Design of a Switched Control Lyapunov Function for Mobile Robots Aggregation

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Abstract: This paper proposes a novel aggregation strategy for a network of mobile wheeled robots with constrained dynamics. The strategy assumes a centralized control architecture, which collects all the robot positions and generates the control signals sent to the robots in the network. To do this a control Lyapunov function (CLF) based approach is designed relying on a switched formulation of the robot models. Such a formulation is in fact made possible by constraining the robot motion only to rotation and roto-translation in the plane. Moreover, a collision avoidance objective is taken into account in the design of the CLF. The approach is analyzed, and simulations as well as experiments with six robots show its effectiveness and practical applicability.

1 INTRODUCTION

Nowadays, distributed autonomous robotic systems are increasingly applied in industrial and field operations, ranging from material delivery to precision farming. In particular, advances and many researches have been devoted to the so-called *swarm robotics*, according to which a large number of mobile robots is coordinated to achieve desired performance, e.g., average consensus and leader-follower tracking (Tzafestas, 2012). The desired collective motion, which is determined by the interaction of the robots among each other or with the environment, is in fact appropriate to artificially emulate the behaviour of many multi-agent systems present in nature.

1.1 Brief literature overview

In the context of distributed robotics, many control methodologies have been developed in order to guarantee stability, taking into account the presence of disturbances, communication constraints among the agents, plug-and-play capabilities, and inaccuracies due to sensors and actuators. Traditionally, the so-called *principle of locality* has been exploited in distributed robotics, relying on the capability of the robots to make use of information from their immediate surroundings (Parker et al., 2016; Brambilla et al., 2013). Specifically, the robots viability property is captured by a function designed to measure the performance, and the goal becomes that of finding the best approach to optimize such a cost. Among many works in the literature, in (Beni, 1988; Bonabeau et al., 1999), motivated by the collective behaviour in social insect colonies and other animal communities, a specific form of collective intelligence has been introduced making the agents capable to self-organize and produce predefined patterns.

Other applications of swarm robotics are devoted to transportation of large objects, surveillance, formation control in autonomous aircrafts or water vehicles, see (Brambilla et al., 2013) for an overview on this topic. Moreover, stability properties of these systems have been formally investigated e.g., in (Gazi and Passino, 2003; Gazi and Passino, 2004). Among the used control approaches, in (Desai et al., 1998) for instance a feedback linearization control technique is proposed, making use only of local information to de-

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rive a controller that exponentially stabilizes the relative distances between the robots of the system. In (Egerstedt and Hu, 2001; Leonard and Fiorelli, 2001; Ogren et al., 2001; Bachmayer and Leonard, 2002), the concept of control Lyapunov functions (CLFs) is introduced to formalize a constrained formation control problem. Moreover, in (Jin et al., 1994; Beni and Liang, 1996), Lyapunov based methods are discussed for one and two dimensional swarm structures, showing synchronous and asynchronous convergence of the network to a predefined configuration. Other works, as (Saska, 2016), rely instead on model predictive control strategies to achieve collision-free and deadlock free navigation by multiple wheeled mobile robots.

1.2 Contribution

Inspired by (Gauci et al., 2014), in this paper we present an original formulation of a multi-robot system recast in the framework of switched systems in order to solve a rendezvous control problem. Specifically, the main goal is to design a control approach capable of making the robots in the network aggregate, while guaranteeing collision avoidance. As in (Gauci et al., 2014), a group of differential wheeled robots aggregate by using only two modes of motion, which represent rotation and roto-translation. Differently from (Gauci et al., 2014), where each robot chooses its mode based on the reading value of a binary line-of-sight sensor, in the present paper, a centralized approach is pursued where however the amount of information sent to each robot is reduced to only one bit. This latter approach offers superior performance, and, by assuming idealised sensing, could provide an upper bound for the performance of systems with concrete sensing implementations. In the present paper, the constrained dynamics is recast into a switched system with two modes. A centralized control architecture is then proposed, so that all system information are collected to generate the control commands sent to each robot in the network. In particular, a CLF is designed proving stabilization of the controlled multi-robot network, and also enabling a collision avoidance property.

1.3 Outline of the paper

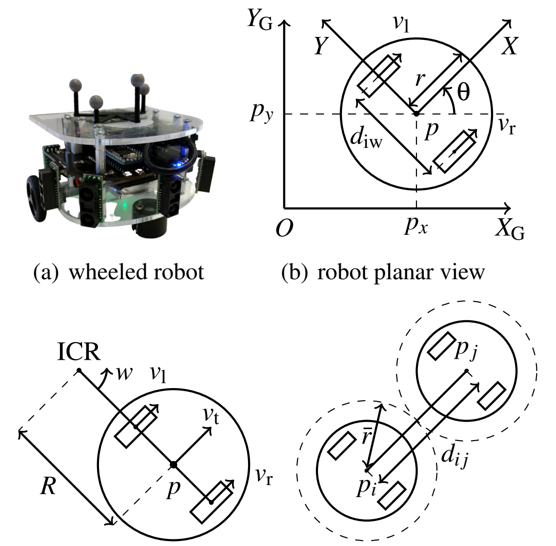
The paper is organized as follows. In Section 2 the model of the considered robot is defined and the control problem is formulated. In Section 3 the switched version of the multi-robot system is derived, while the proposed CLF based aggregation approach is introduced and analyzed in Section 4. Simulation and experimental results are illustrated in Section 5, and some conclusions are finally drawn in Section 6.

2 MODELLING AND PROBLEM FORMULATION

In this section the model of the considered wheeled robots is described. Specifically, our proposal is based on the kinematic unicycle-like model of the robot.

2.1 Robot model

Consider a wheeled robot in the plane as illustrated in Figure 1(a). It consists of two individually controlled



(a) wheeled robot (b) robot planar view

(c) ICR point (d) robot aggregation

Figure 1: Wheeled robot (a), schematic rendering in the plane (b) with instantaneous center of rotation (c), and aggregation of two robots.

wheels, and a third passive castor wheel to provide stabilization of the structure. Such a configuration allows the robot to move in the plane indicated with the global frame $O - X_G, Y_G$, while the robot frame is given by $p - X, Y$ (see Figure 1(b)). Note that, for the sake of simplicity, a symmetric circular body with center of mass and centroid coinciding with the robot position $p = [p_x \, p_y]^\top$ is considered. Moreover, let d_{iw} be the inter-wheel distance, v_l and v_r be the left and right wheel linear velocities, r the body radius and θ the robot orientation. To reasonably reduce the complexity of the model, the passive castor wheel is ignored and the rolling motion of the main wheels is instead constrained by the existence of the instantaneous center of rotation (ICR), that is the point lying on the common lateral axis of the wheels (see Figure 1(c)). The angular speed is indicated as w, such that $v_\mathrm{r} := w\left(R + \frac{d_\mathrm{iw}}{2}\right)$ and $v_\mathrm{l} := w\left(R - \frac{d_\mathrm{iw}}{2}\right)$, with R being the distance between the ICR and the centroid p.

Now, consider the robot centroid p and its linear velocity v_t projected in the global frame as

$$v_\mathrm{t} = wR = \frac{v_\mathrm{r} + v_\mathrm{l}}{2}, \tag{1a}$$

$$v_{\mathrm{t}x} = v_\mathrm{t} \cos\theta, \tag{1b}$$

$$v_{\mathrm{t}y} = v_\mathrm{t} \sin\theta. \tag{1c}$$

In order to write a state-space representation of the system, we define the state vector as the robot pose (i.e., position and orientation) $x = [p_x\, p_y\, \theta]^\top$, while the input vector is given by $u = [v_\mathrm{r}\, v_\mathrm{l}]^\top$. Therefore, one can write (1) as the time-invariant nonlinear system

$$\dot{x} = \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{\theta} \end{bmatrix} = f(x,u) = \begin{bmatrix} \left(\frac{v_\mathrm{r}+v_\mathrm{l}}{2}\right)\cos\theta \\ \left(\frac{v_\mathrm{r}+v_\mathrm{l}}{2}\right)\sin\theta \\ \frac{1}{d_\mathrm{iw}}(v_\mathrm{r} - v_\mathrm{l}) \end{bmatrix}. \tag{2}$$

Note that, the dependence of all the variables on time is omitted when obvious for the sake of simplicity.

2.2 Problem statement

Consider a network of N identical wheeled robots with model as in (2). The problem to solve consists of designing a control strategy constrained to dynamics (2) with two modes, capable of making the robots aggregate, and avoiding collisions among them (see Figure 1(d), as an example where the aggregation of two robots without collision is illustrated).

3 SWITCHED ROBOT SYSTEM

Now, by virtue of the robot dynamics constrained to only two modes (rotation and roto-translation, in analogy with (Gauci et al., 2014)), model (2) can be recast in the framework of switched systems and extended to the case of a switched multi-robot system. In the following, let $x^{[i]}$ be the state of the ith robot in the network, with $i = 1, \ldots, N$. Consider the rotation around the center of the robot and around its ICR determined by the two pairs of velocities $(v_{\mathrm{r}0}, v_{\mathrm{l}0})$ and $(v_{\mathrm{r}1}, v_{\mathrm{l}1})$, respectively. Then, let $\sigma^{[i]}(t) \in \mathcal{M}$, with $\mathcal{M} := \{0,1\}$, be the so-called switching signal, so that the motion mode is realized based on the following input variables,

$$u^{[i]} = [v_\mathrm{r}^{[i]}\, v_\mathrm{l}^{[i]}]^\top := \begin{cases} [v_{\mathrm{r}0}, v_{\mathrm{l}0}]^\top, & \sigma^{[i]} = 0 \\ [v_{\mathrm{r}1}, v_{\mathrm{l}1}]^\top, & \sigma^{[i]} = 1. \end{cases} \tag{3}$$

Therefore, letting $f^{[i]}_{\sigma^{[i]}(u^{[i]})}(x^{[i]}) := f(x,u)$, the switched nonlinear time-invariant system corresponding to (2) is

$$\dot{x}^{[i]} = f^{[i]}_{\sigma^{[i]}(u^{[i]})}(x^{[i]}), \quad x^{[i]}(0) = x_0^{[i]}, \tag{4}$$

where $f^{[i]}_{\sigma^{[i]}(u^{[i]})}$ belongs to the set of vector fields $\{f_0^{[i]}, f_1^{[i]}\}$, with

$$f_0^{[i]} := \begin{bmatrix} 0 \\ 0 \\ \frac{1}{d_\mathrm{iw}}(v_{\mathrm{r}0} - v_{\mathrm{l}0}) \end{bmatrix},$$

$$f_1^{[i]} := \begin{bmatrix} \left(\frac{v_{\mathrm{r}1}+v_{\mathrm{l}1}}{2}\right)\cos\theta^{[i]} \\ \left(\frac{v_{\mathrm{r}1}+v_{\mathrm{l}1}}{2}\right)\sin\theta^{[i]} \\ \frac{1}{d_\mathrm{iw}}(v_{\mathrm{r}1} - v_{\mathrm{l}1}) \end{bmatrix},$$

indicating rotation and roto-translation, respectively. Finally, the robot network model is obtained by re-defining $x = [x^{[1]\top} \ldots x^{[N]\top}]^\top$, $v = [u^{[1]\top} \ldots u^{[N]\top}]^\top$, and a switching string $\Sigma(v) \in \mathcal{M}^N$, with $\mathcal{M}^N$ containing all the possible 2^N switching signals configurations, so that

$$\dot{x} = f_{\Sigma(v(t))}(x), \quad x(0) = x_0, \tag{5}$$

where $f_{\Sigma(v(t))}$ belongs to the set $\{f_1, \ldots, f_{2^N}\}$, with

$$f_1 := \begin{bmatrix} f_0^{[1]} \\ \vdots \\ f_0^{[N]} \end{bmatrix},\ f_2 := \begin{bmatrix} f_1^{[1]} \\ \vdots \\ f_0^{[N]} \end{bmatrix}, \ldots,\ f_{2^N} := \begin{bmatrix} f_1^{[1]} \\ \vdots \\ f_1^{[N]} \end{bmatrix}.$$

4 THE PROPOSED STRATEGY

We are now in a position to introduce the proposed control strategy to solve the problem stated in §2.

More precisely, our approach is based on the notion of CLF (Sun and Zhao, 2001; Freeman and Kokotovic, 2008). In general, given a system of the form $\dot{x} = f_\sigma(x)$, with the vector field Lipschitz continuous and $\sigma \in \mathcal{M}$, a function $V(x)$ is a CLF for the switched system if $V(x)$ is continuously differentiable, positive definite and $V(0) = 0$, and for any $x \neq 0$, there always exists σ such that $\dot{V}(x) = \nabla V(x) f_\sigma(x) < 0$.

4.1 CLF aggregation strategy

In order to simplify the design of CLF, we take into account only the robot positions without orientation, hence the robots are considered as point masses. It is worth mentioning that the embodied property of the robots is a posteriori contemplated by the insertion of a proper weighting matrix into the control law. Moreover, we consider a centralized architecture capable of retrieving and sending all the needed information to the robots.

Consider Figure 1(d), and let $d_{ij}(x^{[i]}, x^{[j]})$, $\forall i, j = 1, \dots, N$, $i \neq j$ be the reciprocal distance function between the centers of two robots, defined as

$$d_{ij} = \sqrt{(p_x^{[i]} - p_x^{[j]})^2 + (p_y^{[i]} - p_y^{[j]})^2}, \tag{6}$$

whose derivatives are given by

$$\dot{d}_{ij} = \frac{1}{d_{ij}} \Big[(p_x^{[i]} - p_x^{[j]})(\dot{p}_x^{[i]} - \dot{p}_x^{[j]}) + (p_y^{[i]} - p_y^{[j]})(\dot{p}_y^{[i]} - \dot{p}_y^{[j]}) \Big]. \tag{7}$$

Now, collect all the distances and their derivatives in the column vectors d and $\dot{d}$, respectively, by omitting the repeated values $d_{ij} = d_{ji}$. The candidate CLF is then selected as

$$V(x) := \frac{1}{2} d^\top d. \tag{8}$$

In order to prove the stability property of the proposed approach, compute the derivative of $\dot{V}_{ij}$, so that one has

$$\begin{aligned}
\dot{V}_{ij}(x) =& d_{ij}\dot{d}_{ij} \\
=& \Big[(p_x^{[i]} - p_x^{[j]})(\dot{p}_x^{[i]} - \dot{p}_x^{[j]}) \\
& + (p_y^{[i]} - p_y^{[j]})(\dot{p}_y^{[i]} - \dot{p}_y^{[j]}) \Big] \\
=& \sigma^{[i]} \Big[\tfrac{1}{2}(v_{\mathrm{r}1}^{[i]} + v_{\mathrm{l}1}^{[i]}) \Big((p_x^{[i]} - p_x^{[j]}) \cos\theta^{[i]} + \\
& (p_y^{[i]} - p_y^{[j]}) \sin\theta^{[i]} \Big) \Big] - \\
& \sigma^{[j]} \Big[\tfrac{1}{2}(v_{\mathrm{r}1}^{[i]} + v_{\mathrm{l}1}^{[i]}) \Big((p_x^{[i]} - p_x^{[j]}) \cos\theta^{[j]} + \\
& (p_y^{[i]} - p_y^{[j]}) \sin\theta^{[j]} \Big) \Big] \\
=& \sigma^{[i]} g_{ij}(x) + \sigma^{[j]} g_{ji}(x),
\end{aligned} \tag{9}$$

where

$$g_{ij}(x) := \Big[\tfrac{1}{2}(v_{\mathrm{r}1}^{[i]} + v_{\mathrm{l}1}^{[i]}) \Big((p_x^{[i]} - p_x^{[j]}) \cos\theta^{[i]} + (p_y^{[i]} - p_y^{[j]}) \sin\theta^{[i]} \Big) \Big]. \tag{10}$$

The whole derivative of V can be therefore expressed in a compact form as

$$\dot{V}(x) = d^\top \dot{d} = \begin{bmatrix} \sigma^{[1]} & \dots & \sigma^{[N]} \end{bmatrix} \begin{bmatrix} g_1 \\ \vdots \\ g_N \end{bmatrix}, \tag{11}$$

with $g_i := \sum_{k=1, k\neq i}^{N} g_{ik}$.

The previous expression leads to a linear combination of state functions weighted by the switching signals. Therefore, by a proper choice of the switching string such that

$$\sigma^{[i]} = \begin{cases} 1, & g_i < 0, \\ 0, & g_i \geq 0. \end{cases}, \tag{12}$$

it can be guaranteed that $\dot{V}(x)$ is semi-negative definite, i.e.,

$$\dot{V}(x) = \sum_{1}^{N} \sigma_i g_i(x) \leq 0. \tag{13}$$

The previous inequality means that when $\dot{V} = 0$, then the robots stop and start to rotate around their centroid.

Remark 4.1 (Tracking control problem)**.** *Note that tracking goal can be achieved by including in* (8) *an additional term d_o depending on the distance between each robot and a target position $p^o = [p_x^o \, p_y^o]^\top$, i.e.,*

$$V(x) := \frac{1}{2} d_o^\top d_o + \frac{1}{2} d^\top d. \tag{14}$$

Following the same previous reasoning, one obtains

$$\begin{aligned}
\dot{V}(x) &= \begin{bmatrix} \sigma^{[1]} & \dots & \sigma^{[N]} \end{bmatrix} \begin{bmatrix} h_1(x) + g_1(x) \\ \vdots \\ h_N(x) + g_N(x) \end{bmatrix} \\
&= \sum_{1}^{N} \sigma^{[i]} \left[h_i(x) + g_i(x) \right] \leq 0,
\end{aligned} \tag{15}$$

where

$$h_i(x) := \Big[\frac{1}{2}(v_{r1}^{[i]} + v_{l1}^{[i]}) \Big((p_x^{[i]} - p_x^{o}) \cos\theta^{[i]} + (p_y^{[i]} - p_y^{o}) \sin\theta^{[i]} \Big) \Big], \forall i = 1, \dots, N. \tag{16}$$

□

4.2 Collision avoidance property

Although the strategy previously presented allows to achieve the desired robot aggregation, it does not take into account possible collisions among the robots. However, having in mind field implementation, it is instrumental to consider the embodied property of the robots and enable a collision avoidance strategy. This property can be formalized as a constraint by imposing that the distance between any two robots has to be $d_{ij} > 2r$. A way to introduce this constraint into the control law consists in the manipulation of the selected CLF (8).

Let $\bar{d}$ be a positive scalar parameter greater than $2r$. The modified CLF $V(x)$ becomes

$$V(x) := \frac{1}{2}(d - \hat{d})^\top (d - \hat{d}). \tag{17}$$

where $\hat{d}$ is a column vector with all components equal to $\bar{d}$.

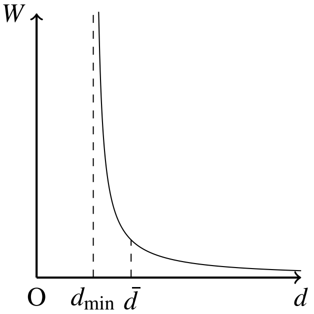


Figure 2: Weighting function W.

Additionally, let us introduce the desired minimum distance between two robots $d_{\min}$, such that $2r \le d_{\min} < \bar{d}$. This allows us to define a state-dependant weighting function $\frac{1}{(d_{ij}-d_{\min})^2}$ (see Figure 2), aimed at penalizing the cost with increasing value as d_{ij} tends to $2r$ when $d_{ij} < \bar{d}$. Such a weighting function is included in the CLF as a diagonal weighting matrix given by

$$W := \begin{bmatrix} \frac{1}{(d_{12}-d_{\min})^2} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{(d_{N-1N}-d_{\min})^2} \end{bmatrix}, \tag{18}$$

so that

$$\begin{aligned} V(x) =& \frac{(d_{12}-\bar{d})^2}{2(d_{12}-d_{\min})^2} + \cdots + \frac{(d_{N-1N}-\bar{d})^2}{2(d_{N-1N}-d_{\min})^2} \\ =& \frac{1}{2}(d-\hat{d})^T W (d-\hat{d}). \end{aligned} \tag{19}$$

Since for $d_{ij} \ge \bar{d}$ the modified CLF (19) according to the designed weighting function W (see Figure 2) would make the robot move away from $\bar{d}$, a state-dependant parameter $\alpha_{ij}(d_{ij})$ is introduced as domain selector for each d_{ij} in order to solve this issue, that is

$$\alpha_{ij}(d_{ij}) = \begin{cases} (d_{ij}-d_{\min})^2, & d_{ij} \ge \bar{d} \\ 1, & d_{ij} < \bar{d} \end{cases}, \tag{20}$$

with $i \neq j$ and $i, j = 1, \ldots, N$. Hence, the new weighting matrix becomes

$$W^{\alpha} := \begin{bmatrix} \frac{\alpha_{12}}{(d_{12}-d_{\min})^2} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{\alpha_{N-1N}}{(d_{N-1N}-d_{\min})^2} \end{bmatrix}. \tag{21}$$

Now, by computing the derivative of the generic entry term $\dot{V}_{ij}$, two different cases need to be distinguished. More specifically, if $d_{ij} \ge \bar{d}$, it yields

$$\begin{aligned} \dot{V}_{ij}(x) =& \dot{d}_{ij}(d_{ij}-\bar{d}) \\ =& \left(1-\frac{\bar{d}}{d_{ij}}\right)\Big[(p_x^{[i]}-p_x^{[j]})(\dot{p}_x^{[i]}-\dot{p}_x^{[j]}) \\ & +(p_y^{[i]}-p_y^{[j]})(\dot{p}_y^{[i]}-\dot{p}_y^{[j]})\Big] \\ =& \begin{bmatrix} \sigma^{[1]} & \ldots & \sigma^{[N]} \end{bmatrix} \begin{bmatrix} \tilde{g}_1(x) \\ \vdots \\ \tilde{g}_N(x) \end{bmatrix}, \end{aligned} \tag{22}$$

with $\tilde{g}_i := \sum_{k=1,k\neq i}^{N} \left(1-\frac{\bar{d}}{d_{ik}}\right) g_{ik}$. Otherwise, if $d_{ij} < \bar{d}$, one has

$$\begin{aligned} \dot{V}_{ij}(x) =& \dot{d}_{ij}(d_{ij}-\bar{d})\frac{(\bar{d}-d_{\min})}{(d_{ij}-d_{\min})^3} \\ =& \left(1-\frac{\bar{d}}{d_{ij}}\right)\frac{(\bar{d}-d_{\min})}{(d_{ij}-d_{\min})^3}\times \\ & \Big[(p_x^{[i]}-p_x^{[j]})(\dot{p}_x^{[i]}-\dot{p}_x^{[j]}) \\ & +(p_y^{[i]}-p_y^{[j]})(\dot{p}_y^{[i]}-\dot{p}_y^{[j]})\Big] \\ =& \begin{bmatrix} \sigma^{[1]} & \ldots & \sigma^{[N]} \end{bmatrix} \begin{bmatrix} \tilde{g}_1(x) \\ \vdots \\ \tilde{g}_N(x) \end{bmatrix}, \end{aligned} \tag{23}$$

with this time $\tilde{g}_i := \sum_{k=1,k\neq i}^{N} \left(1-\frac{\bar{d}}{d_{ik}}\right)\frac{(\bar{d}-d_{\min})}{(d_{ij}-d_{\min})^3} g_{ik}$.

In both cases, analogously to (13), the CLF derivative $\dot{V}(x)$ can be rewritten as a linear combination of state functions weighted by the switching signals, and as such it can be made semi-negative definite by a proper choice of the switching signal. Moreover, as mentioned in Remark 4.1, the tracking control problem can be addressed by defining $V(x)$ as

$$V(x) := \frac{1}{2}d_{\mathrm{o}}^{\top} d_{\mathrm{o}} + \frac{1}{2}(d-\hat{d})^{\top} W^{\alpha}(d-\hat{d}). \tag{24}$$

Finally, the corresponding switching law is given by

$$\Sigma = \arg\min_{\Sigma\in\mathcal{M}^N} \dot{V}. \tag{25}$$

5 CASE STUDY

In this section simulation and experimental results achieved by applying the proposed CLF based aggregation strategy are illustrated.

5.1 Settings and performance metric

A network of $N = 6$ robots with random initial distribution in the plane is considered. The main parameters of the robots are listed in Table 1.

Table 1: Robot settings

Parameter	Value
r	$0.055\,\mathrm{m}$
d_{iw}	$0.105\,\mathrm{m}$
$\bar{d}$	$2.9r$
$d_{\min}$	$2.1r$
R	$0.2975\,\mathrm{m}$
v_{r_0}	$-0.2\,\mathrm{m\,s^{-1}}$
v_{l_0}	$0.2\,\mathrm{m\,s^{-1}}$
v_{r_1}	$-0.2\,\mathrm{m\,s^{-1}}$
v_{l_1}	$-0.14\,\mathrm{m\,s^{-1}}$

Moreover, in order to assess the behaviour of the proposed approach, some performance metrics are hereafter introduced. The first used index is given by the ratio between the maximum number of robots aggregated in a cluster, namely N_{c}, and the total number of robots in the network N, i.e.,

$$\kappa(t) := \frac{N_{\mathrm{c}}(t)}{N}. \tag{26}$$

Note that two robots are considered adjacent if the distance between their centers is less than $4r$. Therefore, the robot network is fully aggregated when the number of robots in a cluster is N. The second index is the dispersion metric of the multi-robot system given by

$$\delta(t) := \frac{1}{4r^2}\sum_{i=1}^{N}\left\|p^{[i]}(t)-\bar{p}(t)\right\|^2, \tag{27}$$

with $\bar{p}(t)$ being the centroid of the robots positions, i.e., $\bar{p} := \frac{\sum_{i=1}^{N} p^{[i]}}{N}$. Then, as term of comparison, we consider the optimal dispersion for circular robots, namely $\delta^{\star}$, reported in (Graham and Sloane, 1990). To fairly compare $\delta^{\star}$ with the actual dispersion of the robots, we introduce the following modified dispersion metric

$$\tilde{\delta}(t) := \frac{1}{\bar{d}^2}\sum_{i=1}^{N}\left\|p^{[i]}(t)-\bar{p}(t)\right\|^2. \tag{28}$$

Note that $\tilde{\delta}$ is computed relying on $\bar{d}$, and takes into account the control objective of reaching a minimum distance between two robots equal to $\bar{d}$. To do this, making reference to (Graham and Sloane, 1990) a fictitious radius equal to $\bar{r} = \frac{\bar{d}}{2}$ is considered (see Figure 1(d)). Finally, the percentage error $\eta_{\tilde{\delta}}$ between the dispersion $\tilde{\delta}$ and its lower bound given by $\delta^{\star}$ is computed, i.e.,

$$\eta_{\tilde{\delta}} = \left|\frac{\tilde{\delta}-\delta^{\star}}{\delta^{\star}}\right| 100\%. \tag{29}$$

In order to assess the validity of the proposal, the average values of the performance metrics are computed over 100 tests of 40 seconds. Simulations have been executed relying on the kinematic model, using MATLAB with sampling time $T = 0.033\,\mathrm{s}$, suitably chosen such that the convergence properties of the proposed control system are preserved.

5.2 Simulation results

The outcome of the simulations is illustrated in the following. Figure 3 shows the final configuration of the robots which aggregate around their centroid without colliding, as expected. In Figure 4, the derivative of the CLF, that is $\dot{V}$, is illustrated, confirming the theoretical analysis reported in §4. In the same figure, the corresponding switching string $\Sigma(t)$ is shown. Note that V is differentiable, and the derivative, i.e., $\dot{V}$, depends on the current operating mode of the robots. As for the performance metrics, Figure 5 shows the dispersion metrics $\kappa(t)$ in (26), $\delta(t)$ in (27) and $\tilde{\delta}$ in (28), respectively, reporting the simulations with maximum and minimum values (red lines), the average value (blue line), the optimal dispersion $\delta^{\star}$ and maximum aggregation $\kappa = 1$ (green line), and the 50th test (black line).

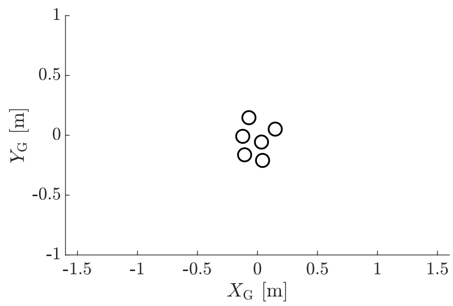
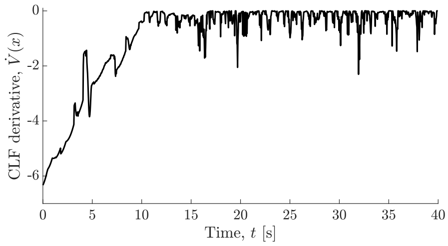


Figure 3: Final aggregation of the robots in the plane.

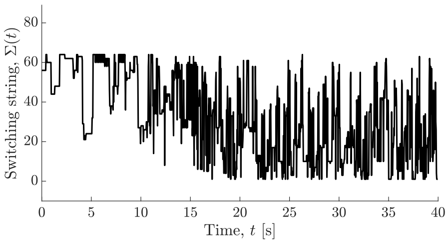
Table 2: Average metrics over 100 simulations.

Strategy	δ	$\tilde{\delta}$	$\eta_{\tilde{\delta}}$
SA	197.4	-	-
CLF	12.2	5.8	20.08

Finally, we compared our proposal with a self-aggregation (SA) method reproducing the one in (Gauci et al., 2014), under the same setting. However, by construction, this method does not intrinsically take into account collisions, but, if a possible collision is detected by the sensors, the robots stop and rotate around their centroid, according to mode 0 in our case. Taking into account the previous performance indexes, the averaged results are reported in Table 2. One can note that, as for the SA approach, the values $\tilde{\delta}$ and the corresponding $\eta_{\tilde{\delta}}$ are not computed since no



(a) CLF derivative



(b) switching string

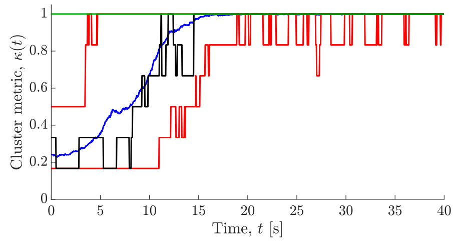
Figure 4: Time evolution of the CLF derivative $\dot{V}(x)$ (a), and of the switching string $\Sigma(t)$ (b) in simulations.

collision avoidance mechanism is taken into account. However, it is evident that in terms of dispersion metric δ, our approach performs better. If we consider the optimal dispersion in (Graham and Sloane, 1990) for 6 robots, which is equal to $\delta^{\star} = 4.83$, one can observe that our proposal guarantees satisfactory results, with a dispersion error equal to 20.08 %.

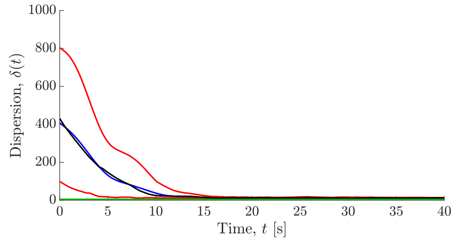
5.3 Experimental results

In order to assess in practice the proposed control approach, it has been implemented by using the remote platform Robotarium (Wilson et al., 2020). The same parameters adopted in simulation are considered, apart from the safety radius set equal to the robot diameter. As for the performance indexes, their average values are computed over 15 tests, and the achieved average dispersion metric is $\delta = 11.59$, that is larger than the network theoretical lower bound, while the average modified dispersion is $\tilde{\delta} = 5.51$, corresponding to a percentage error of the 14% with respect to the optimal dispersion. These results confirm the validity of the model used in simulation and the feasibility in practice of the proposed approach.

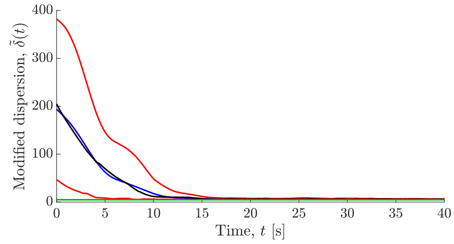
Figure 6 shows the dispersion metrics $\kappa(t)$ in (26), $\delta(t)$ in (27) and $\tilde{\delta}$ in (28), respectively, while a frame of the experimental test is reported in Figure 7.



(a) cluster metric



(b) dispersion metric

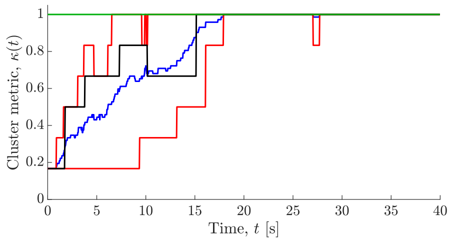


(c) modified dispersion metric

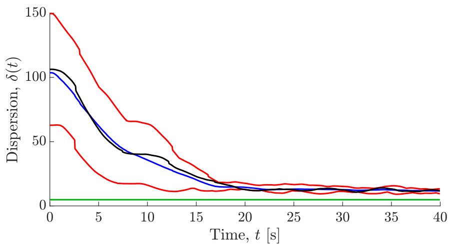
Figure 5: Time evolution of the performance metrics κ (a), δ (b), and $\tilde{\delta}$ (c) in simulations: average value over 100 tests (blue line), minimum and maximum metric values (red lines), optimal dispersion $\delta^{\star}$ and maximum aggregation $\kappa = 1$ (green line), evolution of the performance metrics for the 50th test (black line).

6 CONCLUSIONS

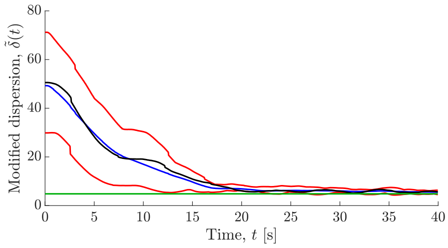
This work has proposed a novel control strategy to solve a rendezvous problem for a network of mobile wheeled robots. The proposed approach is based on a CLF suitably designed relying on a switched formulation of the robot model with two modes. A centralized control architecture allows to retrieve the robot location in the environment, and to generate the best sequence of modes capable of making the robot aggregate without colliding among each other. Simulation and experimental results have shown the effectiveness of the proposed approach. Future works could be devoted to the extension of the proposal to predictive control and distributed control methods, and to the case of more than two modes and more complex scenarios, for example, with communications delays.



(a) cluster metric



(b) dispersion metric



(c) modified dispersion metric

Figure 6: Time evolution of the performance metrics κ (a), δ (b), and $\tilde{\delta}$ (c) in the experimental tests: average value over 15 tests (blue line), minimum and maximum metric values (red lines), optimal dispersion $\delta^{\star}$ and maximum aggregation $\kappa = 1$ (green line), evolution of the performance metrics for the 7th test (black line).

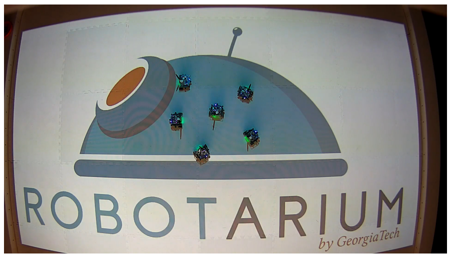


Figure 7: Experimental Robotarium platform with 6 robots.

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