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# Abductive Inference within a Pragmatic Framework

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**Abstract**

This paper presents an enrichment of the Gabbay–Woods schema of Peirce's 1903 logical form of abduction with illocutionary acts, drawing from logic for pragmatics and its resources to model justified assertions. It analyses the enriched schema and puts it into the perspective of Peirce's logic and philosophy.

**Keywords:** Abduction, Peirce, Gabbay–Woods Schema, Logic for Pragmatics, Illocutionary Acts.

## 1 Introduction

Peirce once remarked to Royce that "the art of making explanatory hypotheses is the supreme branch of logic" (Charles Peirce to Josiah Royce, 30 June 1913). This statement conceals an important and hitherto understudied fact. In *abduction*—a congenial part of logic in the wider sense—this scientific art concerns not only being able to conclude certain conjectures, but also being able to *make* them. That is, the art of abduction concerns scientists' *becoming justified in asserting* those hypotheses as plausible scientific conjectures.

Peirce presented his famous and broadly logical form of abduction in his 1903 *Harvard Lectures on Pragmatism* (Peirce 1997). The present paper gives an enrichment of what has become the standard elaboration of Peirce's 1903 form, namely the Gabbay–Woods (G–W) ignorance schema, by taking into account this illocutionary dimension of abduction-making. While the G–W schema has presented us what may be to date the best, or at least the most widely acknowledged[1] explication of Peirce's 1903 logical form, we believe that there is room for its improvement precisely in the direction that Peirce hinted at in his letter to Royce.

We begin this task by first presenting a logic of assertions that can handle the pragmatic senses of logical connectives. We then extend this *logic of pragmatics* to the *logic of hypotheses*. At this point, we will be ready to also present the G–W schema of abduction, to explain its main features, and to proceed

[1]We can safely ignore the IBE-accounts, as they are not what Peirce means by abduction (Campos 2011; Maucliffe 2015).

presenting our strategy of how to enrich it with illocutionary forces. The final section presents this enriched schema, analyses all of its elements, and puts the new schema into the perspective of Peirce's logical philosophy and his theory of assertions.

## 2 Pragmatic Logic for Assertions

In this section a pragmatic logic for assertion (LP) (Dalla Pozza 1991; Dalla Pozza & Garola 1995) is introduced.[2] In LP it is possible to distinguish the propositional content of assertions expressed by radical formulæ and sentential formulæ expressed by asserted propositions such as $\vdash \gamma$, where "$\vdash$" is the sign for assertion. The former radical formulæ are propositions, while the latter are elementary sentential formulæ which result by the application of the Fregean assertion sign prefixed to radical formulæ. Two restrictions are that (i) there can be no nested occurrences of the assertion sign and that (ii) truth-functional connectives cannot be applied to formulæ expressing judgments of assertions. In order to formulate complex sentential formulæ expressing assertions, we introduce pragmatic connectives which are not truth-conditional. Such pragmatic connectives have intuitionistic-like behaviour.

The pragmatic language LP = RAD ∪ SENT, the union of the set of radical formulæ and the set of sentential formulæ:

RAD: $\gamma ::= | \, p \, | \neg\gamma \, | \, \gamma_1 \wedge \gamma_2 \, | \, \gamma_1 \vee \gamma_2 \, | \, \gamma_1 \rightarrow \gamma_2 \, | \, \gamma_1 \leftrightarrow \gamma_2 \, |$

SENT:
(i) Elementary sentential formulæ $\theta ::= \vdash \gamma$
(ii) Sentential formulæ $\delta ::= | \, \theta \, | {\sim}\delta \, | \, \delta_1 \cap \delta_2 | \delta_1 \cup \delta_2 \, | \, \delta_1 \supset \delta_2 \, | \, \delta_1 \equiv \delta_2 \, |$.

The pragmatic system LP is composed of two categories of logico-pragmatic signs:

1. the signs of pragmatic illocutionary force ("$\vdash$", for assertion);
2. the pragmatic connectives: pragmatic negation $\sim$, pragmatic conjunction $\cap$, pragmatic disjunction $\cup$, pragmatic implication $\supset$ and pragmatic equivalence $\equiv$.

All radical formulæ of LP have a *truth-value* $\in \{\textbf{true}, \textbf{false}\}$. Sentential formulæ have a *justification value* $\in \{J, U\}$ ("$J$" for **justified**, "$U$" for **unjustified**), defined in terms of the notion of *proof* (or construction, verification, transformation). These values depend on the truth-value of the radical sub-formulæ of sentential formulæ.

The semantics of the radical formulæ of LP is classical and it provides only the interpretation of the radical formulæ, assigning them a truth-value and

[2] Applications of LP to philosophical questions are provided in Carrara, Chiffi & De Florio 2017; Carrara & Chiffi 2014; Carrara, Chiffi & Sergio 2014, 2017). See Bellucci & Pietarinen 2017 on Peirce on the logic of assertions.

interpreting propositional connectives as truth-functions in the usual way. The semantic rules for radical formulæ are the usual Tarskian ones and they specify the truth-conditions (but only for the radical formulæ) through an assignment function $\sigma$, thus regulating the semantic interpretation of $\mathsf{LP}$. Let $\gamma_1, \gamma_2$ be radical formulæ and let $1 = \textbf{true}$ and $0 = \textbf{false}$. Then:

1. $\sigma(\neg\gamma_1) = 1$ iff $\sigma(\gamma_1) = 0$
2. $\sigma(\gamma_1 \wedge \gamma_2) = 1$ iff $\sigma(\gamma_1) = 1$ and $\sigma(\gamma_2) = 1$
3. $\sigma(\gamma_1 \vee \gamma_2) = 1$ iff $\sigma(\gamma_1) = 1$ or $\sigma(\gamma_2) = 1$
4. $\sigma(\gamma_1 \rightarrow \gamma_2) = 1$ iff $\sigma(\gamma_1) = 0$ or $\sigma(\gamma_2) = 1$
5. $\sigma(\gamma_1 \leftrightarrow \gamma_2) = 1$ iff $\sigma(\gamma_1) = \sigma(\gamma_2)$.

Pragmatic connectives have a meaning that dates from the BHK (Brouwer–Heyting–Kolmogorov) interpretation of intuitionistic logical constants. In the present paper, the illocutionary force of assertion is taken to play a key role in determining the *pragmatic* component of the meaning of an elementary expression, together with the *semantic* component expressed in the radical formulæ.

Justification rules regulate the pragmatic evaluation $\pi$, and they specify the justification conditions for the assertive formulæ as a function of the $\sigma$-assignments of the truth-values for their radical sub-formulæ:

**JR1** – Let $\gamma$ be a radical formula. Then:

(i) $\pi(\vdash \gamma) = J$ iff a proof exists that $\gamma$ is true, i.e. that $\sigma$ assigns to $\gamma$ the value 1.

(ii) $\pi(\vdash \gamma) = U$ iff no proof exists that $\gamma$ is true.

**JR2** – Let $\delta$ be an assertive formula. Then:

(i) $\pi(\sim\delta) = J$ iff a proof exists that $\delta$ is unjustified, i.e. that $\pi(\delta) = U$.

**JR3** – Let $\delta_1$ and $\delta_2$ be assertive formulæ. Then:

(i) $\pi(\delta_1 \cap \delta_2) = J$ iff $\pi(\delta_1) = J$ and $\pi(\delta_2) = J$;

(ii) $\pi(\delta_1 \cup \delta_2) = J$ iff $\pi(\delta_1) = J$ or $\pi(\delta_2) = J$;

(iii) $\pi(\delta_1 \supset \delta_2) = J$ iff a proof exists that $\pi(\delta_2) = J$ whenever $\pi(\delta_1) = J$;

(iv) $\pi(\delta_1 \equiv \delta_2) = J$ iff $\pi(\delta_1 \supset \delta_2) = J$ and $\pi(\delta_2 \supset \delta_1) = J$.

The *Soundness Criterion* (SC) is the following:

Let $\gamma \in \mathsf{RAD}$. Then $\pi(\vdash \gamma) = J$ implies that $\sigma(\gamma) = 1$.

SC states that if an assertion is justified, then the content of the assertion is true. We can now define

Pragmatic Validity for Assertive Formulæ: A formula $\delta$ is *pragmatically valid* if and only if for every Tarskian semantic interpretation $\sigma$ and for every pragmatic justification function $\pi$, $\pi(\delta) = J$.

An intuitionistic fragment is obtained by limiting the language of LP to complex formulæ that are pragmatically valid with respect to atomic radicals.

Call the intuitionistic fragment ILP. The classical fragment corresponds to the fragment of sentential formulæ without pragmatic connectives (Dalla Pozza 1991). It is worth noting that the justification rules give a *partial interpretation* since they do not determine in every case what the justification value of a complex sentential formula is when all justification values of its components are known. In particular:

**NR1:** $\pi(\delta) = J$ implies $\pi(\sim\delta) = U$,

**NR2:** $\pi(\delta) = U$ does not imply $\pi(\sim\delta) = J$,

**NR3:** $\pi(\sim\delta) = J$ implies $\pi(\delta) = U$,

**NR4:** $\pi(\sim\delta) = U$ does not imply $\pi(\delta) = J$.

For example, **NR2** states that being unjustified does not imply that the pragmatic negation of an unjustified formula is justified.

A modal translation of LP can be given by a function $(\ )^*$ from assertive formulæ to the corresponding modal ones in the system S4. Here $\Box\gamma$ could be taken to mean that there is a *proof* or *conclusive evidence* for $\gamma$:

$$(\vdash \gamma)^* = \Box\gamma$$

$$(\sim \delta)^* = \Box\neg(\delta)^*$$

$$(\delta_1 \cap \delta_2)^* = (\delta_1)^* \wedge (\delta_2)^*$$

$$(\delta_1 \cup \delta_2)^* = (\delta_1)^* \vee (\delta_2)^*$$

$$(\delta_1 \supset \delta_2)^* = \Box((\delta_1)^* \rightarrow (\delta_2)^*)$$

$$(\delta_1 \equiv \delta_2)^* = \Box((\delta_1)^* \leftrightarrow (\delta_2)^*)$$

Connectives for radical and sentential formulæ are related by the following five bridge principles:

(i) $(\vdash \neg\gamma) \supset (\sim\vdash \gamma)$

(ii) $((\vdash \gamma_1) \cap (\vdash \gamma_2)) \equiv (\vdash (\gamma_1 \wedge \gamma_2))$

(iii) $((\vdash \gamma_1) \cup (\vdash \gamma_2)) \supset (\vdash (\gamma_1 \vee \gamma_2))$

(iv) $(\vdash (\gamma_1 \rightarrow \gamma_2)) \supset (\vdash \gamma_1 \supset \vdash \gamma_2)$

(v) $(\vdash (\gamma_1 \leftrightarrow \gamma_2)) \supset (\vdash \gamma_1 \equiv \vdash \gamma_2)$.

The bridge principles (i)–(v) show the formal relations between pragmatic connectives and connectives in the radicals. The principle (i) states that from the assertion of a negation of $\gamma$ the non-assertibility of $\gamma$ can be inferred.[3] The principle (ii) expresses that the conjunction of two assertions is equivalent to the assertion of a conjunction; (iii) in turn states that from the disjunction of two assertions the assertion of a disjunction is inferrable. The principle (iv) expresses the fact that from the assertion of material implication the pragmatic implication between two assertions follows pragmatically. The principle (v) shows that the assertion of a biconditional pragmatically entails the equivalence of assertions.

In the next section, we will explore the logical and pragmatic framework for hypotheses.

# 3 Pragmatic Logic for Hypotheses

We consider hypotheses as having primitive illocutionary force, indicated by $\mathcal{H}$, and which is justified by means of a *scintilla of evidence* [4] (Carrara, Chiffi & De Florio 2017; Chiffi & Schang 2017). What counts as evidence is left here unspecified and it should be contextually decided. The language of hypothetical logic for pragmatics ($\mathsf{HLP}$) is the union of $\mathsf{RAD}$ and the set of hypothetical formulæ $\mathcal{HF}$.[5]

$\mathsf{RAD}$ $\quad \gamma ::= \mid p \mid \neg\gamma \mid \gamma_1 \wedge \gamma_2 \mid \gamma_1 \vee \gamma_2 \mid \gamma_1 \to \gamma_2 \mid \gamma_1 \leftrightarrow \gamma_2 \mid$

$\mathcal{HF}$ (i) Elementary hypothetical formulæ: $\eta ::= \mathcal{H}\gamma$

(ii) Hypothetical formulæ:

$\kappa ::= \mid \eta \mid \frown\kappa \mid \kappa_1 \sqcap \kappa_2 \mid \kappa_1 \sqcup \kappa_2 \mid \kappa_1 \sqsupset \kappa_2 \mid \kappa_1 \sqsubset\sqsupset \kappa_2 \mid$

Analogously to the case of assertions, there are ordinary connectives in the radical formulæ which express their propositional content. Moreover, there are hypothetical connectives for hypothetical formulæ which have a formal behaviour in accordance with the following justification rules. Notice that $\varepsilon$ is a function of *evidence* from hypothetical formulæ to justification values.[6]

**HJR1:** Let $\gamma$ be a radical formula.

(i) $\varepsilon(\mathcal{H}\gamma) = J$ if and only if there is a *scintilla of evidence* that $\gamma$ is true.

[3] The contrary does not hold: "The absence of assertibility is not assertibility of absence".

[4] A variety of standards of evidence is used, for instance, in legal argumentation, namely: *scintilla of evidence*, *preponderance of evidence*, *clear and convincing evidence*, *beyond reasonable doubt.* See (Gordon and Walton 2009). *Scintilla of evidence* means any form of weak and indirect evidence.

[5] Further extensions of $\mathsf{LP}$ are given in Bellin (2014); Bellin, Carrara & Chiffi (2014; 2015).

[6] Considerations of different weights of evidence to refute or cause doubt on different kinds of hypotheses are not taken into account.

(ii) $\varepsilon(\mathcal{H}\gamma) = U$ if and only if no *scintilla of evidence* exists that $\gamma$ is true.

**HJR2:** Let $\kappa$ be a hypothetical formula. Then:

(i) $\varepsilon(\frown\kappa) = J$ if and only if the evidence that $\varepsilon(\kappa) = J$ is smaller than the evidence justifying the opposite hypothesis.[7]

**HJR3:** Let $\kappa_1$ and $\kappa_2$ be hypothetical formulæ. Then:

(i) $\varepsilon(\kappa_1 \sqcap \kappa_2) = J$ iff $\varepsilon(\kappa_1) = J$ and $\varepsilon(\kappa_2) = J$;[8]

(ii) $\varepsilon(\kappa_1 \sqcup \kappa_2) = J$ iff $\varepsilon(\kappa_1) = J$ or $\varepsilon(\kappa_2) = J$;

(iii) $\varepsilon(\kappa_1 \sqsupset \kappa_2) = J$ iff there is evidence that $\kappa_2$ is justified whenever there is evidence that $\kappa_1$ is justified.

(iv) $\varepsilon(\delta_1 \sqsubset\sqsupset \delta_2) = J$ iff $\varepsilon(k_1 \sqsupset k_2) = J$ and $\varepsilon(k_2 \sqsupset k_1) = J$.

The soundness criterion (SC-H) for hypotheses is the following:

> Let be $\gamma \in \mathsf{RAD}$. Then $\varepsilon(\mathcal{H}\gamma) = J$ implies that there is a *scintilla of evidence* that $\sigma(\gamma) = 1$.

Let us consider now some principles regarding the negation of hypotheses:

**HNR1:** $\varepsilon(\kappa) = J$ does not imply that $\varepsilon(\frown\kappa) = U$,[9]

**HNR2:** $\varepsilon(\kappa) = U$ implies that $\varepsilon(\frown\kappa) = J$,

**HNR3:** $\varepsilon(\frown\kappa) = J$ does not imply that $\varepsilon(\kappa) = U$,

**HNR4:** $\varepsilon(\frown\kappa) = U$ implies that $\varepsilon(\kappa) = J$.

We propose a neutral, 'fuzzy' approach towards the content of hypotheses. This is in order to handle certain insights concerning them, especially *fundamental uncertainty*, which goes beyond standard probabilistic frameworks.[10] Radical formulæ can be interpreted in a fuzzy logic, whose truth-values indicated by $|\,|$

[7] That is, by this comparative we mean the justifiability iff we are more justified in doubting $k$ rather than in believing it, after a duly consideration of total evidence. A closely related notion would be to think of the justifiability of the former as less compelling than the justifiability of the latter.

[8] We assume that $\kappa_1$ and $\kappa_2$ are independent, uncorrelated formulæ. Otherwise adjoining them might yield misleading predictions.

[9] The idea here is that the agent keeps an open mind, as evidence may easily get reinterpreted.

[10] For instance, we may want to be able to reason about plausibility of hypotheses. Unlike probability, plausibility measures are not required to be additive, among other things.

range in a closed interval $[0, 1]$, in the following way:

$$
\begin{aligned}
|\neg\gamma| &= 1 - |\gamma| \\
|\gamma_1 \vee \gamma_2| &= \max(|\gamma_1|, |\gamma_2|) \\
|\gamma_1 \wedge \gamma_2| &= \min(|\gamma_1|, |\gamma_2|) \\
|\gamma_1 \rightarrow \gamma_2| &= 1, \quad \text{if } |\gamma_1| \leq |\gamma_2| \\
|\gamma_1 \rightarrow \gamma_2| &= 1 - (|\gamma_1| - |\gamma_2|), \quad \text{if } |\gamma_1| > |\gamma_2| \\
|\gamma_1 \leftrightarrow \gamma_2| &= 1, \quad \text{if } |\gamma_1| = |\gamma_2| \\
|\gamma_1 \leftrightarrow \gamma_2| &= 1 - (|\gamma_1| - |\gamma_2|), \quad \text{if } |\gamma_1| \neq |\gamma_2|.
\end{aligned}
$$

The following fuzzy interpretation[11] shows how hypothetical formulæ can be interpreted in our framework. These fuzzy values can for instance be understood as *plausibility values* for hypothesis.

| | |
|---|---|
| $\mathcal{H}\gamma_1 = J$ | $\lvert\gamma_1\rvert \neq 0$ |
| $\mathcal{H}\gamma_1 = U$ | $\lvert\gamma_1\rvert = 0$ |
| $\frown\mathcal{H}\gamma_1 = J$ | $1 - \lvert\gamma_1\rvert > \lvert\gamma_1\rvert$ |
| $\frown\mathcal{H}\gamma_1 = U$ | $\lvert\gamma_1\rvert \geq 1 - \lvert\gamma_1\rvert$ |
| $(\mathcal{H}\gamma_1 \sqsupset \mathcal{H}\gamma_2) = J$ | $\lvert\gamma_1\rvert \leq \lvert\gamma_2\rvert$ |
| $(\mathcal{H}\gamma_1 \sqsupset \mathcal{H}\gamma_2) = U$ | $\lvert\gamma_1\rvert > \lvert\gamma_2\rvert$ |
| $(\mathcal{H}\gamma_1 \sqcap \mathcal{H}\gamma_2) = J$ | $\lvert\gamma_1\rvert \neq 0$ and $\lvert\gamma_2\rvert \neq 0$ |
| $(\mathcal{H}\gamma_1 \sqcap \mathcal{H}\gamma_2) = U$ | $\lvert\gamma_1\rvert = 0$ or $\lvert\gamma_2\rvert = 0$ |
| $(\mathcal{H}\gamma_1 \sqcup \mathcal{H}\gamma_2) = J$ | $\lvert\gamma_1\rvert \neq 0$ or $\lvert\gamma_2\rvert \neq 0$ |
| $(\mathcal{H}\gamma_1 \sqcup \mathcal{H}\gamma_2) = U$ | $\lvert\gamma_1\rvert = 0$ and $\lvert\gamma_2\rvert = 0$ |
| $(\mathcal{H}\gamma_1 \sqsubset\sqsupset \mathcal{H}\gamma_2) = J$ | $\lvert\gamma_1\rvert = \lvert\gamma_2\rvert$ |
| $(\mathcal{H}\gamma_1 \sqsubset\sqsupset \mathcal{H}\gamma_2) = U$ | $\lvert\gamma_1\rvert \neq \lvert\gamma_2\rvert$ |

We can now define the PRAGMATIC VALIDITY FOR HYPOTHETICAL FORMULÆ:

> A formula $k$ is *pragmatically valid* if and only if for every $|\ |$ and $\varepsilon$, $k$ is justified.

Analogously to the case of assertions, the modal translation in **S4** of hypothetical formulæ is the following. Let $(\ )^{**}$ be a function from hypothetical formulæ to the corresponding modal ones:[12]

$$
\begin{aligned}
(\mathcal{H}\gamma)^{**} &= \Diamond\gamma \\
(\frown\kappa)^{**} &= \Diamond\neg(\kappa)^{**} \\
(\kappa_1 \sqcap \kappa_2)^{**} &= (\kappa_1)^{**} \wedge (\kappa_2)^{**} \\
(\kappa_1 \sqcup \kappa_2)^{**} &= (\kappa_1)^{**} \vee (\kappa_2)^{**} \\
(\kappa_1 \sqsupset \kappa_2)^{**} &= ((\kappa_1)^{**} \rightarrow (\kappa_2)^{**})
\end{aligned}
$$

[11] When unnecessary, the indication of the justification function will be omitted.

[12] Similar modal translations have been recently analyzed by Shramko (2005, 2016).

$(\kappa_1 \sqsubset\sqsupset \kappa_2)^{**} = ((\kappa_1)^{**} \leftrightarrow (\kappa_2)^{**})$

The bridge principles between connectives for radical and hypothetical formulæ are the following five:

(a) $(\frown \mathcal{H}\gamma) \sqsupset (\mathcal{H}\neg\gamma)$

(b) $\mathcal{H}(\gamma_1 \wedge \gamma_2) \sqsupset (\mathcal{H}(\gamma_1) \sqcap \mathcal{H}(\gamma_2))$

(c) $\mathcal{H}(\gamma_1 \vee \gamma_2) \sqsupset (\mathcal{H}(\gamma_1) \sqcup \mathcal{H}(\gamma_2))$

(d) $(\mathcal{H}\gamma_1 \sqsupset \mathcal{H}\gamma_2) \sqsupset \mathcal{H}(\gamma_1 \rightarrow \gamma_2)$

(e) $(\mathcal{H}\gamma_1 \sqsubset\sqsupset \mathcal{H}\gamma_2) \sqsupset \mathcal{H}(\gamma_1 \leftrightarrow \gamma_2)$

Finally, the following general principles connect assertions and hypotheses:

**(GP1a):** $\vdash \neg\gamma = J$ iff $\mathcal{H}\gamma = U$

**(GP1b):** $\vdash \neg\gamma = U$ iff $\mathcal{H}\gamma = J$

**(GP2):** From the justification of $\vdash \gamma$ follows the justification of $\mathcal{H}\gamma$.

**(GP1a)** states that a propositional content cannot be part of a justified hypothesis when the assertion of its negation is justified. **(GP1b)** is its converse. **(GP2)** indicates that the ground justifying an assertion of $\gamma$ is sufficient to justify the hypothesis of $\gamma$. In fuzzy terms, this means that if $|\gamma| = 1$, then it is certainly different from 0.

# 4 Abductive Inference

In our logical framework it now becomes possible to provide a formal pragmatic treatment not only of the plausibility of hypotheses but also their assertibility. Indeed the combination of the pragmatic and illocutionary analysis with the abductive inference schemas strikes us as a promising and under-exploited strategy to be further pursued. According to the well-known definition of abduction that dates back to Peirce's 1903 Harvard Lectures (Peirce 1997; CP 5.189), the general structure of abductive inference can be schematicized as follows:

1. The surprising fact, C, is observed.
2. But if A were true, C would be a matter of course.
3. Hence, there is reason to suspect that A is true.

This schema does not represent the only, and not even the ultimate, attempt by Peirce to propose a logical form for abduction. It nevertheless has become the standard springboard for many discussions, including the Gabbay–Woods (G–W) interpretation of abductive reasoning. Since the aim of the present paper is to provide an improvement and an enrichment of that interpretation by the injection of the pragmatic and illocutionary analysis into the G–W interpretation, we will stick to what our analysis of that 1903 formulation would look like.[13]

The first premise tells that what is involved in abduction is the idea of the surprise with an observational fact. The second premise formulates the relation between a hypothesis A and a fact C, as expressed by means of a subjunctive conditional. The third clause, which is the conclusion, states that in virtue of these two premises, a certain plausibility concerning A is taken to obtain.

In his later writings, Peirce emphasized the interrogative aspect of abduction and clarified that it is really its conjectural nature that the logical schemas of abduction are supposed to capture. He pointed out that there is a specific mood (called the "co-hortative mood" in Ma & Pietarinen 2017, and "investigand" by Peirce 1905) by which the abductive conclusion is expressed, such as making it worthy of further investigation, given the situation that obtains at the moment of deriving it.

The G–W seminal view on abductive inference[14] assumes that abduction is a response to an ignorance problem. Abductive inference is treated as an *ignorance preserving* or *ignorance mitigation* process. The insight is that an abductive piece of inference can yield a hypothesis to be conjectured (or presumptively asserted), but it cannot be yet to be stated to be known in any sensible meaning of that term, fallible or infallible alike. Given this ignorance we moreover are not entitled to any knowledge whether the hypothesis will remain a conjecture or whether it will become converted into a confident assertion and thus something beyond conjectures and conjecturability, namely something that could possibly be known.

Numerous other noteworthy accounts and variations on the theme of abduction exist, such as those in which abductive processes are in fact viewed as knowledge-enhancing contributions (see e.g. Magnani 2017). Nevertheless, there seems to exist a relative consensus on the felicity and merits of the ignorance-preserving or mitigative character of those epistemic situations that the general schema is intended to bring out.

The ignorance-preserving character of abduction is well represented by the G–W schema of abduction (Gabbay & Woods 2005, 2006; Woods 2013). We will present its main characteristics here next. Let us assume that $\alpha$ is a proposition towards which an agent stands in an ignorance relation. This is the ignorance-related problem. $T$ is taken to represent the epistemic goal of verifying the truth of $\alpha$, and $R$ is the attainment relation for $T$. $K$ indicates the knowledge base that the epistemic agent has and $K^*$ is an immediate knowledge step that the agent can reconstruct in a timely way, for instance by making a quick online search or drawing a look-up on a dictionary, such as to know how a word

[13] See Ma & Pietarinen (2016, 2017) for a logical elaboration of Peirce's later, interrogative schemas of abduction, and Pietarinen & Bellucci (2014) on retroduction.

[14] The G–W schema is intended to replace what may be called the *standard schema for abduction*, since the previous standard (or obsolete) schemas were not able to handle the requisite features of Peirce's abductive reasoning, such as subjunctive conditions. On the details of this line of criticism, see Woods (2012; 2013, pp. 366–377) and Pietarinen (2014).

translates into another language. The wiggly arrow $\rightsquigarrow$ stands for a subjunctive conditional, $H$ is the hypothesis that the agent holds, $K(H)$ is the revision of $H$ in virtue of $K$, $C(H)$ stands for the agent's justified conjecture of the hypothesis,[15] and $H^C$ is the discharge of $H$. It is said that $H$ is discharged when "it is forwarded *assertively*" (Gabbay & Woods 2005, p. 47).[16]

In sum, the G–W schema is customarily formulated as involving the following steps:[17]

| | | |
|---|---|---|
| (1) | $T!\alpha$ | epistemic target $T$ for $\alpha$ |
| (2) | $\neg(R(K, T))$ | fact |
| (3) | $\neg(R(K*, T))$ | fact |
| (4) | $H \notin K$ | fact |
| (5) | $H \notin K*$ | fact |
| (6) | $\neg(R(H, T))$ | fact |
| (7) | $\neg(R(K(H), T))$ | fact |
| (8) | $H \rightsquigarrow (R(K(H), T))$ | fact |
| (9) | $H$ meets further conditions $S_1, S_2, \ldots, S_n$ | fact |
| (10) | Therefore, $C(H)$ | sub-conclusion, (1–7) |
| (11) | Therefore, $H^C$ | conclusion, (1–8). |

In words: Step 1 fixes the epistemic goal. Step 2 states the fact[18] that agent's knowledge base is insufficient to attain the epistemic goal. Step 3 affirms that the same holds true for agent's successive and extended knowledge bases. Step 4 postulates that $H$ is not part of agent's knowledge base and its successive extensions, as expressed by Step 5. Moreover, Step 6 states that $H$ does not attain its epistemic goal, and the same holds for the revisions of $H$ in the light of $K$ and as expressed by Step 7. What these steps add up to at this point is the important observation that the ignorance towards $\alpha$ is not a contingent feat, but goes deeper than that. Step 8 then points out, subjunctively, that if $H$ *were* true, then the epistemic goal *would be* attained when $K$ is revised upon the addition of $H$. Formula in Step 9 brings in further conditions that are required to increase the plausibility of the hypothesis $H$. They are elaborated below. The statement in Step 10 affirms that $H$ can be conjectured based on the previous conditions in (1)–(7), whereas such conjectures can be asserted and activated to have pragmatic consequences if also the subjunctive clause in Step 8 is taken into account.

It is quite evident that the crux of the matter is represented by the conditions in Steps 8 and 9 and the central notions of hypothesis, conjecture and

[15] "$C(H)$ is read 'it is justified (or reasonable to) conjecture that $H$' " (Gabbay and Woods 2005, p. 47).

[16] Here we can notice the germs of illocutionary force that are present in the original formulation of the G–W schema. But they have not been made explicit, let alone incorporated into the logical schema.

[17] There are some variations and renditions of the standard schematism. For example, for an interpretation of the G–W schema in dialogical logic see (Barés Gómez & Fontaine 2017).

[18] The word "fact" occurs in the G-W schema and it is intended to indicate (in a general sense) those conditions that hypotheses should meet in reality.

assertion involved in the G–W schematism.[19] Statement in Step 8 expresses the subjunctive conditional, while Step 9 indicates that *certain other conditions* are required in the abductive inference in order for an agent or a group of agents to conclude with an assertion.

Since these other conditions have not been explicitly stated in the original schema, and since the elements exhibited in Step 9 of the story have not received much attention in the literature elsewhere either, we will next provide a brief explanation of where and how in Peirce's logic and epistemology we will find the proper conditions to guide agents in their ignorance-mitigating endeavours to stand for $S_1, S_2, \ldots, S_n$.

Our explanation derives from the insight that these further conditions are *scientific values*, and that they are for that reason congenial to abduction. Such values can be viewed as regulative methodological principles for scientific inquiry. They are thus to be incorporated in its logical schematism to represent the conditions $S_1, S_2, \ldots, S_n$. There are essentially three such important values in Peirce's logical philosophy that cannot be avoided in abduction and scientific reasoning:

**Tychism:** Tychism is a value that states that *nature pursues randomness.* Nature is, in Peirce's words, "constantly receiving excessively minute accessions of variety" (R 292a). Laws, including laws of nature and laws of logic, are results of evolutionary processes. Tychism implies that there are *real possibilities*, which are governed by tendencies that are not perfectly actualized. Universe is not uniform at bottom; it is the uniformity and laws that call for explanation. But chance does not call for explanation, it is the explanation.

So it appears that tychism pertains to those classes of future conditions that abductive reasoning aspires to exploit.

**Synechism:** Synechism is a value (rather than a metaphysical principle), which insists on the "idea of continuity as being of prime importance in phi-

[19]Since in Step 8 the conditional is in a subjunctive mood it may strike one as an epistemic variant of doxastic adherence conditions, such as "If $p$ were true, the subject would believe that $p$", as used by Nozick (1981) in his tracking theory of knowledge, for example. In Nozick's analysis of knowledge, there is also the *sensitivity condition*, which states that "If $p$ weren't true, the subject wouldn't believe that $p$. But as soon as we consider an epistemic variant of the sensitivity condition in the abductive schema, as something like $(8^{\S})\ \neg H \rightsquigarrow \neg(R(K(H), T))$, we notice that the clause $(8^{\S})$ now states that if $H$ were untrue, then the epistemic goal would be attained when $K$ is revised upon the addition of $H$. In $(8^{\S})$, when we say that "if $H$ were untrue", it is not clear whether it is the act of hypothesizing or the content of the hypothesis that is to be rejected. The formalism used in the schema does not clearly distinguish the propositional content from the illocutionary force of the hypothesis. At any rate, neither reading yields any principle relevant for abductive inference. Thus, unlike what happens in Nozick's tracking theory of knowledge it is not fruitful to have anything like (an epistemic version of) the *sensitivity condition* in the G–W schema. Nozick's tracking theory is intended to suggest a certain analysis of knowledge, while abduction is an ignorance-preserving or mitigation procedure related to presumptive forms of reasoning. It is thus reasonable and natural to think that the two have very different epistemic and formal properties.

losophy" (CP 6.169, 1902).[20] It regulates what logical hypotheses are to be entertained, and which are those that are to be further examined.

So it also appears that synechism has an important regulative role in effecting the processes of abduction.

**Uberty:** Uberty is a value of hypotheses that tells when they would be suspected to be good in their character of productiveness.[21] In Peirce's terms, a hypothesis is uberous, if it is "*gravid* with young truth" (EP 2:472), that is, it encourages invention and discovery in its capability of suggesting certain other, new and connected hypotheses in case the original, low-security one fails to survive for long. Uberty is the converse of security, which characterizes deductive reasoning. If the value of the security of reasoning goes up, then the value of uberty goes down, and *vice versa*.

Thus also uberty belongs to this prime list of values that abduction makes good use of.

As can be seen from this characterization of scientific values, these three values are not purely or even predominantly things that would claim to rule epistemic domains. They are related to issues that sometimes have been grouped under the umbrella of non-epistemic values in science. They are primarily of methodological importance in elaborating, analysing and enriching Peirce's abductive schema. In fact, the G–W elaboration of the 1903 abductive schema is precisely the place where the distinction between epistemic and non-epistemic values is beginning to evaporate.[22]

# 5 Abductive Schema in a Pragmatic Framework

The previous section prepared grounds for the case that the distinction between the content of an illocutionary act and its linguistic force enables a fine-grained analysis of hypothetical reasoning in its abductive modes.[23] Given a propositional content $\alpha$, the target of the abductive inference is to justifiably assert it. However, what may actually happen is that, by means of an abductive procedure, $\alpha$ may indeed be conjectured yet only plausibly asserted. The idea is

[20]Peirce tells that synechism is a "synthesis of tychism and of pragmatism" (Pietarinen 2015). This is worth a good notice. Pragmatism, in turn, concerns the significance of "what *can* become actual" (MS 280; Pietarinen 2008).

[21]Peirce wrote to F. A. Woods (6 November 1913): "I think logicians should have two principle aims: First, to bring out the amount and kind of security (approach to certainty) of each kind of reasoning, and second, to bring out the possible and esperable uberty, or *value* in productiveness, of each kind".

[22]There are other important and related considerations which we cannot take up in the present context, such as the fundamental question of the justification of abduction. Although this matter is related to the discussion of scientific values, the justification concerns the leading principles of abduction—that nature is explainable—and although value-laden we need not go further in that topic in the present context.

[23]See (Searle and Vanderveken 2005) on formalizing speech acts and illocutionary forces, and (Bellucci 2018) for a detailed exposition of how various speech acts arise from Peirce's later classification of signs.

that even if $\alpha$ cannot be fully asserted in earnest and $\alpha$ can nonetheless be conjectured (or plausibly asserted) rather than merely hypothesized, since that hypothesis fulfills the conditions required by the ordinary G–W schema. This fact is in accordance with the idea that abduction is ignorance preserving or mitigating: in the course of abduction we gain no new knowledge but we may increase the evidential support for $\alpha$ and its plausibility.

In order to enrich the G–W schema with these notions we need to introduce some further logical concepts. We take the proposition $\beta$ to stand for a surprising fact. The formula $\vdash_K \alpha$ expresses the assertion of $\alpha$ in agent's knowledge base $K$. Analogously, $\vdash_{K^*} \alpha$ means that the assertion of $\alpha$ is in $K^*$ (that is, in what is in the neighbourhood of knowledge that the agent can reconstruct in a reasonable, timely and commonly expected way). We take $|\,\alpha\,|$ to be the fuzzy value (say, its "uberty") associated to a hypothetical propositional content of $\alpha$, and $|\,\alpha\,|_{K(\mathcal{H}\alpha)}$ to be the fuzzy value associated to a hypothetical propositional content that comes to be revised in $K$.

Notice that $(\vdash_K \alpha) = J$ when there is complete evidence but it is limited to agent's knowledge base $K$. For instance, if $(\vdash \alpha) = J$ then it does not necessarily follow that $(\vdash_K \alpha) = J$, since there may not be enough resouces in $K$ to make a justified assertion out of $\alpha$. For instance, a method can be contingently lacking, such as a good method of proof, construction or verification, none of which may be reasonably available in agent's knowledge base. However, such method can exist *in line of principle* even if it is not contigently available. Basically, from the mere existence of a potential method we cannot conclude to the existence of an actual method (of proof or verification) available in agent's knowledge base, even if, of course, the opposite holds true. Moreover, if $(\vdash \alpha) = J$ then $(\mathcal{H}\alpha) = J$, as required by the principle **(GP2)**. So, $\mathcal{H}\alpha$ and $\vdash_K \alpha$ have non-equivalent justification conditions since the former can be derived by $(\vdash \alpha) = J$ but not necessarily the latter. $(\vdash_{K(\mathcal{H}\alpha)} \beta) = J$ means that the proposition expressing the surprising fact $\beta$ can be justifiably asserted once $\beta$ is evaluated in the context of $K(\mathcal{H}\alpha)$.

Finally, $\mathbb{C}\alpha$ is the conjecture of $\alpha$,[24] and $\vdash^{\mathbb{C}} \alpha$ expresses the target of asserting $\alpha$ in what Gabbay and Woods call "a way that reflects its conjectural origins" (Gabbay & Woods 2005, p. 47).

Our pragmatic overhaul of the G–W schema now comes to look like the following:

[24]From a pragmatic perspective, a justified conjecture expresses the possibility that a propositional content may be asserted (see e.g. Bellin 2014). This may be too weak, since the conclusion of the abductive schema is not only that something *may* be the case, nor that we would have gained some confidence to assert that possibility, but that there really is further content in the conclusion to justify why it would be worth engaging in further investigation of that promise. In Peirce's terms, such hypotheses are "investigands" (Ma and Pietarinen 2017; Peirce 1905). They are connected to Peirce's theory of the economy of research.

| | | |
|---|---|---|
| (0˚) | $\beta$ | surprising fact |
| (1˚) | $(\vdash \alpha) = J$ | epistemic target |
| (2˚) | $(\vdash_K \alpha) = U$ | fact |
| (3˚) | $(\vdash_{K^*} \alpha) = U$ | fact |
| (4˚) | $\mathcal{H}\alpha \notin K$ | fact |
| (5˚) | $\mathcal{H}\alpha \notin K^*$ | fact |
| (6˚) | $\mathcal{H}\alpha = (0 \leq \|\alpha\| < 1)$ | fact |
| (7˚) | $\mathcal{H}\alpha = (0 \leq \|\alpha\|_{K(\mathcal{H}\alpha)} < 1)$ | fact |
| (8˚) | $((\mathcal{H}\alpha) = J) \rightsquigarrow ((\vdash_{K(\mathcal{H}\alpha)} \beta) = J)$ | fact |
| (9˚) | $\mathcal{H}\alpha$ meets further conditions $S_1, S_2, \ldots, S_n$ | fact |
| (10˚) | Therefore, $(\mathbb{C}\alpha) = J$ | sub-conclusion, (1˚–7˚) |
| (11˚) | Therefore, it is plausible that$(\vdash^{\mathbb{C}} \alpha)$=$J$ | conclusion, (1˚–8˚). |

Two promissory notes are in order before having a run-through of this enriched schema.

First, instead of having to read the last line invariably as the plausibility of the assertion of conjectured hypotheses, we could read it just as well along Peirce's own lines as there being sufficient and compelling "reason to suspect" that such conjectures are assertible.

Second—and this indeed is the main import—the content of a hypothesis can be justifiably conjectured, and hence plausibly or reasonably asserted, when (together with certain other and further conditions) the following holds:

- If it *were* assumed that the hypothesis holds, then it *would be* possible to make the justified assertion—in the agent's or a group of agents' knowledge base which is revised with the hypothesis—of the proposition that expresses the surprising fact.

In the light of the reconstruction of the G–W schema along these lines, and the enrichment of abductive reason by illocutionary force, the propositional content of the hypothesis can come to be conjectured and plausibly asserted in a justified way.

Let us next give an analytic account of our pragmatic rendering of the G–W schema.

Step (0˚) states that $\beta$ is a surprising fact and that it cannot be asserted since at this stage we do not possess evidence for its justification. This interpretation is in accordance with Peirce's 1903 Harvard account of abductive inference, with that significant element of scientific surprise that sets mind into motion. Step (1˚) clarifies the epistemic status of the situation: the target of the abductive inference is to be able to justifiably assert $\alpha$. It does not merely describe the inability in the prevailing situation of the agents to make it so that they would attain their epistemic goals. Rather, this enriched clause now expresses the idea that the conceivability of the testability of a hypothesis is a necessary condition for its (scientific) admissibility (cf. Gabbay & Woods 2006). Hypotheses are inadmissible and unassertible when no conceivable experiential consequences could be foreseen or imagined when entertaining the hypothesis. This is essentially the content of Peirce's maxim of pragmaticism. Step (2˚) indicates that the assertion of $\alpha$ is unjustified in the knowledge base $K$, namely that we fail to possess a method (such as a proof, experimental verification or constructions in our thoughts) of $\alpha$ in that $K$. This can mean two things: first, that there in fact is no proof (or verification etc.) of $\alpha$ in $K$, but there might be (yet ultimately fail to be) one in some different knowledge base $K'$. Second, it could mean that a refutation, argument or construction is readily available that would conceivably suggest discrediting $\alpha$ in $K$. Step (3°) states that the assertion of $\alpha$ is unjustified also in the immediate epistemic vicinity of $K$, namely $K^*$. Step (4°) and (5°) both affirm that $\mathcal{H}\alpha$ belongs neither to $K$ nor to $K^*$. Step (6°) states that $\mathcal{H}\alpha$ is a genuine hypothesis, that is, it receives a certain and initially perhaps a very small plausibility value, and so its propositional content is, for the time being, far from being fully established. This makes Step (6°) markedly different from Step (2°), namely the absence of a proof or the presence of a refutation. Analogously, Step (7°) is similar to Step (6°), but with the addition that the fuzzy value of the hypothesis (such as its value in uberty or plausibility) is evaluated upon $K$. Consequently, this seventh step imposes a condition that is different also from the condition in Step (3°).[25] The subjunctive conditional expressed in the condition in Step (8°) then affirms that, if $\mathcal{H}\alpha$ *were* justified then it *would be* plausible to justifiably assert $\beta$, given $K(\mathcal{H}_\alpha)$. This would now mean that assuming the justification of $\mathcal{H}\alpha$, the justified assertion of $\beta$ which is evaluated in $K(\mathcal{H}_\alpha)$ would, in terms of Peirce's 1903 schema, "be a matter of course".

Let us notice that the justification of a hypothesis is associated with the condition concerning the existence of a *scintilla of evidence* for $\alpha$. The fulfillment of this condition does not impose that $\alpha$ should be true. The fact that the antecedent of the subjunctive conditional should be *justified* rather than *true* is indeed an important and desirable feature of our pragmatic enrichment of the G–W schema: it now allows hypotheses with potentially false propositional content to contribute to the abductive inference in (8°), as soon as they have some plausibility or further value (such as those that follow from the principles of the economics of research) given by some (even minimal) *scintilla of evidence* (Magnani 2017). Then, Step (9°) states that certain other conditions be satisfied by $\mathcal{H}\alpha$. Three such conditions are of prime importance and were delineated in the previous section, namely tychism, synechism and uberty. Naturally these are the required further conditions or values also in this revised schema. In particular, as already noted in the analysis of the previous step, false hypotheses (hypotheses with potentially false propositional content) may and perhaps often do contribute to the abductive inference at Step (8°). Such hypotheses are high in uberty but low in security. Step (10°) then makes it evident that a conjecture of $\alpha$ can indeed be justifiably formulated. Step (11°) finally states that the attempts of asserting $\alpha$ are justified.

Starting from the hypothesis $\alpha$, as soon as the conditions in Steps (1°–11°)

[25] There is an important update process going on here. For an elaboration of this and the adjacent steps, see Ma & Pietarinen (2017), which sets the abductive process within the framework of dynamic epistemic logic for sub-beliefs, using a modification of neighborhood semantics.

are fulfilled, then *it becomes plausible* to justifiably assert $\alpha$. This is certainly stronger than just idly entertaining an idea. From the point of view of illocutionary analysis, the condition in Step (9˚) is particularly relevant, since if there were (a scintilla of) evidence associated to the hypothesis of $\alpha$, then the assertion of $\alpha$ in $K$ would in fact be justified. Hence, if there is the possibility for a propositional content to switch from the illocutionary force of hypothesis to that of assertion (and even if it were to be restricted to some specific knowledge base), then it becomes plausible to justifiably assert such propositional content. The three illocutionary forces—those of *hypothesizing*, *conjecturing* and *asserting*—are thus all involved in the pragmatistic unearthing of the meanings involved in the fine structure of abductive reasoning. Likewise, the three scientific (non-epistemic) values, tychism, synechism and uberty, are involved as essential further conditions that such reasoning is expected to meet. They are not extraneous to the logic of abduction but form a congenial part of what the very logic of abduction would ultimately have to look like.

# 6 Conclusion

This, in sum, explains the felicity and the value of our pragmaticist overhaul of the G–W schema of abduction. Our analysis makes the fundamental role of illocutionary forces in the G–W schema explicit. Moreover, our pragmatic interpretation of the G–W schema shows how to reconstruct abductive inferences together with justificationist elements. These justification conditions for illocutionary acts are distinguished from truth-conditions for propositional contents. However, justification is a strictly pragmatic undertaking related to illocutionary acts. It is not to be confused with epistemic justification. What are justified in our framework are (linguistic) acts, not knowledge. Therefore also our enriched G–W schema remains ignorance-preserving or mitigating, because it does not have as its direct aim to justify knowledge. The aim is to justify actions, or intentions to act, that hypotheses trigger when they are the products of this due abductive process. The notion of justification in our new abductive schema is not an epistemic justification or justification of knowing the truth of a hypothesis. It is the justification of asserting the hypothesis given its suitability to experimental testing. Justification concerns our actions of being prepared to subject the hypothesis to further inquiry.[26] This is what Peirce in 1905 meant in his letter to Welby when characterizing the mood of the conclusion of abductive reasoning as the "investigand".

What was proposed naturally points only at the certain beginnings by which to pursue new perspectives to Peirce's original schemas, and to explicate the possible intentions and unfulfilled ideas that he may have had concerning abduction. One could, for example, go on to see how well Peirce's *responsibility theory of assertions* (see e.g. Boyd 2016) agrees with scientific assertions involved in abductive reasoning. Clearly the assertions that scientists are inclined

[26]For a brilliant discussion on abduction in relation and contradistinction to theories of knowledge, see (Woods 2017).

to make imply the presence of some special scientific attitudes concerning commitments and assertoric responsibilities. Second, one might take there to be a certain Deweyan character to the proposed reinterpretation, as we are bringing something like the "warranted assertibility", in terms of justified assertions, to the picture. However, our enrichment does not amount to the replacement of "truth" with another quality such as "warranted (or justified) assertibility", since justification conditions are separated from truth-conditions in our framework. Third, just to be mentioned here as a future line of development, it might be useful to apply a graphical logic of assertion, initially proposed in (Bellucci, Chiffi & Pietarinen 2017) and (Pietarinen & Chiffi 2018), and based on Peirce's graphical logic (Bellucci & Pietarinen 2016, 2017), in the analysis of abduction with its illocutionary content. This could pave a way for a new diagrammatic (graphical) logic of abduction, which in fact was also one of Peirce's ultimate yet unfulfilled goals.

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