

Trajectory Design, Stability, and Maintenance Analysis of Quasi-Satellite-Orbits in Martian Moons Environment

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Abstract. This paper presents advanced methods for designing scientific trajectories in the unique dynamical environment of the Martian moons, with a particular focus on stability analysis and station-keeping strategies. Such trajectories, aimed at deepening the understanding of Phobos and Deimos characteristics, have been proposed in different mission concepts over the past decade. The Japan Aerospace Exploration Agency's Martian Moons eXploration (MMX) mission, scheduled to visit Phobos in 2026, will mark the first practical implementation of these concepts. To build upon and complement the preparatory studies conducted for MMX, this work provides an in-depth analysis of orbital dynamics and long-term trajectory design specifically around Deimos. The results contribute to a broader framework for mission planning in irregular and weak-gravity environments, with implications for future exploration of small celestial bodies.

Introduction

Due to the strong gravity pull of Mars and the mass of Deimos being too small to capture a satellite, it is not possible to orbit the Martian moonlet in the usual two-body sense. Quasi-Satellite-Orbits (QSOs) [1] are a special kind of stable retrograde orbits in the restricted three-body problem that have gained attention as a viable candidate for future deep-space missions towards remote planetary satellites due to their mapping capabilities and high stability, coupled with low station-keeping costs.

These orbits can be found by iterative differential correction starting from a desired value of orbital radius. Additionally, out-of-plane motion can be also induced, in order to increase the surface coverage. However, the entity of the out-of-plane component must be limited, as it tends to destabilize the QSO, leading to an impact on Deimos or to an escape far from it. This work studies QSOs around Deimos, assessing their stability, their sensitivity to injection error and proposing a station-keeping strategy.

Dynamics

Due to the very low gravitational pull of Deimos, Hill's approximation of the Circular Restricted Three-Body Problem can be used to describe the dynamics [2], with the equations of motion that read:

$$\begin{aligned}\ddot{x} - 2\dot{y} &= g_x + 3x \\ \ddot{y} + 2\dot{x} &= g_y \\ \ddot{z} &= g_z - z\end{aligned}\quad (1)$$

where g represents the gravitational acceleration of Deimos, computed accordingly to a high-fidelity polyhedron shape model. Indeed, at the distances of the chosen QSOs from the surface of the moonlet, the error between the simple spherical gravity approximation and the polyhedron



model is up to 10-20%, as visible in Figure 1. The polyhedron approximation is computed as derived in [3] using the reference shape model of [4].

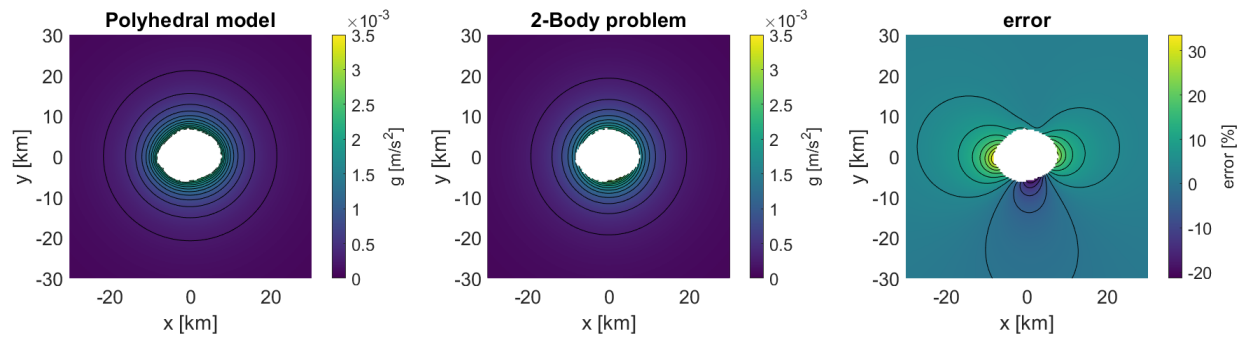


Figure 1: Difference between irregular and spherical gravity field models.

In addition, also a full ephemerides model should be considered to accurately describe the motion in the proximity of the small body. Consequently, two different dynamic models have been used in this work:

- A simplified model, in the Hill's reference frame, considering Martian gravity and a shape model to compute Deimos' gravitational attraction.
- A high fidelity model, propagated in an inertial reference frame centered at Mars. Here, Martian point mass and J_2 gravitational attraction are considered, together with Deimos shape model gravity. The motion of Deimos with respect to Mars is recovered using NASA spice ephemerides, with the mar097 and de432 kernels.

Stability analysis

The in-plane and out-of-plane periodicity of QSOs is assessed for a wide range of initial conditions. In particular, QSOs have been generated starting from an offset from Deimos on the x-axis of the Hill's reference frame ranging from 15 to 100 km. The prediction-correction is carried out in the simplified Hill model, with each orbit that is then re-propagated in full-ephemeris. More precisely, each QSO is computed considering only an in-plane initial velocity vector, then an out-of-plane component is included, progressively increasing the angle with respect to the x-y plane.

The objective of this analysis is to assess what is the limit on the out-of-plane component of the velocity vector, that tends to destabilize QSO motion as visible in Figure 2. Going further from Deimos the non-spherical gravitational attraction has a less important perturbing effect, with out-of-plane QSOs that are possible for higher inclinations of the initial velocity vector. This is outlined by Figure 3, in which the motion is bounded also for an out-of-plane angle of 20° .

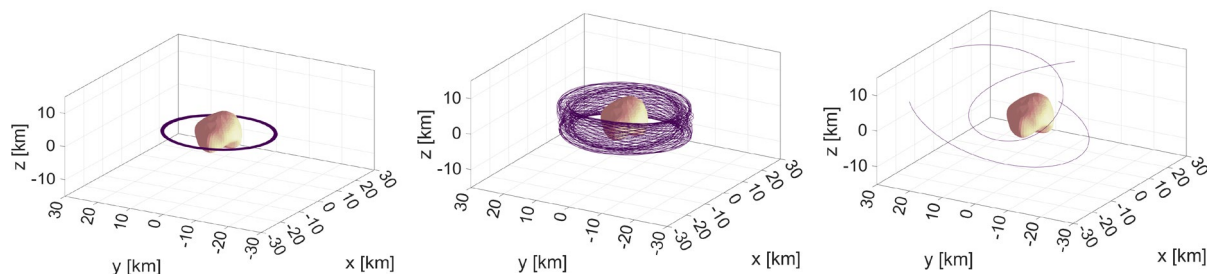


Figure 2: 60 km radius QSO. Motion computed with an in-plane initial velocity vector (left), and with a 10° (center) and 20° degree angle (right) with respect to the plane.

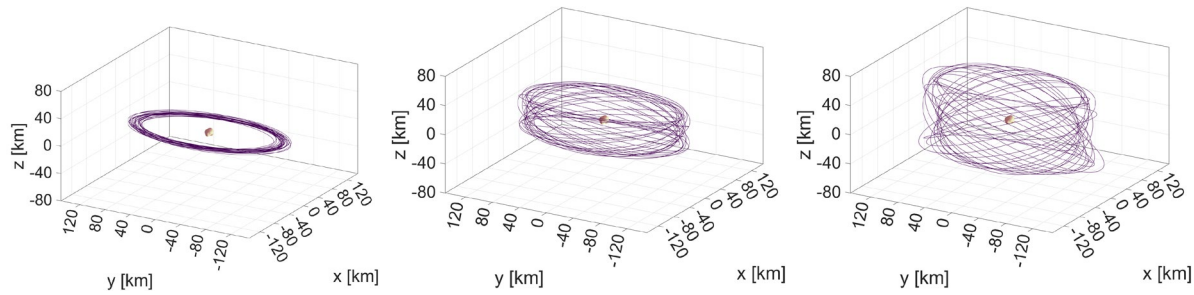
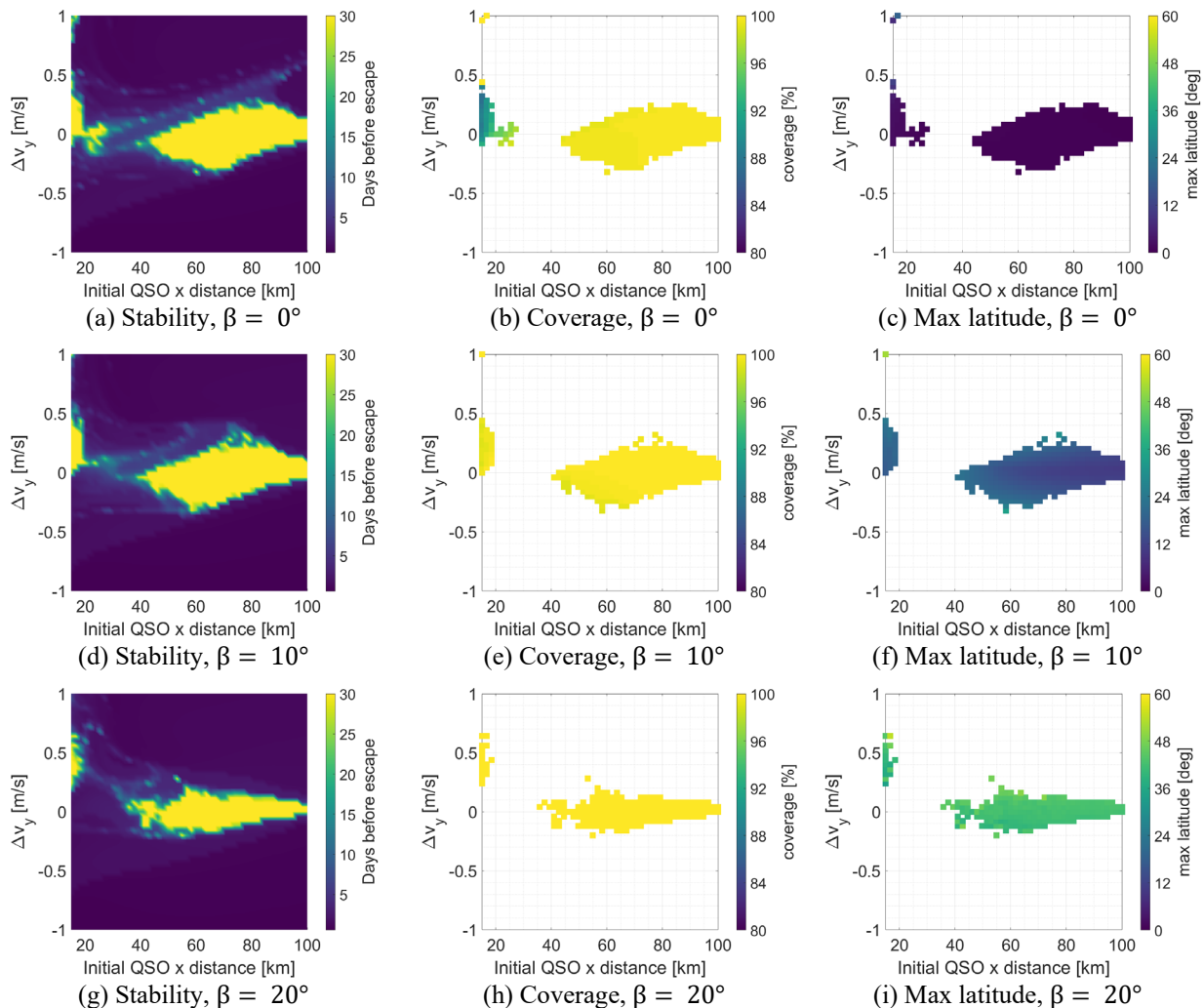


Figure 3: 60 km radius QSO. Motion computed with an in-plane initial velocity vector (left), and with a 10° (center) and 20° degree angle (right) with respect to the plane.

These results have then been generalized, by means of a sensitivity analysis of the stability of QSOs with respect to their initial distance and velocity with respect to Deimos. In this work, the stability analysis is carried out by propagating the orbit for a large number of revolutions in the high fidelity model, and by checking if the motion is bounded around Deimos or if the spacecraft escapes from the moonlet. Approaches based on the eigenvalues of the monodromy matrix are also possible [5], however, the necessity of using the full-ephemeris dynamics with non-spherical Martian moon gravity drove the choice towards a numeric approach for this work, with high-fidelity propagation.



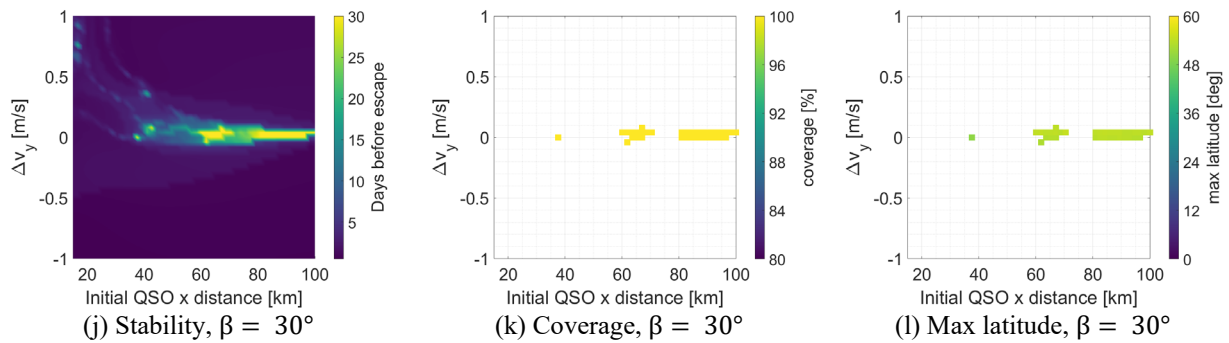


Figure 4: QSO stability and coverage analysis at different values of out-of-plane velocity component.

The result for in plane motion is shown in Figure 4a, which reports the days of bounded QSO motion over a maximum propagation interval of 30 days, in the full-ephemeris model. The plot presents the distance from Deimos along the Mars-Deimos line on the x-axis and possible tangential perturbations on the initial velocity on the y-axis. It can be noted that planar bounded QSO motion exists for almost the whole range of distances considered, with the exception of the region from 25 to 40 km. This difference with respect to the simplified Hill motion, for which bounded motion exists for all ranges, is probably due to the effects of the actual shape of Deimos' orbit around Mars, coupled with the highly irregular gravity field. Furthermore, it can be seen from the figures how the stability region shrinks for increasing value of initial out-of-plane velocity component.

Naturally, an additional limiting factor is the Δv_y perturbation on the initial velocity, that shall always remain within around 0.5 m/s to have a bounded QSO. Scientific performance of bounded orbits is also reported in Figure 4b, showing a percentage of visible surface up to 100%. This shows that from a sufficient distance almost the entire surface of Deimos can be covered, even with planar QSOs. The last plot on the right (Figure 4c) is the maximum latitude of the ground track of the spacecraft on Deimos. For planar motion it is obviously always zero. An exception arises for some perturbed low altitude orbits, that end up having an out-of-plane motion due to the non-symmetrical gravitational attraction of Deimos.

For out-of-plane QSOs the situation is similar, as visible from Figure 4d to Figure 4l. With the increase of the out-of-plane component of the initial velocity, the region of bounded motion in the $x - v_y$ plane tends to shrink and less QSOs are found for lower values of x offset. Again, this is due to the non-symmetrical shape of Deimos, that has a more important impact on the dynamics if the spacecraft is close to the moonlet.

Scientific orbit maintenance

In this Section, a station-keeping strategy for QSOs is explored, based on convex optimization techniques, which have found application in a wide variety of trajectory computation problems in the space domain [6, 7].

In this case, Sequential Convex Programming (SCP) techniques have been put in place, incorporating the effects of the irregular gravity field of the body in the trajectory generation. More in detail, an iterative station keeping algorithm is designed to compute the optimal control actions to maintain the nominal QSO, computed according to Hill three-body problem dynamics. In particular, each time the distance between the chosen QSO in the simplified model and the full-ephemeris propagation goes over 3 km, the GuSTO algorithm is used to bring the error to zero while minimizing $\Delta \mathbf{V}$, leaving a free maneuvering time. The example demonstrates the applicability of this strategy for a 30 km QSO with an initial velocity off-plane angle of 5° , which

would be unstable in the full ephemeris model. In particular, Figure 5 shows that the strategy is successful, being able to periodically correct the orbit for periods larger than one week with an average daily ΔV equal to 1.12 m/s.

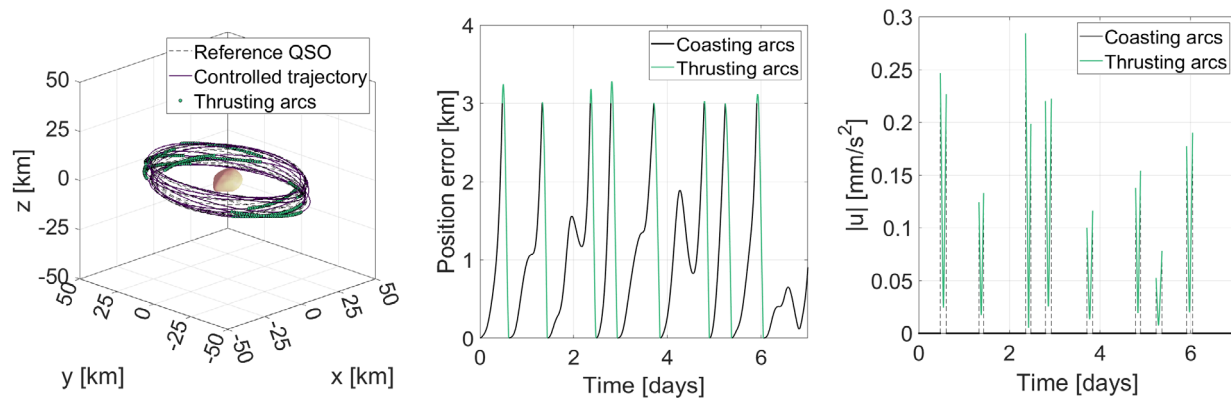


Figure 5: Results of station keeping analysis over a week of controlled motion in unstable 3D QSO region.

Conclusions

The analyses presented in this paper show that QSOs can be viable choices to meet high level requirements of typical scientific missions exploring Martian moon environment, while guaranteeing high orbital stability. Additionally, operational safety with respect to navigation and injection errors is also assessed for both planar and 3D orbit, in order to understand the applicability of the latter for high latitude observations. Finally, a SCP algorithm is proposed to iteratively compute dynamically feasible low- ΔV trajectory correction maneuvers to maintain periodic off-plane motion.

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