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Pulse Buckling of a Thin-Walled CFRP Cylindrical Shell – A Numerical Approach

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Abstract

The present study investigates the behaviour of a CFRP cylindrical shell under compressive pulse loading. The in-pulse analyses are performed numerically, using the finite element code ABAQUS. The dynamic buckling load is determined using the Budiansky & Hutchinson criterion. Parameters like the shape of pulse loading and pulse duration were varied, and their influence on the Dynamic Load Factor (DLF) was investigated. The investigation shows that DLF tends to increase well above unity for short duration impulses, while for the larger duration the value is decreasing towards unity. The shape of the pulse also has a significant influence on the DLF value. DLF<1 was found only for a trapezoidal pulse. For sinusoidal pulse shape, the static buckling load of the CFRP shell was consistently below the dynamic one.

1. Introduction

The topic of applying an axially time-dependent load onto a column, thus inducing lateral vibrations and eventually causing the buckling of the column, was studied for many years. Sometimes this is called vibration buckling, as proposed by Lindberg [1]. As described in his fundamental report [1], the axial oscillating load might lead to unacceptable large vibrations amplitudes at a critical combination of the frequency and amplitude of the axial load and the inherent damping of the column. This behaviour is presented in Fig. 1a, where an oscillating axial load induces bending moments that cause lateral vibrations of the column. As described in [1], the column will laterally vibrate at a large amplitude when the loading frequency will be twice the natural lateral bending frequency of the column. The term used by Lindberg, vibration buckling, presents some kind of similarity to vibration resonance. However, in the case of vibration resonance, the applied load is in the same direction as the motion, namely lateral to the column, and the resonance will occur when the loading frequency equals the natural frequency of the column. This type of vibration buckling was called by Lindberg as: *dynamic stability of vibrations induced by oscillating parametric loading*. This type of resonance is also named in the literature as *parametric resonance* (see [2] and [3]).

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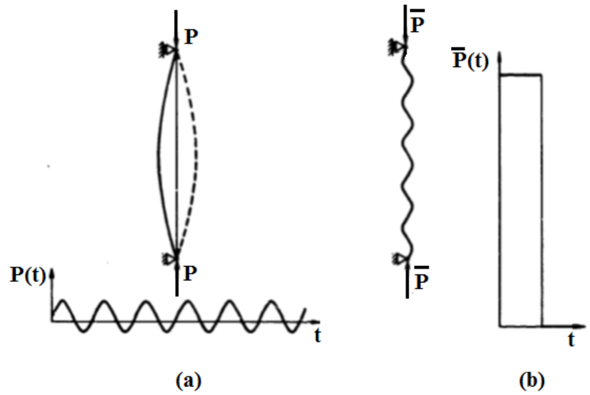

Figure 1: (a) Buckling under parametric resonance, (b) Pulse-type buckling [1].

Another type of vibration is sometimes also called pulse buckling. The structure will be deformed to unacceptably large amplitudes as a result of the transient response of the structure to the dynamic axially applied load [1]. One should note that the suddenly applied load might cause a permanent deformation due to the plastic response of the column, a snap to a larger post-buckling deformation or simply a return to its undeformed state. An example can be found in Fig. 1b, where the high-order buckling mode under short load duration is presented.

Loss of stability under pulse loading is associated with the rapid increase of structure deformations (e.g. Volmir or Budiansky-Hutchinson criteria) or achieving a given stress level (e.g. Petry-Fahlbusch criterion). It was observed that the structure could withstand a higher axial load before reaching the buckling condition, provided the load duration is short enough. Petry and Fahlbusch [2] observed an almost fourfold increase of dynamic buckling load when a very short load duration is analyzed. However, with the rise of load duration, it was observed that the structure is less resistant to pulse load than the static one [3].

The dynamic buckling of structures has been widely addressed in the literature. It started with the famous paper by Budiansky and Roth [5], through Hegglin's report on dynamic buckling of columns [6] and continued with Budiansky & Hutchinson [7] and Hutchinson & Budiansky [8] in the mid-sixties.

It is difficult to define a criterion of the critical load causing the structure to buckle under the subjected pulse loading. As presented by Kubiak [9] and also by Ari Gur [10], [11],[12] a new quantity is introduced called DLF (Dynamic Load Factor) to enable the use of the dynamic buckling criteria. It is defined as:

$$DLF \equiv \frac{Pulse \quad Buckling \quad Amplitude}{Static \quad Buckling \quad Amplitude} \equiv \frac{\left(P_{cr}\right)_{dyn.}}{\left(P_{cr}\right)_{static}} \tag{1}$$

According to Kubiak [9], the most popular criterion had been proposed by Volmir for plates subjected to in-plane pulse loading. As quoted in [9], Volmir proposed the following criterion:

"Dynamic critical load corresponds to the amplitude of pulse load (of constant duration) at which the maximum plate deflection is equal to some constant value k (k - half or one plate thickness").

Another very widely used criterion has been formulated and proposed by Budiansky &Hutchinson [5],[7],[8]. Originally, the criterion was formulated for shell-type structures but was also used for columns and plates. The criterion claims that:" *Dynamic stability loss occurs when the maximum deflection grows rapidly with the small variation of the load amplitude".*

2. FE model and methodology

The analyses were performed on a CFRP cylindrical shell with the laminate stacking sequence [$\pm 45^0/0^0/90^0$]s. The considered cylindrical shell has a radius R = 300 mm, the length L = 705 mm and the total thickness t = 1.448 mm (eight plies, each with the thickness of 0.181 mm). The mechanical properties of CFRP are listed in Table 1.

Table 1: Mechanical properties of AS4/8552 CFRP.

	E_1 (GPa)	E_2 (GPa)	ν_{12} (-)	G_{12} (GPa)	ρ (g/cm^3)
AS4/8552 CFRP	145	10.3	0.3	4.5	1.58

The numerical model of the cylindrical shell was created in ABAQUS 2017 using S4R shell elements. The finite element length is equal to 7.5 mm and was determined by the convergence analysis. The boundary conditions were applied to the nodes localized in both shell edges: in the bottom edge, all degrees of freedom are removed, while on the top one, only the axial displacement was possible. The loading was applied as a compressive force to one point localized on the upper edge and then transferred to all nodes at this edge. The assumed boundary conditions correspond to conditions assured during the laboratory test [12, 13].

In the investigation of the CFRP shell resistance to pulse loading, the following studies were performed:

a) *Static buckling analysis* – the analysis aims to determine the static buckling load and corresponding buckling shape. The study was performed numerically using the eigenvalue buckling analysis and dynamic explicit analysis. The numerical investigations were compared with the analytical calculations and with the results of the laboratory test [12, 13].
b) *Modal analysis* – modal analysis was carried out to define the natural frequency of the shell, and next - the natural bending period of the shell T_b. The outcomes of numerical calculations were confronted with laboratory test [12, 13].
c) *Dynamic buckling analysis* – pulse buckling analyses were performed numerically using the Explicit method. The shell is subjected to the pulse axially compressive loading, with the amplitude being a fraction or a multiplier of the static buckling load. The structure is loaded with various pulse shape (trapezoidal and sinusoidal), with the time being a fraction or a multiplier of the natural period of the shell. The numerical analyses were performed for the shell with initial geometric imperfections that corresponds to the lowest buckling mode.

3. Static buckling and modal analysis

The buckling load P_{cr} was determined analytically according to the following formula [15]:

$$P_{cr} = \frac{2\pi^3 R D_{11}}{L^2}\left[m^2\left(1+2\frac{D_{12}}{D_{11}}\beta^2+\frac{D_{22}}{D_{11}}\beta^4\right)+\frac{\gamma^2 L^4}{D_{11}\pi^4 m^2 R^4}\frac{A_{11}A_{22}-A_{12}^2}{A_{11}+\left(\frac{A_{11}A_{22}-A_{12}^2}{A_{66}}-2A_{12}\right)\beta^2+A_{22}\beta^4}\right] \tag{2}$$

where: *L, R* – length and the radius of the cylinder, respectively
n, m – number of half-waves in the circumferential and axial directions,
β – buckle aspect ratio ($\beta = \frac{nL}{\pi R m}$),
Aij, Dij – elements of the extensional stiffness matrix and bending stiffness matrix
γ – a correction factor, in the considered case γ=0.446

The static buckling load was also determined numerically and compared with the experimental data. In the numerical calculations, the eigenvalue analysis was performed. Next, the non-linear analysis for the model with initial geometric imperfections with the shape corresponding to the lowest buckling mode and the imperfections amplitude equal to 20% of the shell thickness (w_0=0.2t) was carried out.

The results comparison is presented in Fig. 1 and Table 2. In Fig. 1, the comparison of buckling modes with the equilibrium paths obtained from laboratory test and non-linear analysis with initial imperfection is presented. In the upper left corner, the lowest buckling mode obtained from LB (Linear Buckling) analysis is also depicted. Significantly high results agreement was obtained in the buckling load estimation. A slight difference in the structure's stiffness in the pre-buckling regime and the buckling modes was obtained. This discrepancy could result from different imperfections, which were not applied to the FE model as thickness imperfection.

The comparison of buckling load obtained from analytical and numerical approach present high correlation. Similarly, the high agreement of the results was obtained from laboratory test and from FE non-linear analysis, where the buckling load of the imperfect structure was studied. The difference between results obtained from the experiment and FE analysis is less than 1% which confirms the correctness of the adopted numerical model and its application to further dynamic buckling analyses.

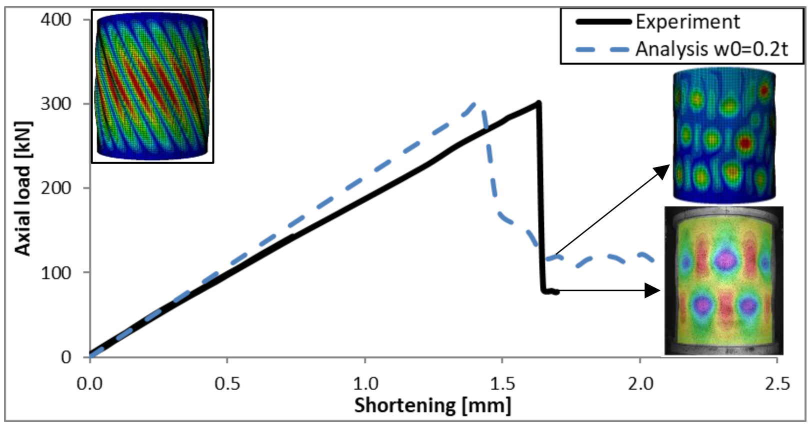

Figure 1: Comparison of equilibrium path obtained from experiment [16] and FE analysis.

Table 2: Static buckling load of the analyzed shell.

Method	Pcr [kN]
Analytical formula	426.2
FEM Linear buckling analysis	402.6
FEM Explicit analysis	299.8
Laboratory test	301.2

To determine the natural period of duration, a modal analysis was performed. The lowest natural frequency obtained from FE analysis f_{FE}=293.8Hz, while from laboratory test f_{EXP}=243Hz. The significantly high difference in natural frequency estimation is a consequence of the applied boundary conditions. The boundary conditions assured during the numerical analysis differs slightly from these assumed in laboratory test.

3. Dynamic buckling analysis

The dynamic buckling analysis was performed for the model with initial geometric imperfections w0=0.2t. From numerical calculations, the first natural bending period T_b = 3.40 ms was determined. The structure was subjected to pulse loading with six different load duration, being a fraction or a multiplier of T_b (T = 0.43ms, T = 0.85ms, T = 1.70ms, T = 3.40ms, T = 6.80ms and T/T = 17.00ms). Different pulse shapes: trapezoidal and sinusoidal, were considered. The trapezoidal pulse shape could be described by the following equation:

$$0 \leq t \leq 0.1T \quad P(t) = \frac{10P_0}{t}$$
$$0.1T \leq t \leq 0.9T \quad P(t) = P_0 \tag{3}$$
$$0.9T \leq t \leq T \quad P(t) = \frac{-10P_0}{t}$$

The sinusoidal pulse shape is defined as:

$$0.T \leq t \leq T \quad P(t) = P_0 \sin\left(\frac{\pi t}{T}\right) \tag{4}$$

To assess the shell resistance to pulse loading, the structure was observed in the time T~60T$_b$. Dynamic Load Factor was estimated by the application of the Budiansky & Hutchinson (B&H) criterion. The details of the B&H criterion application are presented in [17]. The high sensitivity to initial imperfections characterizes shell structures [18, 19]. Thus, DLF was calculated according to eq. 1, assuming that the static buckling load P_{cr} is the buckling load obtained from non-linear analysis for a model with initial geometric imperfection (P_{cr} =299.8kN).

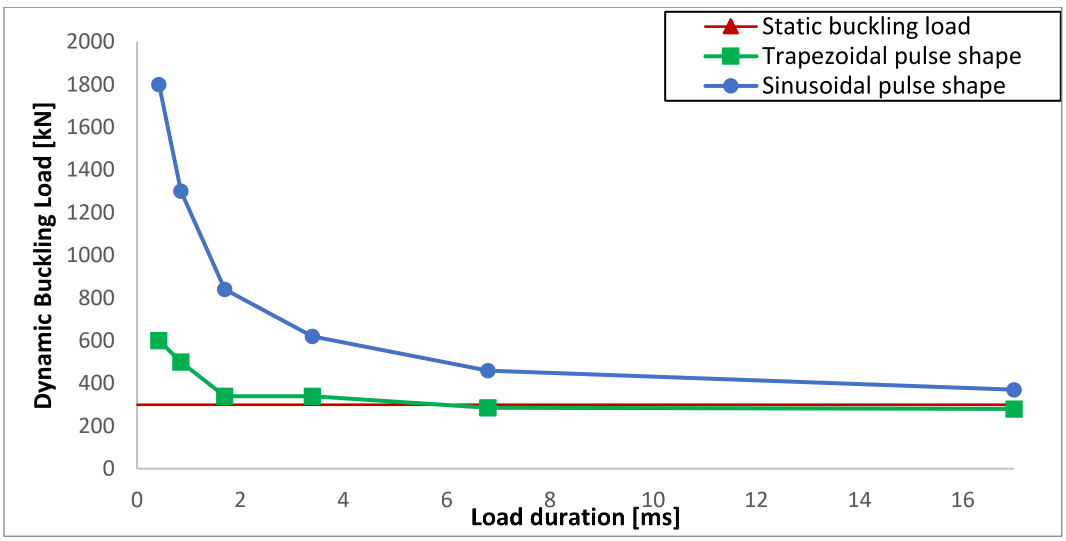

Figure 2: The effect of pulse duration and pulse shape on the Dynamic Buckling Load.

In Fig. 2, the change of Dynamic Buckling Load with the increase of load duration for the shell subjected to trapezoidal- and sinusoidal-shaped pulse load is presented. For load duration lower than the first natural bending period of the structure (T<3.40ms), for both considered pulse shapes, the shell is significantly more resistant to pulse load than the static one. In this regime, the resistance increases with the decrease of the load duration (up to P_{dyn}=1800kN for sinusoidal pulse load and P_{dyn}=600kN for trapezoidal pulse load; for load duration T=0.43ms). A different tendency is observed for T>3.40ms (T>T$_b$), where the Dynamic Buckling Load is almost unchanged for 3.40ms<T <17ms.For trapezoidal pulse shape, the dynamic buckling load is near the static buckling load in that load duration regime. Comparison of two pulse shapes reveals higher resistance to pulse loading for sinusoidal pulse shape than the trapezoidal one. The pulse shape has the most significant effect on the Dynamic Buckling Load for a short load duration (T=0.43ms). For that load duration, the Dynamic Buckling Load is three times higher for sinusoidal pulse load than trapezoidal load shape. With the increase of load duration, the influence of load shape on the buckling resistance decreases slightly. Nevertheless, the dynamic buckling load for sinusoidal pulse shape is at least thirty percent higher than the trapezoidal one. The lower dynamic buckling load obtained for the trapezoidal pulse shape could be explained by the high value of the pulse energy [19]. Pulse energy is described as the area under the curve representing the time dependence of load. The trapezoidal pulse shape is characterized with higher pulse energy, than the sinusoidal one. This tendency is reflected in Dynamic Pulse Load (P_{dyn}) and Dynamic Load Factor (DLF)(Table 3).

Table 3: Dynamic Buckling Load and Dynamic Load Factor for analyzed pulse shapes and load duration

		Load duration [ms]					
		0.43	0.85	1.70	3.40	6.80	17.00
Trapezoidal pulse shape	P_{dyn} [kN]	600	500	340	340	285	280
	DLF [-]	2.00	1.67	1.13	1.13	0.95	0.93
Sinusoidal pulse shape	P_{dyn} [kN]	1800	1300	840	620	460	370
	DLF [-]	6.01	4.34	2.80	2.07	1.54	1.23

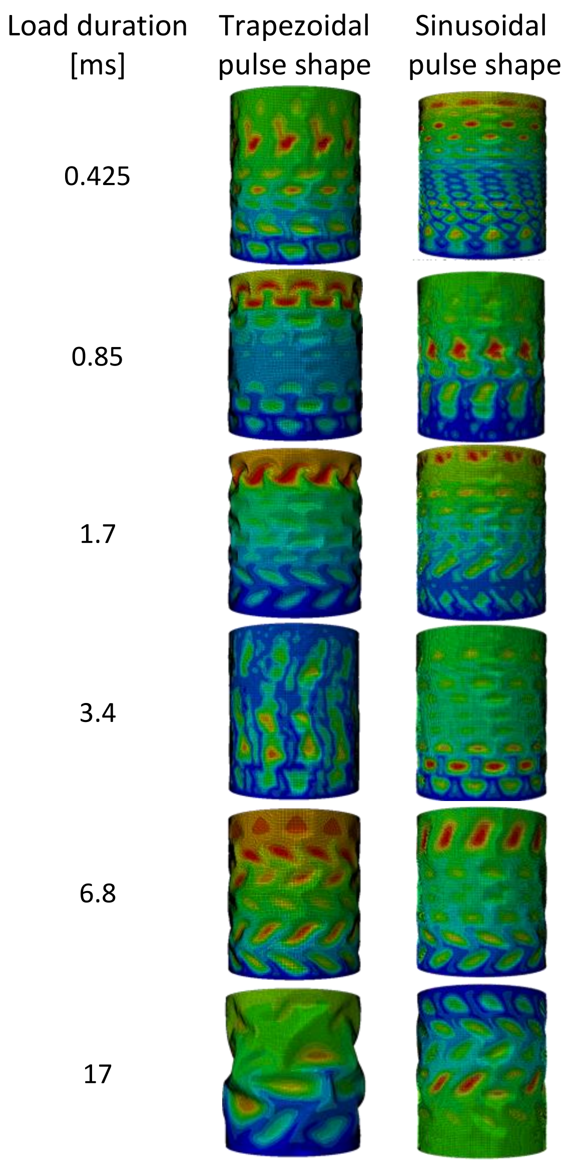

Figure 3: Buckling modes for trapezoidal pulse shape.

The effect of pulse duration and pulse shape on the buckling mode was also studied. The results are presented in Fig. 3. High sensitiveness of load duration on the buckling mode was obtained. A diamond shape is noticed for lower load duration, while the increase of the load duration leads to the appearance of the oblique waves.

4. Conclusions

The dynamic buckling behaviour of a composite cylindrical shell has been studied numerically using the ABAQUS code. The numerical model was validated experimentally on the shell subjected to static axial compressive loading. To assess the resistance of the structure to pulse loading, the Budiansky & Hutchinson criterion was applied. The effect of the load duration and pulse shape was investigated. The high impact of load duration on dynamic buckling resistance was observed. For load duration lower than the natural period of duration, the shell structure is few time more resistant to pulse loading compared to the static load. For duration longer than the natural bending period of the shell the Dynamic Buckling Load is almost unchanged with the increase of load duration. The high effect of the pulse shape was also analyzed. Two different shapes of pulse load were studied: sinusoidal and trapezoidal. Higher value of the Dynamic Load Factor was obtained for sinusoidal pulse shape than for trapezoidal shape.

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