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Instability Mechanisms in the Compressible Falkner–Skan Boundary Layers with A Cooled Wall

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Abstract. We study the stability of compressible Falkner Skan boundary layers over a wedge at hypersonic speeds (target location $Ma_e = 7$). For a cooled wall with temperature ratio $T_{ratio} = 0.25$ (wall temperature scaled by the corresponding adiabatic value), we compute accurate basic solution via Laguerre spectral method and perform a temporal linear stability analysis to locate the critical Reynolds number Re_c and streamwise wavenumber α_c. A key result is quite unexpected: as the favorable pressure-gradient parameter β increases, Re_c decreases (from 115.3 at $\beta = 0$ to 83.5 at $\beta = 0.3$), opposite to the classical incompressible Falkner-Skan boundary layer. In contrast, for an adiabatic wall ($T_{ratio} = 1$), Re_c increases with β (from 107.7 to 153.1), recovering the classical behavior. A fluid–thermodynamic energy decomposition shows that vortical contributions to the instability are predominantly stabilizing, whereas the thermal and acoustic contributions strengthen with β under cooled wall conditions, thereby lowering the stability threshold.

Introduction

The Falkner–Skan equation, introduced by Falkner and Skan in 1930 [1], provides similarity solutions work for boundary layers over wedges subject to pressure gradients depending on the parameter β. While the incompressible case is classical, its compressible counterpart in hypersonic regimes remains less explored. We employ a Laguerre spectral method based on modified Laguerre bases [2] to solve both the base flow as well as the linear stability equations in a temporal setting. By satisfying the far-field boundary condition naturally, this method avoids the singularities of mapping [3] and the domain-size-dependent truncation error [4], delivering spectral accuracy on semi-infinite domains. After a high-accuracy basic solution is obtained, we apply the temporal linear stability analysis [5] to determine Re_c and α_c for adiabatic and cooled walls. For the latter, we interpret the observed trends using a fluid–thermodynamic energy decomposition [6] that separates the perturbation energy into vortical, thermal, and acoustic energy components.

Theory and Numerical Methods

Referring to the work of A. F. Stephansen and L. Quartapelle [7], the compressible Falkner- Skan boundary layer equation is shown as follows:

$$\begin{cases} f'''(\eta) + f(\eta)f''(\eta) + \beta\left(g(\eta) - f'^2(\eta)\right) = 0, \\ g''(\eta) + Prf(\eta)g'(\eta) = 2(1-Pr)\big(f'(\eta)f''(\eta)\big)', \end{cases} \tag{1}$$

where $f(\eta)$ and $g(\eta)$ represent the dimensionless stream function and total enthalpy, respectively. The parameter β denotes the pressure-gradient factor, while the Prandtl number Pr is fixed at $Pr = 0.72$ in the present study. The corresponding boundary conditions are:

$$\begin{cases} f(0) = 0, f'(0) = 0, \lim_{\eta\to\infty} f'(\eta) = 1, \\ g(0) = g_{\text{wall}}(\text{isothermal wall}), \lim_{\eta\to\infty} g(\eta) = 1. \end{cases} \tag{2}$$

After computing the basic solution, a linear stability analysis is carried out following Malik [5] to extract the critical curve (Re, α). To interpret the physics, we adopt the fluid– thermodynamic decomposition of Unnikrishnan and Gaitonde [6].

Results and Discussion

Table 1: Summary of flow parameters at the inlet for different pressure-gradient parameters β. (The subscript 'e0' indicates external conditions evaluated at the inlet).

β	ρ_e [kg/m^3]	μ_{e0} [kg/[m·s]]	T_{e0} [K]	Ma_{e0}	P_{e0} [Pa]	U_{e0} [m/s]
0.000	0.0175	0.00000504	74.0740	7.000	371.626	1207.636
0.100	0.1169	0.00001082	158.416	4.500	5315.808	1135.316
0.200	0.3031	0.00001505	231.884	3.500	20170.761	1068.337
0.300	0.8824	0.00002099	355.556	2.500	90043.072	944.928

The inlet parameters listed in Table 1 correspond to different pressure-gradient factors β. They are selected so that, for each case, the external flow conditions at the target location ($Ma_e = 7$) match those of the Blasius flow ($\beta = 0$). Particularly, the wall temperature T_{wall} (cold wall) is obtained by $T_{wall} = 0.25\ T_{adb}$, where T_{adb} is defined as$T_{adb} = T_e\,(1 + Pr^{1/2}(\gamma\text{-}1)\ Ma_e^2/2)$. With a cooled wall ($T_{ratio} = 0.25$), increasing the favorable pressure-gradient parameter from $\beta = 0$ to 0.3 lowers the critical Rynolds number from $Re_c = 115.3$ to 83.5 (Fig.1), in contrast with the classical incompressible Falkner-Skan boundary layer where Re_c increases with β. The critical wavenumber α_c also decreases with β, indicating longer wavelengths at the onset of instability, while the shape of the neutral curves is independent of β, suggesting a common instability mechanism. For an adiabatic wall ($T_{ratio} = 1$) opposite trend is observed as shown in Table 2. Re_c rises from 107.7 to 153.1 as β increases from 0 to 0.3 (≈42%). α_c grows slightly (0.082→0.085), in line with the incompressible case, highlighting the important role of wall temperature in hypersonic boundary-layer flows.

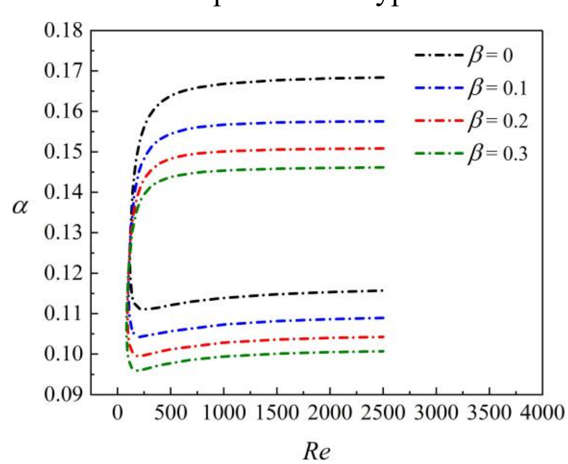
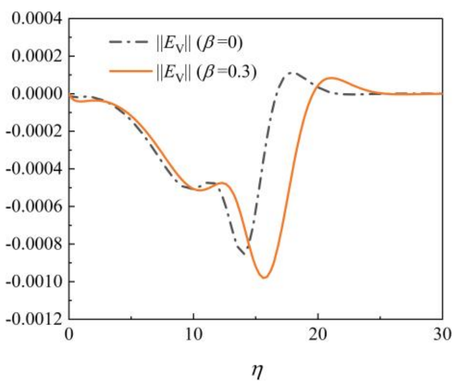


Figure 1: Neutral stability curves for four pressure-gradient parameters β, plotted in theReynolds number–streamwise wavenumber (Re-α) plane (T_{ratio} = 0.25, Pr = 0.72, Ma_e = 7)

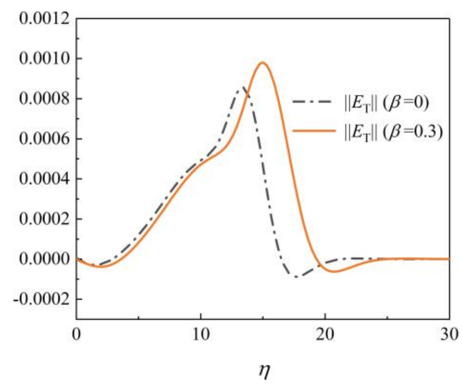
Table 2: Critical parameters for an adiabatic wall T_{ratio} = 1 with pressure-gradient parameters β = 0, 0.1, 0.2, and 0.3.

β	Re_c	α_c
0.000	107.7	0.082
0.100	125.3	0.083
0.200	140.3	0.084
0.300	153.1	0.085

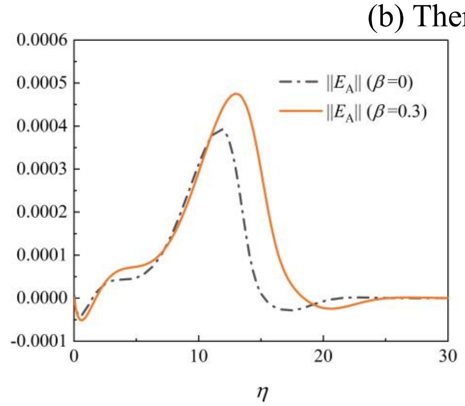
To elucidate the physics behind the surprising decrease of Re_c with increasing β under a cooled-wall condition, we employ a fluid-thermodynamic decomposition as shown in Fig. 2, which compares the spatial distributions of the perturbation-energy components: vortical (E_V), thermal (E_T), and acoustic (E_A). The vortical contribution E_V is negative over most of the boundary-layer thickness and reaches its largest magnitude in the outer region, indicating a stabilizing effect. By contrast, the compressible E_T and E_A are predominantly positive and peak toward the outer edge, thereby supplying energy to the disturbance and enhancing the instability. Under the cooled-wall condition (T_{ratio} = 0.25), increasing the favorable pressure-gradient from β = 0 to β = 0.3 amplifies the compressible components: both E_T and E_A show higher peaks and wider positive support. This shift increases the perturbation energy gain and leads to the reduction in Re_c observed as β grows.



(a) Vortical energy



(b) Thermal energy



(c) Acoustic energy

Figure 2: Comparison of the perturbation energy components E_V (vortical), E_T (thermal), and E_A (acoustic) for β = 0 and β = 0.3 (cooled wall with T_{ratio}=0.25).

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Summary
We analyzed hypersonic ($Ma_e = 7$) compressible Falkner-Skan boundary layers under cooled ($T_{ratio} = 0.25$) and adiabatic ($T_{ratio} = 1$) wall conditions. Using a spectral discretization for the basic solution and linear stability analysis, we mapped the neutral curves in the Re-α plane for $\beta \in [0, 0.3]$. A robust unexpected trend emerges for cooled walls, where Re_c decreases when β increases—while in the adiabatic case the classical increase is observed. The decomposition of perturbation energy indicates stabilization of the vortical contribution and a destabilization of the thermal/acoustic contributions with increasing β, which explains the decreases in the instability threshold.

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