15th World Congress on Computational Mechanics (WCCM-XV)
8th Asian Pacific Congress on Computational Mechanics (APCOM-VIII)
Virtual Congress: 31 July – 5 August 2022
S. Koshizuka (Ed.)

A NUMERICAL APPROACH TO THE DESIGN OF GRIDSHELLS FOR WAAM

M. BRUGGI[1], V. LAGHI[2] AND T. TROMBETTI[2]

[1] Department of Civil and Environmental Engineering, Politecnico di Milano,
P.zza Leonardo da Vinci 32, 20133 Milano, Italy
matteo.bruggi@polimi.it

[2] Department of Civil, Chemical, Environmental and Materials Engineering, Università di Bologna,
Via Zamboni 33, 40126 Bologna, Italy
vittoria.laghi2@unibo.it, tomaso.trombetti@unibo.it

Key words: Form Finding, Wire-and-Arc Additive Manufacturing, Funicular Analysis

Abstract. *A novel approach based on funicular analysis is investigated to cope with the design of spatial truss networks fabricated by Wire-and-Arc Additive Manufacturing (WAAM). The minimization of the horizontal thrusts of networks with fixed plan geometry is stated both in terms of any independent subset of the force densities and in terms of the height of the restrained nodes. Local enforcements are formulated to prescribe lower and upper bounds for the vertical coordinates of the nodes, and to control the stress regime in the branches. This allows also for a straightforward control of the length and maximum force magnitude in each branch. Constraints are such that sequential convex programming can be conveniently exploited to handle grids with general topology and boundary conditions. Optimal networks for WAAM are preliminary investigated, accounting for different sets of the above prescriptions.*

1 INTRODUCTION

Among different Additive Manufacturing (AM) processes, Wire-and-Arc Additive Manufacturing (WAAM) results particularly suitable for applications in structural engineering. The WAAM process, which consists of off-the-shelf welding equipment mounted on top of a numerically controlled robotic arm, allows realizing large-scale structural elements (up to few meters), with limited constraints in terms of forms and shapes. The WAAM process can be used to fabricate two-dimensional specimens using a layer-by-layer deposition in a so-called "continuous" printing process. The WAAM technique employing "dot-by-dot" printed stainless steel bars is herein considered, see Figure 1 and the investigations in [1,2]. High strength performances are reported for specimens fabricated using the WAAM process, although a more complex mechanical response is pointed out with respect to those traditionally manufactured.

Gridshells take their strength from their double curvature, being constructed from members that mainly undergo axial forces [3]. A numerical approach based on funicular analysis, see e.g. [4-6], is proposed to cope with the design of spatial truss networks fabricated by "dot-by-dot" WAAM.

Figure 1: A specimen fabricated using the “dot-by-dot” WAAM process

The equilibrium of funicular networks can be conveniently handled through the force density method, i.e. writing the problem in terms of the ratio of force to length in each branch of the network [7]. As investigated in the literature for the case of vertical loads, independent sets of branches can be detected for networks with fixed plan geometry [8].

In this contribution, following [9], the minimization of the horizontal thrusts in networks with fixed plan geometry is stated both in terms of any independent subset of the force densities and in terms of the height of the restrained nodes. Local enforcements are formulated to prescribe lower and upper bounds for the vertical coordinates of the nodes, and to control the force densities in the branches. This allows also for a straightforward control of the force magnitude and of the length of each branch of the gridshell, in view of prescriptions arising from the results of the experimental tests. Constraints are such that sequential convex programming can be conveniently exploited to handle grids with general topology and boundary conditions.

Preliminary numerical simulations are shown concerning the optimal design of gridshells with minimum thrusts, under the combined effect of different sets of the above prescriptions. The ongoing research is mainly devoted towards endowing this design formulation with buckling constraints.

2 THE NUMERICAL APPROACH

Funicular analysis is widely adopted to cope with the design of arcuated structures, see e.g. [3]. Following this approach, spatial structures such as three-dimensional trusses and gridshells can be modelled as statically indeterminate networks of vertices and edges with given topology. Boundary supports are prescribed at the restrained nodes of the network; unrestrained ones are in equilibrium with the prescribed vertical and horizontal loads.

The equilibrium of funicular networks can be handled by means of the force density method, that consists in writing the problem in terms of the ratio of force to length in each branch of the network [6]. Considering a network with m branches, and denoting by $\boldsymbol{L} = diag(\boldsymbol{l})$ the diagonal matrix gathering the length of the branches stored in the vector $\boldsymbol{l}$, the force density vector may be defined as:

$$\boldsymbol{q} = \boldsymbol{L}^{-1}\boldsymbol{s}, \tag{1}$$

being $\boldsymbol{s}$ the vector collecting the force in the branches.

As investigated in the literature for the case of vertical loads, independent sets of branches can be detected when addressing networks with fixed plan geometry [8]. However, enforcing the nodes to lie within a prescribed design domain (that is a range of heights) is not straightforward.

To this goal, a minimization problem with multiple constraints was formulated in [9] to enforce lower and upper bounds for the vertical coordinates of the vertices of the network.

At first, the equations that link dependent and independent branches in the network with fixed plan projection are presented. The horizontal equilibrium of the nodes with prescribed horizontal coordinates $\boldsymbol{x}_{s0}$ and $\boldsymbol{y}_{s0}$ reads:

$$\begin{bmatrix} \boldsymbol{C}^T diag(\boldsymbol{C}_s \boldsymbol{x}_{s0}) \\ \boldsymbol{C}^T diag(\boldsymbol{C}_s \boldsymbol{y}_{s0}) \end{bmatrix} \boldsymbol{q} = \begin{bmatrix} \boldsymbol{p}_x \\ \boldsymbol{p}_y \end{bmatrix} \tag{2}$$

In the above equations, $\boldsymbol{C}_s$ is the connectivity matrix of the network and $\boldsymbol{C}$ is its subset referring to the unrestrained nodes, whereas $\boldsymbol{p}_x$ and $\boldsymbol{p}_y$ are the components along the cartesian axes x and y of the point loads applied at the unrestrained ones. Indeed, by applying Gauss-Jordan elimination to Eqn. (2), see also [9], the r dependent force densities $\tilde{\boldsymbol{q}}$ may be re-written in terms of the $m - r$ independent ones $\bar{\boldsymbol{q}}$ as it follows:

$$\tilde{\boldsymbol{q}} = \boldsymbol{B}\bar{\boldsymbol{q}} + \boldsymbol{d}, \tag{3}$$

where $\boldsymbol{B}$ and $\boldsymbol{d}$ have entries that are known.

The vertical equilibrium of the n unrestrained nodes of the network reads:

$$\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}\boldsymbol{z} + \boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}_f\boldsymbol{z}_f = \boldsymbol{p}_z, \tag{4}$$

where $\boldsymbol{z}$ and $\boldsymbol{z}_f$ gather the vertical coordinates of the unrestrained and restrained nodes, respectively, $\boldsymbol{C}_f$ is the subset of $\boldsymbol{C}$ for the restrained nodes; $\boldsymbol{p}_z$ are the vertical components of the point loads and $\boldsymbol{Q} = diag(\boldsymbol{q})$.

Hence, an optimization problem with multiple constraints is formulated in terms of the independent force densities $\bar{\boldsymbol{q}}$ and the vertical coordinates of the restrained nodes $\boldsymbol{z}_f$ as:

$$\begin{cases} \min\limits_{\bar{\boldsymbol{q}},\ \boldsymbol{z}_f} f(\bar{\boldsymbol{q}}) & & (5.1) \\ s.t.\quad \tilde{\boldsymbol{q}} = \boldsymbol{B}\bar{\boldsymbol{q}} + \boldsymbol{d} & & (5.2) \\ \boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}\boldsymbol{z} + \boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}_f\boldsymbol{z}_f = \boldsymbol{p}_z & & (5.3) \\ z_j(\bar{\boldsymbol{q}}, \boldsymbol{z}_f) \geq z_j^{min} & j = 1 \dots n & (5.4) \\ z_j(\bar{\boldsymbol{q}}, \boldsymbol{z}_f) \leq z_j^{max} & j = 1 \dots n & (5.5) \\ |s_i(\bar{\boldsymbol{q}}, \boldsymbol{z}_f)| \leq s_i^{max} & i = 1 \dots m & (5.6) \\ l_i(\bar{\boldsymbol{q}}, \boldsymbol{z}_f) \leq l_i^{max} & i = 1 \dots m & (5.7) \\ z_{fh}^{min} \leq z_{fh} \leq z_{fh}^{max} & h = 1 \dots n_f & (5.8) \end{cases},$$

A norm of the horizontal thrusts is adopted as objective function, i.e.

$$f(\overline{\boldsymbol{q}}) = \sum \sqrt{R_{xh}^2 + R_{yh}^2}, \tag{6}$$

where $R_{xh}^2$and R_{yh}^2 are the squared values of the component of the reaction along the x and y direction, respectively, for the h-th of the n_f restrained nodes. By using the equilibrium in Eqn. (5.3) and Eqn. (5.2), the vertical coordinates of the unrestrained nodes may be written in terms of the minimization unknowns (the independent force densities $\overline{\boldsymbol{q}}$ and the vertical coordinates of the restrained nodes $\boldsymbol{z}_f$). Suitable set of constraints can be enforced to prescribe the limits of the design domain, see Eqn. (5.4) and (5.5). Side constraints on $\boldsymbol{z}_f$ are used to enforce similar prescriptions at the restrained nodes, see Eqn. (5.8). Constraints on the magnitude of the force in each branch of the network may be accounted for, see Eqns. (5.6), as well as enforcements on the maximum length of each branch, see Eqn. (5.7). It must be remarked that the length vector may be straightforwardly computed working on the given coordinates for the prescribed plan projection of the network along with the variable height of the nodes. The force vector depends both on the force density vector and the vector gathering the length of the branches, see Eqn. (1).

Due to its form, the optimization problem in Eqn. (5) can be solved efficiently by means of techniques of sequential convex programming [11]. These were originally conceived to handle large scale multi-constrained formulations of size optimization for elastic structures. In a stress-constrained minimum weight problem of truss design, the area of the sections is sought such that the volume is minimized, subject to strength limits. In a statically determinate truss, the objective function is linear in the unknowns, whereas the constrained stress may be written in terms of the inverse of the unknowns. In [9], the formulation of Eqn. (5), skipping the constraints in Eqns. (5.6)-(5.7), was used to find the funicular polygon of an arch on which vertical loads are exerted: it is shown that the thrust is linear in the only independent force density $\bar{q}$, whereas the constrained vertical coordinates of the unrestrained nodes $\boldsymbol{z}$ are linear in the vertical coordinate of the restrained nodes $\boldsymbol{z}_f$ and in the reciprocal variable $1/\bar{q}$. Approaches of sequential convex programming such as the Method of Moving Asymptotes (MMA) [12] implement approximations of the objective functions and constraints in the direct or the reciprocal variable, depending on the sign of the gradient. These gradient-based methods can be conveniently adopted to handle the considered minimization problem. Reference is also made to the application of the multi-constrained formulation herein considered to thrust networks with multiple layers, see [13]. It must be remarked that MMA is a versatile tool for structural optimization and is extensively used in topology optimization. The solution of a displacement-constrained formulation to seek simultaneously for the optimal shape of two-dimensional structural elements and for the optimal printing orientation in the "continuous" WAAM process has been recently tackled by means of MMA in [14].

3 NUMERICAL SIMULATIONS

A preliminary assessment of the proposed numerical approach for form-finding is shown, addressing gridshells that span a $2m\, x\, 3m$ bay. The nodes along the perimeters are fully restrained. A distributed load per unit of area, measured in the projection of the gridshell onto the horizontal plane, is considered. The intensity assumed in the simulations is $2\ kN/m^2$.

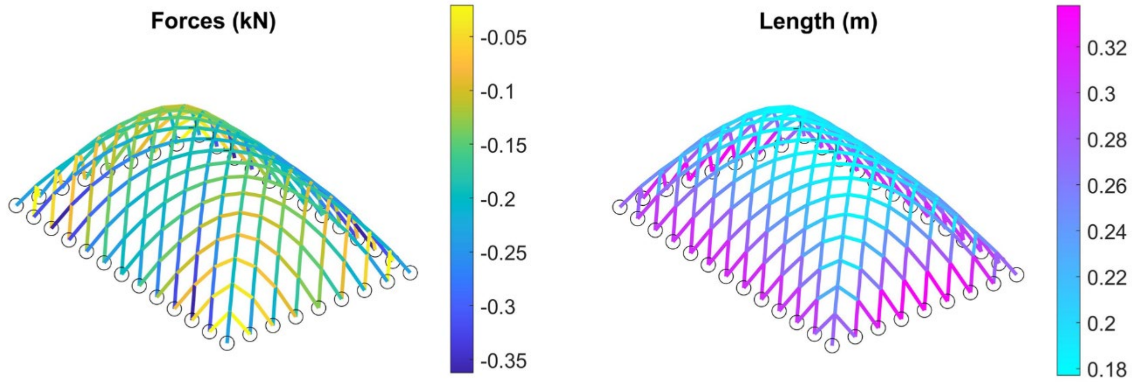

Figure 2: Optimal design for minimum thrust

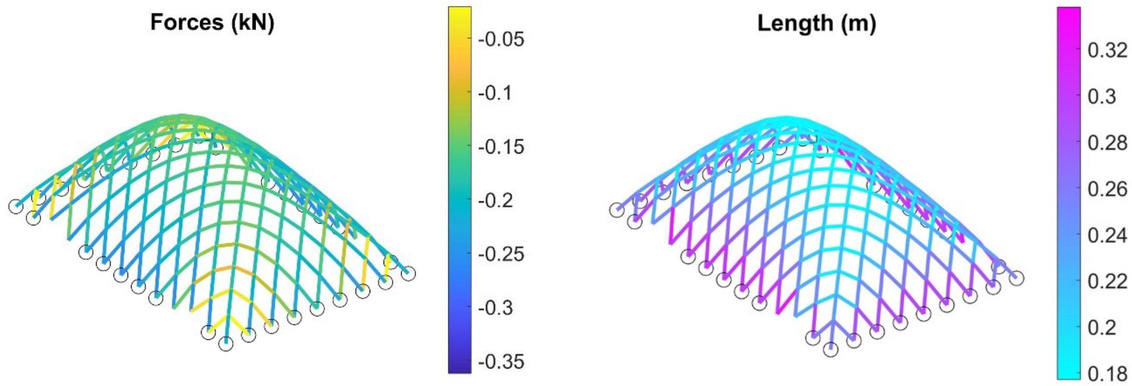

Figure 3: Optimal design for minimum thrust, with constraints on the maximum value of the force in the branches of the network

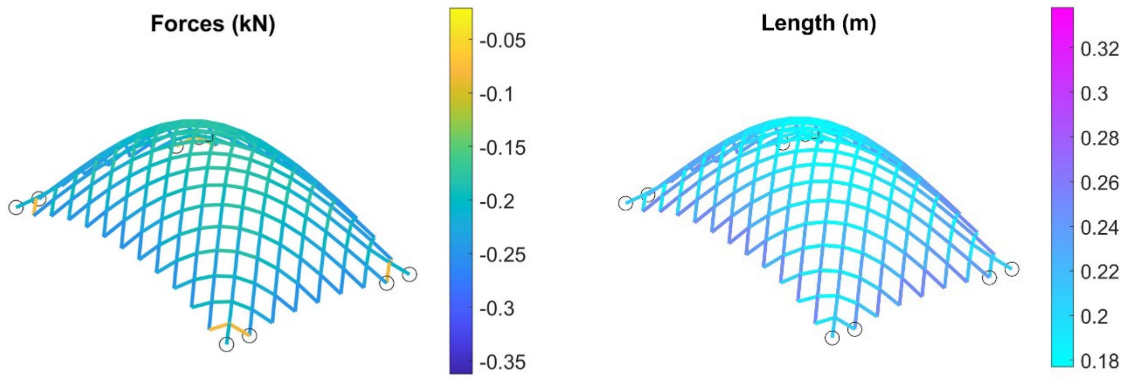

Figure 4: Optimal design for minimum thrust, with constraints on the maximum value of the force and of length for each one of the branches of the network

The number of branches in the network is $m = 384$, but the independent ones (to preserve the fixed plan projection) are only 38. This means that the number of unknowns for the optimization procedure is limited to 78, being 40 the number of restrained nodes (where supports are given).

At first, an optimal design is sought seeking for gridshells whose nodes coordinate lie in the interval $[0, 1]\ m$: $2\ x\ 213$ constraints of the type in Eqn. (5.4) and (5.5) are enforced, whereas constraints in Eqn. (5.6) and (5.7) are disregarded. The achieved gridshell is given in Figure 2. Crosses and circles stand for nodes whose heights match the prescribed upper and lower bounds of the design domain, respectively. Force and length maps are provided in the two relevant pictures.

Then, the optimization is run accounting also for the constraints in Eqn. (5.8). The absolute value of the force acting in each branch of the network is limited by prescribing a threshold equal to $0.25\ kN$. The achieved layout, which is fully compliant with the enforced local prescriptions, is given in Figure 3. With respect to the solution depicted in Figure 2, small variations of the geometry are reported, also affecting the vertical coordinate of some of the restrained nodes. The objective function at convergence is 6% more than in the previous case. Indeed, this is due to the introduction of the new set of constraints.

A last run of the optimization is performed accounting for the entire set of constraints given in Eqn. (5). This includes control over the maximum length of each branch in the network: the maximum allowed value is set to $0.25\ m$. As reported in Figure 4, in each branch, the maximum stress and maximum length do not exceed the enforced limit. The network still lies within the prescribed design domain. A noticeable variation of the shape of the entire gridshell may be observed in comparison to the results presented in the previous figures. This last run ends with an objective function that is one and half the value found for the reference solution given in Figure 2.

4 CONCLUSIONS

Wire-and-Arc Additive Manufacturing (WAAM) technology allows realizing metal-based free forms and shapes, introducing very few fabrication constraints. The “dot-by-dot” technique employs stainless steel rods to build spatial truss-like structures with high strength performances, although experimental results point out a more complex mechanical response with respect to those that are traditionally manufactured.

A design approach for “dot-by-dot” WAAM is proposed in this contribution that searches among spatial truss networks fulfilling equilibrium using funicular analysis. The minimization of the horizontal thrusts of a spatial network with given plan geometry is formulated not only in terms of an independent set of force densities, but also in the height of the restrained nodes. Constraints are enforced on the height of the vertical coordinates of the nodes, as well as on the stress regime and on the length of each branch. Multi-constrained solutions are achieved using sequential convex programming.

The proposed approach should be regarded as a preliminary step towards the automatic generation of gridshell accounting for prescriptions peculiar to the WAAM fabrication process. The ongoing research is devoted to endowing the design formulation with buckling constraints derived from an ongoing experimental campaign on bars printed through the “dot-by-dot” WAAM process.

REFERENCES

[1] Laghi, V., Palermo, M., Gasparini, G., and Trombetti, T. Computational design and manufacturing of a half-scaled 3D-printed stainless steel diagrid column, *Addit Manuf*. (2020) **36:** 101505.

[2] Laghi, V., Palermo, M., Tonelli, L., Gasparini, G., Girelli, V. A., Ceschini, L., and Trombetti, T. Mechanical response of dot-by-dot wire-and-arc additively manufactured 304L stainless steel bars under tensile loading, *Constr and Build Mater*. (2022) **318**: 125925.

[3] Adriaenssens, S., Block, P., Veenendaal, D., and Williams, C. *Shell structures for architecture: Form finding and optimization*. Routledge, (2014).

[4] O'Dwyer, D.W. Funicular analysis of masonry vaults, *Comput. Struct.* (1999) **73**: 187-197.

[5] Block, P. and Lachauer, L. Three-dimensional funicular analysis of masonry vaults, *Mech. Res. Commun.* (2012) **56**: 53-60.

[6] Marmo, F. and Rosati, L. Reformulation and extension of the thrust network analysis, *Comput. Struct.* (2017) **182**: 104-118.

[7] Schek, H.J. The force density method for form finding and computation of general networks, *Comput. Methods Appl. Mech. Eng.* (1974) **3**:115-134.

[8] Liew, A., Pagonakis, D., Van Mele, T., and Block, P. Load-path optimisation of funicular networks, *Mecc.* (2018) **53**: 279-294.

[9] Bruggi, M. A constrained force density method for the funicular analysis and design of arches, domes and vaults. *Int. J. Solids Struct.* (2020) **193-194**: 251-269.

[10] Pellegrino S. and Calladine, C.R. Matrix analysis of statically and kinematically indeterminate frameworks. Int. J. Solids Struct. (1986) **22**:409-428.

[11] Christensen P. and Klarbring, A., *An Introduction to Structural Optimization*. Springer (2009).

[12] Svanberg, K. The method of moving asymptotes - a new method for structural optimization, *Int. J. Numer. Methods Eng.* (1987) **24**(2):359-373.

[13] Bruggi, M., Lógó, B.A., Deák, Z. Funicular analysis of ribbed masonry vaults: a case study, *Int. J. Archit. Herit*. (2021) https://doi.org/10.1080/15583058.2021.1910879.

[14] Bruggi, M., Laghi, V., Trombetti, T. Simultaneous design of the topology and the build orientation of Wire-and-Arc Additively Manufactured structural elements. *Comput. Struct.* (2021) **242**:106370.