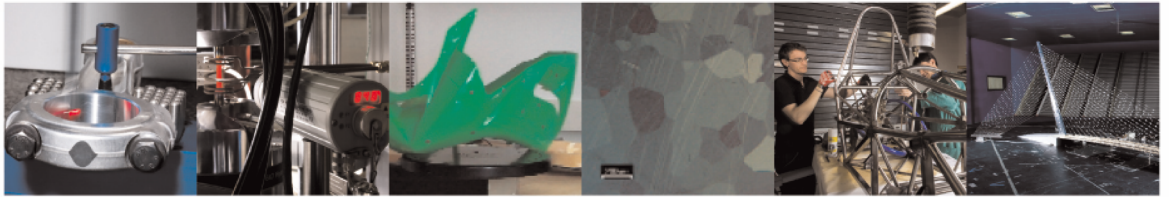




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## An integrated algorithm for ego-vehicle and obstacles state estimation for autonomous driving

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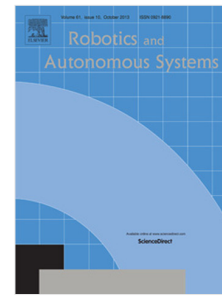
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## Journal Pre-proof

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# An integrated algorithm for ego-vehicle and obstacles state estimation for autonomous driving

Mattia Bersani<sup>a,\*</sup>, Simone Mentasti<sup>b</sup>, Pragyan Dahal<sup>a</sup>, Stefano Arrigoni<sup>a</sup>, Michele Vignati<sup>a</sup>, Federico Cheli<sup>a</sup>, Matteo Matteucci<sup>b</sup>

<sup>a</sup>*Department of Mechanical Engineering, Politecnico di Milano, Italy*

<sup>b</sup>*Department of Electronics, Information Engineering and Bioengineering, Politecnico di Milano, Italy*

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## Abstract

Understanding of the driving scenario represents a necessary condition for autonomous driving. Within the control routine of an autonomous vehicle, it represents the preliminary step for the motion planning system. Estimation algorithms hence need to handle a considerable number of information coming from multiple sensors, to provide estimates regarding the motion of ego-vehicle and surrounding obstacles. Furthermore, tracking is crucial in obstacles state estimation, because it ensures obstacles recognition during time. This paper presents an integrated algorithm for the estimation of ego-vehicle and obstacles' positioning and motion along a given road, modeled in curvilinear coordinates. Sensor fusion deals with information coming from two Radars and a Lidar to identify and track obstacles. The algorithm has been validated through experimental tests carried on a prototype of an autonomous vehicle.

*Keywords:* Obstacles tracking, Sensor fusion, State estimation, Autonomous driving

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## 1. Introduction

State estimation represents an essential part of the control routine of an autonomous vehicle. Together with the behavioral layer and the higher-level route planner, it provides the initial and boundary conditions for the motion planning system, and it feeds information to the trajectory planner and the low-level trajectory follower, which actuates the vehicle [1]. Initial and boundary conditions (*IC*, *BC*) are usually provided in terms of road geometry, limitations given by regulations, ego-vehicle and obstacles current positions, and velocities. This overall architecture is schematized in the control loop presented in Fig.1.

The importance of vehicle state estimation has increased starting from 90s, when it became a fundamental task for the incoming active safety systems like

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\*corresponding author

*Email address:* [mattia.bersani@polimi.it](mailto:mattia.bersani@polimi.it) (Mattia Bersani)

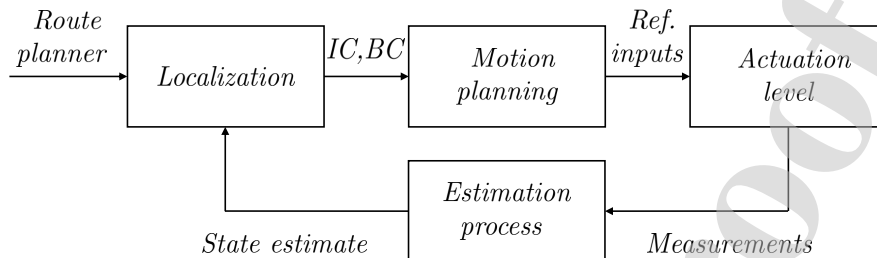


Figure 1. Representation of the control loop for an autonomous vehicle

12 ABS and ESP [2]. The measurements of yaw rate, wheels rotational speeds, and  
 13 oil pressure in the brake circuit were provided to ensure the feedback control  
 14 for vehicle handling. Moreover, kinematic quantities like vehicle sideslip were  
 15 estimated to account for saturation effects in the tire contact patch. However,  
 16 an autonomous vehicle's control routine requires further information about its  
 17 relative motion with respect to road bounds and other vehicles.

18 For these reasons, one of the biggest challenges for the development and  
 19 deployment of autonomous driving has been understanding the environment  
 20 it operates in, which is extremely dynamic and uncertain. Various perception  
 21 sensors have been developed and then used for this scope: ranging from stand-  
 22 alone ones to full-suites, allowing localizing and perceiving the environment  
 23 around the vehicle. Devices like Radars, Lidars, and cameras are very popular  
 24 in this field, even though they provide different performances and information in  
 25 terms of perception. Hence, various cost-effective combinations of sensor suites  
 26 have been proposed to perceive the surrounding environment. The use of Radars  
 27 [3], stereo cameras [4, 5] and Lidars [6] as stand-alone sensors has been done  
 28 in the past for obstacle state estimation. Numerous studies have then been  
 29 conducted based on the fusion of information coming from multiple sensors:  
 30 camera, Lidar, and Radar [7, 8, 9], Radar and Lidar [10, 11]. Each of those  
 31 sensors can provide heterogeneous information with different accuracy levels,  
 32 which explains why they are usually combined to provide a fused representation  
 33 of the environment. Among them, Radar is considered the most accurate sensor  
 34 for what concerns the measurement of velocities as it exploits the Doppler effect.  
 35 About positioning, the accuracy of Lidar measurements are considered better  
 36 [10], while object classification is usually performed by cameras thanks to the  
 37 high semantic content they provide[12].

38 Perception involves two major tasks: Simultaneous Localization and Map-  
 39 ping (SLAM) and Detection and Tracking of Moving Objects (DATMO). SLAM  
 40 allows the map generation around the ego-vehicle while it simultaneously local-  
 41 izes itself through the sensor measurements. DATMO requires the ego-vehicle to  
 42 detect any obstacle within the road bounds and keeps track of them in time, en-  
 43 abling the control system to account for each one's behavior within the current  
 44 driving scenario. This must be guaranteed even during sensors malfunctioning,  
 45 lack of sensors measurements due to asynchronous time sampling, abnormal

46 weather conditions, occlusions, and any other circumstance leading to missing  
 47 measurements that could cause blackouts. Hence, the estimation routine has to  
 48 guarantee that this lack of information does not induce the motion planner to  
 49 make wrong decisions.

50 Moreover, a proper modeling of the road environment close to the au-  
 51 tonomous vehicle, besides allowing navigation, guarantees an efficient prediction  
 52 of the relative positioning with respect to pedestrians, bicycles, other vehicles,  
 53 and road bounds. The most common road definition models are: poly-line  
 54 model, lane-let model, and Hermite spline model with increasing complexity  
 55 and computational need in given order [13]. According to the different motion  
 56 planners presented in [14, 15], the road map model of the track can be approx-  
 57 imated through cubic Hermite spline interpolation [16]. The most important  
 58 advantage of curvilinear coordinates ( $s - n$ ) with respect to Cartesian coordi-  
 59 nates ( $X - Y$ ) is that each road characteristic can be described as a function of  
 60 only one parameter (i.e., the abscissa  $s$ ); thus, each function that approximates  
 61 the centerline is at least surjective.

62 This paper focuses on state estimation for autonomous driving and presents  
 63 an integrated algorithm that provides state estimates for the ego-vehicle and the  
 64 surrounding obstacles. For the former, information about positioning, heading  
 65 angle, and velocity of the vehicle itself are provided by two GPS receivers,  
 66 inertial units, and odometry. About the latter, measurements are provided  
 67 by a multi-sensor framework, which includes two Radars located within the  
 68 vehicle front and rear bumpers and a Lidar mounted on the vehicle's top in  
 69 correspondence of the center of gravity. Information about the surroundings is  
 70 fused and provided to the tracking routine, according to DATMO. Exploiting  
 71 the knowledge of the road map, the ego-vehicle is localized along the track  
 72 within the road's local reference frame, from which the relative positioning and  
 73 motion of each obstacle can be derived in curvilinear coordinates as shown in  
 74 Fig. 2. Throughout this work, information about road boundaries and road  
 75 shape is considered as known. This integrated algorithm has been implemented  
 76 on the prototype of an autonomous vehicle presented in [17], and it has been  
 77 validated through experimental tests carried in the Monza Eni Circuit [18].  
 78 The algorithm works at 20 Hz on a soft real-time system based on ROS (Robot  
 79 Operating System) [19], which allows dealing with asynchronous sensors.

80 This paper is articulated through the following sections: the state of the art  
 81 on sensor fusion, state estimation and DATMO is reported in Section 2 while the  
 82 general structure of the algorithm is presented in Section 3. Section 4 presents  
 83 the ego-vehicle state estimation procedure, while obstacles state estimation and  
 84 tracking are described in Sections 5 and 6. The validation of the estimation  
 85 procedure is given in Section 7, where the experimental framework is presented  
 86 together with results.

## 87 2. Related works

88 As stated in the previous section, DATMO can provide the estimate for each  
 89 obstacle close to a vehicle even in uncertain conditions. Measurements filtering,

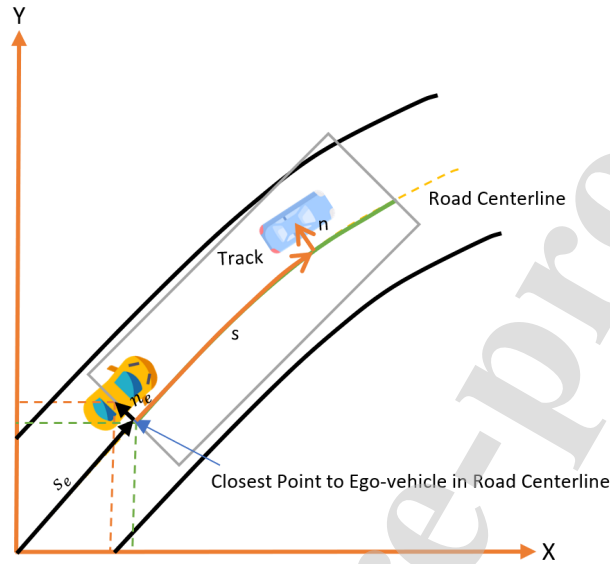


Figure 2. Coordinates transformation

90 data association, and data fusion are the main tasks for a multi-sensors and  
 91 multi-objects estimation problem: this section presents a summary of the state  
 92 of the art for each of them.

### 93 2.1. Filtering techniques

94 The measurements obtained from sensors in a real scenario are affected by  
 95 noise and a high degree of uncertainty. Hence, filtering procedures are usu-  
 96 ally applied to ensure accurate estimates and tracking. Bayesian filters are  
 97 widespread in literature: these filters exploit the Chapman-Kolmogorov the-  
 98 orem through the system transition density to achieve predicted probability  
 99 density functions (PDF) for the objects under consideration. Measurements are  
 100 then used to update the predicted PDF to find the posterior PDF, from which  
 101 the estimates can be obtained. Prediction and update steps in Bayesian filtering  
 102 involve complicated integrals that lead to a high computational burden. When  
 103 the model of the observed system is linear, and noise is Gaussian distributed,  
 104 the integrals can be computed analytically: in these conditions, Kalman filter-  
 105 ing [3, 5, 12, 20] provides the optimal solution, that can also be derived through  
 106 the minimization of the mean squared error. However, if the system behavior is  
 107 nonlinear, Extended Kalman Filter (EKF) [7, 8] and Unscented Kalman Filter  
 108 (UKF) [21, 22] are favored solutions. In particular, when nonlinearities become  
 109 huge, EKF provides less accurate solutions due to the first-order linearization  
 110 of the system's equations through Taylor-series expansion. Conversely, UKF is  
 111 based on the so-called unscented transformation, which approximately provides

112 Gaussian distributed outputs even when dealing with nonlinear transforma-  
113 tions. Particle filters or Sequential Monte Carlo (SMC) filters are other variants  
114 of Bayesian Filters that can be used for nonlinear systems and non-Gaussian  
115 Noise Distributions. As the name suggests, they use weighted particles, each  
116 represented by possible state estimation and posterior distribution. The use-  
117 age of Random Finite Set (RFS) statistics is common in Multi-Object-Tracking  
118 (MOT). In particular, RFS enables MOT without a priori measurement asso-  
119 ciation through the implementation of recursive Bayes filtering. When dealing  
120 with scenarios in which the birth and death of objects are regular, with a sig-  
121 nificant amount of clutter and false positives, the association process provided  
122 by traditional Bayes filters leads to erroneous results. Conversely, RFS allows  
123 accounting for objects birth (regular or spawning), occlusions, misdetections,  
124 and disappearances by taking the number of objects under consideration as a  
125 stochastic variable. Gaussian Mean-Probability Hypotheses Density (GM-PHD)  
126 Filter [23], Multi-Bernoulli Mixture (MBM) Filter, Poisson Multi-Bernoulli Mix-  
127 ture (PMBM) Filter, etc. are other filters adopted in the literature. A detailed  
128 study for these filters is presented in [24].

## 129 2.2. Pointcloud elaboration

130 As anticipated in the previous section, throughout this work, multi-sensors  
131 data fusion is considered to be done between two Radars and a Lidar. Contrary  
132 to the general case, Radars used in this work already provide preprocessed clus-  
133 tered point detections coming from an object. The processing for the 3D point-  
134 cloud coming from a Lidar sensor represents a more complex task. Referring to  
135 the robotics literature, obstacle detection from 3D pointcloud can be provided  
136 through a map-based approach or with deep learning-based techniques.

137 Authors in [25] implemented an occupancy grid for the space surrounding a  
138 robot in which each cell is labeled as empty or occupied. Scenarios with a large  
139 number of sensors usually employ multilayers based solution [26, 27], where each  
140 sensor provides a different occupancy grid, then fused to retrieve a representa-  
141 tion of the environment. Other scenarios, where the terrain presents significant  
142 changes in height, require instead using more complex maps, which also con-  
143 sider changes in elevation [28, 29]. More straightforward solutions, based on  
144 2.5D maps [30, 31], merge the reduced dimensions and limited computational  
145 requirements of a 2D grid with the height of the 3D approach. Recent ap-  
146 proaches, specially designed for autonomous driving scenarios, also implement  
147 combinations of 2D and 3D based processing [32] using the original pointcloud  
148 to label the obstacles and the grid to perform planning. In general, most of the  
149 classification oriented systems prefer 3D pointcloud to identify and label the  
150 obstacles [33, 34, 35].

151 In the last years, a different approach to pointcloud elaboration has emerged,  
152 the usage of deep learning techniques, particularly Convolutional Neural Net-  
153 work (CNN). The most successful solutions in the autonomous driving field do  
154 not try to label each pixel of the pointcloud but predict 3D bounding box around  
155 obstacles [36], [37], [38]. This guarantees low processing time and the ability to

run in real-time. Nevertheless some approaches of 3D points semantic segmentation, mainly based on the PointNet [39] and PointNet++ [40] architecture, has emerged. Lastly, hybrid approaches, which combine the usage of occupancy grids and CNN has been proposed [41] to reduce the required computational power. Those solutions first reduce the pointcloud to a 2D occupancy grid and then process it with a classical 2D neural network; in such a way, the input size is considerably reduced compared to the original 3D pointcloud, and it can be processed much faster. The main disadvantage of those solutions is the need for dense pointcloud to feed the network with feature-rich data. This is possible with 32 and 64 planes Lidars. Still, with smaller sensors, with fewer planes, the obstacles become less defined, and the networks are generally not able to extract enough features to detect the obstacles, as shown in [42].

### 2.3. Sensor fusion

Data Association is one of the crucial steps in MOT problems. A critical assumption for this task, among others, is that the number of objects ( $n$ ) is not a random variable, but it is considered as known during each filtering iteration. Global Nearest Neighbour (GNN) filters, Joint Probabilistic Data Association (JPDA) filters, and Multi Hypothesis Tracking (MHT) filters are the most commonly adopted approaches in MOT. These filters are presented in detail in [24, 43]. Kalman filtering and its advanced versions (EKF and UKF) are usually employed to ensure objects tracking. GNN filters perform association of measurements and estimates under the best association hypotheses (i.e., the one with the lowest association cost is considered while others are pruned). Although computationally cheap and fairly accurate in case of high Signal to Noise Ratio (SNR), performances can degrade in moderate or low SNR. However, JPDA considers a certain number of best assignment hypotheses for the association and computes marginal posterior densities with corresponding marginal association probability. Weighted merging of these posterior densities is done to extract the estimated state. With increased computational burden, JPDA performs better in low to medium SNR scenarios compared to GNN. MHT filter requires calculating a pre-defined number of best association hypotheses while pruning all others: in this way, posterior densities retains a certain number of most probable hypotheses. This allows for corrections in previous association decisions when new information from sensors is given.

Multi-sensors data fusion for autonomous driving can be described as centralized, decentralized, or hybrid architectures [44]. In centralized data fusion, also referenced as central level fusion, the sensors' raw data are minimally pre-processed at sensor level and then forwarded to be fused in the central module. Object discrimination and tracking are handled at central level. In decentralized data fusion, each sensor is tasked to identify and track objects. Fusion of these tracks is done in a centralized module and may involve feedback to the sensor module. Hybrid data fusion architectures are a combination of previous approaches. Two sets of information are conveyed from the sensor module: minimally pre-processed data to the central module and simultaneously tracks



200 to decentralized fusion modules. The outputs of the decentralized modules are  
201 fed again to the central module for fusion purposes.

#### 202 *2.4. State estimation*

203 In obstacles state estimation, algorithms have to deal with a large number of  
204 measurements collected by sensors. Hence, any possibility of filtering in advance  
205 any unwanted noise or false positives may help in reducing the computational  
206 burden. For this reason, the filtering process can be improved by exploiting the  
207 knowledge of road bounds leading to higher estimates accuracy [3, 13, 20]. In  
208 particular, authors in [20] have studied the possibility of providing estimates in  
209 curvilinear coordinates by tracking fusion and behavioral reasoning of obstacles  
210 within the road bounds. As anticipated in the previous section, conversion  
211 from Cartesian to curvilinear coordinates can be beneficial in multiple aspects  
212 of autonomous driving but even for communication systems between different  
213 vehicles [13]. Authors in [3] have presented estimates in curvilinear coordinates  
214 to analyze obstacles motion close to the ego-vehicle. Estimation and tracking  
215 are given through decentralized fusion mode based on a Radar sensor, while  
216 nearest neighbor filter ensures track-measurement association. Obstacles state  
217 estimation is done in Cartesian coordinates and later converted into curvilinear  
218 ones through traditional Kalman filtering. However, this conversion process is  
219 highly nonlinear, so estimates can be vulnerable to faulty results.

220 As presented in [45], road definition adaptation in the estimation process  
221 has allowed for the development of a cooperative algorithm between two vehi-  
222 cles expediting their lane level localization. Authors in [46] have represented the  
223 status of ego-vehicle, objects, and traffic participants in road coordinates (i.e.,  
224 Curvilinear Coordinates). This conversion enabled them to accelerate and sim-  
225 plify the trajectory planning of the ego vehicle. Knowledge of the road curvature  
226 and geometry allows the planning task to be performed in a simplified environ-  
227 ment by eliminating the road curvature and performing the planning task in a  
228 straight line. This reduced the computational burden and time consumed for  
229 performing an optimization task. The planned motion is again interpreted in a  
230 road environment for defining the designated motion.

231 For what concerns the ego-vehicle positioning, GPS sensors with RTK cor-  
232 rection systems are becoming widespread in autonomous and intelligent vehicles.  
233 These sensors can be equipped with 6-DOFs inertial units (IMU), ensuring a  
234 cheap setup for the inertial navigation system. Authors in [47, 48] integrate  
235 a GPS receiver in the estimation process based on a kinematic vehicle model.  
236 They demonstrate how these sensors can improve the estimate accuracy even for  
237 the vehicle lateral velocity. This consideration, applied to autonomous driving,  
238 allows avoiding a complex reverse engineering process to tune parameters like  
239 tire cornering stiffness and relaxation lengths, mass, and moment of inertia (at  
240 least along the vertical axis) [49, 50]. Moreover, a couple of GPS receivers can  
241 be installed on the same vehicle to provide an estimation of the absolute heading  
242 angle [48, 51]. Accuracy increases if the receivers are located on the longitu-  
243 dinal axis. About the vehicle motion, lateral velocity in the center of gravity  
244 (CoG) can be derived by kinematic relationships assuming pure rolling contact

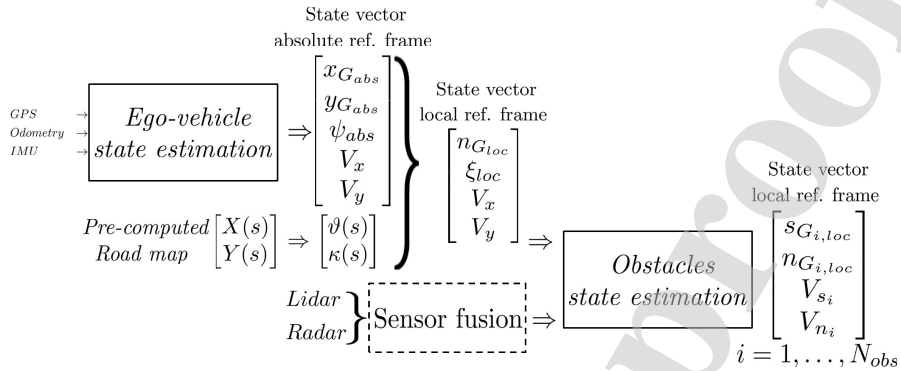


Figure 3. Scheme of the integrated estimation algorithm

245 and low longitudinal speed [52, 53]. Then, the estimated accuracy can be im-  
 246 proved, accounting for vehicle lateral dynamics. In the literature, performances  
 247 related to these two different modeling approaches have been compared many  
 248 times [54, 55, 56, 57]. In general, dynamic and physical vehicle models ensure  
 249 more accurate estimates as the vehicle speed increases, but a higher number of  
 250 parameters must be tuned, and the computational cost increases. Authors in  
 251 [51] implemented an EKF to provide positioning, heading angle, and lateral ve-  
 252 locity for autonomous vehicles based on a kinematic single-track vehicle model.  
 253 Authors in [58] compared performances between EKF and UKF for a similar  
 254 estimation procedure. Results assess that UKF provides more accurate results,  
 255 ensuring fast computational time.

256 Compared to the current state of the art, the presented work aims to esti-  
 257 mate obstacles positioning and relative motion referenced to the ego-vehicle in  
 258 curvilinear coordinates, which involves a highly non-linear measurement model.  
 259 Hence, a UKF has been implemented as it represents a compromise between  
 260 accuracy, computational effort, and ease of implementation.

### 261 3. Architecture of the estimation system

262 As stated in previous sections, the aim of the presented estimation system  
 263 is to compute ego-vehicle and obstacles state vectors in curvilinear coordinates.  
 264 As shown in Fig.3, measurements for ego-vehicle state estimation are given by  
 265 GPS receivers, inertial units, and odometry. The estimation algorithm is then  
 266 based on a UKF to provide vehicle positioning and heading angle in the global  
 267 Cartesian reference frame. At the same time, longitudinal and lateral velocities  
 268 are given according to the moving reference system centered with the vehicle  
 269 (i.e., the vehicle reference frame, VRF).

$$x_{e,abs} = [x_{G_{abs}} \quad y_{G_{abs}} \quad \psi_{abs} \quad V_x \quad V_y]^T \quad (1)$$

270 The G-subscript in (1) means that positioning is given in the vehicle center  
 271 of gravity, which also represents the center of the moving reference system, in  
 272 which velocities  $V_x$  and  $V_y$  are estimated. As stated in the previous section, the  
 273 curvilinear framework provides many advantages compared to the Cartesian  
 274 one when applied to autonomous driving. As shown in Fig. 2, through the pre-  
 275 computed road map description it is possible to move from global coordinates  
 276 to the local reference frame along the road centerline. Doing so, ego-vehicle  
 277 positioning in (1) can be converted to curvilinear coordinates:

$$x_{e,loc} = [n_{G_{loc}} \quad \xi_{loc} \quad V_x \quad V_y]^T \quad (2)$$

278 where  $\xi_{loc} = \psi_{abs} - \theta_e$  represents the current relative heading direction of the  
 279 ego-vehicle with respect to the road angle ( $\theta_e$ ). Once ego-vehicle positioning  
 280 is computed with respect to the road centerline, the pre-computed road map  
 281 provides the road description in terms of road angle and curvature ( $\theta(s)$  and  
 282  $\kappa(s)$ , respectively) for the current local reference frame. The road description  
 283 is given in terms of Hermite spline curves for the following 50 m, which corre-  
 284 sponds to the overall estimation process's field of view (FoV). The state vector  
 285 in (2) does not include the vehicle's absolute position along the track (i.e., the  
 286 curvilinear abscissa  $s_G$ ). Aiming to provide obstacles' positioning with respect  
 287 to the ego-vehicle, this variable is not required because the vehicle is localized  
 288 at any time step in a different local reference frame, to which the road map  
 289 associates the corresponding road description.

290 Measurements of obstacles are provided by two Radar sensors and by a Lidar  
 291 in VRF (i.e., the same one in which longitudinal and lateral velocities are given).  
 292 For each tracked obstacle  $i = 1, \dots, N_{obs}$ , state estimation (3) is provided in local  
 293 reference frame in terms of longitudinal distance with respect to ego-vehicle  $s_{i,loc}$   
 294 and lateral distance with respect to road centre line  $n_{i,loc}$ . Moreover, absolute  
 295 velocities are given according to road tangential and orthogonal directions ( $V_{s_i}$   
 296 and  $V_{n_i}$ , respectively).

$$x_{o_i,loc} = [s_{i,loc} \quad n_{i,loc} \quad V_{s_i} \quad V_{n_i}]^T \quad (3)$$

297 Throughout this work, the small objects assumption is adopted (i.e., an object  
 298 is represented by a point, and its state is defined with positional and velocity  
 299 values only, neglecting the orientation information). Hence, the relative orien-  
 300 tation of obstacles with respect to the road is not included in the state vector.  
 301 Even though an obstacle's orientation is an important information in the over-  
 302 all perception module, it is not considered in this application to speed up the  
 303 implementation and ensure real-time. However, all the estimates are provided  
 304 in the road reference, whose direction and limits are known in advance, and the  
 305 algorithm computes magnitude and direction of the velocity's vector for each  
 306 detected obstacle. Thus, if coupled together, these pieces of information can  
 307 eventually provide a motion planner with an estimate of the obstacle's trajec-  
 308 tory.

#### 309 4. Ego-vehicle state estimation

310 Kalman filtering usage for vehicle state estimation, with EKF and UKF,  
 311 is well-established to account for model nonlinearities. Furthermore, Kalman  
 312 filtering requires a reasonable computational effort and allows managing differ-  
 313 ent sampling frequencies from sensors: this guarantees that estimates can be  
 314 provided even in a real-time control routine.

315 As stated in previous sections, ego-vehicle state estimation is provided in  
 316 terms of positioning and velocity. According to the works presented in Section  
 317 2, a kinematic single-track vehicle model can be implemented within a range of  
 318 speed typical of urban driving scenarios. Although a simple kinematic model  
 319 guarantees fast implementation and interchangeability on different vehicles, the  
 320 lack of accuracy can lead to estimation errors. These errors are mainly related  
 321 to the lateral velocity estimation, which is strongly affected by tire cornering  
 322 stiffness, geometry of the suspensions, saturation of friction in the tire contact  
 323 patch, and load transfers. Even the vehicle's longitudinal dynamic is crucial  
 324 when dealing with strong braking maneuvers that are very common, especially  
 325 in the urban environment. Despite this, the estimate accuracy can be improved  
 326 utilizing a GPS receiver with real-time kinematic (RTK) correction. In this  
 327 way, the motion planning system will continuously receive precise and accurate  
 328 estimates, at least in terms of positioning. Furthermore, including the vehicle's  
 329 heading angle in (1) can lead to the motion planner to account for the car's  
 330 mutual direction with respect to the road and other obstacles.

331 The discrete time definition of the UKF is based on the nonlinear systems of  
 332 equations (4) and (5), where process disturbance  $w_k$  and measurement noise  $v_k$   
 333 are assumed to be additive and zero mean distributed with covariance matrices  
 334  $Q_k$  and  $R_k$  as indicated in (6).

$$x_k = x_{k-1} + f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1})\delta t \quad (4)$$

$$y_k = h_k(x_k, v_k) \quad (5)$$

335

$$w_{k-1} \sim (0, Q_{k-1}) \quad (6)$$

$$v_k \sim (0, R_k)$$

336 The system is modeled based on a kinematic single-track vehicle model, which  
 337 considers the IMU measurements as input with included disturbances (7). These  
 338 measurements are the longitudinal and lateral accelerations in the vehicle CoG  
 339 ( $a_{G,x}$  and  $a_{G,y}$ ), and the yaw rate  $\omega$ . The sensor bias is eliminated during the  
 340 initialization phase when the vehicle is standstill.

$$f_{k-1} = \begin{cases} \dot{x}_G = V_x \cos\psi - V_y \sin\psi \\ \dot{y}_G = V_x \sin\psi + V_y \cos\psi \\ \dot{\psi} = \omega \\ \dot{V}_x = V_y \dot{\psi} + a_{x,G} \\ \dot{V}_y = -V_x \dot{\psi} + a_{y,G} \end{cases} \quad (7)$$

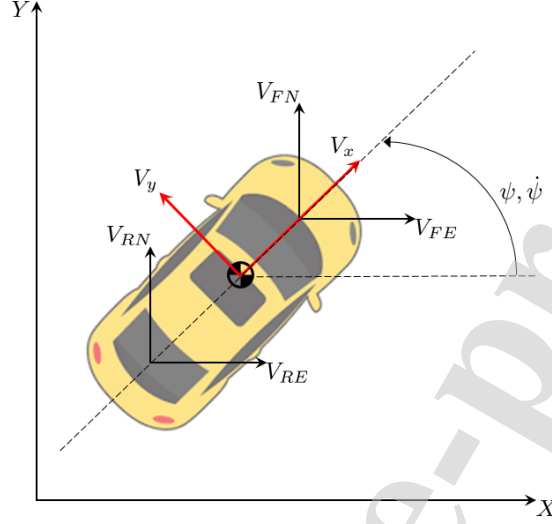


Figure 4. Representation of sensors orientation on the vehicle

341 The filter update equations integrate velocities and positions provided by  
 342 the GPS receivers together with odometry (8). GPS measures velocities in the  
 343 absolute reference system (ENU) through the Doppler effect, while odometry  
 344 can be considered as given from exciters and encoders located on the ego-vehicle.

$$h_k = \begin{cases} V_{FE} = V_x \cos \psi - (V_y + l_f \dot{\psi}) \sin \psi \\ V_{FN} = V_x \sin \psi + (V_y + l_f \dot{\psi}) \cos \psi \\ V_{RE} = V_x \cos \psi - (V_y - l_r \dot{\psi}) \sin \psi \\ V_{RN} = V_x \sin \psi + (V_y - l_r \dot{\psi}) \cos \psi \\ V_{x, odom} = V_x \\ x_G = (x_F l_R + x_R l_F) / (l_F + l_R) \\ y_G = (y_F l_R + y_R l_F) / (l_F + l_R) \end{cases} \quad (8)$$

345 Parameters  $l_i$  and  $l_I$ , with  $i \in [f, r]$  and  $I \in [F, R]$  refer respectively to: distance  
 346 between vehicle CoG and vehicle front and rear axis and distance between vehicle  
 347 CoG and front and rear GPS receiver. Then,  $V_{FE}$ ,  $V_{FN}$ ,  $V_{RE}$ , and  $V_{RN}$  are the  
 348 velocities in ENU coordinates measured by the GPS receivers, while  $V_{x, odom}$   
 349 is the longitudinal speed of the vehicle given by odometry. The measurement  
 350 model is represented by Fig. 4.

351 The unscented transformation (9) is applied to the estimated state vector

352  $\tilde{x}_{k-1}^+$  based on the state covariance matrix  $P_{k-1}^+$ .

$$\begin{aligned}\hat{x}_{k-1}^{(i)} &= \tilde{x}_{k-1}^+ + \chi^{(i)} \quad i \in [1, \dots, 2n] \\ \chi^{(i)} &= \left( \sqrt{nP_{k-1}^+} \right)_i^T \quad i \in [1, \dots, n] \\ \chi^{(n+i)} &= -\left( \sqrt{nP_{k-1}^+} \right)_i^T \quad i \in [1, \dots, n]\end{aligned}\quad (9)$$

353 The number of sigma points  $n$  can double the length of the state vector to  
354 speed-up calculations. Sigma points are passed through (4) to compute the ma-  
355 trix  $\hat{x}_k^{(i)}$ , which is used to evaluate the predicted state vector  $\tilde{x}_k^-$  and covariance  
356 matrix  $P_k^-$  as indicated in (10) and (11). In both the equations, each sigma  
357 point is properly weighted through the parameter  $w_i = 1/2n$ .

$$\tilde{x}_k^- = \sum_{i=1}^{2n} w_i \hat{x}_k^{(i)} \quad (10)$$

$$P_k^- = \sum_{i=1}^{2n} w_i [\hat{x}_k^{(i)} - \tilde{x}_k^-][\hat{x}_k^{(i)} - \tilde{x}_k^-]^T + Q_{k-1} \quad (11)$$

358 A further unscented transformation (9) based on  $\tilde{x}_k^-$  and  $P_k^-$  is required to  
359 evaluate a new set of sigma points ( $\hat{x}_k^{(i)}$ ) to update the state vector prediction.  
360 This set of points is then propagated through the update equations of the filter  
361 (8) to calculate the predicted measurement matrix  $\hat{y}_k^{(i)}$  from which the predicted  
362 measurements vector and the innovation covariance matrix  $P_y$  are evaluated  
363 according to (12) and (13).

$$\tilde{y}_k = \sum_{i=1}^{2n} w_i \hat{y}_k^{(i)} \quad (12)$$

$$P_y = \sum_{i=1}^{2n} w_i [\hat{y}_k^{(i)} - \tilde{y}_k][\hat{y}_k^{(i)} - \tilde{y}_k]^T + R_k \quad (13)$$

$$P_{xy} = \sum_{i=1}^{2n} w_i [\hat{x}_k^{(i)} - \tilde{x}_k^-][\hat{y}_k^{(i)} - \tilde{y}_k]^T \quad (14)$$

364 To conclude, the measurement update of the state estimates can be per-  
365 formed accounting for the cross covariance matrix given by (14), that is required  
366 to compute the Kalman gain matrix as indicated in (15). The updated state  
367 vector ( $\tilde{x}_k^+$ ) and covariance ( $P_k^+$ ) are obtained from equations (16) and (17).

$$K_k = P_{xy} P_y^{-1} \quad (15)$$

$$\tilde{x}_k^+ = \tilde{x}_k^- + K_k [y_k - \tilde{y}_k] \quad (16)$$

$$P_k^+ = P_k^- - K_k P_y K_k^T \quad (17)$$

368 The estimation process presented in this section provides the vehicle position-  
 369 ing and heading in global coordinates (i.e., in the absolute reference frame).  
 370 Longitudinal and lateral velocities in the state vector (1) are given in the moving  
 371 reference system centered with the vehicle CoG. Since positioning must be  
 372 provided in the road local reference frame, as indicated by the state vector in  
 373 (2), we have to solve an optimization problem before performing the estimation  
 374 to position the vehicle within the road map. The pre-computed road map  
 375 describes the road centerline in terms of heading and curvature in curvilinear  
 376 coordinates with a discretization step of  $ds = 0.5 m$ . A minimization algorithm  
 377 computes the two smallest distances between the ego-vehicle position and each  
 378 of the sampled map points through the euclidean norm. This brute force approach  
 379 is performed only during the filter initialization phase: starting from  
 380 the second iteration, a warm start is used to account for the previous vehicle  
 381 position, to reduce the computational effort. Once the closest points are found,  
 382 the algorithm computes the tangent to the road centerline close to the vehicle  
 383  $\theta_e$  to provide the lateral position in this local reference frame  $n_{G_{loc}}$  and the  
 384 relative heading angle  $\xi_{loc}$ . Thus, the experimental setup required to provide  
 385 initial conditions to a motion planner shall include a GPS receiver coupled with  
 386 an inertial unit (IMU), an encoder on the steering shaft, and a couple of exciters  
 387 for the measurement of the longitudinal vehicle's speed. Then, as explained in  
 388 section 2, adding a GPS receiver located along the longitudinal axis of the car,  
 389 it becomes possible to give an accurate estimate of the absolute heading angle.

## 390 5. Data processing and sensor fusion

391 The sensing architecture for obstacles state estimation consists of two Radar  
 392 sensors mounted respectively on the front and the rear bumpers of the car, and  
 393 a Lidar mounted on the roof. The *Continental ARS 408-21* long-range Radar  
 394 sensor retains a  $180^\circ$  field of view in the horizontal plane, while a *Velodyne VLP-*  
 395 *16* Lidar guarantees a  $360^\circ$  coverage.

### 396 5.1. Radar data

397 Data coming from the Radar sensors are already pre-processed and provided  
 398 as clusters of detections in VRF. Those clusters give information on real objects  
 399 and not single points. For each of them, the Radar measures the longitudinal  
 400 and lateral distance in the VRF. Moreover, in the same reference frame, it also  
 401 returns the longitudinal and lateral components of the relative velocity with  
 402 respect to the ego-vehicle ( $V_{x,rel}^{VRF}$  and  $V_{y,rel}^{VRF}$  respectively). To compute these  
 403 two velocities, the Radar sensors require the current ego-vehicle longitudinal

404 speed and yaw rate, received through CAN-bus communication. Hence, the raw  
405 measurements available for each object can be summarized in the following:

$$y_{r, o_i}^{VRF} = [x_{P_i}^{VRF} \quad y_{P_i}^{VRF} \quad V_{x_i}^{VRF} \quad V_{y_i}^{VRF}]^T \quad (18)$$

406 where  $V_{x_i}^{VRF}$  and  $V_{y_i}^{VRF}$  are respectively the longitudinal and later components  
407 in the VRF of the  $i$ -object absolute velocity. These components can be com-  
408 puted as reported in (19) because Radar sensors evaluate each detected object's  
409 relative velocity with respect to the ego-vehicle accounting for the yaw rate of  
410 the VRF (i.e., accordingly to the relative motions theorem).

$$\begin{cases} V_{x_i}^{VRF} = V_x^{VRF} + V_{x, rel}^{VRF} \\ V_{y_i}^{VRF} = V_y^{VRF} + V_{y, rel}^{VRF} \end{cases} \quad (19)$$

411 For what concerns every single object's relative positioning, the measures can  
412 be considered related to the closest part of the leading (or following) vehicle.  
413 As indicated in (20), longitudinal and lateral distances from objects are derived  
414 accounting for the displacement between the sensors and the ego-vehicle CoG,  
415 i.e.  $l_{R_i}$  with  $i \in [front, rear]$ .

$$\begin{cases} x_{P_{i, front}}^{VRF} = x_{P_i, front} + l_{R_{front}} \\ x_{P_{i, rear}}^{VRF} = -x_{P_i, rear} - l_{R_{rear}} \end{cases} \quad (20)$$

416 A complete list of the provided data for each cluster identified by the Radar  
417 is provided in Table 1. As reported, the internal pre-processing of raw Radar  
418 detections guarantees not only objects measurements in VRF, but also tracking  
419 of the cluster in time, and an estimation of the related probability of existence  
420 and class.

421 Object tracking is already performed by the Radar sensor ( $ID(o_i)$ ), but this in-  
422 formation is not considered within the presented algorithm because it is strongly  
423 affected by noise. Nevertheless, the related probability of existence is used to  
424 filter out objects characterized by  $p(o_i) \leq 99\%$ . Indeed, some preliminary ex-  
425 perimental tests demonstrated that lower probability measures are mostly due  
426 to misleading and false positive. For this reason, all clusters with a probabili-  
427 ty of existence lower than this threshold are removed. The filtering process is  
428 completed by clustering all the object detections within a pre-defined spatial  
429 threshold, whose value changes according to the object class indicated by the  
430 Radar sensor. During clustering, all measurements related to positioning and  
431 relative motion of each object are mediated between them. To conclude, this  
432 sensor provides an estimation of the standard deviation for each measurement of  
433 a given cluster ( $\sigma_{Vmeas}$ ). This information is used during the following filtering  
434 process to account for the noise that affects measurements. During clustering,  
435 only the largest standard deviation for any different measurement is considered.

## 436 5.2. Lidar elaboration

437 Lidar measurements are provided as 3D pointclouds referenced to the sensor  
438 position, located in the ego-vehicle CoG. Thus, pointclouds processing is re-  
439 quired to derive objects information in a similar form to that given by the Radar



data	description
$ID(o_i)$	ID of the tracked object
$x_{P_i}^{VRF}$	longitudinal distance in VRF
$y_{P_i}^{VRF}$	lateral distance in VRF
$V_{x,rel}^{VRF}$	longitudinal relative speed in VRF
$V_{y,rel}^{VRF}$	lateral relative speed in VRF
$\sigma_{\forall meas}$	standard deviation for all measurements
$Class(o_i)$	object typology (pedestrian, motorcycle, car)
$p(o_i)$	probability of existence

Table 1. Information provided by the Radar for each identified cluster

440 to ensure sensor fusion and state estimation. The low number of planes of the  
441 VLP-16 Lidar made impossible to implement one of the deep learning-based  
442 approaches presented in Section 2.2, due to the sparseness of the Pointcloud  
443 and the low number of features. Thus, a solution based on a 2D occupancy  
444 grid, similar to the one used in mobile robotics, has been adapted. Unlike the  
445 classical robotics scenarios, where the area of interest is limited to only a few  
446 meters around the robot, and the ground plane is generally flat. In this case,  
447 the obstacles can be at a high distance (i.e., 20 meters), move at high speed, and  
448 be as big as a truck. For all these reasons, we had to implement our pipeline for  
449 Lidar obstacle detection, leveraging on the classical occupancy grid approach,  
450 but adapting it to this new scenario. The pipelines in Fig. 5 and 6 show the  
451 operations required to convert a set of 3D points into a list of obstacles on the  
452 horizontal plane. In particular, this pipeline can be divided into two blocks: the  
453 first one concerns the conversion from 3D points to a 2D occupancy grid, while  
454 the latter deals with obstacle identification and tracking on the bi-dimensional  
455 grid.

456 The conversion of a 3D pointcloud into a 2D occupancy grid can be divided  
457 into some fundamental steps, shown in Fig. 5. The first block consists of the ro-  
458 tation of the pointcloud and ground plane fitting. The sensor is indeed mounted  
459 on the ego-vehicle roof with a slightly negative pitch to cover the frontal area  
460 better. Ground plane removal allows excluding all points belonging to the road  
461 surface to reject false positive. To perform this task, an approach similar to the  
462 one presented in [59] is implemented, in which the plane fitting problem is based  
463 on RANSAC (RANdom SAMple Consensus). An initial guess for the normal  
464 direction to the horizontal plane can be derived in standstill conditions by mea-  
465 suring the projection of the acceleration of gravity along each dimension of the  
466 triaxial accelerometer of the vehicle’s inertial unit (IMU); while the distance of  
467 the plane has been previously measured in a controlled environment. During

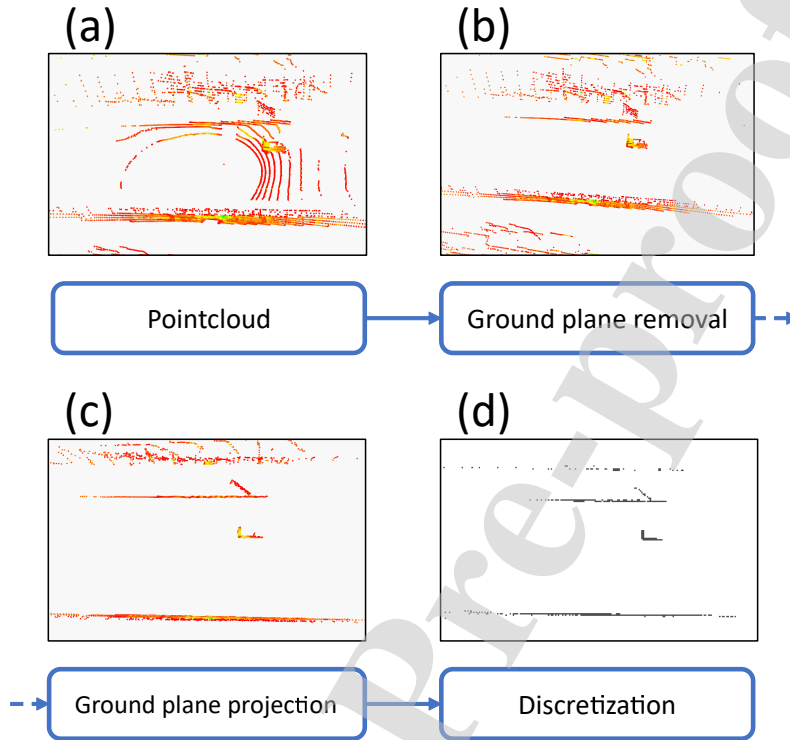


Figure 5. Pointcloud elaboration pipeline. The input pointcloud (a) is first processed to remove the ground plane point (b). Then is projected on a 2D plane parallel to the ground (c), and lastly converted into a binary grid map (d).

468 this step, all points above a fixed threshold, in our case  $4.0\text{ m}$ , are also removed.  
 469 This operation prevents the projection of noise from trees or traffic sign above  
 470 the car clearance on the occupancy grid.

471 Once the ground plane is removed, the pointcloud includes only points be-  
 472 longing to obstacles. Thus, it is possible to project each one of them on a 2D  
 473 plane using the normal direction retrieved in the previous step: the result is  
 474 a set of 2D points on a plane parallel to the road surface. Although this pro-  
 475 cess provides a significant simplification of the data, the set of measurements is  
 476 still too complex to be directly used. Discretization is then carried out through  
 477 the application of a grid on the identified horizontal plane. In particular, the  
 478 grid is divided into square cells with side equal to  $0.3\text{ m}$ : by iterating through  
 479 each element, if the number of points in the cell is higher than a pre-computed  
 480 threshold, the cell is set to occupied. The squared cells' size, equal to  $0.3\text{ m}$ , is  
 481 a good trade-off between accuracy and computational power. This value allows  
 482 us to have a small occupancy grid, which can be computed and processed in  
 483 real-time, but also retrieve the obstacle position with low error. The output of  
 484 this filtering phase is a binary grid that describes the ego-vehicle surroundings

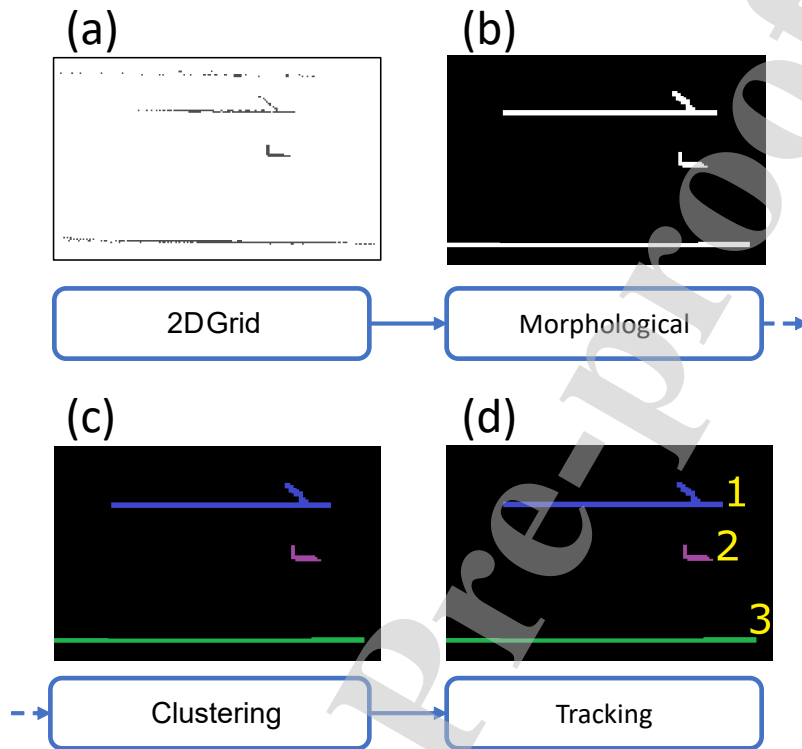


Figure 6. Occupancy grid elaboration pipeline. The occupancy grid (a) is first elaborated using morphological operations to remove noise and connect components (b). Then is processed using a clustering algorithm to identify all objects (c). Lastly, tracking is performed through consecutive frames of the identified obstacles (d).

485 with only a few thousand cells. The use of a threshold parameter is needed  
 486 as it allows to reduce further the detection of false positives related to noise.  
 487 Its value can be tuned based on experimental measurements with a decreasing  
 488 value depending on the radial distance to consider the variable density of the  
 489 pointcloud, as shown in [27].

490 The previous phase's output is a simplified representation of the area sur-  
 491 rounding the ego-vehicle compared to a 3D dense pointcloud. However, this in-  
 492 formation cannot be directly supplied to the control routine of an autonomous  
 493 vehicle. For this reason, a further elaboration block takes as input the 2D  
 494 occupancy grid to return a small list of fully characterized obstacles.

495 The occupancy grid provides information regarding objects in each cell, but  
 496 contiguous elements, which are parts of the same object, are considered sep-  
 497 arately. Thus, clustering is required to merge elements in the 2D-grid. As a  
 498 preliminary step, a set of morphological operations is needed to connect areas  
 499 that might belong to the same object but are not directly connected. This

500 might happen due to some obstructions or the particular shape of the object  
 501 itself, causing the number of points belonging to a specific cell to be lower than  
 502 the filtering threshold explained before. This operation also filters single points  
 503 in the occupancy grid, which are imputable to noise in the sensors, and can  
 504 easily generate false positives. The result is still an occupancy grid where all  
 505 elements belonging to an obstacle are connected. Further filtering is performed  
 506 by discharging merged elements that are detected more than  $50\text{ m}$  meters ahead  
 507 of the ego-vehicle, and  $\pm 15\text{ m}$  in the lateral direction.

508 Clustering is based on the OpenCV [60] implementation of SAUF (Scan  
 509 plus Arraybased Union-Find) [61]; the output is a list of all the connected  
 510 components in the occupancy grid which belong to real obstacles, defined by  
 511 the relative position of the respective center of symmetry (CoS) with respect to  
 512 the ego-vehicle and its equivalent dimension  $\rho_{o_i}$ . The length and width of each  
 513 identified object are not considered because the mesh adopted for the 2D grid  
 514 is not sufficiently fine to provide a measure of the heading. This causes a loss of  
 515 accuracy in the estimation routine but allows us to provide obstacle measures  
 516 at high frequency.

517 Obstacles tracking ensures accurate state estimation for many reasons. It  
 518 allows to predict the relative positioning of obstacles with respect to the ego-  
 519 vehicle also if measurements are not available; moreover, data coming from  
 520 sensors that are not synchronized can be used for sensor fusion. For what con-  
 521 cerns Lidar data processing, a feature-based approach guarantees preliminary  
 522 obstacle tracking. In particular, for this stage, we use an object descriptor  
 523 built using the obstacle dimensions and position. The first time the algorithm  
 524 detects a specific obstacle, it assigns a unique ID and the respective features  
 525 (i.e., dimensions and position). At each successive Lidar reading, the algorithm  
 526 compares the previously detected obstacles with the current ones starting from  
 527 the previously known locations. Warm starts are used to speed up calculations,  
 528 together with a growing window that expands from given locations to search in  
 529 the neighborhoods for objects with similar sizes. If a candidate tracked obstacle  
 530 is found close enough to the previous one and with comparable dimensions, the  
 531 same obstacle ID is assigned. When this process is completed for all obstacles,  
 532 different IDs are automatically set for all elements coming from new readings  
 533 that have not yet been tracked. Moreover, to account for noisy measurements  
 534 or sensor misreadings, the algorithm keeps track of the older obstacles for which  
 535 the matching has not been satisfied for 5 iterations (i.e.,  $0.25\text{ s}$ ). Doing so, the  
 536 algorithm can reassign IDs to un-tracked obstacles. To conclude, Lidar data  
 537 processing provides a list of tracked obstacles characterized by relative posi-  
 538 tions with respect to ego-vehicle, size, and ID.

### 539 *5.3. Sensor fusion*

540 All measurements obtained through the processing of raw data from the  
 541 two Radar sensors and the Lidar are expressed in VRF. The knowledge of the  
 542 road limits is exploited for clutter removal, applied to all the processed mea-  
 543 surements. Any measurement out of the road bounds is assumed to be clutter

544 and hence removed. Radar measurements reported in (18) provide relative po-  
 545 sitioning and motion of the clustered detections belonging to the same obstacle  
 546 with respect to the ego-vehicle. On the other hand, Lidar measurements pro-  
 547 vide relative positioning of each obstacle with respect to the ego-vehicle, and  
 548 information about obstacle identification during time. Although the presented  
 549 pre-processing of Lidar data already ensures tracking, the multi-sensors data  
 550 fusion architecture proposed in this work can be considered centralized. Indeed,  
 551 multi-sensors data pre-processing represents the input for a central module in  
 552 which object discrimination and tracking are performed basing on the complete  
 553 set of data.

554 Measurements from the two Radar sensors are synchronized with respect to  
 555 each other, while they are asynchronous with respect to the data coming from  
 556 Lidar. Thus, they are received associated with different timestamps. More-  
 557 over, as explained in Section 7, the estimation routine is driven at 20 Hz, while  
 558 Radar and Lidar data processing are provided respectively at 14 and 16 Hz.  
 559 Thus, it can happen that both sensors measurements do not retain the same  
 560 timestamp and that no new measurements are available at a given time instant.  
 561 For these reasons, sensor fusion is based on a LIFO routine (last in first out)  
 562 in case of different timestamps. If Radar and Lidar measurements are available  
 563 at the same time, fusion is performed through weighted averaging. In this case,  
 564 the fused obstacle retains velocity measurements from the Radar, while posi-  
 565 tioning is computed assuming that Lidar measurements are more accurate, as  
 566 they are related to the obstacle CoS. For a Radar measurement  $y_{r,o_j}^{VRF}$ , a Lidar  
 567 measurement  $y_{l,o_i}^{VRF}$  is considered for fusion if two criteria are satisfied:

- 568 1. For all  $i \in [1, 2, \dots, N_{obs,l}]$  the Euclidean norm between  $y_{r,o_j}^{VRF}$  and  $y_{l,o_i}^{VRF}$  is  
 569 minimum;
- 570 2. This minimum distance is smaller than the size of the object  $\rho_{o_i}$  estimated  
 571 through Lidar processing.

$$\begin{cases} x_{F_j}^{VRF} = 0.8 x_{l,o_i}^{VRF} + 0.2 x_{r,o_j}^{VRF} \\ y_{F_j}^{VRF} = 0.8 y_{l,o_i}^{VRF} + 0.2 y_{r,o_j}^{VRF} \\ V_{x,F_j}^{VRF} = V_{x,r_j}^{VRF} \\ V_{y,F_j}^{VRF} = V_{y,r_j}^{VRF} \end{cases} \quad (21)$$

572 **Sensor Id Assignment:** To each measurement is assigned an *Id* based  
 573 on the sensor it was originated from. The knowledge of the origin of the mea-  
 574 surements was deemed helpful to perform gating task and measurement to track  
 575 association. Since Lidar measurements do not provide information regarding ve-  
 576 locity of detections, the gating task needs to be customized completely, basing  
 577 it only on positional values. Similarly, the predicted state update by measure-  
 578 ments needs to consider the unavailability of velocity measurement from the  
 579 Lidar sensor as only positional values are used for update.

$$\begin{cases} Id_k^i = A, z_k^i \text{ Radar object detection without fusion with Lidar} \\ Id_k^i = AB, z_k^i \text{ Radar and Lidar Fused object detection} \\ Id_k^i = B, z_k^i \text{ is Lidar object detection without fusion with Radar} \end{cases} \quad (22)$$

580 Where, A, AB and B are some numerical constants used to identify the mea-  
 581 surement origination in further estimation steps. Numerical values of these  
 582 constants are not relevant as they are used solely for the purpose of identifica-  
 583 tion of measurement origination. Whenever the measurement vector is specified  
 584 in following sections, it must be implicitly understood that the Sensor *ID* comes  
 585 assigned to it.

586 The fused measurement vector is calculated as in Equation (21). The objects  
 587 identified by the Lidar sensor that do not satisfy these fusion criteria mentioned  
 588 above, with respect to an object found by a Radar sensor, are assigned with  
 589 different fusion *Id*, signifying that the measurement was obtained from Lidar  
 590 only and was not fused with Radar data.

## 591 6. Obstacles state estimation and tracking

592 Tracking obstacles in autonomous driving allows establishing a control rou-  
 593 tine that considers the same obstacle during time to define proper control poli-  
 594 cies. This is mandatory both during vehicle following and overtaking maneuvers.  
 595 Tracking can be performed only once the state estimation routine has provided  
 596 measurement prediction for each obstacle, that must be defined in VRF ac-  
 597 cording to Radar and Lidar data processing algorithms. For each obstacle, the  
 598 state vector (3) defined in curvilinear coordinates according to the local ref-  
 599 erence frame of the road requires a highly nonlinear transformation to move  
 600 each measurement prediction to VRF. For this reason, Unscented Kalman Fil-  
 601 tering has been adopted to provide obstacles state estimates, which represents  
 602 a compromise between accuracy and computational effort. The discrete-time  
 603 implementation of the UKF is equal to the one defined in equations (4) and (5)  
 604 with process disturbance and measurement noise assumed as additive and zero  
 605 mean distributed (6). In the following, the model and measurement equations  
 606 are presented.

607 The state variable  $s_{i,loc}$  represents the distance computed along the road  
 608 centerline in the local reference frame, between the ego-vehicle and the obstacle.  
 609 While the variable  $n_{i,loc}$  represents a measure of how much the obstacle is  
 610 displaced with respect to the centerline, and  $V_{s_i}$  and  $V_{n_i}$  are the components of  
 611 the obstacle absolute velocity in curvilinear coordinates. Since the small objects'  
 612 hypothesis is adopted throughout this work, each obstacle is considered a single  
 613 point (i.e., its heading angle is not estimated).

614 The two different reference frames are represented in Fig. 7, where  $\theta_e$  and  
 615  $\theta_o$  are the heading angles of the road centerline in correspondence of ego-vehicle  
 616 and obstacle respectively. About  $\vec{s}_e$  and  $\vec{n}_e$ , they are the tangential and nor-  
 617 mal directions to the local reference frame of the road, that is centered in the

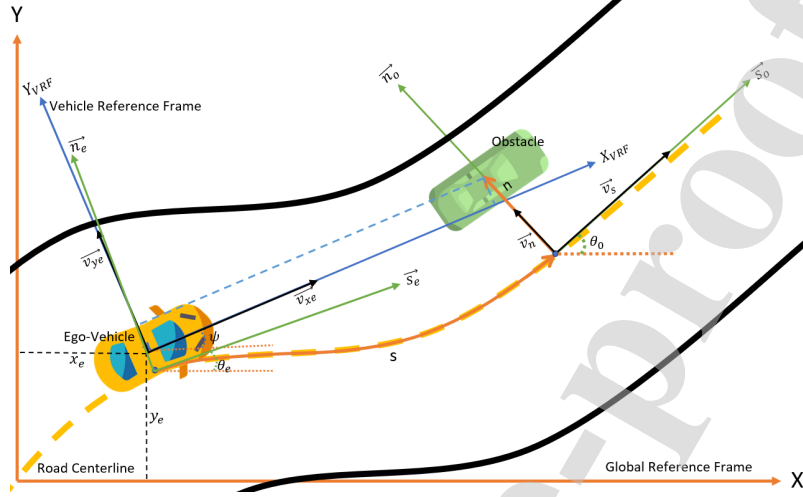


Figure 7. Representation of road and vehicle reference frame

618 closest point corresponding to the ego-vehicle belonging to the road centerline.  
 619 Similarly,  $\vec{s}_o$  and  $\vec{n}_o$  are the main directions of the road in correspondence of  
 620 the obstacle. Finally,  $\psi$  is the estimated heading angle of the ego-vehicle in the  
 621 absolute reference frame.

622 The definition of the nonlinear transformation that allows moving from curvi-  
 623 linear to Cartesian coordinates in VRF is required to ensure the measurement  
 624 prediction during the filtering process. For this purpose, an Euler-based conver-  
 625 sion model is devised. In particular, this model allows computing the Cartesian  
 626 coordinates  $(x_c, y_c)$  corresponding to the point that is  $s_{i,loc}$  away from the ego-  
 627 vehicle, measured along the road centreline (Fig. 8). The calculation is based  
 628 on Euler integration with step size equal to  $\delta s = 0.5 m$ . Once the ego-vehicle  
 629 is localized on the track, the road map provides the road heading for the next  
 630  $50 m$ . Given  $N$  the required number of steps, with  $N = s_{i,loc}/\delta s$ , the model  
 631 computes:

$$\begin{cases} x^{k+1} = x^k + \cos(\theta_{s_k})\delta s \\ y^{k+1} = y^k + \sin(\theta_{s_k})\delta s \end{cases} \quad (23)$$

632 where the point  $(x^N, y^N)$  is approximately equal to  $(x_c, y_c)$ , and  $\theta_{s_k}$  is the road  
 633 heading angle for each step. Then, the lateral displacement of the obstacle from  
 634 the road centerline  $n_{i,loc}$  is used to compute its position in the Global Reference  
 635 Frame.

$$\begin{cases} x_g = x_c + \cos(\frac{\pi}{2} - \theta_0)n_{i,loc} \\ y_g = y_c + \sin(\frac{\pi}{2} - \theta_0)n_{i,loc} \end{cases} \quad (24)$$

636 Finally, the rotation matrix based on the heading angle of the ego vehicle  $\psi_{abs}$

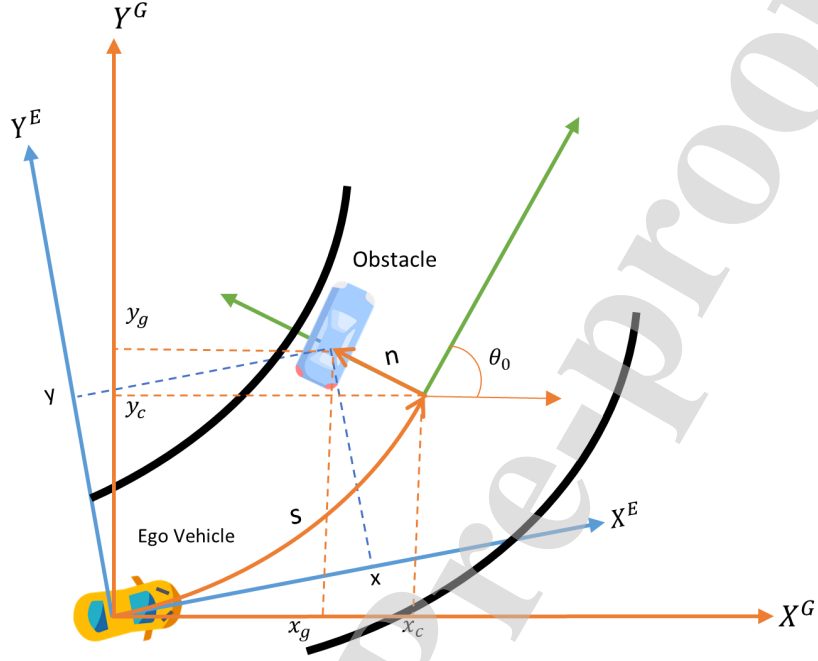


Figure 8. Coordinate conversion between curvilinear and Cartesian reference frame

637 allows computing the relative positioning in VRF.

$$\begin{bmatrix} x_{o_i}^{VRF} \\ y_{o_i}^{VRF} \end{bmatrix} = \begin{bmatrix} \cos(\psi_{abs}) & -\sin(\psi_{abs}) \\ \sin(\psi_{abs}) & \cos(\psi_{abs}) \end{bmatrix} \begin{bmatrix} x_g \\ y_g \end{bmatrix} \quad (25)$$

638 The presented mathematical model assumes that the ego-vehicle is located on  
 639 the road centerline. However, the estimated lateral displacement is considered  
 640 through the following equation:

$$y_{o_i}^{VRF} = y_{o_i}^{VRF} + \cos(\psi_{abs} - \theta_e) n_{Gloc} \quad (26)$$

641 The conversion of the obstacle absolute velocity from curvilinear coordinates  
 642 to VRF can be done rotating the velocity vector two times as in (27). The  
 643 former accounts for the road's heading angle in correspondence of the obstacle  
 644  $\theta_o$  to transform velocity components from road to Cartesian global reference  
 645 frame. About the latter one, it moves the two components in VRF through the  
 646 ego-vehicle absolute heading angle ( $\psi_{abs}$ ).

$$\begin{bmatrix} V_{x, o_i}^{VRF} \\ V_{y, o_i}^{VRF} \end{bmatrix} = \begin{bmatrix} \cos(\psi_{abs}) & -\sin(\psi_{abs}) \\ \sin(\psi_{abs}) & \cos(\psi_{abs}) \end{bmatrix} \begin{bmatrix} \cos(\theta_o) & \sin(\theta_o) \\ -\sin(\theta_o) & \cos(\theta_o) \end{bmatrix} \begin{bmatrix} V_{s_i} \\ V_{n_i} \end{bmatrix} \quad (27)$$



647 The filter initialization is performed with the measurements obtained from  
 648 sensor fusion. During the first iteration, these processed measurements in VRF  
 649 are equated into Curvilinear Co-ordinate frame to initialize tentative tracks.  
 650 Concurrently, initialization of the tracking routine is done using these tentative  
 651 tracks. If they are retained during the next second (i.e., for 20 iterations), the  
 652 tracked hypothesis is converted to a confirmed tracked obstacle. If not, any  
 653 other tentative track is deleted.

654 Once initialization is completed, state prediction is performed based directly  
 655 on the previously tracked obstacles state estimates and covariance. Indeed, the  
 656 constant velocity lane changing model (CVLC) [20], which defines the obstacles  
 657 motion in curvilinear coordinates, it is a linear model, as shown in (28).

$$\tilde{x}_{o_i k}^- = \begin{bmatrix} s_{i, loc} \\ n_{i, loc} \\ V_{s_i} \\ V_{n_i} \end{bmatrix}_k = \underbrace{\begin{bmatrix} 1 & 0 & \delta t & 0 \\ 0 & 1 & 0 & \delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{F_{k-1}} \begin{bmatrix} s_{i, loc} \\ n_{i, loc} \\ V_{s_i} - V_s \\ V_{n_i} \end{bmatrix}_{k-1} + \begin{bmatrix} \omega_{a_s}(\delta t^2/2) \\ \omega_{a_n}(\delta t^2/2) \\ V_{s_{k-1}} + \omega_{a_s}\delta t \\ \omega_{a_n}\delta t \end{bmatrix} \quad (28)$$

658 The terms  $\omega_{a_s}$  and  $\omega_{a_n}$  are used to add Gaussian noise within the linear  
 659 model that describes obstacle motion in curvilinear coordinates. They can be  
 660 considered with zero mean and associated to the standard deviation of accelera-  
 661 tions in curvilinear coordinates respectively as  $N(0, \sigma_{a_s}^2)$  and  $N(0, \sigma_{a_n}^2)$ . More-  
 662 over, according to linear Kalman filtering in discrete time, the state prediction  
 663 covariance for each obstacle can be computed as in (29), where  $F_{k-1}$  is the ma-  
 664 trix of the linear model and  $P_{o_i k-1}^+$  is the covariance matrix of the state updated  
 665 by the measurements at the previous step. Nevertheless, the unscented trans-  
 666 formation is still performed as in (9) to allow computing the cross covariance  
 667 matrix (14). In practice, among all the  $2n + 1$  sigma points, only one is used to  
 668 perform state prediction and covariance, while the remaining  $2n$  are required to  
 669 compute  $P_{xy, o_i}$ .

$$P_{o_i k}^- = F_{k-1} P_{o_i k-1}^+ F_{k-1}^T + Q_{k-1} \quad i = 1, \dots, N_{obs} \quad (29)$$

670 It is important to notice that while obstacle positioning is relative to the  
 671 road reference frame,  $V_{s_i}$  and  $V_{n_i}$  are the components of the absolute velocity  
 672 of a tracked obstacle. Thus, to ensure the correct prediction of  $s_{i, loc}$  at the  
 673 current time step, it is required to consider the difference between obstacle  
 674 and vehicle velocity along the road direction  $V_s$ . This is valid in case of both  
 675 positive and negative values of  $s_{i, loc}$ . Equation (30) summarizes the clockwise  
 676 rotation required to obtain the components of the ego-vehicle absolute velocity  
 677 in curvilinear coordinates.

$$V_s = \cos(\xi_{loc})V_x + \sin(\xi_{loc})V_y \quad (30)$$

678 The predicted state vector  $\tilde{x}_{o_i k}^-$  for each tracked obstacle is used to perform  
 679 the unscented transformation (9) through the covariance matrix  $P_{o_i k}^-$ . The new  
 680 sigma points ( $2n$ ) are then used to compute the predicted measurements matrix

681  $\hat{y}_{o_i k}^{(i)}$ . As shown in Section 4, this matrix is computed by feeding measurements  
 682 equations (23) to (27) with sigma points to obtain the predicted measurements  
 683 vectors for each obstacle  $\tilde{y}_{o_i k}$  and the related innovation covariance matrix  $P_{y, o_i}$ .  
 684 This is shown in (12) and (13).

685 To reduce the number of association hypotheses required to compare pre-  
 686 dicted measurements with the ones received from sensor fusion module  $y_{o_i k}$ ,  
 687 fused measurements are taken into account only if they fall within a gate cre-  
 688 ated around predicted measurements  $\tilde{y}_{o_i k}$ . Under the assumption of Gaussian  
 689 distributed noise, it is possible to adopt ellipsoidal gates [24]. In particular,  
 690 an ellipsoidal gate is defined through a gating probability  $P_G$ , which represents  
 691 the probability that the object measurement is inside the gate, together with a  
 692 cumulative distribution  $\chi^2(n)$  required to compute the gate size  $G$ . Then, the  
 693 so-called *Mahalanobis distance* can be calculated as in (31) to find which fused  
 694 measurements are inside the gates:

$$D^2(y_{o_j k}, \tilde{y}_{o_i k}) = [y_{o_j k} - \tilde{y}_{o_i k}]^T (P_{y, o_i})^{-1} [y_{o_j k} - \tilde{y}_{o_i k}] \quad (31)$$

695 for  $i = 1, \dots, n$  and  $j = 1, \dots, m$ . About  $n$  and  $m$ , they indicate respectively the  
 696 number of predicted measurements during the current time step and the number  
 697 of tracked objects at the previous one. Any measurement  $y_{o_j k}$  that does not  
 698 satisfy the criterion (32) is hence disregarded from the association set and will  
 699 be used to initialize new tentative obstacles. Conversely, all the measurements  
 700 included in ellipsoidal gates are collected and used for association.

$$D^2(y_{o_j k}, \tilde{y}_{o_i k}) < G \quad (32)$$

701 Association is done gathering all the selected measurements in one single  
 702 matrix. Although grouping by gating is computationally cheaper, for a moder-  
 703 ate number of tracked obstacles the exhaustive method does not reduce perfor-  
 704 mances. Association is then performed through a GNN algorithm that considers  
 705 only the best association hypotheses due to the lowest cost while discharging all  
 706 the others. To do so, the cost matrix  $L$  is defined through the likelihoods of as-  
 707 sociation between tracked objects and measurements inside gates, together with  
 708 the likelihoods of misdetection. These likelihoods can be calculated by know-  
 709 ing the probability of detection  $p(d)$  as in (33), assuming that the one assigned  
 710 by the Radar ( $p(o_i) > 0.99$ ) is much lower with respect to the one guaranteed  
 711 through the processing of Lidar data.

$$l_k^{i,0} = \log(1 - p(d)) \quad (33)$$

$$l_k^{i,j} = \log\left(\frac{p(d)}{\lambda(c)}\right) - \frac{1}{2} \log(\det(2\pi P_{y, o_i})) + \quad (34)$$

$$- \frac{1}{2} [y_{o_j k} - \tilde{y}_{o_i k}]^T (P_{y, o_i})^{-1} [y_{o_j k} - \tilde{y}_{o_i k}]$$

712 This formulation is valid only if the value of  $p(d)$  is assumed as constant and  
 713 the clutter intensity  $\lambda(c) = \lambda/FoV$  is positive and constant, where  $\lambda(c)$  can

714 be considered as the average number of clutters along the bounded FoV per  
 715 time step. The average number of clutters for each time step  $\lambda = 2$  has been  
 716 determined through simulation based on real data, processed as stated in Section  
 717 5.

718 The cost matrix  $L$  is hence a  $[n \cdot (m+n)]$  rectangular matrix, as shown in (35),  
 719 in which the  $[n \cdot m]$  left sub-matrix considers only real detections and is defined  
 720 by likelihoods of association between tracked objects and measurements. On the  
 721 other hand, the  $[n \cdot n]$  right sub-matrix collects all the misdetections determined  
 722 by the corresponding likelihoods.

$$L = \left[ \begin{array}{cccc|cccc} -l^{1,1} & -l^{1,2} & \dots & -l^{1,m} & -l^{1,0} & \infty & \dots & \infty \\ -l^{2,1} & -l^{2,2} & \dots & -l^{2,m} & \infty & -l^{2,0} & \dots & \infty \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -l^{n,1} & -l^{n,2} & \dots & -l^{n,m} & \infty & \infty & \dots & -l^{n,0} \end{array} \right] \quad (35)$$

723 Moreover, given the assignment matrix  $A$ , the corresponding assignment cost  
 724 can be defined by solving the optimization problem (36). The solution to this  
 725 problem is found adopting the 2D assignment algorithm described in [62].

$$\min tr(A^T L) = \sum_{i=1}^n \sum_{j=1}^{m+n} A^{i,j} L^{i,j} \quad (36a)$$

subject to:

$$A^{i,j} \in \{0 \ 1\} \quad (36b)$$

$$\sum_{j=1}^{n+m} A^{i,j} = 1 \quad (36c)$$

$$\sum_{i=1}^n A^{i,j} \in \{0 \ 1\} \quad (36d)$$

726 The optimal solution ensures the optimal correspondence between tracked  
 727 objects and measurements required to update state prediction and covariance  
 728 for each obstacle as in (16) and (17). If no measurements are provided from  
 729 sensor fusion or there are no measurements inside any gate, equations (37) and  
 730 (38) are adopted.

$$\tilde{x}_k^+ = \tilde{x}_k^- \quad (37)$$

$$P_k^+ = P_k^- \quad (38)$$

731 As previously stated, non associated measurements are used as new genera-  
 732 tions to initialize the tracking process. If a tentative obstacle is updated with  
 733 the assigned measurement throughout 1 second (i.e., for 20 iterations), it is



Figure 9. Picture of the vehicle, the Lidar sensor is visible on the roof, while the two Radar are incorporated in the rear and front bumpers

734 confirmed as a real obstacle. Moreover, a warning can be provided to the con-  
 735 troller if an object suddenly appears close to the ego-vehicle but then it is not  
 736 confirmed. Although this could be justified to enhance safety, simulation tests  
 737 carried on experimental data showed that objects that suddenly appear close  
 738 to the ego-vehicle without being tracked earlier could be considered as clutters.  
 739 On the other hand, if no measurements are assigned to a tentative obstacle  
 740 during the following 5 iterations (i.e., 0.25 s), this is deleted. An obstacle that  
 741 has already been confirmed is kept in record for 10 iterations (i.e., 0.5 s): if any  
 742 measurement is associated with it, this obstacle is still seen as confirmed and  
 743 state estimation provided.

## 744 7. Experimental results

745 The presented algorithm provides ego-vehicle and obstacles state estimation  
 746 in curvilinear coordinates for an autonomous vehicle. Ego-vehicle state estima-  
 747 tion is computed in the global reference frame and then collocated in the road's  
 748 local reference frame. This is done by exploiting the map's knowledge, which  
 749 associates to each point of the centerline the description of the road heading and  
 750 curvature along the considered FoV. Once the ego-vehicle is collocated within  
 751 the road map, raw data coming from Radar sensors and Lidar are processed and  
 752 fused in VRF to provide tentative obstacles to the tracker. Then, state estima-  
 753 tion is performed in curvilinear coordinates. The algorithm has been validated  
 754 during some experimental campaigns carried on Monza Eni Circuit.

755 The instrumented vehicle, showed in Fig. 9, is a prototype for an autonomous  
 756 driving car [17] equipped with sensors for the measurement of absolute position-  
 757 ing, odometry, and motion. In particular, the sensor suite includes:

- 758 • two *Piksi Multi* GPS receivers are located along the vehicle's longitudinal  
759 axis, coupled with a ground station through 4G connection. They provide  
760 positioning in absolute coordinates with RTK correction and velocities in  
761 East-Nord-UP (ENU) reference frame. As shown in section 4, velocities  
762 allow predicting the measurement of the ego-vehicle heading angle within  
763 the UKF. Measurements are provided at  $10\text{ Hz}$ ;
- 764 • an IMU located in correspondence of the vehicle CoG, which measures  
765 linear accelerations and angular velocities on the three principal axes.  
766 Measurements are available at  $100\text{ Hz}$ .
- 767 • odometry is given at  $20\text{ Hz}$  by an encoder mounted on the steering wheel  
768 to measure the steering angle, while two exciters on the rear axle provide  
769 the longitudinal speed of the vehicle;
- 770 • two *Continental ARS 408-21* Radar sensors provide relative positioning  
771 and motion of obstacles in VRF at  $14\text{ Hz}$ . They are located in the front  
772 and rear bumpers of the vehicle;
- 773 • a *Velodyne VLP-16* Lidar mounted on the roof provides 3D pointclouds  
774 at  $20\text{ Hz}$ .

775 The overall estimation routine runs at  $20\text{ Hz}$  on a soft real-time system based  
776 on ROS (Robot Operating System). This allows managing the different sam-  
777 pling frequencies, because triggering can be based on ROS timestamps. If no  
778 measurements arrive from the GPS receivers state prediction is used instead of  
779 state estimation ((37), (38)).

780 Concerning ego-vehicle estimates, accuracy can be assessed by analyzing the  
781 predicted heading angle and the lateral speed in the vehicle CoG, which are not  
782 measured by any sensor included within the listed suite. To do so, a further  
783 automotive optical sensor has been mounted on the vehicle during some exper-  
784 imental campaigns to collect ground truth data regarding vehicle sideslip. The  
785 comparison between measured and estimated longitudinal and lateral speeds is  
786 presented in Fig.10. The figure points out the comparison between measure-  
787 ments and estimates during a steering pad maneuver completed on a circle with  
788 a radius equal to  $27\text{ m}$ . As shown in the first two subplots, the vehicle's longi-  
789 tudinal speed increases approximately from  $20$  to  $40\text{ km/h}$ , while the steering  
790 angle is worth about  $100\text{ deg}$ . The third and last subplot points out a strong  
791 correlation between estimated and measured lateral speed. Moreover, during  
792 the presented maneuver the vehicle is close to the tires' friction saturation: this  
793 is highlighted to assess the effectiveness of the estimation algorithm.

794 For what concerns the estimation of heading angle, it is not possible to define  
795 a ground truth basing on the angle between the horizontal and the straight line  
796 that connects the measures given by the GPS receivers at the same time step.  
797 Although the RTK correction ensures that the measurement error for positioning  
798 decreases up to a few centimeters, this still affects the heading angle's estimate  
799 with an error that depends on the distance between the two receivers. For  
800 the presented vehicle, these drift effects produce an error that varies in the

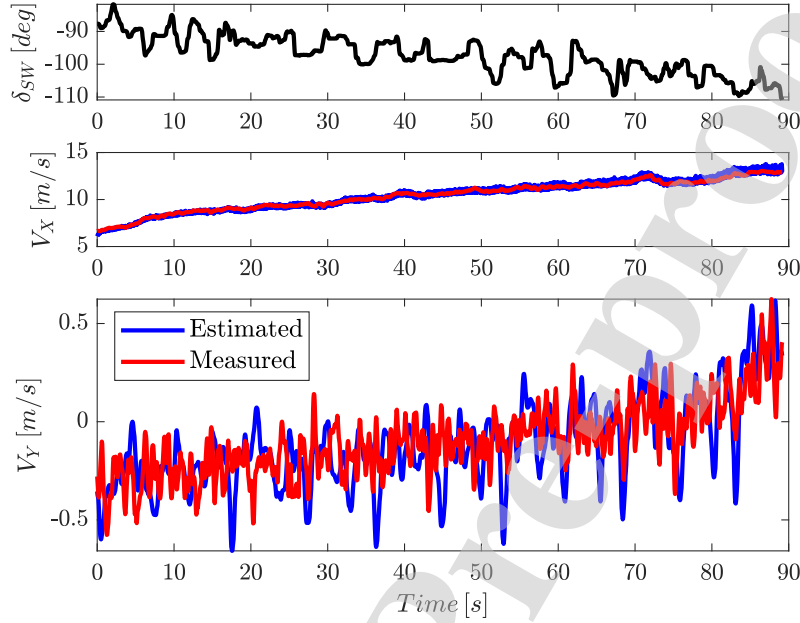


Figure 10. Comparison between estimated and measured lateral speed during a steering pad manoeuvre performed at increasing speed

801 range  $\pm 10 \text{ deg}$ , which is too high to guarantee a significant ground truth. A  
 802 further possibility is to analyze in time the angle found by tracking subsequent  
 803 positions of the rear GPS receiver (i.e., the one less affected by steering effects).  
 804 However, this angle is the tangent to the trajectory completed by the rear part  
 805 of the vehicle ( $\gamma_R$ ), which is related to the vehicle heading angle as indicated by  
 806 Eq.(39).

$$\begin{cases} \gamma_R - \beta_R = \psi_{abs} \\ \beta_R = \text{atan}((V_y - \psi l_R)/V_x) \end{cases} \quad (39)$$

807 Here, the vehicle's sideslip angle is reported to the rear's GPS receiver, consid-  
 808 ering the variation of lateral speed. This is done accounting for the distance to  
 809 the vehicle CoG and the yaw rate. Given that the heading angle  $\psi_{abs}$  is con-  
 810 stant along the vehicle, it is possible to state that the estimate is correct if the  
 811 difference  $\gamma_R - \psi_{abs} - \beta_R$  is null for any time instant. This difference is reported  
 812 for the aforementioned steering pad in Fig.11, whose offset from null is constant  
 813 and equal to  $+0.06 \text{ deg}$ . This result assesses the performance of the estimation  
 814 also during a challenging driving maneuver. Indeed, although the sideslip angle  
 815  $\beta_R$  increases from 1 to 5  $\text{deg}$ , the offset remains constant. Regarding the high  
 816 level of noise in the plot, this is due to the lateral speed measurements provided

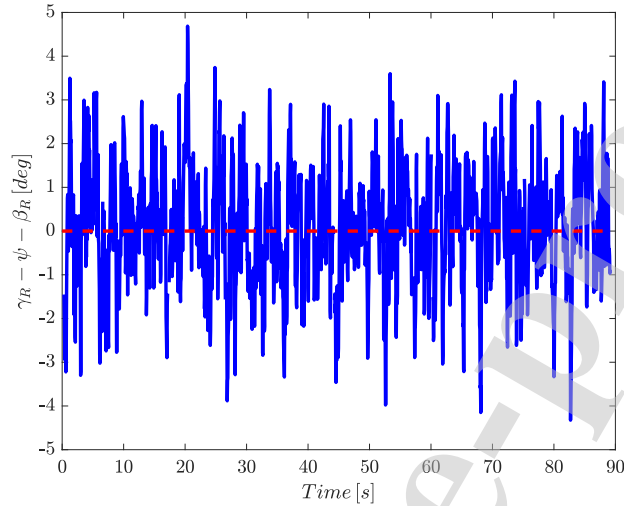


Figure 11. Validation of the estimation of the ego-vehicle heading angle. The difference between trajectory angle and sideslip angle at the rear, and the heading angle must be null

817 by the automotive optical sensor. Furthermore, a qualitative representation of  
 818 the vehicle's estimated heading angle is reported in Fig. 12. Two different  
 819 plots point out the vehicle's direction during the first two tight chicanes of the  
 820 track that the vehicle performs respectively from the bottom to the top of the  
 821 first plot, and from left to right in the second one. The quality of the estimate  
 822 can be evaluated observing the direction during straights, superimposed to the  
 823 predicted position of the vehicle CoG. At the same time, during curving, the  
 824 heading angle is comparable to the tangent to the trajectory.

825 The validation of the obstacles' state estimation module is allowed by a set  
 826 of experimental data collected in some significant mutual maneuvers between  
 827 the ego-vehicle and a designated obstacle vehicle (FIAT Talento, a van whose  
 828 dimensions are  $5 \times 2 \times 2 \text{ m}$ ). To assess the accuracy of the algorithm, the absence of  
 829 false positives, and the accuracy of the estimated state vector are analyzed. The  
 830 results discussed in this section derives from a vehicle-following maneuver: the  
 831 obstacle is driving ahead of the autonomous vehicle between turn 3 and turn 6,  
 832 hence the road curvature changes significantly during the test. The algorithm  
 833 performs well in filtering clutters within and out of road bounds. Moreover,  
 834 the presented results prove that it performs well also during tight curvature  
 835 scenarios.

836 A snapshot from the described scenario is reported in Fig. 13. For ease of  
 837 viewing, the overall framework with ego-vehicle, measurements, and obstacles  
 838 is shown in Cartesian coordinates, in the global reference frame. Nevertheless,  
 839 the plot reports the estimated positioning of the obstacle in curvilinear coordi-  
 840 nates in the road reference frame, i.e., the longitudinal distance  $s_{i, loc}$ , and the

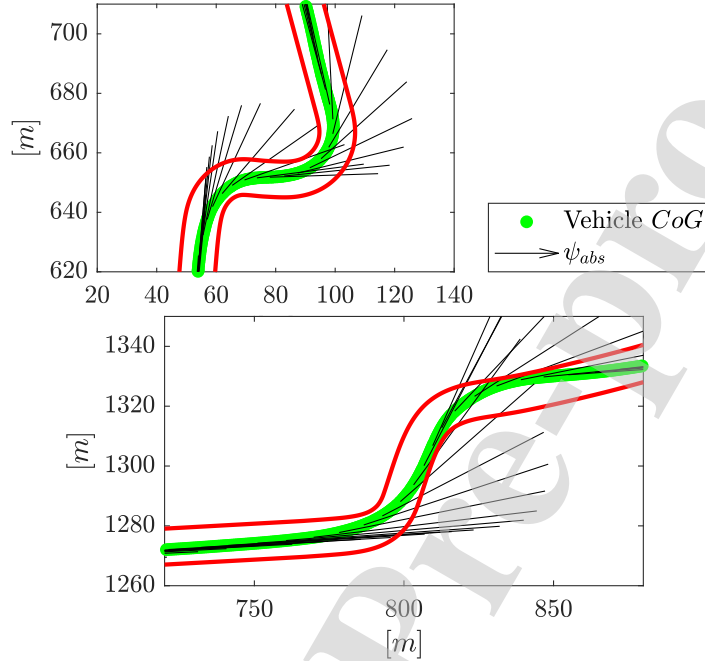


Figure 12. Heading angle estimate for the ego-vehicle

841 lateral displacement to centreline  $n_{i,loc}$ . The ego-vehicle estimated position is  
 842 represented by a  $(\odot)$ .

843 The plot shows multiple measurements obtained from Radar and Lidar sensors.  
 844 The  $(\odot)$  marks are clustered Radar objects fed to the road filtering module  
 845 which provides  $(*)$  as output. These results are the inputs to the next sensor  
 846 fusion module. Similarly, the  $(\odot)$  mark represents the output of the Lidar  
 847 processing module and  $(*)$  are the Lidar measurements within road boundaries  
 848 fed as inputs to the sensor fusion module. As shown in the plot, information  
 849 about road width allows filtering all the measurements related to any obstacle  
 850 or object out of interest. In this particular instance, measurements coming from  
 851 both the Lidar and the Radar processing modules are simultaneously available  
 852 for the fusion module. Thus, fused measurements computed by Eq. 21, are  
 853 represented by the  $(\odot)$  mark. As explained in previous sections, this output is  
 854 used for object initialization and association in the remaining steps of the esti-  
 855 mation routine. The algorithm is also able to ensure the accuracy of the object  
 856 cardinality, which in this scenario is consistently equal to one, by implement-  
 857 ing a track confirmation and removal routine. Although the figure illustrates  
 858 two detections from the sensor fusion module, the tracking algorithm accurately  
 859 confirms a single object while providing its state estimate as confirmed. The



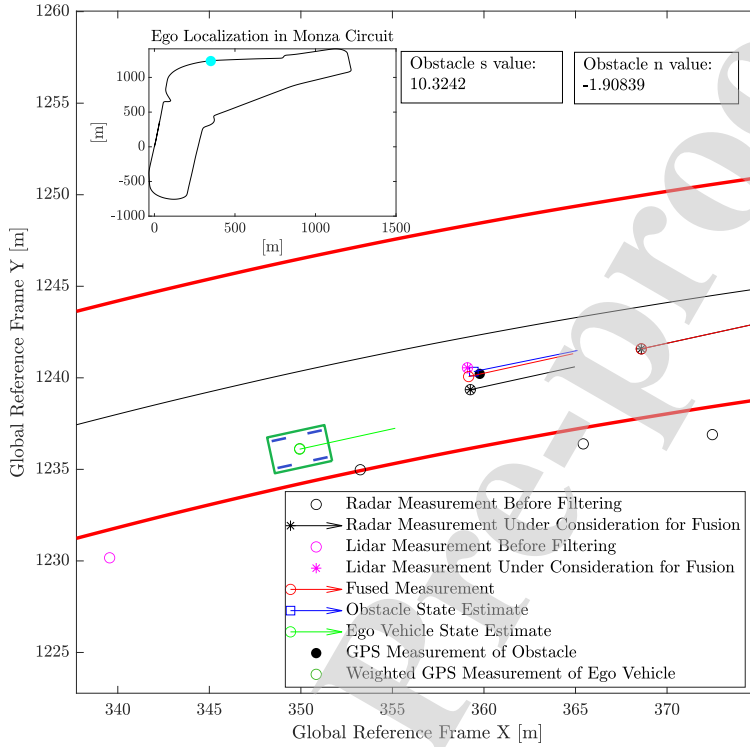


Figure 13. Visual representation of obstacle identification and tracking. The green box represents the ego-vehicle, while the circles represent the obstacles identified by the different sensors. (attached video V1.avi)

obstacle state estimate is represented by ( $\square$ ), while the estimated distances in  
 860 curvilinear co-ordinates are mentioned in the bottom part of the figure. More-  
 861 over, for those clusters or estimates whose velocity is known or computed, the  
 862 plot points out a vector that represents its magnitude and direction.  
 863

To conclude, Fig. (14) illustrates the comparison of the estimated obstacle's  
 864 state vector with the ground truth given by the GPS receiver installed on the  
 865 tracked obstacle vehicle with RTK correction. Both the GPS measurements  
 866 and the estimates are represented in vehicle reference frame (VRF). Due to un-  
 867 availability of ground truth in curvilinear co-ordinates, estimates are converted  
 868 from curvilinear coordinates to VRF by applying the Euler model presented  
 869 in the previous section. The root mean square error ( $RMSE$ ) is computed  
 870 as the distance between the estimated position of the obstacle vehicle and the  
 871 real one. In the described scenario, the algorithm performs the estimation with  
 872  $RMSE = 0.6039 m$ , that is reasonable compared to the size of the obstacle.  
 873

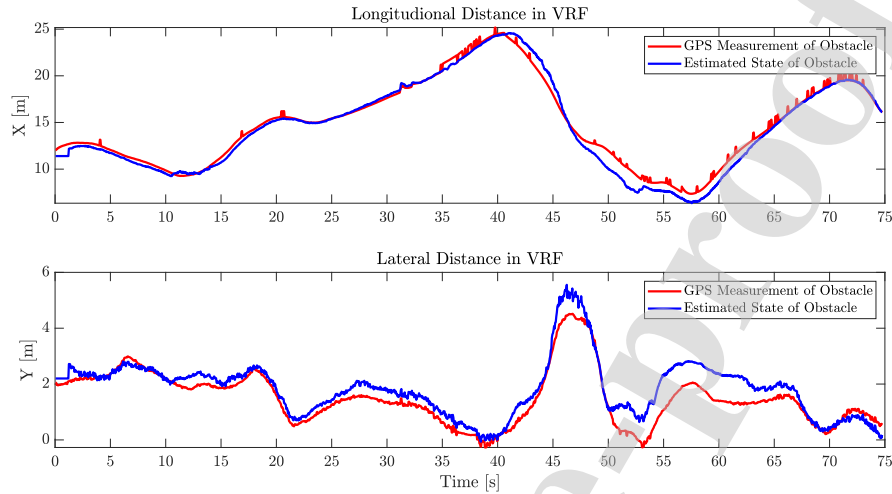


Figure 14. Comparison between the estimated relative position between the obstacle and the ego-vehicle, and the ground truth given by the GPS receiver

## 874 8. Conclusions

875 The presented paper focuses on state estimation applied to autonomous vehi-  
 876 cles. It describes an integrated algorithm that computes ego-vehicle and obsta-  
 877 cles' state estimation in curvilinear coordinates, according to the road reference  
 878 frame. The ego-vehicle's state vector includes positioning, heading angle, and  
 879 the longitudinal and lateral components of velocity in the vehicle reference  
 880 (VRF). Estimates are provided in Cartesian coordinates and then converted to  
 881 the local reference frame of the road. About the obstacles in the surround-  
 882 ing of the ego-vehicle, the presented algorithm computes their relative position  
 883 and absolute velocity in curvilinear coordinates according to the road reference  
 884 frame, under the assumption of small dimensions. Measurements of obstacles  
 885 are provided by a multi-sensor framework, which includes two Radars located  
 886 within the vehicle front and rear bumpers and a Lidar mounted on the vehicle  
 887 top in correspondence of the center of gravity. Sensor fusion provides the track-  
 888 ing module with filtered measurements, allowing to associate each of them to  
 889 the respective obstacle. Association is performed through GNN. Due to strong  
 890 nonlinearities in each measurement model of the two filters, both the estimation  
 891 routines are based on Unscented Kalman Filters. The integrated algorithm has  
 892 been validated through experimental tests carried in the Monza ENI circuit.  
 893 The overall estimation routine runs at  $20\text{ Hz}$  on a soft real-time system based  
 894 on ROS: this allows managing the different sampling frequencies of each sensor.

895 To conclude, the presented estimation algorithm provides a detailed set of  
 896 initial conditions for any motion planning routine for autonomous vehicles. In  
 897 future works, ego-vehicle dynamic behavior will be considered at least in the

898 lateral direction; moreover, a camera will be installed on the car, to improve  
899 sensor fusion and object tracking, basing on the high semantic content of images.

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## 909 References

- 910 [1] B. Paden, M. Čáp, S. Z. Yong, D. Yershov, E. Frazzoli, A survey of mo-  
911 tion planning and control techniques for self-driving urban vehicles, *IEEE*  
912 *Transactions on intelligent vehicles* 1 (2016) 33–55.
- 913 [2] A. Van Zanten, R. Bosch Gmbh, Evolution of electronic control systems for  
914 improving vehicle dynamic behaviour, in: *Proceedings of the International*  
915 *Symposium on Advanced Vehicle Control (AVEC)*, 2002.
- 916 [3] J. Kim, K. Jo, W. Lim, M. Lee, M. Sunwoo, Curvilinear-coordinate-  
917 based object and situation assessment for highly automated vehicles,  
918 *IEEE Transactions on Intelligent Transportation Systems* 16 (2015) 1–17.  
919 doi:10.1109/TITS.2014.2369737.
- 920 [4] R. Danescu, F. Oniga, S. Nedevschi, Modeling and tracking the driv-  
921 ing environment with a particle-based occupancy grid, *IEEE Trans-*  
922 *actions on Intelligent Transportation Systems* 12 (2011) 1331–1342.  
923 doi:10.1109/TITS.2011.2158097.
- 924 [5] A. Barth, U. Franke, Tracking oncoming and turning vehicles at intersec-  
925 tions, in: *13th International IEEE Conference on Intelligent Transportation*  
926 *Systems*, 2010, pp. 861–868. doi:10.1109/ITSC.2010.5624969.
- 927 [6] B. Fortin, R. Lherbier, J. Noyer, A model-based joint detection and  
928 tracking approach for multi-vehicle tracking with lidar sensor, *IEEE*  
929 *Transactions on Intelligent Transportation Systems* 16 (2015) 1883–1895.  
930 doi:10.1109/TITS.2015.2391131.
- 931 [7] B. Jahromi, T. Tulabandhula, S. Cetin, Real-time hybrid multi-sensor  
932 fusion framework for perception in autonomous vehicles, *Sensors* 19 (2019)  
933 4357. doi:10.3390/s19204357.

- 934 [8] H. Cho, Y. Seo, B. V. K. V. Kumar, R. R. Rajkumar, A multi-sensor fusion  
935 system for moving object detection and tracking in urban driving environ-  
936 ments, in: 2014 IEEE International Conference on Robotics and Automa-  
937 tion (ICRA), 2014, pp. 1836–1843. doi:10.1109/ICRA.2014.6907100.
- 938 [9] H. Zhu, K.-V. Yuen, L. Mihaylova, H. Leung, Overview of environment  
939 perception for intelligent vehicles, *IEEE Transactions on Intelligent Trans-  
940 portation Systems PP* (2017). doi:10.1109/TITS.2017.2658662.
- 941 [10] D. Göhring, M. Wang, M. Schnürmacher, T. Ganjineh, Radar/lidar sen-  
942 sor fusion for car-following on highways, in: *The 5th International Con-  
943 ference on Automation, Robotics and Applications*, 2011, pp. 407–412.  
944 doi:10.1109/ICARA.2011.6144918.
- 945 [11] J. Hollinger, B. Kutscher, R. Close, Fusion of lidar and radar for detection  
946 of partially obscured objects, 2015, p. 946806. doi:10.1117/12.2177050.
- 947 [12] C. Yi, K. Zhang, N. Peng, A multi-sensor fusion and object tracking algo-  
948 rithm for self-driving vehicles, *Proceedings of the Institution of Mechanical  
949 Engineers, Part D: Journal of Automobile Engineering* 233 (2019) 2293–  
950 2300. doi:10.1177/0954407019867492.
- 951 [13] E. Héry, S. Masi, P. Xu, P. Bonnifait, Map-based curvilinear coordinates  
952 for autonomous vehicles, 2017, pp. 1–7. doi:10.1109/ITSC.2017.8317775.
- 953 [14] G. Inghilterra, S. Arrigoni, F. Braghin, F. Cheli, Firefly algorithm-based  
954 nonlinear mpc trajectory planner for autonomous driving, in: *2018 Interna-  
955 tional Conference of Electrical and Electronic Technologies for Automotive,  
956 2018*, pp. 1–6.
- 957 [15] S. Arrigoni, E. Trbalzini, M. Bersani, F. Braghin, F. Cheli, Non-  
958 linear mpc motion planner for autonomous vehicles based on ac-  
959 celerated particle swarm optimization algorithm, 2019, pp. 1–6.  
960 doi:10.23919/EETA.2019.8804561.
- 961 [16] L. Zheng, B. Li, B. Yang, H. Song, Z. Lu, Lane-level  
962 road network generation techniques for lane-level maps of au-  
963 tonomous vehicles: A survey, *Sustainability* 11 (2019) 4511. URL:  
964 <http://dx.doi.org/10.3390/su11164511>. doi:10.3390/su11164511.
- 965 [17] M. Vignati, D. Tarsitano, M. Bersani, F. Cheli, Autonomous steer ac-  
966 tuation for an urban quadricycle, in: *2018 International Conference of  
967 Electrical and Electronic Technologies for Automotive, 2018*, pp. 1–5.
- 968 [18] Monza eni circuit, 2020. <https://www.monzanet.it/>.
- 969 [19] M. Quigley, K. Conley, B. Gerkey, J. Faust, T. Foote, J. Leibs, R. Wheeler,  
970 A. Y. Ng, Ros: an open-source robot operating system, in: *ICRA workshop  
971 on open source software, volume 3, Kobe, Japan, 2009*, p. 5.

- 972 [20] K. Jo, M. Lee, J. Kim, M. Sunwoo, Tracking and behavior reason-  
973 ing of moving vehicles based on roadway geometry constraints, *IEEE*  
974 *Transactions on Intelligent Transportation Systems PP* (2016) 1–17.  
975 doi:10.1109/TITS.2016.2605163.
- 976 [21] D. Simon, *Optimal State Estimation: Kalman, H Infinity, and Nonlinear*  
977 *Approaches*, Wiley, 2006.
- 978 [22] S. J. Julier, J. K. Uhlmann, Unscented filtering and nonlinear estimation,  
979 in: *Proceedings of the IEEE*, volume 92, 2004.
- 980 [23] R. Juang, P. Burlina, Comparative performance evaluation of gm-phd filter  
981 in clutter, in: *2009 12th International Conference on Information Fusion*,  
982 2009, pp. 1195–1202.
- 983 [24] Y. X. Lennart Svensson, Karl Granstrom, Multi-object track-  
984 ing for automotive systems, [https://courses.edx.org/courses/course-](https://courses.edx.org/courses/course-v1:ChalmersX+ChM013x+3T2019/course/)  
985 [v1:ChalmersX+ChM013x+3T2019/course/](https://courses.edx.org/courses/course-v1:ChalmersX+ChM013x+3T2019/course/), 2019.
- 986 [25] P. Tripathi, K. Nagla, H. Singh, S. Mahajan, Occupancy grid mapping  
987 for mobile robot using sensor fusion, in: *2014 International Conference on*  
988 *Issues and Challenges in Intelligent Computing Techniques (ICICT)*, IEEE,  
989 2014, pp. 47–51.
- 990 [26] D. V. Lu, D. Hershberger, W. D. Smart, Layered costmaps for context-  
991 sensitive navigation, in: *2014 IEEE/RSJ International Conference on In-*  
992 *telligent Robots and Systems*, IEEE, 2014, pp. 709–715.
- 993 [27] S. Mentasti, M. Matteucci, Multi-layer occupancy grid mapping for au-  
994 tonomous vehicles navigation, in: *2019 AEIT International Conference of*  
995 *Electrical and Electronic Technologies for Automotive (AEIT AUTOMO-*  
996 *TIVE)*, IEEE, 2019, pp. 1–6.
- 997 [28] P. Fankhauser, M. Hutter, A universal grid map library: Implementation  
998 and use case for rough terrain navigation, in: *Robot Operating System*  
999 *(ROS)*, Springer, 2016, pp. 99–120.
- 1000 [29] P. Fankhauser, M. Bloesch, M. Hutter, Probabilistic terrain mapping for  
1001 mobile robots with uncertain localization, *IEEE Robotics and Automation*  
1002 *Letters (RA-L)* 3 (2018) 3019–3026. doi:10.1109/LRA.2018.2849506.
- 1003 [30] J. Gu, Q. Cao, Path planning for mobile robot in a 2.5-dimensional grid-  
1004 based map, *Industrial Robot: An International Journal* 38 (2011) 315–321.
- 1005 [31] A. A. Souza, L. M. Goncalves, 2.5-dimensional grid mapping from stereo  
1006 vision for robotic navigation, in: *2012 Brazilian Robotics Symposium and*  
1007 *Latin American Robotics Symposium*, IEEE, 2012, pp. 39–44.
- 1008 [32] M. Himmelsbach, A. Mueller, T. Lüttel, H.-J. Wünsche, Lidar-based 3d ob-  
1009 ject perception, in: *Proceedings of 1st international workshop on cognition*  
1010 *for technical systems*, volume 1, 2008.

- 1011 [33] D. Anguelov, B. Taskarf, V. Chatalbashev, D. Koller, D. Gupta, G. Heitz,  
1012 A. Ng, Discriminative learning of markov random fields for segmentation of  
1013 3d scan data, in: 2005 IEEE Computer Society Conference on Computer  
1014 Vision and Pattern Recognition (CVPR'05), volume 2, IEEE, 2005, pp.  
1015 169–176.
- 1016 [34] J.-F. Lalonde, N. Vandapel, D. F. Huber, M. Hebert, Natural terrain  
1017 classification using three-dimensional lidar data for ground robot mobility,  
1018 *Journal of field robotics* 23 (2006) 839–861.
- 1019 [35] Y. Zeng, Y. Hu, S. Liu, J. Ye, Y. Han, X. Li, N. Sun, Rt3d: Real-time  
1020 3-d vehicle detection in lidar point cloud for autonomous driving, *IEEE*  
1021 *Robotics and Automation Letters* 3 (2018) 3434–3440.
- 1022 [36] B. Zhu, Z. Jiang, X. Zhou, Z. Li, G. Yu, Class-balanced grouping and sam-  
1023 pling for point cloud 3d object detection, *arXiv preprint arXiv:1908.09492*  
1024 (2019).
- 1025 [37] Y. Yan, Y. Mao, B. Li, Second: Sparsely embedded convolutional detection,  
1026 *Sensors* 18 (2018) 3337.
- 1027 [38] A. H. Lang, S. Vora, H. Caesar, L. Zhou, J. Yang, O. Beijbom, Pointpillars:  
1028 Fast encoders for object detection from point clouds, in: *Proceedings of*  
1029 *the IEEE Conference on Computer Vision and Pattern Recognition, 2019,*  
1030 pp. 12697–12705.
- 1031 [39] C. R. Qi, H. Su, K. Mo, L. J. Guibas, Pointnet: Deep learning on point  
1032 sets for 3d classification and segmentation, in: *Proceedings of the IEEE*  
1033 *conference on computer vision and pattern recognition, 2017,* pp. 652–660.
- 1034 [40] C. R. Qi, L. Yi, H. Su, L. J. Guibas, Pointnet++: Deep hierarchical feature  
1035 learning on point sets in a metric space, *arXiv preprint arXiv:1706.02413*  
1036 (2017).
- 1037 [41] B. Li, T. Zhang, T. Xia, Vehicle detection from 3d lidar using fully convo-  
1038 lutional network, *arXiv preprint arXiv:1608.07916* (2016).
- 1039 [42] G. Yang, S. Mentasti, M. Bersani, Y. Wang, F. Braghin, F. Cheli, Lidar  
1040 point-cloud processing based on projection methods: a comparison, 2020.  
1041 *arXiv:2008.00706*.
- 1042 [43] J. van Genderen, Tracking and data fusion: a handbook of al-  
1043 gorithms, by yaakov bar-shalom, peter k. willett and xin tian,  
1044 *International Journal of Image and Data Fusion* 4 (2013) 102–  
1045 104. URL: <https://doi.org/10.1080/19479832.2012.749304>.  
1046 doi:10.1080/19479832.2012.749304.
- 1047 [44] H. Winner, S. Hakuli, F. Lotz, C. Singer, Data fusion of environment-  
1048 perception sensors for adas, in: *Handbook of Driver Assistance Systems -*  
1049 *2016,* 2016.

- 1050 [45] E. Héry, P. Xu, P. Bonnifait, Along-track localization for cooperative au-  
1051 tonomous vehicles, in: 2017 IEEE Intelligent Vehicles Symposium (IV),  
1052 2017, pp. 511–516.
- 1053 [46] J. Hudecek, L. Eckstein, Improving and simplifying the generation of  
1054 reference trajectories by usage of road-aligned coordinate systems, in:  
1055 2014 IEEE Intelligent Vehicles Symposium Proceedings, 2014, pp. 504–509.  
1056 doi:10.1109/IVS.2014.6856502.
- 1057 [47] J. Ryu, J. Christian Gerdes, Integrating inertial sensors with global po-  
1058 sitioning system (gps) for vehicle dynamics control, *Journal of Dynamic*  
1059 *Systems Measurement and Control-transactions of The Asme - J DYN*  
1060 *SYST MEAS CONTR* 126 (2004).
- 1061 [48] D. M. Bevly, J. Ryu, J. C. Gerdes, Integrating ins sensors with gps mea-  
1062 surements for continuous estimation of vehicle sideslip, roll, and tire cor-  
1063 nering stiffness, *IEEE Transactions on Intelligent Transportation Systems*  
1064 7 (2006) 483–493.
- 1065 [49] T. Wenzel, K. Burnham, M. Blundell, R. Williams, Dual extended kalman  
1066 filter for vehicle state and parameter estimation, *Vehicle System Dynamics*  
1067 - *VEH SYST DYN* 44 (2006) 153–171.
- 1068 [50] J. Chen, J. Song, L. Li, G. Jia, X. Ran, C. Yang, Ukf-based adaptive  
1069 variable structure observer for vehicle sideslip with dynamic correction,  
1070 *IET Control Theory Applications* 10 (2016) 1641–1652.
- 1071 [51] J. Yoon, H. Peng, A cost-effective sideslip estimation method using velocity  
1072 measurements from two gps receivers, *IEEE Transactions on Vehicular*  
1073 *Technology* 63 (2014) 2589–2599.
- 1074 [52] R. Rajamani, *Vehicle Dynamics and Control*, Springer, 2012.
- 1075 [53] J. Villagra, B. d’Andrea-Novell, M. Fliess, H. Mounier, Estimation of lon-  
1076 gitudinal and lateral vehicle velocities: An algebraic approach, in: 2008  
1077 American Control Conference, 2008.
- 1078 [54] A. Y. Ungoren, H. Peng, A study on lateral speed estimation, *International*  
1079 *Journal of Vehicle Autonomous Systems*, Vol. 2 (2004).
- 1080 [55] J. Farrelly, P. Wellstead, Estimation of vehicle lateral velocity, in: Pro-  
1081 ceeding of the 1996 IEEE International Conference on Control Applications  
1082 IEEE International Conference on Control Applications held together with  
1083 IEEE International Symposium on Intelligent Control, 1996, pp. 552–557.
- 1084 [56] L. Imslund, T. A. Johansen, T. I. Fossen, J. C. Kalkkuhl, A. Suissa, Vehicle  
1085 velocity estimation using modular nonlinear observers, in: Proceedings of  
1086 the 44th IEEE Conference on Decision and Control, 2005, pp. 6728–6733.

- 1087 [57] D. Selmanaj, M. Como, G. Panzani, S. Savaresi, Robust Vehicle Sideslip  
1088 Estimation Based on Kinematic Considerations, in: IFAC PapersOnline,  
1089 Volume 50 Issue 1, 2017.
- 1090 [58] M. Bersani, M. Vignati, S. Mentasti, S. Arrigoni, F. Cheli, Vehicle state es-  
1091 timation based on kalman filters, in: 2019 AEIT International Conference  
1092 of Electrical and Electronic Technologies for Automotive (AEIT AUTO-  
1093 MOTIVE), 2019, pp. 1–6. doi:10.23919/EETA.2019.8804527.
- 1094 [59] D. Zermas, I. Izzat, N. Papanikolopoulos, Fast segmentation of 3d point  
1095 clouds: A paradigm on lidar data for autonomous vehicle applications, in:  
1096 2017 IEEE International Conference on Robotics and Automation (ICRA),  
1097 IEEE, 2017, pp. 5067–5073.
- 1098 [60] G. Bradski, The OpenCV Library, Dr. Dobb's Journal of Software Tools  
1099 (2000).
- 1100 [61] K. Wu, E. Otoo, K. Suzuki, Optimizing two-pass connected-component  
1101 labeling algorithms, Pattern Analysis and Applications 12 (2009) 117–135.
- 1102 [62] D. F. Crouse, On implementing 2d rectangular assignment algorithms,  
1103 IEEE Transactions on Aerospace and Electronic Systems 52 (2016) 1679–  
1104 1696. doi:10.1109/TAES.2016.140952.



An integrated algorithm for ego-vehicle and obstacles state estimation for autonomous driving

Highlights:

- The estimation process is a fundamental task for autonomous driving.
- Estimates are related to the ego-vehicle and the surrounding obstacles.
- The estimation routine handles in proper way the model nonlinearities.
- Estimates are provided in the local reference frame of the road.
- The algorithm performs sensor-fusion and estimation in real-time.

Mattia Bersani} is currently a doctoral candidate at Politecnico di Milano. He obtained the M.S. degree in mechanical engineering from Politecnico di Milano, Milano, Italy, in 2017. His research activity is focused on the definition and implementation of control and state estimation algorithms for autonomous and remote-driven vehicles.

Journal Pre-proof

Simone Mentasti is a PhD student at Dipartimento di Elettronica Informazione e Bioingegneria of Politecnico di Milano, Italy. He obtained the M.S. degree in computer science from Università Statale di Milano, Italy, in 2017. His interest focuses on robotics, perception, sensor fusion, sensor calibration and deep learning for autonomous driving cars. His research concerns the development of a sensor fusion framework for autonomous vehicles able to retrieve a uniform representation of the environment surrounding the car.

Stefano Arrigoni} is currently an Assistant Professor at Politecnico di Milano. He received the M.S. degree in mechanical engineering and the Ph.D. degree in applied mechanics both from Politecnico di Milano, Milano, Italy, in 2013 and 2017, respectively. His research interests lie in the area of autonomous vehicles with a focus on trajectory planning techniques and V2V communication.

Journal Pre-proof

Federico Cheli received the M.S. degree in mechanical engineering from the Politecnico di Milano, Milano, Italy, in 1981. He is currently a Full Professor at the Department of Mechanical Engineering, Politecnico di Milano and author of more than 380 publications on international journals and conferences. His scientific activity concerns research on vehicle performance, handling and comfort problems, active control, ADAS, and electric and autonomous vehicles. He is a member of the editorial board of the International Journal of Vehicle Performance and International Journal of Vehicle Systems Modeling and Testing.

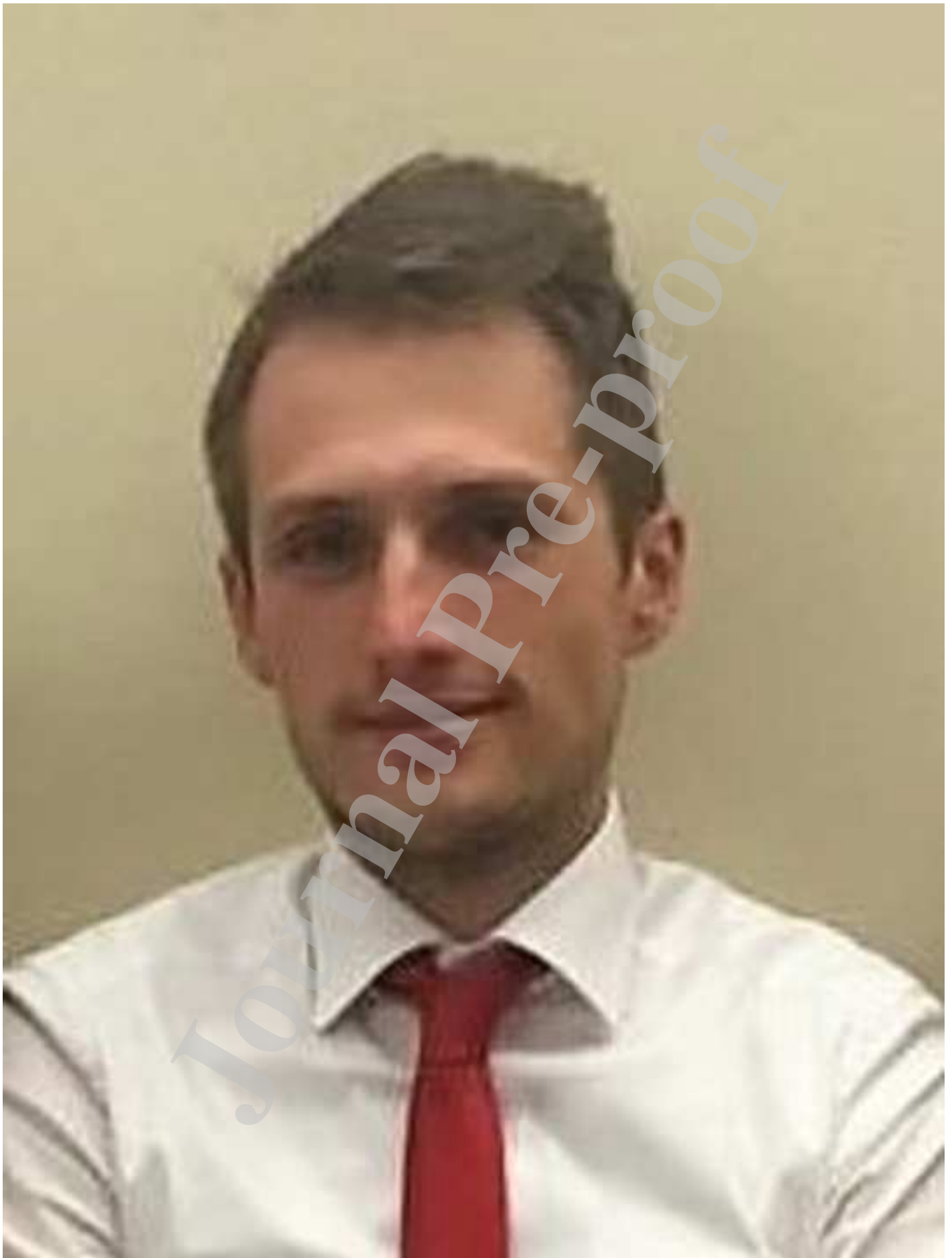
Matteo Matteucci (PhD) is Associate Professor at Dipartimento di Elettronica Informazione e Bioingegneria of Politecnico di Milano, Italy. In 1999 he got a Laurea degree in Computer Engineering at Politecnico di Milano, in 2002 he got a Master of Science in Knowledge Discovery and Data Mining at Carnegie Mellon University (Pittsburgh, PA), and in 2003 he got a PhD in Computer Engineering and Automation at Politecnico di Milano (Milan, Italy). His main research topics are pattern recognition, machine learning, machine perception, robotics, computer vision and signal processing. His main research interest is in developing, evaluating and applying, in a practical way, techniques for adaptation and learning to autonomous systems interacting with the physical world. He has co-authored more than 150 scientific international publications and he has been the principal investigator in national and international funded research projects on machine learning, autonomous robots, sensor fusion and benchmarking of autonomous and intelligent systems

Pragyan Dahal received B. Tech in Mechanical Engineering from Suresh Gyan Vihar University, India in 2015. Currently he is working towards completing MSc in Mechanical Engineering with Mechatronics specialization at Politecnico Di Milano, Italy. His main research interests include Environment Perception, Multi Object Tracking (MOT) algorithms, Control and Path Planning etc of Autonomous Vehicle.

Journal Pre-proof

Michele Vignati received his master degree (2013) and PhD (2017) in mechanical engineering in Politecnico di Milano. From 2019 he is Assistant Professor (RTDA) in the research field of applied mechanics. In particular, he focuses on mechanical systems dynamics and control applied in the automotive field of which he has more than 20 publications in international journals and conferences. In 2018 he won the best paper award for a paper presented at the AVEC'18 international conference.

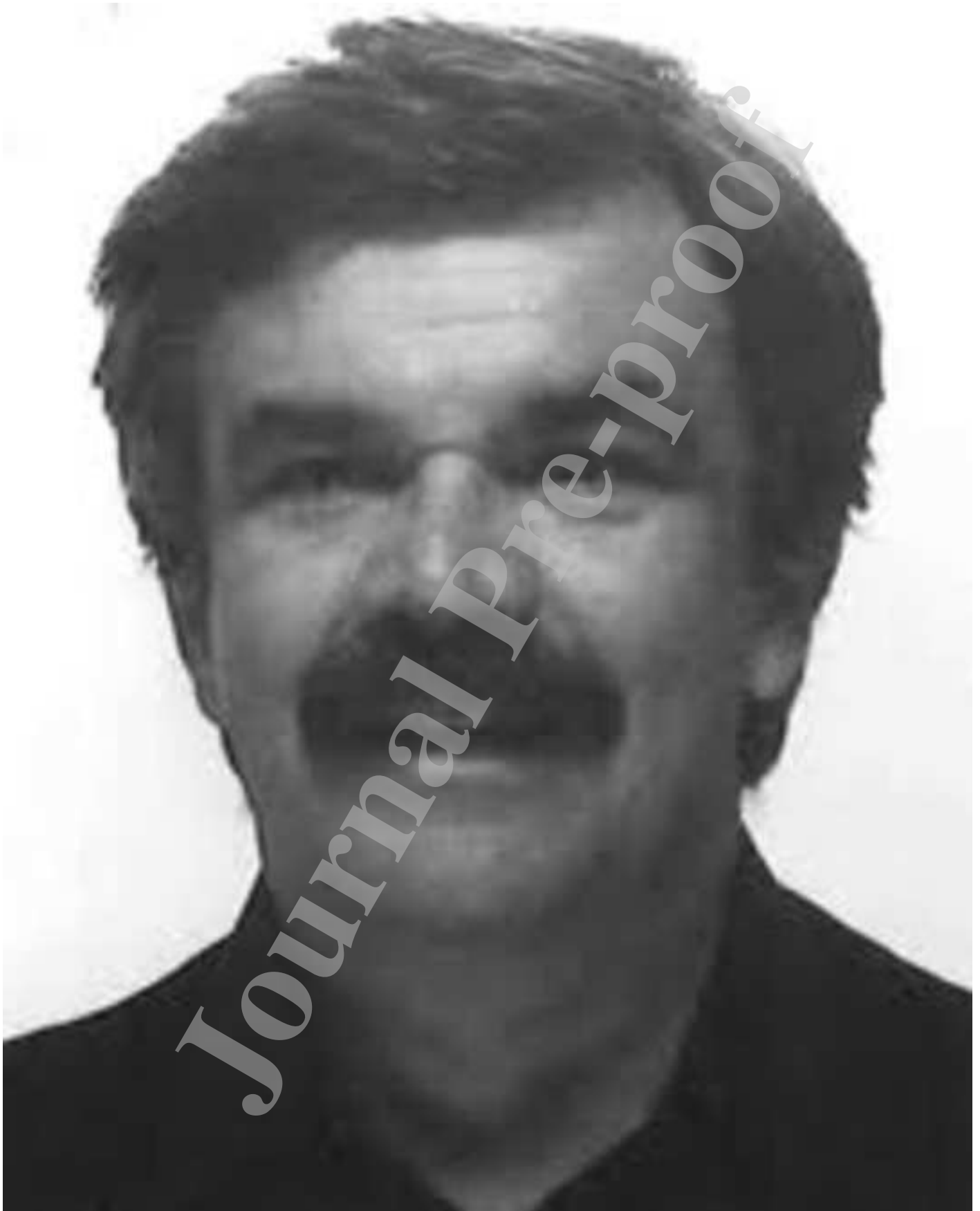




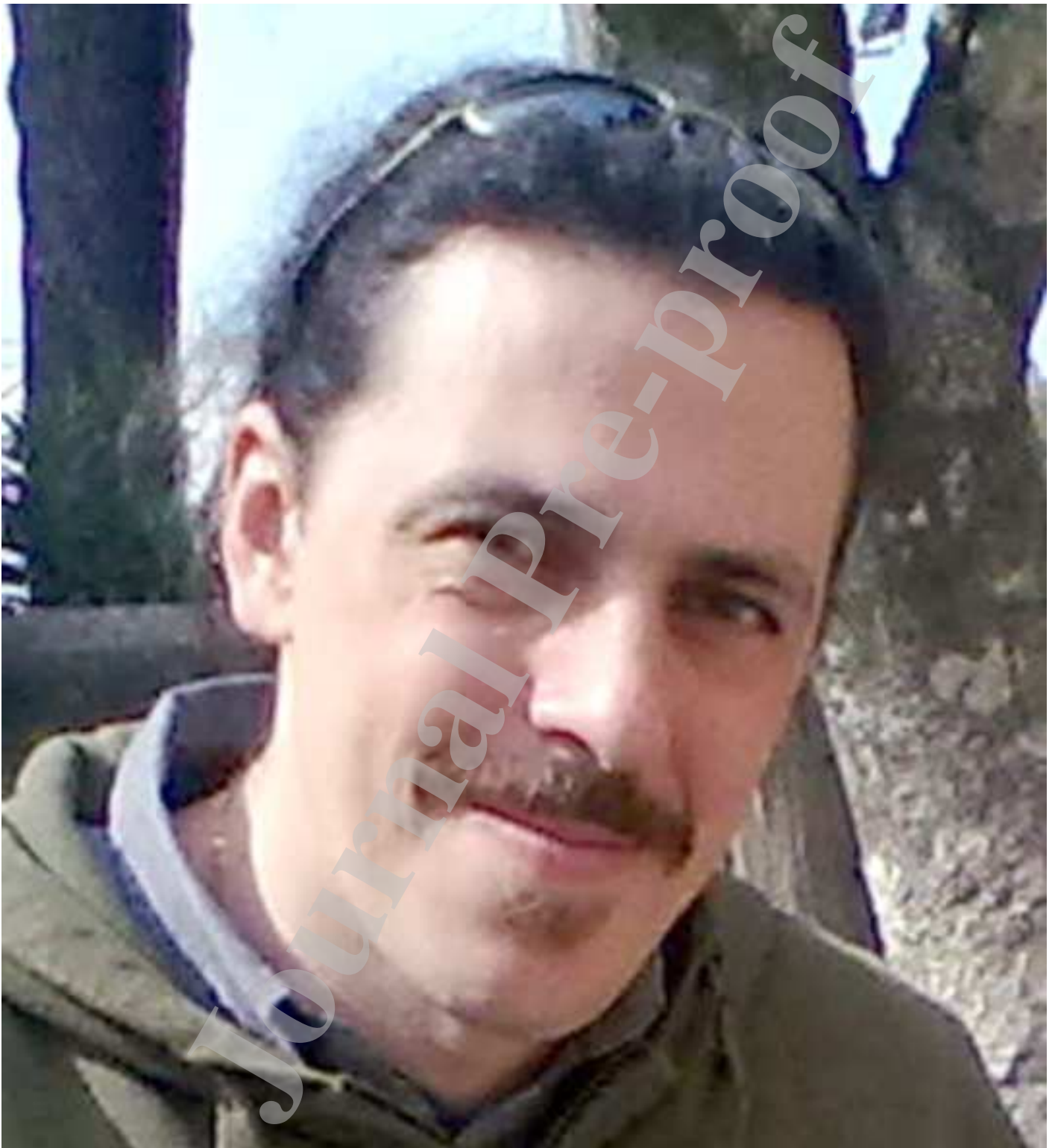














**Declaration of interests**

✘ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

✘ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

	Mattia Bersani
	Simone Mentasti
	Pragyan Dahal
	Stefano Arrigoni
	Michele Vignati
	Federico Cheli
	Matteo Matteucci

Journal Pre-proof