

Asynchronous Networked MPC with ISM for Uncertain Nonlinear Systems

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Abstract—A model-based event-triggered control scheme for nonlinear constrained continuous-time uncertain systems in networked configuration is presented in this paper. It is based on the combined use of Model Predictive Control (MPC) and Integral Sliding Mode (ISM) control, and it is oriented to reduce the packets transmission over the network both in the direct path and in the feedback path, in order to avoid network congestion. The key elements of the proposed control scheme are the ISM local control law, the MPC remote controller, a smart sensor and a smart actuator, both containing a copy of the nominal model of the plant. The role of the ISM control law is to compensate matched uncertainties, without amplifying the unmatched ones. The MPC controller with tightened constraints generates the control component oriented to comply with state and control requirements, and is asynchronous since the underlying constrained optimization problem is solved only when a triggering event occurs. In the paper, the robustness properties of the controlled system are theoretically analyzed, proving the regional input-to-state practical stability of the overall control scheme.

Index Terms—Model predictive control, sliding mode control, networked control systems, event-triggered control, nonlinear systems, uncertain systems.

I. INTRODUCTION

The last decades progress in telecommunication has brought along the development of new control configurations, named Networked Control Systems (NCSs), which are a reality by virtue of their cost, effectiveness and flexibility (see, for an overview, [1], [2], and [3]). Yet, several technical and theoretical problems arise because of the presence of a network in the control system. These are related to the fact that a communication network is a band-limited channel which can feature delays, packet loss and jitter. To overcome these drawbacks, different approaches have been proposed in the literature, with the objective of designing control algorithms capable of coping with communication imperfections and constraints. Among these, algorithms designed according to the so-called event-triggered control approach (see, among others, [4]–[9], and the references therein cited) are surely effective solutions. In event-triggered control, the state of the plant is transmitted over the network only if a pre-specified triggering condition holds.

A similar approach is adopted in the so-called model-based event-triggered control (see, for instance, [10]–[12]),

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which uses an explicit model of the plant asynchronously updated with the actual plant state of the system, when this is transmitted through the network. Both basic and model-based event-triggered controllers significantly reduce the transmission rate, though guaranteeing satisfactory performance, as also observed in applications (see, for instance, [13]–[15]).

In this paper, a Model Predictive Control (MPC) based approach is proposed for nonlinear networked uncertain systems [16], [17]. In the considered context, the model-based event-triggered strategy results in being the more natural way to face the problem. In order to reduce the conservativeness inherent in any robust MPC, an Integral Sliding Mode (ISM) control has been adopted to reject at least the matched uncertainty [18]. Since the ISM component is very simple from a computational viewpoint, it is locally implemented so that it can run at a higher rate than the MPC controller, and it is continuously fed by the actual state of the plant. Note that MPC and sliding mode control have been used in a combined scheme in [19], [20], where the MPC has been applied to update the parameters of the so-called sliding manifold. A different idea to combine MPC and ISM has already been investigated in [21]–[23] in a conventional, i.e., non NCS, framework. The first event-triggered version of a sliding mode control scheme has been discussed in [24]. In the context of MPC, the problem to reduce the energy consumption due to data transmission is considered in [25], where a min-max MPC has been adopted for discrete time linear systems. In [25] only sensor data are remotely transmitted so that the control law is computed at any discrete time instant. Finally, note that a very preliminary version of the present paper, with no network between controller and plant, only smart sensor and conventional actuator, and no proofs of the theoretical results was presented in [26].

In this paper, a multi-rate control law for nonlinear constrained continuous-time uncertain NCSs is designed. The proposed hierarchical control scheme, illustrated in Figure 1, consists of the following key elements: the remote MPC controller, the ISM local control law, a smart actuator and a smart sensor. The controller generates the MPC component, by using the nominal model of the plant to predict the future evolution of the system state. The smart actuator and the smart sensor both include a copy of the nominal model of the plant. The smart actuator provides the MPC component to the system, and is capable of checking if all the elements of the last transmitted control sequence have been used as inputs to the plant, and, if this is the case, computing an auxiliary control law relying on the nominal model. The smart sensor continuously checks a triggering condition, function of

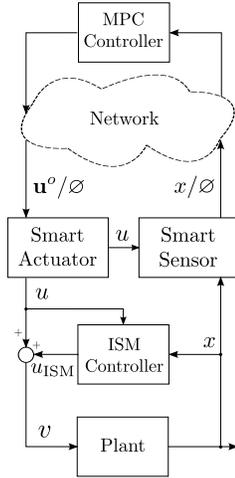


Figure 1. Representation of the proposed hierarchical model-based event-triggered MPC/ISM control scheme.

the plant state, on the basis of which decides whether it is necessary to transmit the measured state to the controller and to update the nominal model or not. The ISM controller is local, in the sense that it is embedded with the plant, and has the role of compensating the matched uncertainty affecting the system.

The motivation for using ISM control, apart from its property of providing robustness versus matched uncertain terms, is also given by its capability of enforcing sliding modes of the controlled system since the initial time instant, without amplifying the remaining unmatched uncertain terms [27], [28]. The presence of the ISM local controller allows MPC controller to solve the optimization problem relying on a system with reduced uncertainties, while fulfilling the state and control constraints. More specifically, in the paper, an asynchronous MPC is used since the optimization is performed only when a triggering event occurs. In this case, the sensor decides to transmit the actual state over the network and the optimal control sequence is sent, packetized, to the plant (i.e., the entire control sequence computed at the current time instant is transmitted). In this operational mode, the triggering condition provides a bound on the mismatch between the plant and the nominal model state, which is suitably exploited to shrink the state admissible region, according to the approach discussed in [29], [30], so as to guarantee feasibility of the solution in a robust way. Until the occurrence of a new triggering event, an auxiliary control law, based on the nominal state, is provided by the smart actuator and applied to the plant. When a new triggering event takes place, the optimization problem is solved again.

More specifically, the main original contributions of the present work are the following: first of all, the design of an asynchronous packetized MPC algorithm suitable for networked control loops involving a system to control which is of uncertain nonlinear affine type, with inequality constraints on both input and state variables; the proposal of the joint use of this new algorithm with an ISM algorithm; the analysis of the robustness features of the controlled system versus matched

and unmatched uncertainties; finally, the proof of the regional Input-to-State practical Stability (ISpS) of the overall control scheme. The performance of the proposed control strategy are assessed in simulation relying on an illustrative example of mechanical type.

The present paper is organized as follows. In Section II, the notation adopted in the paper is introduced, while in Section III, the considered control problem is formulated. In Section IV, the proposed model-based event-triggered control scheme is presented, illustrating the event-triggered strategy, the ISM and the MPC components. The stability of the proposed control scheme is analyzed in Section V. Section VI is devoted to present simulation results obtained by applying the proposed control approach to a cart moving on a plane. The paper ends with some concluding remarks in Section VII, and appendices that contain the proofs of the theoretical results.

II. NOTATIONS

The Euclidean norm is denoted as $|\cdot|$, while the infinity norm as $|\cdot|_\infty$. For any symmetric matrix A , $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the largest and the smallest eigenvalue of matrix A , respectively. Given a generic signal w , let $\mathbf{w}_{[t_1, t_2]}$ be a signal defined from time t_1 to time t_2 . In order to simplify the notation, when it is obvious from the context, the subscript $[t_1, t_2]$ is omitted. The set of signals w , the values of which belong to a compact set $\mathcal{W} \subseteq \mathbb{R}^m$, is denoted by $\mathcal{M}_{\mathcal{W}}$, while $\mathcal{W}^{\sup} \triangleq \sup_{w \in \mathcal{W}} \{|w|\}$. Given two sets $\mathcal{A} \in \mathbb{R}^n$, $\mathcal{B} \in \mathbb{R}^n$, then the Pontryagin difference set \mathcal{C} is defined as $\mathcal{C} = \mathcal{A} \sim \mathcal{B} \triangleq \{x \in \mathbb{R}^n : x + \xi \in \mathcal{A}, \forall \xi \in \mathcal{B}\}$. Given a matrix $M \in \mathbb{R}^{n \times m}$ with $n > m$, then, its orthogonal complement is $M^\perp \in \mathbb{R}^{n \times (n-m)}$. Given a vector $s \in \mathbb{R}^m$, the sign function $\text{sgn}(s)$ is defined as $\text{sgn}(s) = [\text{sgn}(s_1), \dots, \text{sgn}(s_m)]^T$.

III. PROBLEM STATEMENT

We consider the dynamics of the plant given by

$$\dot{x}(t) = h(x(t)) + Bv(t) + \eta(t), \quad t \geq 0 \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $v \in \mathbb{R}^m$ is the control variable, and $\eta \in \mathbb{R}^n$ is the disturbance term. Given system (1), which is assumed to be forward complete, also assume that the plant nominal model is

$$\dot{\hat{x}}(t) = h(\hat{x}(t)) + Bv(t) \quad (2)$$

where $\hat{x} \in \mathbb{R}^n$ is the state of the nominal model, $h : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $B \in \mathbb{R}^{n \times m}$, and $\text{rank}(B) = m$. Moreover, in (1), the term η denotes the additive uncertainty such that

$$\eta(t) = B\eta_m(t) + B^\perp\eta_u(t) \quad (3)$$

where $B^\perp \in \mathbb{R}^{n \times (n-m)}$ is the orthogonal complement matrix. Note that $\eta_m(\cdot) \in \mathbb{R}^m$ and $\eta_u(\cdot) \in \mathbb{R}^{n-m}$ represent the so-called ‘‘matched’’ and ‘‘unmatched’’ uncertainty, respectively [31]. They are due to unavoidable unmodelled dynamics, parameter uncertainties and disturbances.

Remark 1: Note that, the control-affine form (1) is required in order to obtain an explicit control law for the ISM strategy, as will be clarified in Subsection IV-B. \square

System (1) is supposed to fulfill the following assumption on state, input and uncertain terms.

Assumption 1:

- 1) System (1) is forward complete.
- 2) $h(0) = 0$.
- 3) The state and control variables are restricted to fulfill the following constraints

$$x \in \mathcal{X} \quad (4)$$

$$v \in \mathcal{U} \quad (5)$$

where \mathcal{X} and \mathcal{U} are compact sets containing the origin as an interior point.

- 4) The uncertainty η is such that

$$\eta \in \mathcal{W} \quad (6)$$

where \mathcal{W} is a compact set containing the origin, with \mathcal{W}^{sup} known. \square

Now, taking into account system (1) with suitable initial conditions, the problem to solve consists in designing a control scheme to guarantee the regional ISpS of the controlled system subject to constraints (4) and (5), and to the uncertainty in (3) with bound (6). Moreover, a further requirement is to limit data transmissions over the network.

IV. MODEL-BASED EVENT-TRIGGERED MPC/ISM: THE PROPOSED CONTROL STRATEGY

In this paper, to solve the problem formulated in Section III, we propose a robust MPC/ISM control strategy relying on a model-based event-triggered approach. It allows to execute the state and control transmission as rarely as possible, thus reducing the communication effort, alleviating the network overload, and decreasing the possible occurrence of packet loss, delays and jitter.

The considered control scheme (see Figure 1) includes four key blocks: the MPC remote controller, the ISM local controller, the smart actuator and the smart sensor. The MPC controller uses a copy of the nominal model (2) as a predictor, and produces, at asynchronous time instants, i.e., when a new state measurement is transmitted over the network, a control law capable of fulfilling with the state and control constraints, while guaranteeing the optimality of the control law. The ISM controller has the task of rejecting the matched uncertainty by using the measure of the state provided locally (i.e., not necessarily transmitted) by the smart sensor at any $t \geq 0$. Apart from the nominal model used as a predictor in the MPC controller, copies of the nominal model are assumed to be accessible also to the smart actuator and the smart sensor. Such copies are reinitialized with the actual plant state, whenever a state transmission occurs.

The MPC remote controller and the ISM local controller together contribute to generate the control variable $v(t)$ as

$$v(t) = u(t) + u_{\text{ISM}}(t) \quad (7)$$

where $u(t)$ and $u_{\text{ISM}}(t)$ are, respectively, the piecewise-constant MPC component and the ISM component.

A. The Event-Triggered Strategy

The smart sensor contains an explicit copy of the nominal model of the plant, which, for any $t \geq 0$, receives as input the MPC component $u(t)$ of the control law (7), and provides the computed state $\hat{x}(t)$ to the triggering condition block. This block, relying on the measured state x , determines the state error $e(t) = \hat{x}(t) - x(t)$, and verifies the so-called ‘‘triggering condition’’.

In the present paper, following the suggestion in [32], we adopt a triggering condition with relative threshold. The threshold is progressively reduced as a function of the measured state, i.e.,

$$|e| < \max\{\varepsilon_1|x|, \varepsilon_2\} \quad (8)$$

where $0 < \varepsilon_1 < 1$ and $\varepsilon_2 \geq 0$ arbitrarily set. If condition (8) is violated, the actual state is sent to the remote controller and the states of the nominal models which are present in the scheme are suitably updated. Thus, the MPC controller, smart actuator and smart sensor are all synchronized at the triggering time instants.

B. The Integral Sliding Mode Component

According to the ISM control theory [18], it is possible to force the system to evolve in sliding mode starting from the initial time instant. This is beneficial for the proposed scheme, since in this way the optimization problem solved by the MPC controller can be stated relying on a system with reduced uncertainties, as will be clarified in the following.

Consider the nominal closed-loop system written as

$$\dot{x}_0(t) = h(x_0(t)) + Bu(t) \quad (9)$$

where x_0 denotes the state evolution of the nominal system under the MPC law u . Now, consider the original system (1), which is assumed to be affected by the matched uncertainty term $B\eta_m(t)$ in (3). Select the sliding variable as

$$\sigma(t) = Sx(t) \quad (10)$$

with $S = B^T \in \mathbb{R}^{m \times n}$, SB invertible, and the auxiliary sliding variable as

$$\Sigma(t) = \sigma(t) + \varphi(t) \quad (11)$$

with φ being the desired transient trajectory specified, with reference to (2), as

$$\dot{\varphi}(t) = -S \{h(x(t)) + Bu(t)\} \quad (12)$$

$$\varphi(0) = -\sigma(0) \quad (13)$$

where $\varphi(0)$ is chosen so as to enforce a sliding mode on the sliding manifold $\mathcal{S} = \{x \in \mathcal{X} : \Sigma(t) = 0\}$ (see [33]) from the initial time instant 0. Then, the discontinuous control u_{ISM} in (7) is designed as

$$u_{\text{ISM}}(t) = -U_{\text{max}} \text{sgn}(\Sigma(x(t))) \quad (14)$$

with $U_{\text{max}} > 0$ suitably chosen so as to satisfy the sliding condition [33] with respect to the auxiliary sliding variable, thus making \mathcal{S} an attractive subspace of the system state space.

Note that, the ISM component allows one to define a new set \mathcal{U}^{ISM} such that

$$\mathcal{U}^{\text{ISM}} = \{u_{\text{ISM}} \in \mathbb{R}^m : |u_{\text{ISM}}|_{\infty} \leq U_{\text{max}}\} \quad (15)$$

Remark 2: Note that, the control law (14) can cause the so-called chattering phenomenon, i.e., high frequency oscillations of the controlled variable due to the discontinuity of the control law [34], [35]. As shown in [18], the equivalent value of the discontinuous control, i.e., the so-called ‘‘equivalent control’’ (see [33], for a definition), $u_{\text{ISM}_{\text{eq}}}(t)$, can be used instead of the discontinuous control law to alleviate this phenomenon. According to [18], if the equivalent control is used, to ensure $\Sigma(t) = 0, \forall t \geq 0$, the transient trajectory φ must be redesigned as follows

$$\dot{\varphi} = -S \{h(x) + B(u + u_{\text{ISM}_{\text{eq}}} - u_{\text{ISM}})\} \quad (16)$$

$$\varphi(0) = -\sigma(0) \quad (17)$$

Note that, the equivalent control cannot be directly computed, because it depends on the uncertain terms. In [18], it is shown that the equivalent control is equal to the average value obtained at the output of a first order linear filter with the real discontinuous control as input. \square

With reference to system (1) with u_{ISM} as in (14), the following invariance property of the locally controlled system with respect to the matched uncertainty can be proved.

Lemma 1: Given the plant (1), with the uncertainty bound in (6), then, by applying the control law (7) with u_{ISM} as in (14), the closed-loop system is invariant with respect to the matched uncertainty in (3). \square

Proof: See Appendix A. \blacksquare

Moreover, it is necessary to prove that the unmatched uncertainty is not amplified by the application of the ISM control.

Lemma 2: Given the plant (1), with the uncertainty bound in (6), then, by applying the control law (7) with u_{ISM} as in (14), and the sliding variable in (10) with $S = B^T$, the locally controlled system results in being

$$\dot{x}(t) = h(x(t)) + Bu(t) + w(t), \quad t \geq 0 \quad (18)$$

with $w(t) = B^{\perp} \eta_u(t)$. \square

Proof: See Appendix A. \blacksquare

Note that the local application of the ISM control law transforms the original uncertain system into system (18), where the matched uncertainty is completely rejected and the unmatched uncertainty is not amplified. As such, the system involved in the optimization problem, which is solved to determine the asynchronous MPC component, has reduced uncertainty with respect to system (1).

C. The Packetized Model Predictive Control Component

By virtue of the rejection of the matched uncertainty produced by the ISM part of the controller, the MPC component can be developed relying on system (18). To this end, it is useful to introduce some preliminary issues.

Let T be a suitable sampling period, and let t_k , with $k \geq 0$, be the sampling time instants. Furthermore, let t_j , with $j > 0$, the asynchronous triggering time instants. Let t_{k_j} , with $k_j > 0$,

be the first sampling time instant after \tilde{t}_j . The solution of system (18) with initial state $x(0) = \bar{x}$ and uncertain input signal \mathbf{w} is denoted by $\varphi(t, \bar{x}, \mathbf{u}, \mathbf{w})$. Moreover, if \mathbf{w} only consists of null values, then $\mathbf{w} = \mathbf{0}$. Now that the ISM controller has been introduced, with reference to system (18), it is possible to define the assumption reported below.

Assumption 2: Considering a generic time instant $\tilde{t} \triangleq t_k + \tau$, $0 \leq \tau \leq T$, system (18) is such that:

- 1) Given two different initial conditions x_1 and $x_2 \in \mathcal{X}$ at time 0, and a signal $\mathbf{u} \in \mathcal{M}_{\mathcal{U}}$, it yields

$$|\varphi(\tilde{t}, x_1, \mathbf{u}, \mathbf{0}) - \varphi(\tilde{t}, x_2, \mathbf{u}, \mathbf{0})| \leq \mathcal{L}_{\tau} \mathcal{L}_T^k |x_1 - x_2| \quad (19)$$

where $\mathcal{L}_{\tau} \triangleq \mathcal{L}(\tau)$ is a nondecreasing continuous function defined in $[0, T]$, such that $\mathcal{L}_0 = 1$ and $\mathcal{L}(\tau_1)\mathcal{L}(\tau_2) \leq \mathcal{L}(\tau_1 + \tau_2)$. The term $\mathcal{L}_T^k = (\mathcal{L}_T)^k$ stands for \mathcal{L}_T raised to the k -th power.

- 2) Given an initial condition \bar{x} at time 0, the signals $\mathbf{u} \in \mathcal{M}_{\mathcal{U}}$ and $\mathbf{w} \in \mathcal{M}_{\mathcal{W}}$, one has that

$$|\varphi(\tau, \bar{x}, \mathbf{u}, \mathbf{0}) - \varphi(\tau, \bar{x}, \mathbf{u}, \mathbf{w})| \leq \gamma \tau \quad (20)$$

where $\gamma \in \mathbb{R}_{\geq 0}$ is a constant value and $\bar{x} \in \mathcal{X}$. \square

Moreover, in order to evaluate the discrepancy between the nominal and perturbed evolutions of the system at a generic time instant, the following lemma can be stated.

Lemma 3: Suppose that Assumptions 1 and 2 hold. Then, given $\tilde{t} \triangleq t_k + \tau$ and $x(0) = \bar{x}$, one has that

$$|\varphi(\tilde{t}, \bar{x}, \mathbf{u}, \mathbf{0}) - \varphi(\tilde{t}, \bar{x}, \mathbf{u}, \mathbf{w})| \leq \gamma \left(\tau + T \mathcal{L}_{\tau} \frac{\mathcal{L}_T^k - 1}{\mathcal{L}_T - 1} \right)$$

for all $\bar{x} \in \mathcal{X}$, all $\mathbf{u} \in \mathcal{M}_{\mathcal{U}}$ and $\mathbf{w} \in \mathcal{M}_{\mathcal{W}}$. \square

Proof: See [21, Lemma 1]. \blacksquare

Now, let us focus on the MPC component $u(t)$. It is a piecewise constant feedback law expressible as

$$u(t) = \kappa(t, \tilde{t}_j, x(\tilde{t}_j)), \quad t \in [\tilde{t}_j, \tilde{t}_{j+1}) \quad (21)$$

In order to describe the hold mechanism implicit in (21), according to [17], a suitable state augmentation is performed. By posing $x_c \triangleq [x^T \hat{x}^T u^T]^T \in \mathbb{R}^{2n+m}$, then the closed-loop system (18), (21) can be written as

$$\dot{x}_c(t) = \begin{bmatrix} h(x(t)) + Bu(t) + w(t) \\ h(\hat{x}(t)) + Bu(t) \\ 0_{m,1} \end{bmatrix}, \quad t \in [t_k, t_{k+1}) \quad (22)$$

with

$$x_c(t_k) = \begin{bmatrix} x(t_k^-) \\ \hat{x}(t_k^-) \\ \kappa(t_k, t_j, x(\tilde{t}_j)) \end{bmatrix} \quad (23)$$

$\forall t \neq \tilde{t}_j$, where \tilde{t}_j is the last execution time before t_k . While, $\forall t = \tilde{t}_j$, one has

$$x_c(\tilde{t}_j) = \begin{bmatrix} x(\tilde{t}_j) \\ x(\tilde{t}_j) \\ \kappa(\tilde{t}_j, \tilde{t}_j, x(\tilde{t}_j)) \end{bmatrix} \quad (24)$$

The solution of (22) from the initial time instant \tilde{t} with initial state $x_c(\tilde{t}) = \bar{x}_c$, will be hereafter denoted by $\varphi_c(t, \bar{x}_c, \mathbf{w})$, $\forall t \geq \tilde{t}$. Moreover, the first $2n$ and the last m components of

$\varphi_c(t, \bar{x}_c, \mathbf{w})$ will be denoted by $\varphi_x(t, \bar{x}_c, \mathbf{w})$, $\varphi_{\hat{x}}(t, \bar{x}_c, \mathbf{w})$ and $\varphi_u(t, \bar{x}_c, \mathbf{w})$, respectively.

Now, to design the MPC controller relying on the event-triggered logic defined by (8), it is necessary to consider that the equivalent system is subject only to the residual uncertainty $w(t)$. Because of the event-based realization of the control scheme, the effect of such uncertainty accumulates. Following the idea behind the control algorithm presented in [30] for discrete-time systems, and considering that system (18), for any $t \geq 0$, is a particular case of system (1), a new robust MPC control algorithm for continuous-time systems in form (18) can be proposed. To this end, define the tightened set

$$\mathcal{X}_{\tau_0 + (\nu_1 + \nu_2)T + \tau_3} = \mathcal{X} \sim \mathcal{B}_{\tau_0 + (\nu_1 + \nu_2)T + \tau_3} \quad (25)$$

where

$$\begin{aligned} \mathcal{B}_{\tau_0 + (\nu_1 + \nu_2)T + \tau_3} \triangleq z \in \mathbb{R}^n : |z| \leq \\ \gamma \left(\bar{\mathcal{L}}_{3\tau_3} \mathcal{L}_1^{\nu_1} \tau_3 + T \bar{\mathcal{L}}_{3\tau_3} \left(\mathcal{L}_2^{\nu_2} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} + \frac{\mathcal{L}_2^{\nu_2} - 1}{\mathcal{L}_2 - 1} \right) \right. \\ \left. + \bar{\mathcal{L}}_{3\tau_3} \mathcal{L}_1^{\nu_1} \mathcal{L}_2^{\nu_2} \tau_0 \right) \quad (26) \end{aligned}$$

with $\tau_0, \tau_3 \in [0, T)$, $\mathcal{L}_1 = \mathcal{L}_T$, $\mathcal{L}_2 = \mathcal{L}_{\kappa_{fT}} > 0$, $\bar{\mathcal{L}}_{3\tau_3} = \max\{1, \mathcal{L}_{3\tau_3}\}$ and $\mathcal{L}_{3\tau_3} = \mathcal{L}_{\tau_3}$ if $\nu_1 < N - 1$ and $\nu_2 = 0$, and $\mathcal{L}_{3\tau_3} = \mathcal{L}_{\kappa_{f\tau_3}}$ if $\nu_1 = N - 1$ and $\nu_2 > 0$, $N \geq 1$ being the prediction horizon. This definition of the tightened set guarantees that, if the nominal state evolution belongs to $\mathcal{X}_{kT + \tau}$, then the perturbed trajectory of the system fulfills (4), as will be proved in the following.

The proposed MPC controller is based on the following Finite-Horizon Optimal Control Problem (FHOCP) that consists in minimizing, at any time instant \tilde{t}_j such that the triggering condition (8) is violated, a suitably defined cost function with respect to the control sequence $\bar{\mathbf{u}}_{[\tilde{t}_j, t_{k_j + N - 1} | \tilde{t}_j]} \triangleq [u_0(\tilde{t}_j), u_1(\tilde{t}_j), \dots, u_N(\tilde{t}_j)]$. The associated finite horizon piecewise-constant control signal $\mathbf{u}_{[\tilde{t}_j, t_{k_j + N} | \tilde{t}_j]}$ is such that

$$\mathbf{u}_{[\tilde{t}_j, t_{k_j} | \tilde{t}_j]}(t) = u_0(\tilde{t}_j)$$

for all $t \in [\tilde{t}_j, t_{k_j})$ and

$$\mathbf{u}_{[t_{k_j}, t_{k_j + N} | \tilde{t}_j]}(t) = u_{i+1}(\tilde{t}_j)$$

for all $t \in [t_{k_j + i}, t_{k_j + i + 1})$ and all $i \in [0, \dots, N - 1]$. Note that because the FHOCP is not solved at any sampling time as usual, but at any triggering time instant \tilde{t}_j , the first value of vector $\bar{\mathbf{u}}_{[\tilde{t}_j, t_{k_j + N - 1} | \tilde{t}_j]}$ is applied only from \tilde{t}_j to t_{k_j} .

Definition 1 (FHOCP): Consider system (18) with $x(t_k) = \bar{x}$. Given the positive integer N , the quadratic stage cost $l(x, u) \triangleq x^T Q x + u^T R u$ (Q and R being positive definite matrices), the quadratic terminal penalty $V_f(x) \triangleq x^T \Pi x$ (being Π a symmetric positive definite matrix), and the terminal set

\mathcal{X}_f , the FHOCP problem consists in minimizing with respect to $\bar{\mathbf{u}}_{[\tilde{t}_j, t_{k_j + N - 1} | \tilde{t}_j]}$ the cost function

$$\begin{aligned} J(\bar{x}, \bar{\mathbf{u}}_{[\tilde{t}_j, t_{k_j + N - 1} | \tilde{t}_j]}, N) = \\ \int_{\tilde{t}_j}^{t_{k_j + N}} l(x(\tau), u(\tau)) d\tau + V_f(x(t_{k_j + N})) \quad (27) \end{aligned}$$

subject to

- 1) the state dynamics (18) with uncertainty term $w(t) = 0$, $\forall t \in [\tilde{t}_j, t_{k_j + N}]$;
- 2) the state tightened constraint $x(t) \in \mathcal{X}_{t - \tilde{t}_j}$, $\forall t \in [\tilde{t}_j, t_{k_j + N}] = [\tilde{t}_j, \tilde{t}_j + \tau_0 + NT]$;
- 3) the control constraint $u \in \bar{\mathcal{U}}$ with $\bar{\mathcal{U}} = \mathcal{U} \sim \mathcal{U}^{\text{ISM}}$;
- 4) the terminal state constraint $x(t_{k_j + N}) \in \mathcal{X}_f$. \square

Remark 3: Since \mathcal{X} and $\bar{\mathcal{U}}$ are bounded, the stage cost is a Lipschitz function with respect to both the state and the control values, i.e., there exist $\mathcal{L}_l > 0$ and $\mathcal{L}_{lu} > 0$ such that

$$|l(x_1, u) - l(x_2, u)| \leq \mathcal{L}_l |x_1 - x_2| \quad (28)$$

$$|l(x, u_1) - l(x, u_2)| \leq \mathcal{L}_{lu} |u_1 - u_2| \quad (29)$$

for all $x_1, x_2 \in \mathcal{X}$ and all $u_1, u_2 \in \bar{\mathcal{U}}$. \square

It is now possible to state the proposed MPC algorithm: at any triggering time instant \tilde{t}_j , apply the control law

$$u(t) = \kappa^{\text{MPC}}(x(\tilde{t}_j), \tilde{t}_j, \tilde{t}_{j+1}) =$$

$$\begin{cases} \mathbf{u}_{[\tilde{t}_j, t_{k_j + N - 1} | \tilde{t}_j]}^o & t \in [\tilde{t}_j, t_{k_j + N}] \\ \kappa_f(\hat{x}(t)) & t > t_{k_j + N} \end{cases}, \tilde{t}_j < t < \tilde{t}_{j+1} \quad (30)$$

where \tilde{t}_{j+1} is the next triggering time instant, $\bar{\mathbf{u}}_{[\tilde{t}_j, t_{k_j + N - 1} | \tilde{t}_j]}^o$ is the optimal control sequence obtained by solving the FHOCP, and κ_f is an auxiliary control law to be specified such that $\kappa_f(\hat{x}(t)) = \kappa_f(\hat{x}(t_k))$ for all $t \in [t_k, t_{k+1}]$.

With reference to (30), denote with $\varphi_{\kappa_f}(t, x, \mathbf{w})$ the solution of (18) when the auxiliary control law κ_f is applied. Then, the following further assumption on the terminal penalty and terminal set of the FHOCP is introduced in order to guarantee closed-loop stability.

Assumption 3: The design elements V_f and \mathcal{X}_f are such that, given a compact set Φ and an auxiliary control law κ_f , the following properties hold:

- 1) $\mathcal{X}_f(x) \triangleq \{x : x^T \Pi x \leq \sigma_f\}$, $\mathcal{X}_f \subseteq \mathcal{X}_{NT}$, such that $\{0\} \subseteq \mathcal{X}_f$ and σ_f is a positive real number;
- 2) $\Phi \triangleq \{x : x^T \Pi x \leq \sigma_\Phi\}$, being $\mathcal{X}_f \subseteq \Phi \subseteq \mathcal{X}_{(N-1)T}$, and $\sigma_\Phi \geq \sigma_f$;
- 3) $\kappa_f(x(t)) \triangleq \kappa_f(x(t_k))$, for all $t \in [t_k, t_{k+1})$, $\kappa_f(0) = 0$ and $x \in \Phi \Rightarrow \kappa_f \in \bar{\mathcal{U}}$;
- 4) κ_f is Lipschitz with respect to the state variable x in the domain Φ with Lipschitz constant $\mathcal{L}_\kappa > 0$, i.e.,

$$|\kappa_f(x_1) - \kappa_f(x_2)| \leq \mathcal{L}_\kappa |x_1 - x_2|, \forall x_1, x_2 \in \Phi \quad (31)$$

- 5) if $x(t_k) \in \Phi$, then $\varphi_{\kappa_f}(t_k + \nu T + \tau, x(t_k), \mathbf{0}) \in \mathcal{X}_{(N + \nu - 1)T + \tau}$ for all $\tau \in [0, T)$ and $\nu \in \mathbb{N}_0^+$. Moreover, one has that $\varphi(t_{k+1}, x(t_k), \kappa_f(x(t_k)), \mathbf{0}) \in \mathcal{X}_f$;

6) Given $\mathcal{L}_{\kappa_{f\tau}}$, with $\mathcal{L}_{\kappa_{fT}} < 1$ it holds

$$|\varphi_{\kappa_f}(\tau, x_1, \mathbf{0}) - \varphi_{\kappa_f}(\tau, x_2, \mathbf{0})| \leq \mathcal{L}_{\kappa_{f\tau}} |x_1 - x_2| \quad (32)$$

with $\tau \in [0, T]$.

7) for any $t \in [t_k, t_{k+1}]$ the following inequality holds

$$V_f(\varphi(t, x(t_k), \kappa_f(x(t_k)), \mathbf{0})) - V_f(x(t_k)) \leq - \int_{t_k}^t l(\varphi(\tau, x(t_k), \kappa_f(x(t_k)), \mathbf{0}), \kappa_f(x(t_k))) d\tau$$

for all $x(t_k) \in \Phi$;

8) consider a generic time instant $t_k + \tau$, $\tau \in [0, T]$; system (1) is such that

$$|\varphi(t_k + \tau, x(t_k), \kappa_f(x(t_k)), \mathbf{0}) - \varphi(t_k + \tau, x(t_k), \mathbf{0}, \mathbf{0})| \leq \mathcal{L}_{u_\tau} |\kappa_f(x(t_k))| \quad (33)$$

for all $x(t_k) \in \Phi$, where $\mathcal{L}_{u_\tau} \triangleq \mathcal{L}_u(\tau)$ is a positive continuous function in $[0, T]$ such that $\mathcal{L}_{u_0} = 0$. \square

Since Φ is a compact set, no additional assumption is needed to state that V_f is Lipschitz with respect to the state variable x in the domain Φ , i.e.,

$$|V_f(x_1) - V_f(x_2)| \leq \mathcal{L}_f |x_1 - x_2|, \quad \forall x_1, x_2 \in \Phi \quad (34)$$

\square

Finally, consider the following assumption on the bound of the uncertainties that the proposed algorithm takes into account.

Assumption 4: Suppose that the parameters involved in Assumptions 1 and 3 have been chosen so as to obtain a value of γ such that

$$\gamma \leq \frac{\sigma_\Phi - \sigma_f (1 + \mathcal{L}_f K_1(\varepsilon_1) \mathcal{X}^{\text{sup}} + \mathcal{L}_f K_2(\varepsilon_2))}{\mathcal{L}_f K_3(\tau_T)} \quad (35)$$

\square

Remark 4: Note that the proposed approach based on the combination of a robust MPC with ISM has a general validity, in the sense that it could be used not only with the constraint tightening method here adopted, but also with any other robust MPC based approach [36], [37]. Yet, the combined use of MPC with ISM provides an advantage over the use of a robust MPC standalone. Indeed, the ISM component, by rejecting the matched uncertainties, allows one to reduce the conservativeness of any robust MPC. \square

V. STABILITY ANALYSIS

In this section the robustness and stability properties of the proposed control strategy are discussed.

A. Tightened Sets

The following lemma is useful to prove the properties of the proposed MPC control law.

Lemma 4: Let $x \in \mathcal{X}_{\tau_0 + (\nu_1 + \bar{k}_1)T + \tau_8 + (\nu_2 + \bar{k}_2)T + \tau_9 + \tau_7}$, $\nu_1, \nu_2, \bar{k}_1, \bar{k}_2 \in N^+$, $\tau_0, \tau_7, \tau_8, \tau_9 \in [0, T]$. Moreover consider two functions $\mathcal{L}_{1\tau}$ and $\mathcal{L}_{2\tau}$ satisfying Assumption 2.1 and Assumption 3.6 respectively, the constants $k_1, k_2, N \in N^+$, $\tau_3, \tau_4 \geq 0$ and

- if $\nu_1 < N$: $\nu_2 = 0$, $\bar{k}_2 = k_2$, $\mathcal{L}_{3\tau_3} = \mathcal{L}_{1\tau_3}$, $\mathcal{L}_{3\tau_4} = \mathcal{L}_{1\tau_4}$, $\tau_9 = 0$, and if $\tau_3 + \tau_4 = T$, $\bar{k}_1 = k_1 + 1$ and $\tau_8 = 0$, while if $\tau_3 + \tau_4 < T$, $\mathcal{L}_{3\tau_8} = \mathcal{L}_{1\tau_8}$, $\bar{k}_1 = k_1 = 0$, $\bar{k}_2 = k_2 = 0$, $\tau_7 = 0$, and $\tau_8 = \tau_3 + \tau_4$;
- if $\nu_1 = N$: $\bar{k}_1 = k_1 = 0$, $\mathcal{L}_{3\tau_3} = \mathcal{L}_{2\tau_3}$, $\mathcal{L}_{3\tau_4} = \mathcal{L}_{2\tau_4}$, $\mathcal{L}_{3\tau_7} = \mathcal{L}_{2\tau_7}$, $\tau_8 = 0$, and if $\tau_3 + \tau_4 = T$, $\bar{k}_2 = k_2 + 1$ and $\tau_9 = 0$, while if $\tau_3 + \tau_4 < T$, $\mathcal{L}_{3\tau_9} = \mathcal{L}_{2\tau_9}$, $\bar{k}_2 = 0$, $\tau_7 = 0$ and $\tau_9 = \tau_3 + \tau_4$.

Hence assuming that $y \in \mathbb{R}^n$ is such that

$$|y - x| \leq \gamma \left(\tau_3 + T \mathcal{L}_{3\tau_3} \left(\mathcal{L}_2^{\nu_2} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} + \frac{\mathcal{L}_2^{\nu_2} - 1}{\mathcal{L}_2 - 1} \right) + \mathcal{L}_{3\tau_3} \mathcal{L}_1^{\nu_1} \mathcal{L}_2^{\nu_2} \tau_0 \right) \mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \quad (36)$$

then, $y \in \mathcal{X}_{\tau_4 + (k_1 + k_2)T + \tau_7}$. \square

Proof: See Appendix B. \blacksquare

B. Feasibility and Input-to-State practical Stability

The Input-to-State practical Stability (ISpS) of the closed-loop system (18), (30) is proved. One can refer to [21], [38] for the concept of continuous-time regional ISpS, that will be used along this section. In the following, let $\mathcal{X}^{\text{MPC}} \subseteq \mathcal{X}$ denote the set of states for which a solution of the FHOCPC exists.

Lemma 5: [Feasibility] Suppose that system (18) satisfies Assumptions 1-4. Then, \mathcal{X}^{MPC} is a robust positively invariant (RPI) set for the closed-loop system (18), (30). \square

Proof: See Appendix C. \blacksquare

Lemma 6: [Regional ISpS] Suppose that system (18) fulfills Assumptions 1-4. Then, the closed-loop system (18), (30) is regional ISpS in \mathcal{X}^{MPC} . \square

Proof: In order to prove the stability properties, one has to show that the following function

$$V(t, x_c(t), N) \triangleq \begin{cases} J \left(x(t), \bar{\mathbf{u}}_{[t, t_{k_j + N - 1} | \tilde{t}_j]}^o, N \right), & \tilde{t}_j \leq t < t_{k_j + N} \\ V_f(x(t)), & t_{k_j + N} \leq t < \tilde{t}_{j+1} \end{cases} \quad (37)$$

is a ISpS Lyapunov function. In view of space limitations, all the technical details are not reported. Most of the steps follow the ideas of the proof of Lemma 4 in [21]. \blacksquare

C. Main Result

We are now in a position to introduce the main stability result for the overall model-based event-triggered MPC/ISM control scheme.

Theorem 1: Given the plant (1), with the uncertainty bound in (6), and the mechanism based on the triggering condition (8), then, supposing that Assumptions 1-4 are fulfilled, by applying the control law (7), (14) and (30), system (1) is regional ISpS in \mathcal{X}^{MPC} . \square

Proof: See Appendix C. \blacksquare

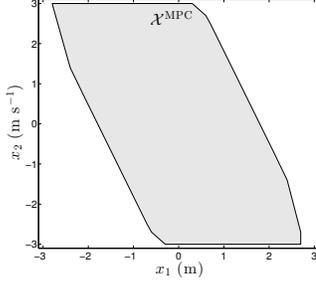


Figure 2. Estimation of the feasible set \mathcal{X}^{MPC} .

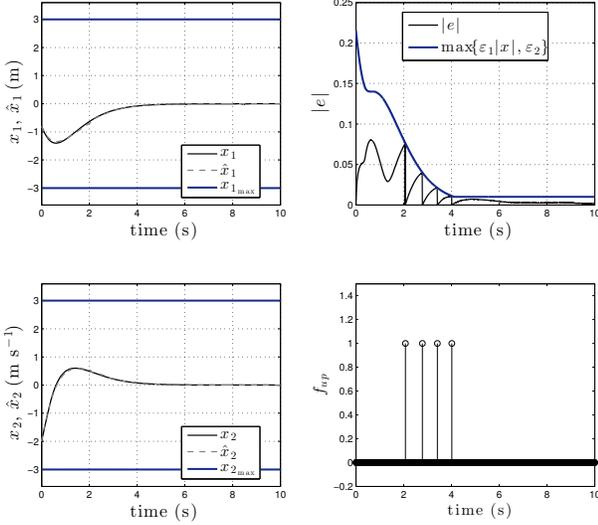


Figure 3. From left to right: time evolution of the state variables of the plant (x_1 : top, x_2 : bottom, solid line), and of the model (\hat{x}_1 : top, \hat{x}_2 : bottom, dashed line); time evolution of the norm of the state error e with the relative threshold (top), and updates of the actual state (bottom).

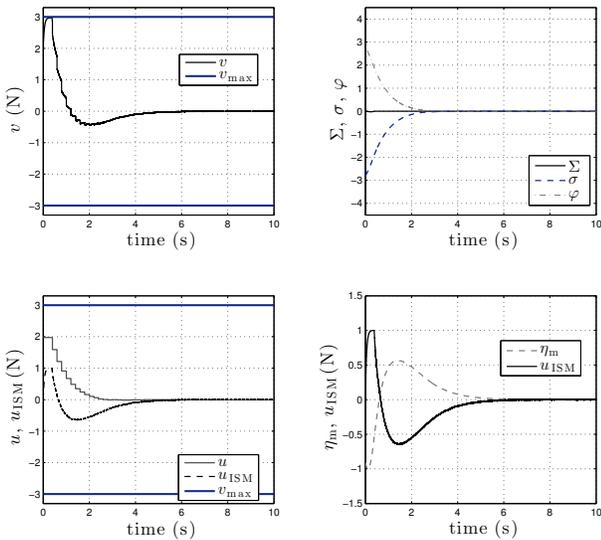


Figure 4. From left to right: time evolution of the control variables $v(t)$ (top), $u(t)$ and $u_{\text{ISM}}(t)$ (bottom); time evolution of the auxiliary sliding variable, of the sliding variable, of the transient trajectory (Σ : solid line, σ : dashed line, φ : dash-dotted line), and of the matched uncertainty with the ISM component (bottom, η_m : dashed line, u_{ISM} : solid line).

VI. ILLUSTRATIVE EXAMPLE

In this section, the proposed control strategy is applied in simulation to a cart moving on a plane. The plant is described by the following equations

$$\begin{cases} \dot{x}_1(t) = x_2(t) + \eta_u(t) \\ \dot{x}_2(t) = \frac{1}{M}(-k_0 x_1(t) - h_0 x_2(t) + v(t) + \eta_m(t)) \end{cases} \quad (38)$$

where the control variable v is the force applied to the cart. Moreover, considering (1),

$$h(x) = \begin{bmatrix} 0 & x_2 \\ -\frac{k_0}{M}x_1 & -\frac{h_0}{M}x_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} \quad (39)$$

where $M = 1 \text{ kg}$ is the mass of the cart, which is assumed to be known, $k_0 = 0.33 \text{ N m}^{-1}$ is the stiffness of the spring, $h_0 = 1.1 \text{ N s m}^{-1}$ is the damping factor, while the matched uncertain term is $\eta_m = W_m \sin(x_2)$, with $|\eta_m|_\infty \leq 1 \text{ N}$. In (38), signal η_u is an unmatched uncertain disturbance, which is generated as the overimposition of a random noise and a sinusoidal function of x_2 such that $|\eta_u|_\infty \leq 0.2 \text{ m s}^{-1}$. Accordingly, the nominal model is

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) \\ \dot{\hat{x}}_2(t) = \frac{1}{M}(-k_0 \hat{x}_1(t) - h_0 \hat{x}_2(t) + v(t)) \end{cases} \quad (40)$$

The initial condition is $x(0) = \hat{x}(0) = [-0.8 \ -2]^T$. In Fig. 2 an estimation of the positive invariant set \mathcal{X}^{MPC} is illustrated.

To perform the simulation tests, the Euler solver is used with a numerical integration step τ_i equal to 0.0002 s , while the MPC sampling time is chosen as $T = 0.2 \text{ s}$. The prediction horizon of the FHOCP is $N = 3$. The quantities Q and R in (27) are chosen respectively as $Q = \mathbb{I}_2$, and $R = 1$, while the auxiliary control law and the matrix Π are equal to

$$\kappa_f(\hat{x}) = -K\hat{x}, \quad K = [0.6413 \ 0.7306] \quad (41)$$

and

$$\Pi = \begin{bmatrix} 8.7647 & 3.6217 \\ 3.6217 & 4.6226 \end{bmatrix} \quad (42)$$

The considered control and state constraints are $|v| \leq 3 \text{ N}$, $|x_1|$, $|\hat{x}_1| \leq 3 \text{ m}$, $|x_2|$, $|\hat{x}_2| \leq 3 \text{ m s}^{-1}$. The relative degree of the system is $r = 1$, since the sliding variable is selected as $\sigma = m_1 x_1 + x_2$, with $m_1 = 1$. The transient trajectory φ is chosen as

$$\begin{aligned} \varphi(t) &= -m_1 x_1(0) - x_2(0) \\ &\quad - \int_0^t m_1 x_2(\tau) - \frac{1}{M}(k_0 x_1(\tau) + h_0 x_2(\tau)) + \frac{1}{M} u(\tau) d\tau \end{aligned} \quad (43)$$

The discontinuous control law in (14) has the amplitude $U_{\text{max}} = 1$. The triggering condition in (8) is specified by choosing $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.01$. The considered tightened set is selected as in (25) and (26) with $\gamma = 1$, $\mathcal{L}_\tau = e^{1.5\tau}$ and $\mathcal{L}_{\kappa_f \tau} = e^{-0.92\tau}$.

In order to evaluate the closed-loop performance with respect to the conventional, i.e., non event-triggered, control, and event-triggered MPC without the ISM controller, that is without any local controller, we consider two indexes: i) the number of updates of the actual state, denoted with n_{up} ; ii)

the root mean square (RMS) value of the plant state, x_{RMS} . These indexes are determined as

$$n_{\text{up}} = \sum_{i=0}^{n_s} f_{\text{up}}(\tau_i), \quad x_{\text{RMS}} = \sqrt{\frac{1}{n_s} \sum_{i=1}^{n_s} \sum_{j=1}^n x_{ji}^2} \quad (44)$$

where $f_{\text{up}}(\cdot)$ is a flag equal to 1 when the actual state is transmitted over the network, equal to zero otherwise, n_s is the number of integration steps during the simulation, and x_{ji} is the j -th component of the state vector at the i -th integration step.

Figure 3 shows the time evolution of the state variables of the plant and of the nominal model, which are both steered to a vicinity of zero depending on the amplitude of the unmatched uncertain term η_u . Moreover, in Figure 3, the relative threshold defined in (8) and the flag values are also reported. Then, Figure 4 shows the time evolution of the control variables $v(t)$, $u(t)$ and $u_{\text{ISM}}(t)$. The behavior of the auxiliary sliding variable, of the sliding variable, of the transient trajectory, and the time evolution of the ISM component with respect to the matched uncertain term η_m are also illustrated in Figure 4. From the analysis, it appears that the RMS values of the state are equal to 5.31783×10^{-2} when the proposed scheme is applied, equal to 5.31436×10^{-2} in case of non event-triggered implementation of the control scheme, and equal to 5.95011×10^{-2} when the standard event-based MPC without ISM component is used. Finally, the number of updates, i.e., state and control transmission, is significantly reduced (4 transmissions) not only with respect to the case in which the state is always transmitted over the network (51 transmissions) but also with respect to a standard event-based MPC (50 transmissions). This confirms the effectiveness of the proposed control approach.

VII. CONCLUSIONS

In this paper, a model-based event-triggered MPC/ISM control scheme for nonlinear constrained uncertain networked systems is proposed. The main objective is to reduce the number of transmissions of the actual plant state over the network, while guaranteeing satisfactory performance of the controlled system. A smart sensor and a smart actuator are included in the scheme. The control law is designed by suitably combining ISM control with MPC. The ISM component, based on the actual state provided by the smart sensor, is used in order to compensate the matched uncertainty affecting the system, so that an asynchronous packetized version of a quasi-infinite horizon MPC with tightened constraints can be designed relying on a plant with reduced uncertainties. The actual plant state is transmitted only when a triggering event occurs. In such time instants the MPC law is updated. As a result, the regional ISpS of the overall control system is proved.

APPENDIX A

RESULTS ON THE INVARIANCE PROPERTY

Proof of Lemma 1: Consider the plant (1), the sliding variable in (10), and the transient trajectory in (12) expressed

as

$$\dot{\phi} = -S \{h(x) + Bu\} \quad (45)$$

Then, the first-time derivative of the auxiliary control variable can be determined as

$$\dot{\Sigma}(t) = S\dot{x} - S \{h(x) + Bu\} \quad (46)$$

Then, one can compute the ‘‘equivalent control’’ [33], by posing $\dot{\Sigma} = 0$, i.e.,

$$\begin{aligned} S(\dot{x} - h(x) - Bu) &= S(h(x) + B(u + u_{\text{ISM}_{\text{eq}}}) \\ &\quad + B\eta_m + B^\perp \eta_u - h(x) - Bu) \\ &= S(Bu_{\text{ISM}_{\text{eq}}} + B\eta_m + B^\perp \eta_u) = 0 \end{aligned}$$

which yields

$$u_{\text{ISM}_{\text{eq}}} = -\eta_m - (SB)^{-1}SB^\perp \eta_u \quad (47)$$

Substituting (47) in (1), the equivalent dynamics of the locally controlled plant results in being

$$\dot{x}(t) = h(x(t)) + Bu(t) + \tilde{w}(t), \quad t \geq 0 \quad (48)$$

with

$$\tilde{w} = (I - B(SB)^{-1}S)B^\perp \eta_u \quad (49)$$

which is invariant with respect to the matched uncertainty. ■

Proof of Lemma 2: This result follows directly from [27, Propositions 2 and 3]. By virtue of Lemma 1, it can be proved that the choice of $S = B^T$ minimizes the norm of \tilde{w} in (48), i.e.,

$$B^T = \arg \min_{S \in \mathbb{R}^{m \times n}} |(I - B(SB)^{-1}S)B^\perp \eta_u| \quad (50)$$

such that the equivalent dynamics is

$$\dot{x}(t) = h(x(t)) + Bu(t) + w(t), \quad t \geq 0 \quad (51)$$

with $w(t) = B^\perp \eta_u(t)$, which concludes the proof. ■

APPENDIX B

RESULTS ON TIGHTENED SETS

Proof of Lemma 4: Let $\alpha_{\tau_4 + (k_1 + k_2)T + \tau_7} \in \mathcal{B}_{\tau_4 + (k_1 + k_2)T + \tau_7}$ and $z = y - x + \alpha_{\tau_4 + (k_1 + k_2)T + \tau_7}$. Then, one

has that $|z| \leq |y - x| + |\alpha_{\tau_4 + (k_1 + k_2)T + \tau_7}|$. It yields

$$\begin{aligned}
|z| &\leq |y - x| + |\alpha_{\tau_4 + (k_1 + k_2)T + \tau_7}| \leq \\
&\gamma \left(\tau_3 + T \mathcal{L}_{3\tau_3} \left(\mathcal{L}_2^{\nu_2} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} + \frac{\mathcal{L}_2^{\nu_2} - 1}{\mathcal{L}_2 - 1} \right) \right. \\
&\quad \left. + \mathcal{L}_{3\tau_3} \mathcal{L}_1^{\nu_1} \mathcal{L}_2^{\nu_2} \tau_0 \right) \mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \\
&\quad + \gamma \left(\bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1} \tau_7 + T \bar{\mathcal{L}}_{3\tau_7} \left(\mathcal{L}_2^{k_2} \frac{\mathcal{L}_1^{k_1} - 1}{\mathcal{L}_1 - 1} + \frac{\mathcal{L}_2^{k_2} - 1}{\mathcal{L}_2 - 1} \right) \right. \\
&\quad \left. + \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \tau_4 \right) \\
&= \gamma \left(\mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \tau_3 + \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1} \tau_7 \right. \\
&\quad \left. + T \mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \mathcal{L}_{3\tau_3} \mathcal{L}_2^{\nu_2} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} + T \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_2^{k_2} \frac{\mathcal{L}_1^{k_1} - 1}{\mathcal{L}_1 - 1} \right. \\
&\quad \left. + T \mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \mathcal{L}_{3\tau_3} \frac{\mathcal{L}_2^{\nu_2} - 1}{\mathcal{L}_2 - 1} + T \bar{\mathcal{L}}_{3\tau_7} \frac{\mathcal{L}_2^{k_2} - 1}{\mathcal{L}_2 - 1} + \right. \\
&\quad \left. + \mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \mathcal{L}_{3\tau_3} \mathcal{L}_1^{\nu_1} \mathcal{L}_2^{\nu_2} \tau_0 \right. \\
&\quad \left. + \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \tau_4 \right)
\end{aligned}$$

Consider now two different cases: i) $\nu_1 < N$, ii) $\nu_1 = N$. For the first case, by assumption, $\nu_2 = 0$, $\bar{\mathcal{L}}_{3\tau_3} = \mathcal{L}_{3\tau_3} = \mathcal{L}_{1\tau_3}$ and $\bar{\mathcal{L}}_{3\tau_4} = \mathcal{L}_{3\tau_4} = \mathcal{L}_{1\tau_4}$,

$$|z| \leq |y - x| + |\alpha_{\tau_4 + (k_1 + k_2)T + \tau_7}| \leq$$

$$\begin{aligned}
&\gamma \left(\mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \tau_3 + \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1} \tau_7 \right. \\
&\quad \left. + T \mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \mathcal{L}_{3\tau_3} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} + T \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_2^{k_2} \frac{\mathcal{L}_1^{k_1} - 1}{\mathcal{L}_1 - 1} \right. \\
&\quad \left. + T \bar{\mathcal{L}}_{3\tau_7} \frac{\mathcal{L}_2^{k_2} - 1}{\mathcal{L}_2 - 1} + \mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \mathcal{L}_{3\tau_3} \mathcal{L}_1^{\nu_1} \tau_0 \right. \\
&\quad \left. + \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \tau_4 \right)
\end{aligned}$$

Moreover note that if $\tau_3 + \tau_4 = T$ the terms

$$\gamma \left(\mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \tau_3 + \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \tau_4 \right)$$

represents the effect of γ during a period T after $k_1T + k_2T + \tau_7$ time. Hence the terms

$$\begin{aligned}
&\gamma \left(\mathcal{L}_{3\tau_4} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \tau_3 + \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1} \mathcal{L}_2^{k_2} \tau_4 \right. \\
&\quad \left. + T \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_2^{k_2} \frac{\mathcal{L}_1^{k_1} - 1}{\mathcal{L}_1 - 1} \right)
\end{aligned}$$

can be rewritten as

$$\gamma \left(T \mathcal{L}_{3\tau_7} \mathcal{L}_2^{k_2} \frac{\mathcal{L}_1^{k_1+1} - 1}{\mathcal{L}_1 - 1} \right) \leq \gamma \left(T \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_2^{k_2} \frac{\mathcal{L}_1^{k_1+1} - 1}{\mathcal{L}_1 - 1} \right)$$

so that

$$\begin{aligned}
|z| &\leq \gamma \left(\bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1} \tau_7 + T \mathcal{L}_1^{k_1+1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} \right. \\
&\quad \left. + T \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_2^{k_2} \frac{\mathcal{L}_1^{k_1+1} - 1}{\mathcal{L}_1 - 1} + T \bar{\mathcal{L}}_{3\tau_7} \frac{\mathcal{L}_2^{k_2} - 1}{\mathcal{L}_2 - 1} \right. \\
&\quad \left. + \mathcal{L}_1^{k_1+1} \mathcal{L}_2^{k_2} \mathcal{L}_{3\tau_7} \mathcal{L}_1^{\nu_1} \tau_0 \right) \\
&\leq \gamma \left(\bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^{k_1+\nu_1+1} \tau_7 + T \mathcal{L}_2^{k_2} \bar{\mathcal{L}}_{3\tau_7} \frac{\mathcal{L}_1^{k_1+\nu_1+1} - 1}{\mathcal{L}_1 - 1} \right. \\
&\quad \left. + T \bar{\mathcal{L}}_{3\tau_7} \frac{\mathcal{L}_2^{k_2} - 1}{\mathcal{L}_2 - 1} + \mathcal{L}_1^{k_1+1+\nu_1} \mathcal{L}_2^{k_2} \bar{\mathcal{L}}_{3\tau_7} \tau_0 \right)
\end{aligned}$$

i.e., according to (26), $z \in \mathcal{B}_{\tau_0 + (\nu_1 + k_1 + 1)T + k_2T + \tau_7}$. Then, since $\bar{k}_1 = k_1 + 1$, $\bar{k}_2 = k_2$, $\tau_8 = \tau_9 = 0$, $\nu_2 = 0$ one has $x \in \mathcal{X}_{\tau_0 + (\nu_1 + k_1 + 1)T + k_2T + \tau_7}$ and $x + z \in \mathcal{X}$, $y + \alpha_{\tau_4 + (k_1 + k_2)T + \tau_7} = x + z$, one can conclude that $y \in \mathcal{X}_{\tau_4 + (k_1 + k_2)T + \tau_7}$. Moreover note that if $\tau_3 + \tau_4 < T$, $k_1 = k_2 = 0$ and $\tau_8 = \tau_3 + \tau_4$, $\tau_7 = 0$, $\bar{\mathcal{L}}_{3\tau_8} = \mathcal{L}_{3\tau_8} = \mathcal{L}_{1\tau_8}$,

$$\begin{aligned}
|z| &\leq |y - x| + |\alpha_{\tau_4}| = \\
&\gamma \left(\mathcal{L}_{3\tau_4} \tau_3 + T \mathcal{L}_{3\tau_3} \mathcal{L}_{3\tau_4} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} + \mathcal{L}_{3\tau_4} \mathcal{L}_{3\tau_3} \mathcal{L}_1^{\nu_1} \tau_0 + \tau_4 \right) \\
&\leq \gamma \left(\mathcal{L}_{3\tau_3 + \tau_4} \mathcal{L}_1^{\nu_1} (\tau_3 + \tau_4) \right. \\
&\quad \left. + T \mathcal{L}_{3\tau_3 + \tau_4} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} + \mathcal{L}_{3\tau_3 + \tau_4} \mathcal{L}_1^{\nu_1} \tau_0 \right) \\
&= \gamma \left(\mathcal{L}_{3\tau_8} \mathcal{L}_1^{\nu_1} \tau_8 + T \mathcal{L}_{3\tau_8} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} + \mathcal{L}_{3\tau_8} \mathcal{L}_1^{\nu_1} \tau_0 \right) \\
&= \gamma \left(\bar{\mathcal{L}}_{3\tau_8} \mathcal{L}_1^{\nu_1} \tau_8 + T \bar{\mathcal{L}}_{3\tau_8} \frac{\mathcal{L}_1^{\nu_1} - 1}{\mathcal{L}_1 - 1} + \bar{\mathcal{L}}_{3\tau_8} \mathcal{L}_1^{\nu_1} \tau_0 \right)
\end{aligned}$$

i.e., according to (26), $z \in \mathcal{B}_{\tau_0 + \nu_1 T + \tau_8}$. Then, since $\bar{k}_1 = k_1 = 0$, $\bar{k}_2 = k_2 = 0$, $\nu_2 = 0$, $\tau_7 = \tau_9 = 0$ one has $x \in \mathcal{X}_{\tau_0 + \nu_1 T + \tau_8}$ and $x + z \in \mathcal{X}$, $y + \alpha_{\tau_4} = x + z$, one can conclude that $y \in \mathcal{X}_{\tau_4}$.

Consider now the second case in which, by assumption, $\nu_1 = N$, $\bar{k}_1 = k_1 = 0$. Analogously to the first case, if $\tau_3 + \tau_4 = T$, $\tau_9 = 0$ and $\bar{k}_2 = k_2 + 1$

$$\begin{aligned} |z| &\leq |y - x| + |\alpha_{\tau_4 + k_2 T + \tau_7}| \leq \\ &\gamma \left(\bar{\mathcal{L}}_{3\tau_7} \tau_7 + T \mathcal{L}_2^{k_2+1} \mathcal{L}_{3\tau_7} \mathcal{L}_2^{\nu_2} \frac{\mathcal{L}_1^N - 1}{\mathcal{L}_1 - 1} \right. \\ &\quad \left. + T \mathcal{L}_2^{k_2+1} \mathcal{L}_{3\tau_7} \frac{\mathcal{L}_2^{\nu_2} - 1}{\mathcal{L}_2 - 1} \right. \\ &\quad \left. + T \bar{\mathcal{L}}_{3\tau_7} \frac{\mathcal{L}_2^{k_2+1} - 1}{\mathcal{L}_2 - 1} + \mathcal{L}_2^{k_2+1} \mathcal{L}_{3\tau_7} \mathcal{L}_1^N \mathcal{L}_2^{\nu_2} \tau_0 \right) \\ &\leq \gamma \left(\bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^N \tau_7 + T \bar{\mathcal{L}}_{3\tau_7} \left(\mathcal{L}_2^{k_2+\nu_2+1} \frac{\mathcal{L}_1^N - 1}{\mathcal{L}_1 - 1} \right. \right. \\ &\quad \left. \left. + \frac{\mathcal{L}_2^{k_2+\nu_2+1} - 1}{\mathcal{L}_2 - 1} \right) + \bar{\mathcal{L}}_{3\tau_7} \mathcal{L}_1^N \mathcal{L}_2^{k_2+\nu_2+1} \tau_0 \right) \end{aligned}$$

i.e., according to (26), $z \in \mathcal{B}_{\tau_0 + NT + (k_2 + \nu_2 + 1)T + \tau_7}$. Then, since $\nu_1 = N$, $\bar{k}_1 = 0$, $\tau_8 = \tau_9 = 0$, $\bar{k}_2 = k_2 + 1$ one has $x \in \mathcal{X}_{\tau_0 + NT + (k_2 + \nu_2 + 1)T + \tau_7}$ and $x + z \in \mathcal{X}$, $y + \alpha_{\tau_4 + k_2 T + \tau_7} = x + z$, one can conclude that $y \in \mathcal{X}_{\tau_4 + k_2 T + \tau_7}$. Moreover note that if $\tau_3 + \tau_4 < T$, $k_1 = k_2 = 0$ and $\tau_9 = \tau_3 + \tau_4$, $\tau_7 = 0$,

$$\begin{aligned} |z| &\leq |y - x| + |\alpha_{\tau_4}| \leq \\ &\gamma \left(\mathcal{L}_{3\tau_4} \tau_3 + T \mathcal{L}_{3\tau_3} \mathcal{L}_{3\tau_4} \mathcal{L}_2^{\nu_2} \frac{\mathcal{L}_1^N - 1}{\mathcal{L}_1 - 1} + T \mathcal{L}_{3\tau_3} \mathcal{L}_{3\tau_4} \frac{\mathcal{L}_2^{\nu_2} - 1}{\mathcal{L}_2 - 1} \right. \\ &\quad \left. + \mathcal{L}_{3\tau_3} \mathcal{L}_{3\tau_4} \mathcal{L}_1^N \mathcal{L}_2^{\nu_2} \tau_0 + \tau_4 \right) \\ &\leq \gamma \left(\bar{\mathcal{L}}_{3\tau_9} \mathcal{L}_1^N \tau_9 + T \bar{\mathcal{L}}_{3\tau_9} \left(\mathcal{L}_2^{\nu_2} \frac{\mathcal{L}_1^N - 1}{\mathcal{L}_1 - 1} + \frac{\mathcal{L}_2^{\nu_2} - 1}{\mathcal{L}_2 - 1} \right) \right. \\ &\quad \left. + \bar{\mathcal{L}}_{3\tau_9} \mathcal{L}_1^N \mathcal{L}_2^{\nu_2} \tau_0 \right) \end{aligned}$$

i.e., according to (26), $z \in \mathcal{B}_{\tau_0 + NT + \nu_2 T + \tau_9}$. Then, since one has $x \in \mathcal{X}_{\tau_0 + NT + \nu_2 T + \tau_9}$ and $x + z \in \mathcal{X}$, $y + \alpha_{\tau_4} = x + z$, one can conclude that $y \in \mathcal{X}_{\tau_4}$. This concludes the proof. ■

APPENDIX C RESULTS ON STABILITY

Proof of Lemma 5: To get the feasibility property, one has to prove that

$$\begin{aligned} x(\tilde{t}_j) \in \mathcal{X}^{\text{MPC}} &\Rightarrow x(\tilde{t}_{j+1}) \in \mathcal{X}^{\text{MPC}}, x(\tilde{t}_{j+1}) \triangleq \\ \varphi(\tilde{t}_{j+1} - \tilde{t}_j, x(\tilde{t}_j), \kappa^{\text{MPC}}(x(\tilde{t}_j), \tilde{t}_j, \tilde{t}_{j+1}), \mathbf{w}_{[\tilde{t}_j, \tilde{t}_{j+1}]}) &\quad (52) \end{aligned}$$

Letting $x(\tilde{t}_j) \in \mathcal{X}^{\text{MPC}}$ and the associated optimal solution $\bar{\mathbf{u}}_{[\tilde{t}_j, t_{k_j+N-1}|\tilde{t}_j]}$ of the FHOCP at time \tilde{t}_j , a possible (sub-

optimal) solution at time \tilde{t}_{j+1} for the FHOCP is

$$\begin{aligned} \bar{\mathbf{u}}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]} &\triangleq \\ &\begin{bmatrix} \bar{\mathbf{u}}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N-1}|\tilde{t}_j]}^o \\ \kappa_f \left(\varphi(t_{k_j+N} - \tilde{t}_{j+1}, x(\tilde{t}_{j+1}|\tilde{t}_j), \right. \\ \quad \left. \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N-1}|\tilde{t}_j]}^o, \mathbf{0}) \right) \\ \vdots \\ \kappa_f \left(\varphi_{\kappa_f}(t_{k_{j+1}+N-1}, \varphi(t_{k_j+N} - \tilde{t}_{j+1}, x(\tilde{t}_{j+1}|\tilde{t}_j), \right. \\ \quad \left. \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N-1}|\tilde{t}_j]}^o, \mathbf{0}, \mathbf{0}) \right) \end{bmatrix} \quad (53) \end{aligned}$$

where

$$x(\tilde{t}_{j+1}|\tilde{t}_j) \triangleq \varphi(\tilde{t}_{j+1} - \tilde{t}_j, x(\tilde{t}_j), \mathbf{u}_{[\tilde{t}_j, \tilde{t}_{j+1}|\tilde{t}_j]}^o, \mathbf{0}) \quad (54)$$

is the value of the state of the MPC nominal (without uncertainties) closed-loop system at time \tilde{t}_{j+1} . To determine the feasibility of such a solution, one must prove the following three steps.

Step 1: It is necessary to show that the state value must lay in \mathcal{X}_f at $t_{k_{j+1}+N}$, i.e.,

$$\varphi((N-1)T + t_{k_{j+1}} - \tilde{t}_j, x(\tilde{t}_{j+1}), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \in \mathcal{X}_f$$

where $\mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N}|\tilde{t}_{j+1}]}^s$ is the signal associated with the control sequence

$$\bar{\mathbf{u}}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s$$

In order to prove this, we show that

$$\begin{aligned} \varphi((N-2)T + t_{k_{j+1}} - \tilde{t}_{j+1}, x(\tilde{t}_{j+1}), \\ \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \in \Phi \end{aligned}$$

To this aim, first define

$$\begin{aligned} \varphi_p \triangleq \varphi((N-2)T + t_{k_{j+1}} - \tilde{t}_{j+1}, x(\tilde{t}_{j+1}), \\ \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \end{aligned}$$

and

$$\begin{aligned} \varphi_n \triangleq \varphi((N-2)T + t_{k_{j+1}} - \tilde{t}_{j+1}, x(\tilde{t}_{j+1}|\tilde{t}_j), \\ \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \end{aligned}$$

Note that Assumption 2 implies

$$|\varphi_p - \varphi_n| \leq \mathcal{L}_{t_{k_{j+1}} - \tilde{t}_{j+1}} \mathcal{L}_T^{N-2} |x(\tilde{t}_{j+1}) - x(\tilde{t}_{j+1}|\tilde{t}_j)|$$

Now denote with \tilde{t}_{j+1}^- the time when the triggering condition is verified before \tilde{t}_{j+1} and define $\tau_T \triangleq \tilde{t}_{j+1} - \tilde{t}_{j+1}^-$. Considering the triggering condition (8), $\forall x \in \mathcal{X}$, $\forall u \in \mathcal{U}$, $\forall w \in \mathcal{W}$, there exists τ_T such that it holds

$$K_{\tau_T} |\varphi(\tau_T, x, \mathbf{u}, \mathbf{w})| \geq |x| \quad (55)$$

with K_{τ_T} positive constant. At time \tilde{t}_{j+1}^- if the triggering rule is violated then $\tilde{t}_{j+1}^- = \tilde{t}_{j+1}$ and $|x(\tilde{t}_j) - x(\tilde{t}_j|\tilde{t}_j)| = 0$, while

if it was satisfied hence

$$|x(\tilde{t}_{j+1}^-) - x(\tilde{t}_{j+1}^-|\tilde{t}_j)| < \max \{ \varepsilon_1 |x(\tilde{t}_{j+1}^-)|, \varepsilon_2 \}$$

Moreover, note that

$$\begin{aligned} |x(\tilde{t}_{j+1}^-)| - |x(\tilde{t}_{j+1}^-|\tilde{t}_j)| &\leq |x(\tilde{t}_{j+1}^-) - x(\tilde{t}_{j+1}^-|\tilde{t}_j)| \\ &< \max \{ \varepsilon_1 |x(\tilde{t}_{j+1}^-)|, \varepsilon_2 \} \\ &\leq \varepsilon_1 |x(\tilde{t}_{j+1}^-)| + \varepsilon_2 \end{aligned}$$

and

$$|x(\tilde{t}_{j+1}^-)| \leq |x(\tilde{t}_{j+1}^-|\tilde{t}_j)| + \varepsilon_1 |x(\tilde{t}_{j+1}^-)| + \varepsilon_2$$

$$(1 - \varepsilon_1) |x(\tilde{t}_{j+1}^-)| \leq |x(\tilde{t}_{j+1}^-|\tilde{t}_j)| + \varepsilon_2$$

$$|x(\tilde{t}_{j+1}^-)| \leq \frac{|x(\tilde{t}_{j+1}^-|\tilde{t}_j)| + \varepsilon_2}{(1 - \varepsilon_1)}$$

only if $0 < \varepsilon_1 < 1$. Hence,

$$\begin{aligned} |x(\tilde{t}_{j+1}^-)| - |x(\tilde{t}_{j+1}^-|\tilde{t}_j)| &\leq \\ &\left(\frac{\mathcal{L}_{\tau_T} \varepsilon_1}{(1 - \varepsilon_1)} (|x(\tilde{t}_{j+1}^-|\tilde{t}_j)| + \varepsilon_2) + \mathcal{L}_{\tau_T} \varepsilon_2 + \gamma \tau_T \right) \quad (56) \\ &\leq \left(\frac{\mathcal{L}_{\tau_T} \varepsilon_1}{(1 - \varepsilon_1)} (K_{\tau_T} |x(\tilde{t}_{j+1}^-|\tilde{t}_j)| + \varepsilon_2) + \mathcal{L}_{\tau_T} \varepsilon_2 + \gamma \tau_T \right) \end{aligned}$$

and

$$|\varphi_p - \varphi_n| \leq K_1(\varepsilon_1) |x(\tilde{t}_{j+1}^-|\tilde{t}_j)| + K_2(\varepsilon_2) + K_3(\tau_T) \gamma \quad (57)$$

with

$$K_1(\varepsilon_1) = \mathcal{L}_{t_{k_{j+1}} - \tilde{t}_{j+1}} \mathcal{L}_T^{N-2} \mathcal{L}_{\tau_T} K_{\tau_T} \frac{\varepsilon_1}{(1 - \varepsilon_1)}$$

$$K_2(\varepsilon_2) = \mathcal{L}_{t_{k_{j+1}} - \tilde{t}_{j+1}} \mathcal{L}_T^{N-2} \mathcal{L}_{\tau_T} \varepsilon_2 \frac{1}{(1 - \varepsilon_1)}$$

$$K_3(\tau_T) = \mathcal{L}_{t_{k_{j+1}} - \tilde{t}_{j+1}} \mathcal{L}_T^{N-2} \tau_T$$

such that $K_1(0) = K_2(0) = 0$. Since $\varphi_n \in \mathcal{X}_f$

$$\begin{aligned} V_f(\varphi_p) &\leq V_f(\varphi_n) + \mathcal{L}_f(K_1(\varepsilon_1) |x(\tilde{t}_{j+1}^-|\tilde{t}_j)| \\ &\quad + K_2(\varepsilon_2) + K_3(\tau_T) \gamma) \\ &\leq \sigma_f + \mathcal{L}_f \left(K_1(\varepsilon_1) \mathcal{X}^{\text{sup}} + K_2(\varepsilon_2) + K_3(\tau_T) \gamma \right) \\ &= \sigma_f (1 + \mathcal{L}_f K_1(\varepsilon_1) \mathcal{X}^{\text{sup}}) + \mathcal{L}_f K_2(\varepsilon_2) \\ &\quad + \mathcal{L}_f K_3(\tau_T) \gamma \leq \sigma_\Phi \end{aligned}$$

At this point, applying κ_f , according to Point 5 in Assumption 3 one obtains

$$\begin{aligned} \varphi((N-1)T + t_{k_{j+1}} - \tilde{t}_{j+1}, x(\tilde{t}_{j+1}^-), \\ \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \in \mathcal{X}_f \end{aligned}$$

Step 2: The control must fulfill the following constraint $\mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N}|\tilde{t}_{j+1}]}^s \in \mathcal{U}$. It follows from the fact that $\mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}+N-1}|\tilde{t}_j]}^o \in \mathcal{U}$ by definition, and since

$\varphi_{\kappa_f}(t_i, \varphi(t_{k_{j+N}} - \tilde{t}_{j+1}), x(\tilde{t}_{j+1}^-|\tilde{t}_j), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+N-1}|\tilde{t}_j]}^o, \mathbf{0})} \in \Phi, \forall t_i > t_{k_{j+N}}$, then $\kappa_f(\varphi_{\kappa_f}) \in \mathcal{U}$.

Step 3: In order to assure the respect of the state constraints, Lemma 4 with $\mathcal{L}_{1\tau} = \mathcal{L}(\tau)$ and $\mathcal{L}_{2\tau} = \mathcal{L}_{\kappa_f \tau}$ is used to verify the following cases.

1) $\nu < N$, $\varphi(\tau_4, x(\tilde{t}_{j+1}^-), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \in \mathcal{X}_{\tau_4}$ with $\tau_4 \in [0, t_{k_{j+1}} - \tilde{t}_{j+1}]$. Since it holds,

$$\varphi(\tau_4, x(\tilde{t}_{j+1}^-|\tilde{t}_j), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \in \mathcal{X}_{\tau_0 + \nu T + \tau_8}$$

where $\tau_0 = t_{k_j} - \tilde{t}_j$, $\tau_8 = \tau_3 + \tau_4 < T$, $\tau_3 = \tilde{t}_{j+1} - t_{k_{j+1}-1}$. According to (19), (20) and Lemma 3, one has

$$\begin{aligned} &|\varphi(\tau_4, x(\tilde{t}_{j+1}^-|\tilde{t}_j), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \\ &\quad - \varphi(\tau_4, x(\tilde{t}_{j+1}^-), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}}|\tilde{t}_{j+1}]}^s, \mathbf{0})| \\ &\leq \gamma \left(\tau_3 + T \mathcal{L}_{\tau_3} \frac{\mathcal{L}_T^\nu - 1}{\mathcal{L}_T - 1} + \mathcal{L}_{\tau_3} \mathcal{L}_T^\nu \tau_0 \right) \mathcal{L}_{\tau_4} \end{aligned}$$

Then, Lemma 4 holds considering $\nu_1 = \nu$, $\nu_2 = k_1 = k_2 = 0$, $\tau_7 = \tau_9 = 0$ which proves this case.

2) $\nu < N$, $\varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}^-), \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \in \mathcal{X}_{\tau_4 + pT + \tau_7}$ with $\tau_4 = t_{k_{j+1}} - \tilde{t}_{j+1}$, and $p = 1, \dots, N - \nu$.

Since it holds,

$$\begin{aligned} &\varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}^-|\tilde{t}_j), \\ &\quad \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \in \mathcal{X}_{\tau_0 + (\nu+1)T + pT + \tau_7} \end{aligned}$$

where $\tau_0 = t_{k_j} - \tilde{t}_j$, $\tau_3 + \tau_4 = T$, $\tau_3 = \tilde{t}_{j+1} - t_{k_{j+1}-1}$, $\tau_7 \in [0, T)$. According to (19), (20) and Lemma 3, one has

$$\begin{aligned} &|\varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}^-|\tilde{t}_j), \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \\ &\quad - \varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}^-), \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s, \mathbf{0})| \\ &\leq \gamma \left(\tau_3 + T \mathcal{L}_{\tau_3} \frac{\mathcal{L}_T^\nu - 1}{\mathcal{L}_T - 1} + \mathcal{L}_{\tau_3} \mathcal{L}_T^\nu \tau_0 \right) \mathcal{L}_{\tau_4} \mathcal{L}_T^p \mathcal{L}_{\tau_7} \end{aligned}$$

Then, Lemma 4 holds considering $\nu_1 = \nu$, $\nu_2 = k_2 = 0$, $k_1 = p$, $\tau_8 = \tau_9 = 0$ which proves this case.

3) $\nu < N$, $\varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}^-), \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \in \mathcal{X}_{\tau_4 + pT + \tau_7}$ with $\tau_4 = t_{k_{j+1}} - \tilde{t}_{j+1}$ and $p = N - \nu + 1, \dots, N$.

Since it holds,

$$\begin{aligned} &\varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}^-|\tilde{t}_j), \\ &\quad \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}]}^s, \mathbf{0}) \in \mathcal{X}_{\tau_0 + (\nu+1)T + pT + \tau_7} \end{aligned}$$

where $\tau_0 = t_{k_j} - \tilde{t}_j$, $\tau_3 + \tau_4 = T$, $\tau_3 = \tilde{t}_{j+1} - t_{k_{j+1}-1}$, $\tau_7 \in [0, T)$. According to (19), (20) and Lemma 3, one has

$$\begin{aligned} & |\varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}|\tilde{t}_j), \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}], \mathbf{0}}^s) \\ & - \varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}), \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}], \mathbf{0}}^s)| \\ & \leq \gamma \left(\tau_3 + T \mathcal{L}_{\tau_3} \frac{\mathcal{L}_T^\nu - 1}{\mathcal{L}_T - 1} \right. \\ & \left. + \mathcal{L}_{\tau_3} \mathcal{L}_T^\nu \tau_0 \right) \mathcal{L}_{\tau_4} \mathcal{L}_T^{(N-\nu)} \mathcal{L}_{\kappa_{fT}}^{(p-(N-\nu))} \mathcal{L}_{\kappa_{f\tau_7}} \end{aligned}$$

Then, Lemma 4 holds considering $\nu_1 = \nu$, $\nu_2 = 0$, $k_1 = N - \nu$, $k_2 = p - (N - \nu)$, which proves this case.

- 4) $\nu \geq N$, $\varphi(\tau_4, x(\tilde{t}_{j+1}), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}}|\tilde{t}_{j+1}], \mathbf{0}}^s) \in \mathcal{X}_{\tau_4}$ with $\tau_4 \in [0, t_{k_{j+1}} - \tilde{t}_{j+1}]$. Since it holds,

$$\varphi(\tau_4, x(\tilde{t}_{j+1}|\tilde{t}_j), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}}|\tilde{t}_{j+1}], \mathbf{0}}^s) \in \mathcal{X}_{\tau_0 + \nu T + \tau_9}$$

where $\tau_0 = t_{k_j} - \tilde{t}_j$, $\tau_9 = \tau_3 + \tau_4 < T$, $\tau_3 = \tilde{t}_{j+1} - t_{k_{j+1}-1}$. According to (19), (20) and Lemma 3, one has

$$\begin{aligned} & |\varphi(\tau_4, x(\tilde{t}_{j+1}|\tilde{t}_j), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}}|\tilde{t}_{j+1}], \mathbf{0}}^s) \\ & - \varphi(\tau, x(\tilde{t}_{j+1}), \mathbf{u}_{[\tilde{t}_{j+1}, t_{k_{j+1}}|\tilde{t}_{j+1}], \mathbf{0}}^s)| \\ & \leq \gamma \left(\tau_3 + T \mathcal{L}_{\kappa_{f\tau_3}} \left(\frac{\mathcal{L}_{\kappa_{fT}}^{\nu-N} \mathcal{L}_T^N - 1}{\mathcal{L}_T - 1} + \frac{\mathcal{L}_{\kappa_{fT}}^{\nu-N} - 1}{\mathcal{L}_{\kappa_{fT}} - 1} \right) \right. \\ & \left. + \mathcal{L}_{\tau_3} \mathcal{L}_T^N \mathcal{L}_{\kappa_{fT}}^{\nu-N} \tau_0 \right) \mathcal{L}_{\kappa_{f\tau_4}} \end{aligned}$$

Then, Lemma 4 holds considering $\nu_1 = N$, $\nu_2 = \nu - N$, $k_1 = k_2 = 0$, which proves this case.

- 5) $\nu \geq N$, $\varphi(\tau_4 + pT + \tau_7, \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}], \mathbf{0}}^s) \in \mathcal{X}_{\tau_4 + pT + \tau_7}$ with $\tau_4 \in [0, t_{k_{j+1}} - \tilde{t}_{j+1}]$, $p = 1, \dots, N$. Since it holds,

$$\begin{aligned} & \varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}|\tilde{t}_j), \\ & \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}], \mathbf{0}}^s) \in \mathcal{X}_{\tau_0 + (\nu+1)T + pT + \tau_7} \end{aligned}$$

where $\tau_0 = t_{k_j} - \tilde{t}_j$, $\tau_3 + \tau_4 = T$, $\tau_3 = \tilde{t}_{j+1} - t_{k_{j+1}-1}$, $\tau_7 \in [0, T)$. According to (19), (20) and Lemma 3, one has

$$\begin{aligned} & |\varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}|\tilde{t}_j), \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}], \mathbf{0}}^s) \\ & - \varphi(\tau_4 + pT + \tau_7, x(\tilde{t}_{j+1}), \mathbf{u}_{[t_{k_{j+1}}, t_{k_{j+1}+N-1}|\tilde{t}_{j+1}], \mathbf{0}}^s)| \\ & \leq \gamma \left(\tau_3 + T \mathcal{L}_{\kappa_{f\tau_3}} \left(\frac{\mathcal{L}_{\kappa_{fT}}^{\nu-N} \mathcal{L}_T^N - 1}{\mathcal{L}_T - 1} + \frac{\mathcal{L}_{\kappa_{fT}}^{\nu-N} - 1}{\mathcal{L}_{\kappa_{fT}} - 1} \right) \right. \\ & \left. + \mathcal{L}_{\tau_3} \mathcal{L}_T^N \mathcal{L}_{\kappa_{fT}}^{\nu-N} \tau_0 \right) \mathcal{L}_{\kappa_{f\tau_4}} \mathcal{L}_{\kappa_{fT}}^p \mathcal{L}_{\kappa_{f\tau_7}} \end{aligned}$$

Then, Lemma 4 holds considering $\nu_1 = N$, $\nu_2 = \nu - N$, $k_1 = 0$, $k_2 = p$, which proves this case. ■

Proof of Theorem 1: By applying the ISM control (14), according to Lemma 1, the equivalent system is (51), i.e., system (1) with reduced uncertainties, that is the unmatched terms $w = B^\perp \eta_u(t)$. Moreover, by applying the ISM inner loop, the control variable in the MPC has to fulfill (5), determined considering that a quantity equal to U_{\max} allocated for the ISM component (see (7)) must be subtracted to the control bounds of the set \mathcal{U} , i.e., which can be determined relying on the Pontryagin difference such that $\bar{\mathcal{U}} = \mathcal{U} \sim \mathcal{U}^{\text{ISM}}$. Then, since Assumptions 1-4 are satisfied, according to Lemma 6 the ISpS of the overall model-based event-triggered control scheme is proved. ■

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