematized. Properly implementing time is therefore crucial for the correct generation of the optimal trajectory.

Taking into consideration the already defined graph problem, it is now necessary to establish the different optimization objectives and constraints that dominate it. The optimization problem can be established as the minimization of

$$\min \left\{ \sum_{i=1}^{N+1} m_{f,i}, t_{tot} \right\} \tag{1}$$

with $m_{f,i}$ being the propellant mass needed for each transfer i, and with t_{tot} being the total mission time. Each one of the arcs is subject to the dynamic equations of motion:[24]

$$\ddot{\boldsymbol{r}} = \frac{\mu}{r^3}\boldsymbol{r} + \frac{T}{M}\boldsymbol{e_T}\delta + \boldsymbol{f}_{dist} \tag{2}$$

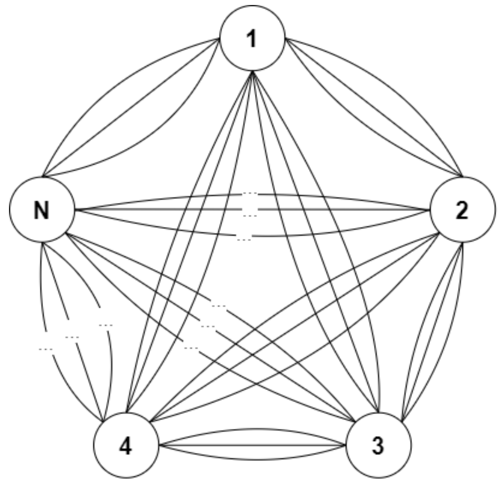

Figure 1. Schematic of the time-dependent TSP

$$\dot{M} = -\frac{T}{I_{sp} g_0}\delta \tag{3}$$

where $\boldsymbol{r}$ is the distance of the spacecraft with respect to the main gravitational body (the Earth in this case) and μ the gravitational parameter of this same body. The second term of the right-hand side is related to the thrust acceleration: T is the thrust magnitude, $\boldsymbol{e_T}$ the thrust vector, M the vehicle's mass (including remaining fuel), δ the relay on-off function of the propulsion system. The last term $\boldsymbol{f}_{dist}$ entails the acceleration provoked by disturbance effects, mainly the components of the Earth's spherical harmonics gravitational model, third-body gravitation, solar radiation pressure, etc. Equation 3 is Tsiolkovski's rocket equation, and determines the vehicle's mass change based on the engine's performance, in particular the specific impulse I_{sp}, and on the gravitational acceleration at the surface of Earth g_0. The solution of these equations of motion is in addition subjected to the boundary conditions of departure and arrival of the vehicle in terms of orbital elements:

$$\boldsymbol{x_k}(\boldsymbol{t_{0,k}}) = \boldsymbol{x_{0,k}};\ \boldsymbol{x_k}(t_{f,k}) = \boldsymbol{x_{f,k}} \tag{4}$$

with

$$\boldsymbol{x_i} = \text{KE}_i = [a,\ e,\ i,\ \Omega,\ \omega]_k$$

with the Kepler elements being the semi-major axis, eccentricity, inclination, right-ascension of ascending node and argument of perigee, respectively in the order of apparition in the above expression, for any *k* transfer. Note how the anomaly is not included, as any point in the orbit can be used for the transfers and it is not fixed.

In addition, since orbits need to be visited only once, the following constraints are imposed:

$$\sum_{i=0}^{N+1} s_{i,j} = 1;\ \sum_{j=0}^{N+1} s_{i,j} = 1 \tag{5}$$

Finally, having a maximum amount of fuel available in the spacecraft, and establishing a limit in the mission time, the constraints related to maxima in the values of the cost are formalized as:

$$\sum_{k=1}^{N+1} m_{f,k} \leq m_{f,\text{max}};\ \sum_{k=1}^{N+1} t_k \leq t_{\text{max}} \tag{6}$$

The mathematical formulation of the multi-rendezvous mission allows for a fast identification of the two differentiated parts already discussed. On the one hand, the combinatorial problem related to the selection of the visitation sequence is established by Equations (5), such that all orbits are visited only once. This type of problem falls into the category of integer programming (IP), as it deals only with integer numbers. On the other hand, the dynamic nature of the transfers is highlighted with Equation (2) and Equation (3). This type of problem falls into the category of nonlinear continuous programming (NLP). Being both tightly coupled, as the cost of the arcs of the elements of the IP are themselves an NLP problem, the full formulation is categorized as a mixed integer nonlinear programming (MINLP) problem. The optimal trajectory is then found only after solving this complex MINLP formulation, for which an algorithm that can deal with both sub-queries has to be developed.

THE OPTIMIZATION TOOL

As stated above, the problem is cataloged as MINLP optimization problem (NP-hard type) and thus cannot be solved deterministically without needing computational times growing factorially with the number of orbits to visit. Therefore, multiple strategies have been proposed in literature making use of heuristic algorithms that need multiple simplifications to solve the problem in feasible times. However, for in-depth analysis from the point of view of both trajectory definition and Guidance, Navigation and Control system design, a more realistic analysis is needed, including modeled disturbances and spacecraft's dynamics.

To solve this problem, the current study proposes a two step optimization algorithm that solves: 1) the sequential problem with a solution on the optimized simplified transfers among the orbits; and 2) a NOC problem related to a selected sequence which uses the results of the previous step for a more refined and realistic optimal trajectory, as well as for a control law. This is summarized in Figure 2. Each one of the two steps are explained in the following lines.

First Step

In the first step, the MINLP problem is first solved by a 2-level algorithm, each one dealing with one of the sub-queries, namely the sequential problem and the transfer problem, separately. The

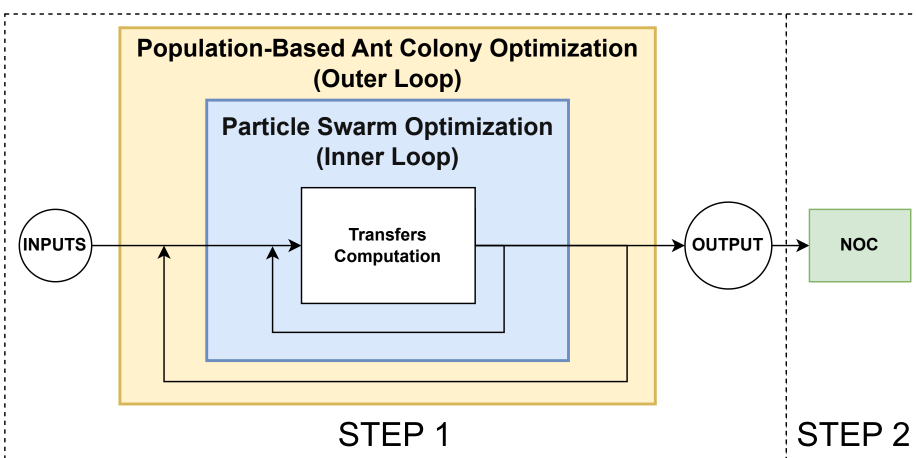

Figure 2. Schematic of the 2-Step Algorithm

process is then divided into: 1) an outer level solving the orbit sequence of visitation; and 2) an inner level optimizing the transfers among a certain sequence of orbits. These two activities are not independent, and need each other to reach the optimal solution. On the one hand, the internal part requires a specific orbit sequence, forwarded by the outer level, for which the transfers are optimized. On the other hand, the cost of this optimized set of transfers is used by the outer level as the quality measure attributed to that specific sequence in the search of the optimal order or visitation. Both levels are then nested and in constant interaction, as shown in Figure 3.

Keeping the connection is, however, complicated as the optimization is bi-objective. Attributing a Pareto front of costs coming from the inner algorithm to a single sequence of visitation is not useful, and thus a single point among these is selected. The tool allows this selection to be done according to different criteria, such as minimum fuel, minimum time, random or by means of a weighted function. An extended explanation on the functional principles of this first step is provided in previous works of the author.[25,26]

Outer Layer To solve the combinatorial problem, an heuristic algorithm that could exploit the natural tree-shape of this type of problems was studied, among which ACO-related algorithms are the most prominent in literature. A Population-based ACO (P-ACO)[27] with a multi-objective expansion[28] was implemented for this purpose. Such an algorithm has already been proven useful in similar multi-rendezvous studies.[29]

ACO algorithms solve sequential problems by the use of ants, which traverse a certain path (or sequence) leaving a trail of pheromones behind (τ). The probability of an ant of a subsequent generation to take a certain path is proportional to the amount of pheromones left by previous ants. However, this node connection is also influenced by a measure of desirability, given by a certain problem-related heuristic (η). In the multi-orbit visitation, this measure is related to the theoretical ΔV needed to change the individual Kepler elements among two orbits.[30] Both factors are weighted to account for their relative importance by means of design exponential factors, such that the probability for an ant to move from node *i* to node *j*, in a set *S* of still unvisited nodes is:

$$p(i,j) = \frac{\tau(i,j)^{\alpha}\eta(i,j)^{\beta}}{\sum_{z \in S} \tau(i,z)^{\alpha}\eta(i,z)^{\beta}} \tag{7}$$

The particularity of this algorithm with respect to a classic ACO is that pheromones are only

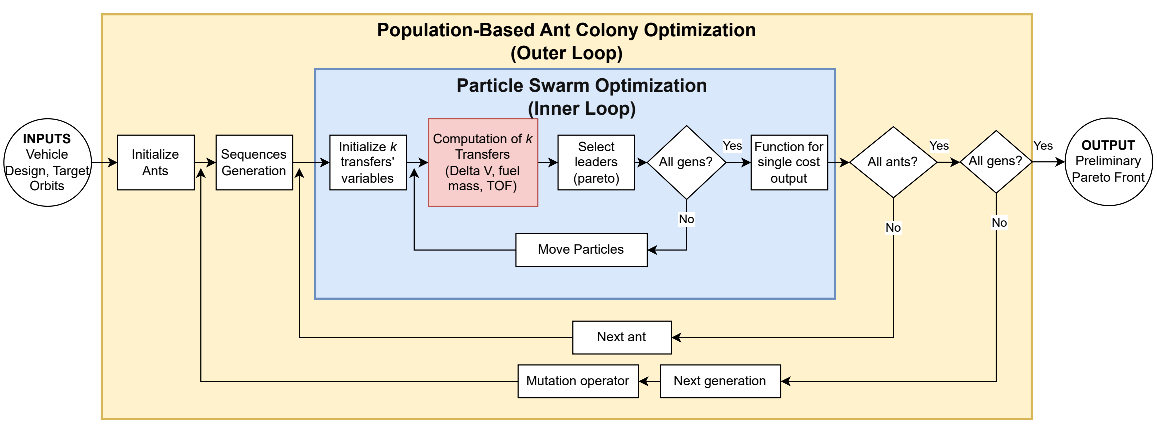

Figure 3. Schematic of the 2-Level First Step

deposited by a subset of the ants' generation, called the Population, composed by the best ants of each iteration (the elite). In the bi-objective case, this elite is composed of the ants belonging to the Pareto front of that generation, which enter the set in a FIFO-queue fashion. The implemented algorithm is a modification of the open source code developed by L. Simões et al.,[29] to which some exploration-promoting strategies have been added.

Inner Layer Each time an ant travels a full tour, it requires of an internal optimization to obtain the optimal transfers and thus the cost associated to such sequence. This is done through another heuristic algorithm which, in this case, exploits the continuous nature of the transfer problem: the Multi-Objective PSO (MOPSO) is selected.[31] Such an algorithm works with the same principles as the classic PSO in which the particles have their own motion across the search space with the influence of their own best result and that of a leader particle. However, in this case, the leader is not a single particle but the complete Pareto front, and in each iteration a random member of the non-dominated set is picked as the leading reference. Daneshjou et al. have already proven the usefulness of this algorithm for the multi-rendezvous problem.[6] The implemented algorithm is a modified version of the open-source code of V. Martínez-Cagigal.[32]

As stated in the previous section, transfers in the 2-level first step are modeled as two-burn Lambert maneuvers for which the burn arcs are simplified to be impulsive ΔVs. The Lambert maneuver is calculated through a hybrid implementation of already developed solvers. On a first calculation, a fast solution is obtained through D. Izzo's formulation.[33] However, this strategy is not extremely robust, and in case of not finding a solution, the slower but more reliable algorithm proposed by Gooding is called.[34] This way, a solution is always found, and the slower solver is only called in case of lack of convergence, speeding up the overall optimization process.[35] For each transfer, both the ΔV and time-of-flight (TOF) are calculated, after ensuring that the transfer is elliptical (such that TOF is always greater than Barker's time[24]) and that the transfer's altitude is above Earth's atmosphere limit. The fuel consumption is approximated at each burn by means of Tsiolkovski's rocket equation (Equation 3). In addition, constraints are accounted for by means of penalty functions of the form;

$$\lambda(f, L) = (\max(0, f - L))^2; \quad f = f + F \cdot \lambda(f, L) \tag{8}$$

where f is the objective, L the constraint, and F a design weight factor.

At each one of the sequence transfers' optimizations, the cost function is dynamically generated according to the order of visitation. this reduces the need to know beforehand the specific set of transfers that is to be solved, allowing for more flexibility of the tool. This dynamic cost function is explained more deeply in a previous work.[25] The output of the inner layer is a set of non-dominated solutions for the transfer sequence given by the outer loop. Nevertheless, only one solution is picked according to a certain design criteria specified beforehand, as stated above.

The output of this first 2-level step is a set of non-dominated tours with their respective costs and optimized maneuvers. However, this is just a preliminary simplified trajectory, and it would be of interest to obtain the full trajectory with the necessary control inputs to reach all orbits for injection.

Second Step

The second step consists on a NOC strategy which converts the results of the first step into a continuous set of control inputs to generate the full trajectory and refine on the cost of the full mission. This way, different environmental disturbances, as well as vehicle dynamics can be implemented and studied. For this purpose, a direct method was selected, due to the simpler implementation and

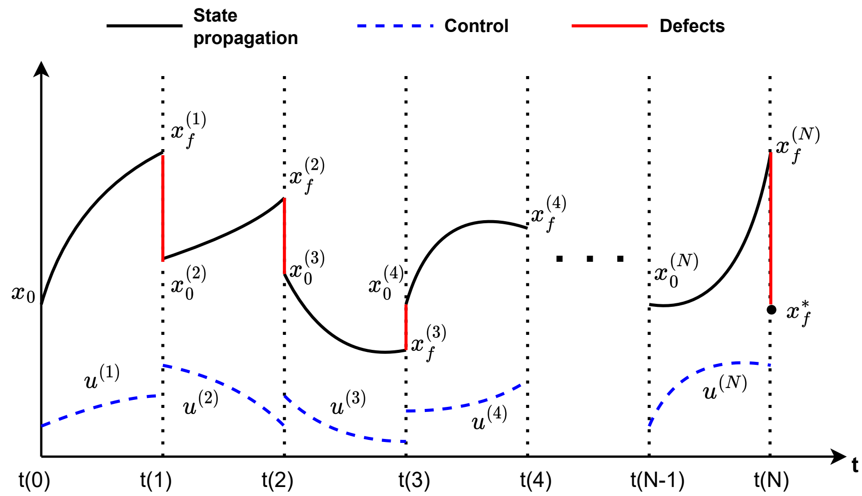

Figure 4. Schematic of the basics of Direct Multiple Shooting Methods

ability to consider the different dynamics in full in a sort of "black-box" fashion.[36] In this manner, the optimal control problem is discretized and transcribed into an NLP problem, for which different techniques exist to solve it. In particular, given the nature of the problem at hand of multiple transfers with, a multiple shooting transcription for the problem is selected. This allows to consider each one of the transfers independently for their optimization, while keeping the continuity among them as part of the constraints (defects), enabling a relatively fast but accurate result,[36] as well as parallel computing. A schematic of this approach is shown in Figure 4.

The idea behind the direct multiple shooting method is to divide the time into $N-1$ intervals, such that each one of these is considered independently for single shooting both in terms of state propagation and control function.[37] The initial states at each one of the sub-intervals (with the exception of the initial trajectory state) are unknown and considered as variables for the optimization. However, these are accounted for in the form of the aforementioned defect constraints by matching at each *t(i)* the initial state x_0^i with the end state of the previous interval x_f^{i-1}. In this manner, state continuity is enforced. While this method does increase the total optimization vector (and thus the complexity of the problem), the generated Jacobian matrix is sparse, which is easy to solve by most numerical NLP solvers.[36] The final states are accounted for in a similar manner by comparing the propagated final state x_f^N with the desired one x_f^*.

In the current problem, each transfer is considered a different phase, which allows also to implement their own different properties. This is especially interesting as the problem is characterized by the discrete change of mass due to satellite injection in-between two consecutive transfers, which is easily accounted for in this multi-phase approach. It also allows to ensure that the final state of each one of the transfers in terms of the injection orbit Kepler elements is reached, by establishing intermediate boundary conditions. Nevertheless, while the Kepler elements are intermediate boundary conditions, the final state within the orbit is still free, and thus must be matched in terms of position and velocity with the next transfer, in the way of multiple shooting.

In addition, due to the coast-burn-coast-burn sequence of each one of the transfers, these are considered as four different phases. This allows to consider the different dynamics in the same transfer shot. For instance, the lack of propulsion of the coast arcs allows for higher integration

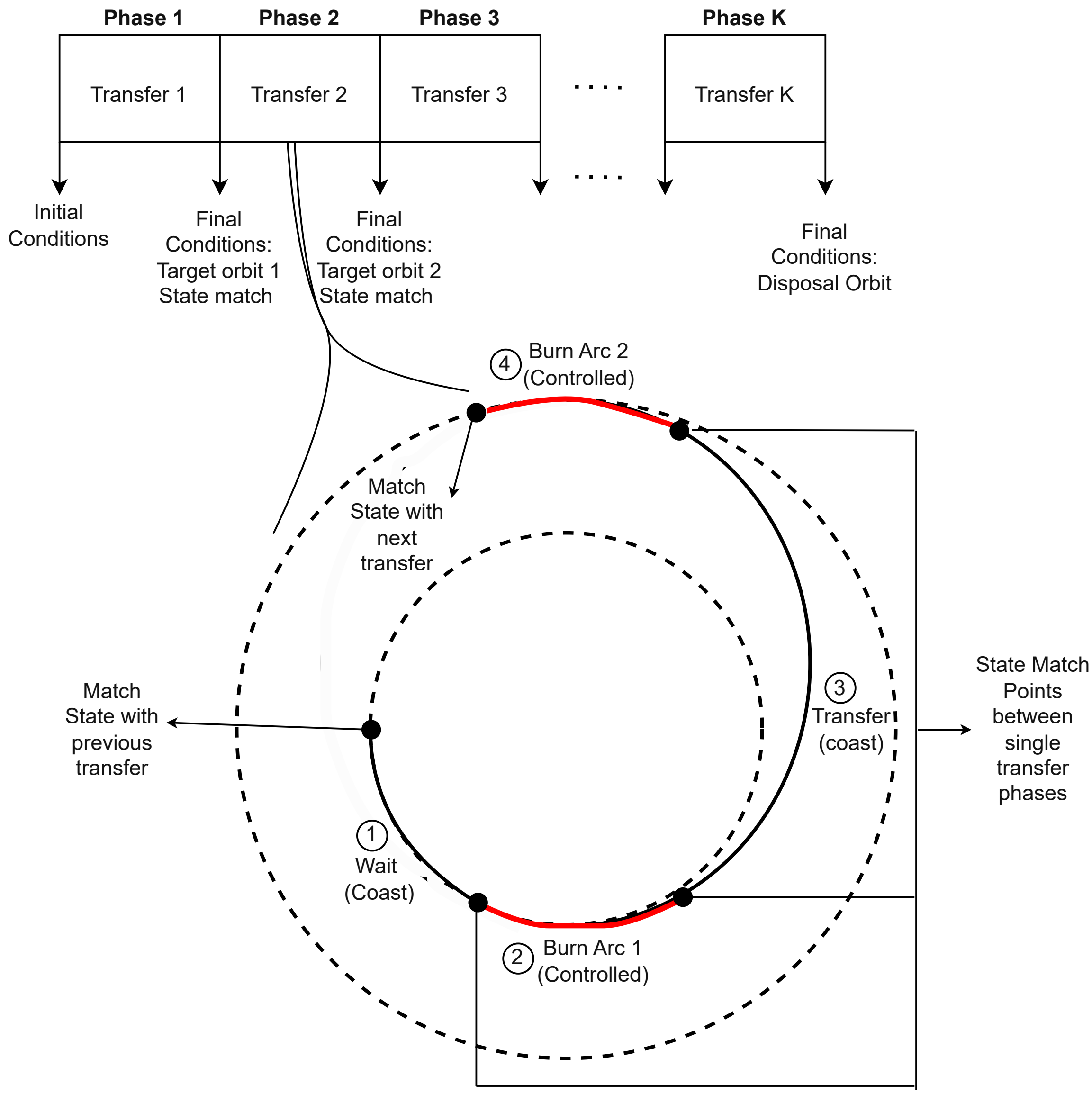

Figure 5. Schematic of the Multi-Phase Approach

steps and, since no control is required (as the engine is turned off), the variables related to the thrust direction can be ignored.[38] On the other hand, for the faster-changing burn phases a lower integration step is needed, as well as the control of the engine direction and the acceleration due to the thrust. The full direct multi-phase multiple shooting approach for the multi-orbit multi-injection mission is summarized in Figure 5, where the divisions between transfers and between arcs within a transfer are easily observed. It must be noted, however, that the total time both for the individual transfers, as for the 4 transfer phases, are not fixed but part of the optimization variables. The overall mission time is not minimized in this NOC step, as it is assumed the work performed by the previous step is sufficient, although it is still considered a requirement and it is checked for infringement.

Taking into consideration the previous description of the transcription of the problem, the following NLP problem is reached (in terms of *k* transfers and 4 arc phases per transfer):

$$
\begin{aligned}
\min \quad & \sum_{k=1}^{N}\sum_{i=1}^{4} m_{f,k,i}; \quad k = 1, ..., N;\ i = 1,2,3,4 \\
\text{s.t.} \quad & \dot{\boldsymbol{x}}_{k,i}(t) = f(t, \boldsymbol{x}_{k,i}, \boldsymbol{u}_{k,i}); \quad k = 1, ..., N;\ i = 1,2,3,4 \\
& \boldsymbol{x}(t=0) = \boldsymbol{x_0} \\
& \boldsymbol{x}_{f,k} - \boldsymbol{x}_{0,k+1} = 0; \quad k = 1, ..., N \\
& \Psi(\boldsymbol{x}_{f,k}) - \Psi(\boldsymbol{x}^*_{f,k}) = \mathrm{KE}_{f,k} - \mathrm{KE}^*_{f,k} = 0; \quad k = 1, ..., N \\
& t_{k,i} \in [t^L_{k,i}, t^U_{k,i}], \quad \boldsymbol{x}_{0,k} \in [\boldsymbol{x}^L_k, \boldsymbol{x}^U_k], \quad \boldsymbol{u}_{k,i} \in [\boldsymbol{u}^L_{k,i}, \boldsymbol{u}^U_{k,i}]; \quad k = 1, ..., N;\ i = 1,2,3,4
\end{aligned}
\tag{9}
$$

where the optimization variables are given for each transfer *k* as:

$$
\boldsymbol{c}^T = (\boldsymbol{x}^T_k,\ t_{k,1},\ \boldsymbol{u}^T_{k,2},\ t_{k,2},\ t_{k,3},\ \boldsymbol{u}^T_{k,4},\ t_{k,3})
$$

It must be noted how for the two coast arcs there are no related control input variables. In addition, as the engine works at either maximum thrust or zero (non-throttling), the control inputs are given in terms of the azimuth angle α and the cant angle β. For the present study, the NLP problem was solved using MATLAB's *fmincon* with Sequential Quadratic Programming (SQP) solver.[39]

The output of this second step is the full 3-dimensional trajectory with the necessary time-stamped control inputs to achieve the complete set of payload injections and final disposal of the vehicle. The solution trajectory is a refined optimization considering more realistic environments. In addition, the direct multiple shooting formulation allows for an easy implementation of the dynamics of the vehicle, allowing to perform analyses on sizing and systems' design.

STUDY OF CASE SCENARIOS

The tool presented is now used to analyze a specific mission scenario in an effort to show its flexibility and usefulness towards future studies, as well as the importance of the added second step.

Mission parameters

For the present study, a set of design parameters are considered, both in terms of mission and delivery vehicle. All the related values are shown in Table 1. The propulsion system considered uses a bi-propellant mixture of monomethyl hydrazine (MMH) and Mixed Oxides of Nitrogen (MON), which is based on decades of heritage and research.[40,41]

Verification of the tool

As a first step, the tool is verified by means of a simple set of transfers among co-planar circular orbits with equal inclination. The satellites to be delivered are all equal in mass. The orbital elements of the orbits to be visited are shown in Table 2, including those for the disposal orbit **D**.

The output of the first step is shown in Figure 6. It is to be noted how the possible sequences range from very fast but expensive trajectories, towards slower but remarkably cheaper ones. For most of these transfers, the foreseen sequence $\{$5-4-3-2-1$\}$ is proposed by the algorithm. As the cost to reach the fifth orbit first is not included in the cost, a sequential orbit altitude reduction from highest to lowest and then into disposal is the most logical output. However, other combinations are proposed by the algorithm. This is an effect of the randomness on which heuristic algorithms rely. While altitude-reduction behavior is obvious in this simple case, it might not be in more complex cases, and so a refinement of these results is necessary. The second step is then applied to some of these transfers to obtain the refined costs in terms of fuel mass and overall mission time. The comparison of the output from step 1 and step 2 for a set of results is shown in Table 3. The trajectory identification numbers are according to their order of appearance in the Pareto Front plot, from left to right. It is observed how for all of these transfers the fuel consumption is reduced drastically, in particular for the cases in which fuel consumption was higher. In fact, some solutions which a priori were average in terms of cost, became better results after this second step. This is the case, for instance, of Trajectory 5, which became the cheapest solution. This trajectory sequence is the previously mentioned {5-4-3-2-1}, which would explain the fact of being the cheapest. On the other hand, while there is a trend of decreasing the overall mission time, it is similar to the ones provided

Table 1. Properties of the baseline delivery spacecraft

Mission Parameters	Parameter	Description	Value
Vehicle			
	M_0	Vehicle dry mass without propulsion system	500 (kg)
	M_{prop}	Propulsion system mass	165 (kg)
	I_{sp}	Specific Impulse	330 (s)
	T_{max}	Maximum thrust	1 (kN)
	g_0	Gravitational acceleration at Earth's surface	9.81 (m/s^2)
Mission requirements			
	t_{serv}	Minimum time in-orbit to deliver a satellite	300 (s)
	h_{min}	Minimum orbital altitude allowed	120 (km)
	t_{max}	Maximum mission time	48 (hours)
	$m_{f,max}$	Maximum fuel mass	1000 (kg)

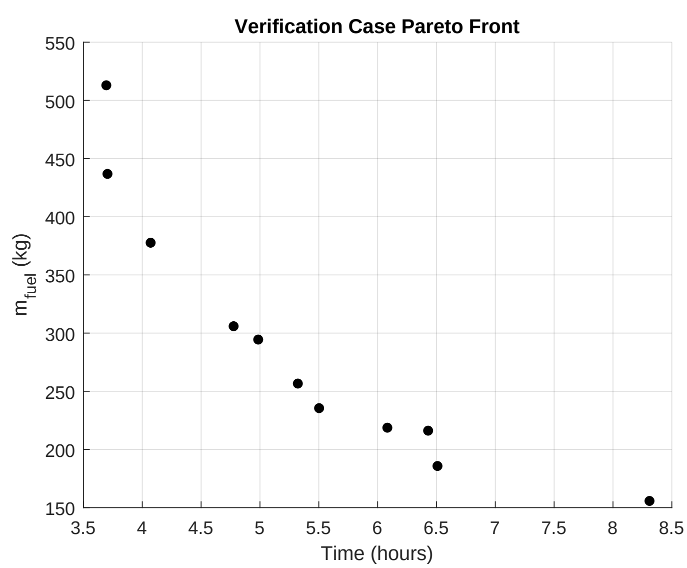

Figure 6. Output of the First Step for the Verification Case

Table 2. **Verification case orbital parameters and payloads**

Orbit ID	h (km)	e	i (rad)	Ω rad	ω (rad)	Payload mass (kg)
1	400	0	1.71	0	0	10
2	450	0	1.71	0	0	10
3	500	0	1.71	0	0	10
4	550	0	1.71	0	0	10
5	600	0	1.71	0	0	10
D	300	0	1.71	0	0	-

Table 3. Comparison of steps in verification

	First Step		Second Step	
	Time (hours)	Fuel mass (kg)	Time (hours)	Fuel mass (kg)
1	3.69	512.98	3.30	166.95
3	4.09	377.66	3.90	183.59
5	4.99	294.43	4.30	89.11
8	6.08	218.70	4.57	116.38
10	6.51	185.77	5.31	110.95
11	8.31	155.74	6.85	111.23

by the first step. In addition, the relative order in mission time among all the sequences is kept the similar. The second optimization refinement shows itself as an interesting second step, as it seems to provide better trajectories and in fact might even change the order of preference among the ones proposed by the first step. This does not invalidate the previous results of step 1, but rather refines the found best sequences, as the primary objective of the first step is to find suitable feasible orders of visitation to then be further studied.

It is interesting to check now the trajectory of one of these optimized sequences to study the performance of the optimizer. Trajectory 5 is picked, given the high decrease in fuel cost. The transfers are shown in Figure 7. Looking at these plots, it is obvious the overall resemblance of the transfers to Hohmann transfers, as one would expect, with the exception of the disposal maneuver for which it waits a full revolution before performing the final burn. This behavior verifies that in the simplest configuration, a set of chained Hohmann transfers is reached by the optimizer, which behaves exactly as the minimal fuel consumption transfer theory states.[24] Now that the optimization tool has been verified, it can be used to solve a more complex multi-satellite delivery mission.

Multi-satellite Delivery Mission

A more complicated case is considered now, involving different payload masses and orbital inclinations. In this case, as before, 5 satellites are to be delivered before the vehicle disposes of itself.

Table 4. Multi-satellite Delivery Mission orbital parameters and payloads

Orbit ID	h (km)	e	i (rad)	Ω rad	ω (rad)	Payload mass (kg)
1	624.5	0.0014	1.7087	0	0	10
2	720.0	0	1.7153	0	0	10
3	638.5	0.0061	1.7061	0	0	12
4	716	8.5e-4	1.7172	0	0	10
5	505	0	1.7006	0	0	40
D	300	0	1.71	0	0	-

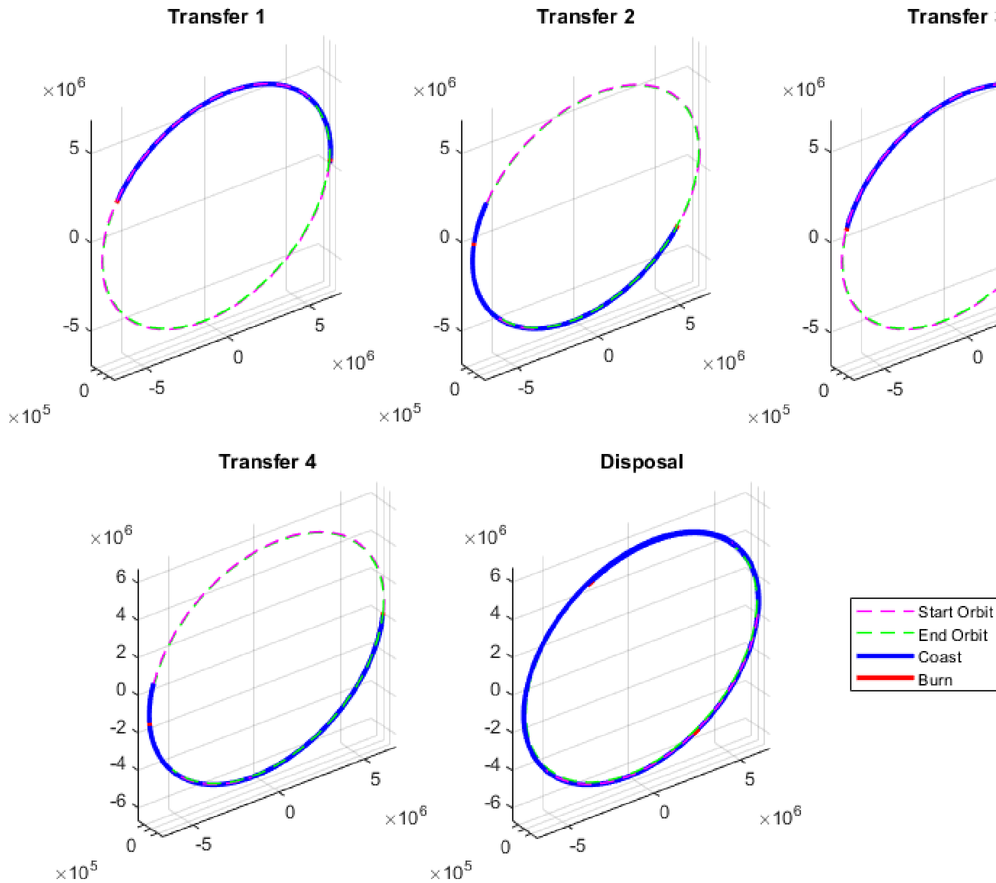

Figure 7. Transfer maneuvers for Trajectory 5 of the Verification Case

The Kepler elements and payload masses are summarized in Table 2, where the same disposal orbit is considered. The selected target orbits were randomly picked from a database generated for university-owned launched and planned satellites.[26] Among these, a subset of near-polar orbits was selected as not to include unfeasible inclination changes into the problem.

As before, the output of the first step is shown in Figure 8. Among the different possible combinations, there is a preferred {2-4-3-1-5} order in which the vehicle balances decreasing in altitude with the inclination change. This order appears up to 5 times in the set of non-dominated solutions. Another cheap alternative is to deliver first the heaviest satellite, in an effort to reduce the necessary fuel to achieve the same ΔV in the following transfers. However, the cheapest solution proposes a sequence for which the altitude is the main driving factor, showing a {4-2-3-1-5} order. As it is observed, when more complex cases are introduced, the variety of options depends on many factors which are not easy to take into account a priori. Thus the second step is done to refine the decisions of the bi-level algorithm. The newly computed costs are shown in Table 5. The trajectory identification numbers are according to their order of appearance in the Pareto Front plot, from left to right. In this case, while the fuel consumption can be greatly reduced, the changes within the order of the non-dominated points is not that drastic, and is done in the intermediate points. On the other hand, the cheapest and longest mission keeps its position, as does the most expensive and fastest one. The

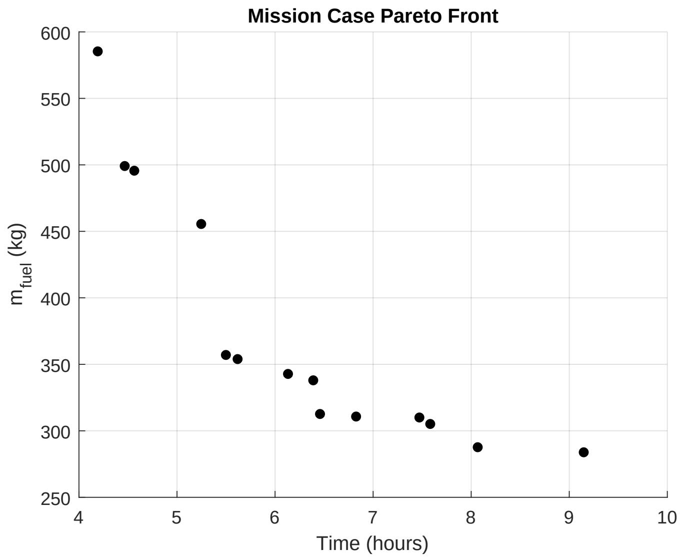

Figure 8. Output of the First Step for the Multi-satellite Delivery Mission

Table 5. Comparison of steps in verification

	First Step		Second Step	
	Time (hours)	Fuel mass (kg)	Time (hours)	Fuel mass (kg)
1	4.19	585.36	3.32	341.72
3	4.56	459.61	3.99	201.34
8	6.39	337.98	5.32	211.32
10	6.83	310.75	5.46	204.27
12	7.58	305.16	6.15	229.57
14	9.15	283.85	7.35	184.06

most interesting case is for Trajectory 3, which becomes the second cheapest after being the second most expensive one. Similarly to the verification case, the time distribution remains similar. Once again, it is proven that the second step is necessary to further refine the solution and filter possible solutions that otherwise might have been overlooked.

Finally, the cheapest solution's trajectory (Trajectory 14) is now checked to observe the proposed maneuvers. The transfers for this trajectory are shown in Figure 9. It is observed how, in general, the tool suggests performing the maneuvers near the equatorial plane, which is where the orbits intersect. Such behavior corresponds to the usual theory for inclination change maneuvers.[24] However, longer times are expected in between the transfer arcs, a behavior attributed to the effect of the first guess of the first step, as the algorithm tends to look around that solution for the optimal one.

With this multi-delivery case scenario, it has been shown the logic behind the decisions performed by the 2-level first step in order to achieve the best sequence order, but also the necessity of the second step to refine those solutions and obtain the true optimal trajectories.

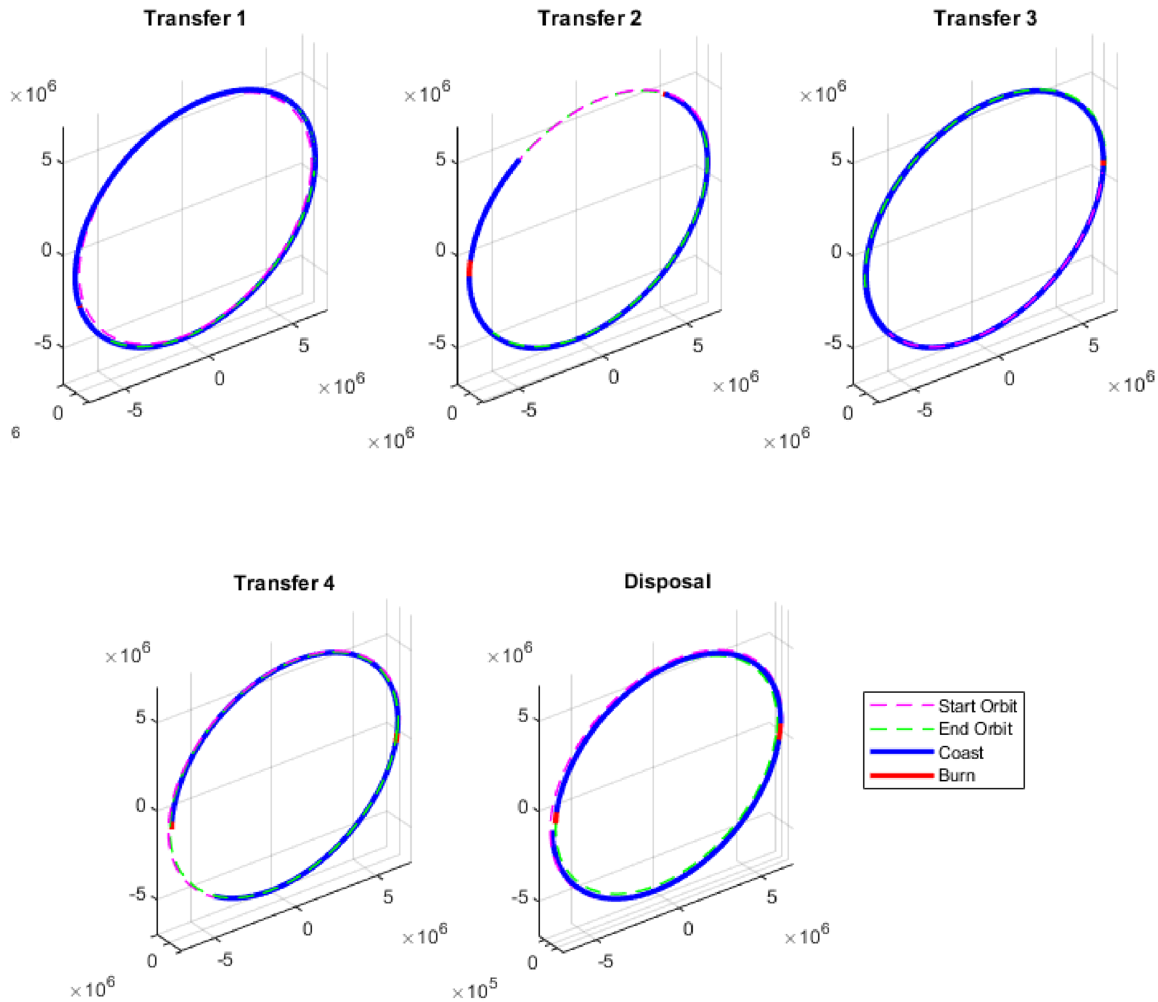

Figure 9. Transfer maneuvers for Trajectory 14 of the Multi-Delivery Mission

CONCLUSIONS

The study proposes a 2-step optimization tool which allows to solve the multi-payload multi-orbit injection problem minimizing both the fuel consumed and the total time of mission. The tool is composed of two steps, such that first it establishes the set of optimal delivery sequences and then it refines the optimal trajectory using more realistic physics modeling. It is constructed of two differentiated phases: 1) a bi-level heuristic optimizer that solves the MINLP providing the sequences and approximated costs; and 2) a NOC algorithm which uses these as a first guess to obtain the full controlled optimal trajectory associated to a sequence.

The first step is composed of a nested structure of two heuristic algorithms, so that the problem is divided into its integer and continuous parts. The outer loop, a multi-objective P-ACO algorithm, solves for the sequence order. The algorithm uses as cost function the output of the inner loop.

This one is composed by a MOPSO algorithm which optimizes the impulsive Lambert maneuvers needed to achieve the orbital sequence given by the outer loop. The algorithm does not require of preliminary knowledge of the orbit sequences, as it can dynamically generate a cost function. The output of this step is a set of non-dominated tours with their corresponding set of preliminary optimized maneuvers. The second step uses this information to convert it into full controlled optimal trajectories by means of a NOC strategy. The implemented algorithm is a direct multi-phase multiple shooting transcription which allows easy implementation of the environmental and vehicle dynamics, allowing for flexibility in the range of possible studies. This formulation allows to convert the NOC into a series of NLP problems. The transfers are considered as different phases, such that parallel computing can be performed and facilitating the discrete mass changes. At the same time, each transfer is considered to have 4 phases, enabling a simple implementation of the different dynamic and control regimes. The output of this second step is the full optimal trajectory and associated control input necessary to reach all desired orbits.

The tool was first verified using a simple case of payload delivery in which only altitude changes were necessary. It was shown that the second step could drastically change the cost of the different sequences, highlighting some of them above the others. In fact, it could improve the internal optimization done by the MOPSO algorithm after its first guess. In addition, the trajectory shown by the cheapest solutions was similar to a set of Hohmann transfers, which is to be expected from changes in altitude between co-planar circular orbits, thus verifying the tool. Then, it was used to solve a more complex multi-delivery mission scenario, with changes in orbit shape and inclination. Once again the second step proved itself useful both to rearrange different sequences among the set of non-dominated solutions, and to refine the costs of the first guesses. The cheapest trajectory was also studied in terms of trajectory, showing that most burn arcs were done around the equator, where the orbits cross. Overall, it was shown that performing the second step is crucial to achieve more realistic and refined optimal solutions.

The result of this paper is a flexible optimization tool which can provide full, accurate, optimal trajectories and the associated control law to the multi-orbit delivery mission scenario. It requires no previous knowledge apart from the orbital parameters of all payloads to be injected, and can easily accommodate environment and vehicle dynamics for their analysis. Ultimately, the tool shows itself to be of usefulness for the mission analysis and vehicle design, as well as for the GNC design, for the multi-rendezvous type of mission. In fact, it is easily implementable in an iterative design process of the upper stage, contributing to the multidisciplinary design approach. In addition, its flexibility allows the tool's usage in other multi-orbit targeting missions such as ADR and OOS, thus contributing to the development tools for the design of these new and increasingly needed types of missions.

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