

Article

# Modeling Cycle and GenAI as Resources for Mathematics Teachers' Professional Development

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## Abstract

This study stems from the need to investigate how GenAI tools, particularly ChatGPT-4o, can support the professional development of mathematics teachers. It explores how Blum's modeling cycle can serve as a conceptual and operational framework for mathematics teachers' instructional design when supported by ChatGPT-4o. Drawing on a qualitative case study within a teacher professional development program, the research analyzes how two upper secondary school teachers engaged with ChatGPT-4o to redesign a mathematical task involving probability and real-world contexts. Data include responses to three modeling-related tasks, teachers' prompts and interactions with ChatGPT-4o, and the final mathematical activity they designed. These materials were analyzed qualitatively according to the modeling cycle and its sub-competencies. The results indicate that the modeling cycle provided teachers with a cognitive and methodological scaffold to guide their interaction with ChatGPT-4o, allowing them to structure, validate, and refine AI-generated ideas through all stages of modeling—from understanding and mathematizing to interpreting and validating. These findings suggest that the modeling cycle can be reinterpreted as a design-oriented framework for integrating ChatGPT-4o in mathematics teacher education. Implications for teacher professional development and future research directions are discussed.

**Keywords:** mathematics teacher education; mathematical modeling; Generative Artificial Intelligence; ChatGPT-4o; instructional design; teacher professional development; digital technologies



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## 1. Introduction

Instructional design plays a central role in improving mathematics teaching and learning: teachers do not simply work in the classroom but, with and without the use of digital technologies, they design activities, learning environments, and assessments that require reflection and professional awareness (e.g., [Gueudet, 2021](#)). In a context where these technologies are becoming an integral part of the school ecosystem, instructional design should be understood not only as content selection but also as the orchestration of technology-related resources, tools, and learning processes ([Drijvers et al., 2010](#)). In mathematics education in particular, digital technologies are not only replacing analog tools but also redefining the ways in which mathematics is explored, represented, and generalized, thus offering opportunities to rethink its design ([Drijvers, 2024](#)).

In this scenario, educational design using digital technologies takes on even greater importance for the training and professional development of mathematics teachers. Indeed, the literature has highlighted how professional development programs that integrate digital technologies should go beyond simple technical training and address issues of instructional adaptation, practice change, and collaborative reflection among teachers (Ratnayake et al., 2020). It is therefore necessary to consciously design professional development paths that enable teachers to adapt, evolve, and scale practices that integrate digital technologies (Getenet, 2020; Thurm & Barzel, 2020).

At the same time, the rapid evolution of Generative Artificial Intelligence (GenAI) technologies opens up new possibilities and scenarios for instructional design. The advent of Large Language Models (LLMs), in particular ChatGPT-4o, has introduced a new kind of support for teachers' work, capable of generating coherent and contextual texts, with potential implications for lesson design and mathematical tasks. These models can support teachers in designing mathematical activities, personalizing assignments, generating simulated learning environments, and potentially lightening some of the design load (Giannakos, 2025).

From a mathematics education point of view, ChatGPT-4o can support concept understanding, lesson planning, and task design, but requires human supervision and thoughtful integration into practice (Pepin et al., 2025). More specifically, in the Special Issue "Mathematics Education in the Era of ChatGPT-4o: Investigating its Meaning and Use for School and University Education" of the journal "Digital Experiences in Mathematics Education" (DEME), recurring uses of LLMs as design assistants for teachers in initial and in-service training emerge. Gurl et al. (2025) show how future teachers use ChatGPT-4o to design "mini-lessons", accurately assessing the pedagogical consistency of the plans generated, but proving less effective in evaluating mathematical correctness, confirming the need for frameworks for disciplinary validation during design. At the same time, Busuttill and Calleja (2025) document positive but cautious attitudes among secondary school teachers: ChatGPT-4o is perceived as a useful tool for generating assignments, assessment tools, and personalized feedback, provided that it is accompanied by professional development that aligns technology with pedagogical values. A consistent result also comes from studies on pre-service teachers by Noster et al. (2024), which show that prompting and quality control skills are crucial in order to avoid accepting answers that appear correct but are not during the design phase.

However, despite its great potential, integrating GenAI into teaching practice requires new skills and critical reflection on issues related to the teacher's role, teachers beliefs, and the design and selection of the generated outputs (Giannakos, 2025). The literature still shows a number of critical issues in the link between instructional design with technology and mathematics teachers' professional development. First, as highlighted by Drijvers (2024), although digital technologies have benefits, issues of equity, curriculum change, and transformation of teaching practices remain. Second, research on the use of GenAI in mathematics teacher education is still unsystematic (Giannakos, 2025). Moreover, as indicated by Ratnayake et al. (2020), even in the teachers' professional development paths with digital technologies, the tasks designed by teachers, although richer, still show limitations in their effective implementation and metadidactic reflection. In light of the above considerations, there is a need to explore how instructional design tools that integrate the capabilities offered by GenAI can support the professional development of mathematics teachers.

This study is part of a larger research project on teachers' professional development with digital technologies in relation to mathematical modeling (Brunetto & Dello Iacono, 2023; Bassi et al., 2024) and focuses on how mathematics teachers design modeling teaching activities using digital technologies, particularly GenAI. This research explores how Blum's

modeling cycle can serve as a conceptual and operational framework for upper secondary school teachers to design modeling activities with the support of ChatGPT-4o. ChatGPT-4o was chosen as the GenAI tool for this study because it is among the most widely accessible and commonly used Large Language Models in educational contexts, and has been the focus of a growing body of research in mathematics education (e.g., Pepin et al., 2025; Gurl et al., 2025). Its widespread availability and familiarity among teachers increase the ecological validity of the study.

## 2. Conceptual Background

### 2.1. Mathematical Modeling

Mathematical modeling is defined as the process of translating between real-world situations and mathematics, in both directions (Blum & Borromeo Ferri, 2009). This process is based on the construction of a simplified and formalized image of a part of the real world (Blum, 2015), which may relate to natural, social, economic, or cultural phenomena. The main objective of developing and using models is to understand and address real-world problems through mathematical tools (Niss et al., 2007). In this sense, modeling is not limited to solving exercises, but involves a complex cognitive activity in which a continuous dialogue is established between reality and mathematical abstraction.

Blum (2015) and Borromeo Ferri (2006) represent this process through the modeling cycle, which describes the successive steps between the two domains: the real domain and the mathematical one. In the seven-step version proposed by Blum (2015), the cycle (see Figure 1) includes:

1. understanding the task: understanding and simplifying the real-world situation;
2. simplifying/structuring: constructing a real-world model;
3. mathematizing: formalizing the model mathematically;
4. working mathematically: solving the problem mathematically;
5. interpreting: interpreting the results;
6. validating: validating the results in the real-world context;
7. presenting: communicating the results.

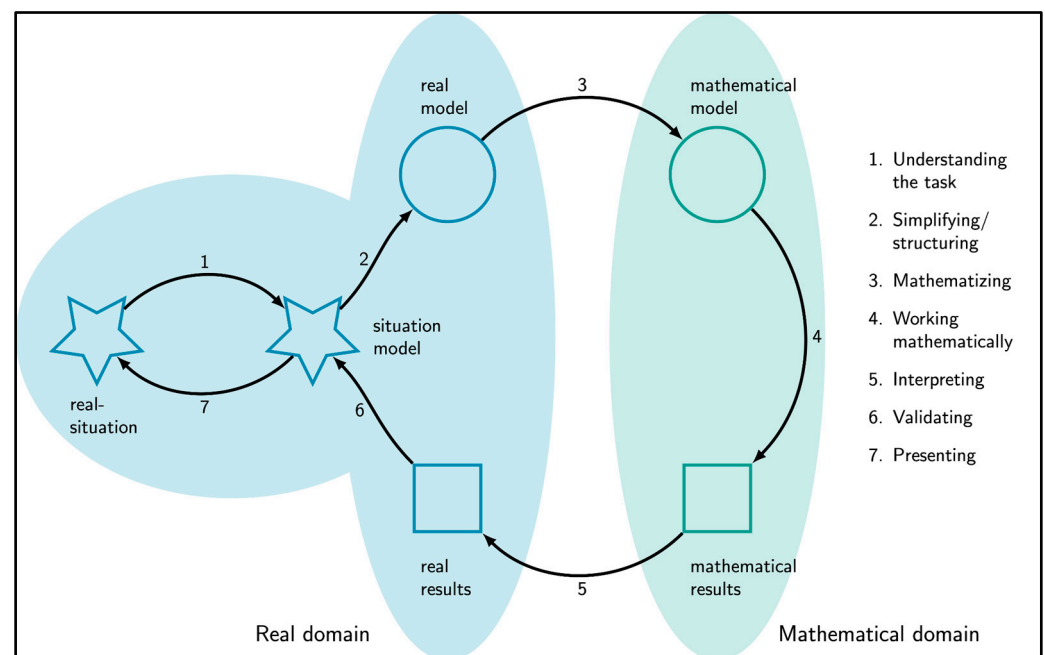


Figure 1. The modeling cycle according to Blum (2015).

This cycle is an idealized and simplified model of the modeling process and is itself a theoretical construct: research shows that students and teachers go through these stages in a non-linear way, sometimes taking shortcuts or retracing their steps (Ärlebäck et al., 2013). Teaching mathematics through modeling therefore involves guiding students through all stages of the cycle in a conscious manner.

From a cognitive point of view, the modeling cycle requires flexible thinking, the ability to represent the problem in different forms (verbal, graphic, symbolic), and to move from the situated context to abstract formalism (Borromeo Ferri, 2018). Niss and Højgaard (2019, p. 16) argue that the modeling competency

“focuses on mathematical models and modeling, i.e., on mathematics being put to use to deal with extra-mathematical questions, contexts and situations. Being able to construct such mathematical models, as well as to critically analyze and evaluate existing or proposed models, whilst taking purposes, data, facts, features and properties of the extra-mathematical domain being modelled into account, are the core of this competency”.

This competency includes specific sub-competencies (see Table 1) associated with each stage of the modeling cycle, as shown by Maass (2006) and further enriched by Bassi and Brunetto (2023), who distinguish between the sub-competencies of understanding/simplifying (step 1–2), mathematizing (step 3), working mathematically (step 4), interpreting (step 5), validating (step 6), and presenting (step 7).

**Table 1.** Sub-competencies for each cycle step.

| Step | [Label] and Sub-Competencies  |
|------|---|
| 1–2  | [S1–2.1] to make assumptions for the problem and to simplify the situation<br>[S1–2.2] to recognize quantities that influence the situation, to name them and to identify key variables<br>[S1–2.3] to construct relations between the variables<br>[S1–2.4] to look for available information and to differentiate between relevant and irrelevant information   |
| 3    | [S3.1] to mathematize relevant quantities and their relations<br>[S3.2] to simplify relevant quantities and their relations if necessary and to reduce their number and complexity<br>[S3.3] to choose appropriate mathematical notations and to represent situations graphically   |
| 4    | [S4.1] to use heuristic strategies such as division of the problem into part problems, establishing relations to similar or analogous problems, rephrasing the problem, viewing the problem in a different form, varying the quantities or the available data etc.<br>[S4.2] to use mathematical knowledge to solve the problem   |
| 5    | [S5.1] to interpret mathematical results in extra-mathematical contexts<br>[S5.2] to generalize solutions that were developed for a special situation   |
| 6    | [S6.1] to critically check and reflect on found solutions<br>[S6.2] to review some parts of the model or again go through the modeling process if solutions do not fit the situation<br>[S6.3] to reflect on other ways of solving the problem or if solutions can be developed differently<br>[S6.4] to generally question the model   |
| 7    | [S7.1] to present the results with an appropriate structure according to the genre and style in which the results are presented<br>[S7.2] to present the results in a way which is appropriate for the addressee<br>[S7.3] to present the results in a rigorous way, in particular to present statements strongly supported by data<br>[S7.4] to present the results in a clear and correct way from the formal point of view |

## 2.2. Teachers' Digital Literacy, GenAI, and Instructional Design Within a Modeling Perspective

The integration of digital technologies into mathematics teaching has long been recognized as a complex professional challenge that goes beyond technical proficiency, requiring

teachers to develop forms of digital literacy that are deeply intertwined with pedagogical and disciplinary knowledge. Research in mathematics education has long emphasized that teachers' use of digital technologies is not merely a technical matter but a deeply pedagogical and epistemological one. Digital literacy for mathematics teachers entails the ability to select, adapt, and orchestrate technological resources in ways that are coherent with mathematical meanings, learning goals, and classroom practices (Drijvers et al., 2010; Gueudet, 2021). In mathematics education, digital literacy is increasingly understood as teachers' capacity to critically select, adapt, and orchestrate digital tools in ways that support mathematical meaning-making rather than merely enhance efficiency or presentation (Drijvers, 2024).

Within this line of research, instructional design is seen as a core component of teachers' professional activity (Gueudet, 2021), particularly when digital technologies are involved. Designing with technology entails anticipating students' mathematical activity, selecting and shaping tasks, and aligning representations, tools, and interactions with intended learning goals (Hoyles et al., 2013). Research in mathematics education shows that professional development programs are most effective when they support teachers in developing design-oriented competencies, rather than focusing exclusively on tool functionality (Ratnayake et al., 2020; Thurm & Barzel, 2020). This is especially relevant in the context of emerging technologies such as GenAI, whose affordances challenge established roles of teachers, tools, and tasks.

Recent contributions in the *Digital Experiences in Mathematics Education* journal highlight that Large Language Models (LLMs) such as ChatGPT-4o introduce a qualitatively new form of digital resource for teachers' instructional design (Pepin et al., 2025). Unlike traditional educational technologies, ChatGPT-4o does not merely mediate mathematical representations or automate calculations; rather, it generates linguistic and semiotic artifacts, such as problem contexts, explanations, and solution strategies, that can directly influence the structure and epistemic quality of mathematical tasks. Research shows that mathematics teachers and pre-service teachers often use ChatGPT-4o as a design assistant, for instance to generate lesson ideas, contextualized problems, or assessment questions (Gurl et al., 2025; Noster et al., 2024). However, these studies also reveal critical issues: AI-generated outputs may appear pedagogically coherent while containing mathematical inaccuracies or oversimplifications, thus requiring strong disciplinary and reflective competencies from teachers.

From this perspective, teachers' digital literacy with GenAI cannot be reduced to prompt-writing skills. Rather, it involves the ability to frame productive questions, interpret AI-generated suggestions, and critically validate them against mathematical and didactical criteria. Meaningful technology integration depends on teachers' capacity to connect digital affordances with mathematical practices and learning theories (Drijvers, 2024; Borromeo Ferri, 2018). GenAI therefore amplifies the need for theoretical frameworks that can guide teachers' design decisions and support critical evaluation during the design process.

In this study, we argue that mathematical modeling, and in particular Blum's modeling cycle, provides such a framework. Traditionally conceived as a model of students' cognitive activity when addressing real-world problems (Blum & Borromeo Ferri, 2009; Blum, 2015), the modeling cycle can be reinterpreted as a design-oriented epistemic structure for teachers working with GenAI. Each stage of the cycle (understanding, simplifying and structuring, mathematizing, working mathematically, interpreting, validating, and presenting) corresponds not only to phases of students' activity but also to key decisions teachers must make when designing modeling tasks. Within this perspective, ChatGPT-4o can be positioned as a mediating design resource whose contributions are embedded within, rather than external to, the modeling cycle. For example, in the early stages of

understanding and structuring a real-world situation, GenAI can support teachers by proposing contextualized scenarios or exploratory questions. However, the modeling framework foregrounds the teacher's responsibility to assess the authenticity and mathematical relevance of these contexts, ensuring that they give rise to a genuine modeling need rather than superficial realism—a distinction widely discussed in the modeling literature (Borromeo Ferri, 2006; Ärlebäck et al., 2013). Similarly, during the mathematizing and working mathematically stages, ChatGPT-4o may suggest representations, variables, or solution approaches. The modeling cycle functions as a disciplinary filter that requires teachers to scrutinize AI-generated mathematical structures, align them with curricular goals, and verify their internal coherence. The interpreting and validating stages further reinforce teachers' critical role, prompting reflection on the plausibility, limitations, and generalizability of models—dimensions that are central to modeling competence (Niss & Højgaard, 2019) and largely absent from automated generation.

### 3. Rationale and Research Question

Blum's (2015) modeling cycle offers a particularly fruitful theoretical perspective. It provides a process diagram that describes the iterative transition between reality and mathematics, which can be reinterpreted as a cognitive map to guide instructional design. The structure of the cycle—with its seven stages of understanding, simplifying, mathematizing, working mathematically, interpreting, validating, and presenting—reflects the mental and pedagogical operations that teachers perform when designing authentic mathematical activities.

More recently, the integration of digital technology into mathematics education has profoundly changed the paradigms of teacher education and professional development (Drijvers, 2024). However, as highlighted by Pepin et al. (2025), the spread of GenAI tools, and in particular large language models (LLMs) such as ChatGPT-4o, opens up new scenarios for instructional design and the role of the teacher as designer, author, and critical evaluator of educational processes. International research indicates that ChatGPT-4o-assisted instructional design can enhance teachers' professional reflection by supporting the creative and anticipatory phase of design (Gurl et al., 2025). However, these tools have obvious limitations related to a lack of semantic understanding, the generation of content that is sometimes mathematically inaccurate, and a tendency to produce standardized lesson plans that lack context (Busuttil & Calleja, 2025). As pointed out by Giannakos (2025), the pedagogically meaningful use of such tools requires theoretical models capable of anchoring design to cognitive and metacognitive processes grounded in the professional practice of teachers. However, this integration is not automatic: it requires a theoretical and professional framework that allows teachers to consciously orchestrate the dialogue between their teaching skills and the generative affordances of GenAI and, in this study, of ChatGPT-4o in particular.

In light of these considerations, the purpose of this study is to explore how Blum's modeling cycle can serve as a conceptual and operational framework to guide ChatGPT-4o-assisted instructional design of learning activities by teachers. We intend to investigate how teachers use the different stages of the cycle to interact critically and reflectively with ChatGPT-4o, assessing their potential and limitations in design practice. We wonder whether ChatGPT-4o could be a resource to support teachers in educational design across the different steps of modeling cycle:

- in the steps of understanding and simplifying the real context, ChatGPT-4o can generate authentic scenarios and exploratory questions;
- during the mathematizing and working mathematically steps, it can suggest mathematical representations or models;

- during the interpreting and validating steps, it can serve as a metacognitive stimulus, helping the teacher to reflect, for example, on the consistency and plausibility of the model.

More precisely, the research question guiding this study is as follows:

(RQ) How can the modeling cycle serve as a reference framework for ChatGPT-4o-assisted instructional design of learning activities by teachers?

## 4. Method

This section first describes the teachers' professional development program, namely the PROMOTED training program, then the participants, and finally the data collection and analysis criteria.

### 4.1. The PROMOTED Training Program

This study was conducted as part of the PROMOTED training program (PROblem solving and mathematical MOdeling: teaching approaches with Digital TEchnologies), under the auspices of the "Fondazione I Lincei per la Scuola" and aimed at upper secondary school mathematics teachers. The program, which was entirely online and lasted 20 hours spread over eight sessions, aimed to provide teachers with theoretical and practical tools for designing digital teaching activities aimed at developing mathematical modeling skills in students. The authors of this paper were the trainers.

The structure of the course integrated three main conceptual axes:

1. mathematical skills and modeling;
2. use of digital technologies;
3. reflection on the conscious use of ChatGPT-4o-4o as a resource for designing educational activities.

At the beginning of the program, an anonymous questionnaire (participants entered their own nickname) was administered to gather information about the participants (e.g., work experience, teaching practices). The data collected through the initial questionnaire played a descriptive and contextual role in the study. In particular, responses were used to construct the participant profiles reported in Section 4.2, providing information on teachers' professional experience, prior training in mathematics education, and familiarity with digital technologies and GenAI tools, in particular ChatGPT-4o. This information supported the characterization of the participants and informed the selection of the case study, without being included in the subsequent analytical phases of the research.

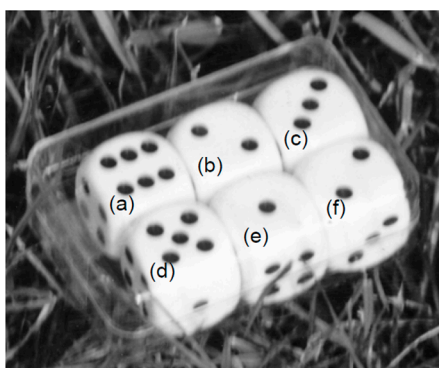
The program began with an analysis by teachers of the OECD-PISA "Dadi" (dice) problem (INVALSI, 2008, see Figure 2).

This "Dadi" problem was chosen by the authors because, although it is related to a real-world domain, namely the dice as physical artefact, the real situation is fictitious as there is no real need or requirement on the part of those addressing the question to know "the number of points on the opposite side of each die". From this point of view, the problem does not seem to favor the start of Blum's modeling cycle. Therefore, the choice of this problem provides an excellent starting point for reflecting on the difference between exercises and modeling situations, highlighting the need to redesign the task to facilitate the start of the modeling cycle.

The teachers, organized into small groups, worked on tasks T1, T2, and T3 (see Table 2).

il numero totale di punti su due facce opposte è sempre sette.

|     |     |     |
|-----|-----|-----|
| (a) | (b) | (c) |
|     |     |     |
| (d) | (e) | (f) |



Scrivi, in ogni casella, il numero di punti della faccia **opposta** di ciascun dado mostrato in fotografia.

**Figure 2.** The OECD-PISA “Dadi” problem (INVALSI, 2008). [Translation: In this photograph there are six playing dice, named with the letters (a–f). The following rule applies to all dice: the total number of points on two opposite faces is always seven. Write, in each box, the number of points on the opposite side of each die shown in the photograph].

**Table 2.** The tasks of the PROMOTED training program.

| Task | Description  | Questions for Teachers   |
|------|--|--|
| T1   | Investigate the characteristics of the “Dadi” problem from the point of view of both the type of problem and the skills that students need to tackle it and that it aims to develop in students. | How much do you agree with the following statements? Is it similar to many math problems proposed at school? Is it an example of the application of mathematics? Is it an example of a real-life problem?<br>What skills do students need to address this task? Justify your answer.<br>What skills does this problem aim to develop in students? Justify your answer.   |
| T2   | Analyze the problem taking into account the steps of the modeling cycle.   | Taking into account Blum’s modeling cycle, how much do you agree with the following statements? The question allows students to go through the steps: understanding the task; simplifying/structuring; mathematizing; working mathematically; interpreting; validating; presenting. Justify your answers.  |
| T3   | With the support of ChatGPT-4o and starting from the original problem, create a real-life situation in accordance with Blum’s model, bringing out the model situation.                           | Starting with the “Dadi” problem, create a real-life situation according to Blum’s model, i.e., write a story that brings out the model situation; design with the support of ChatGPT-4o; create a presentation in which you report the real situation, the chats with the chatbot (the sharing link), and your group’s reflections on the activity carried out, with particular reference to the use of ChatGPT-4o as a design support. |

Tasks T1, T2, and T3 were deliberately designed by the researchers as a coherent sequence aimed at fostering teachers’ critical reflection on the original “Dadi” problem. In particular, T1 was intended to foreground potential limitations of the task in terms of the mathematical skills it promotes and the type of student activity it elicits, while T2 explicitly invited teachers to analyze these limitations through the lens of Blum’s modeling cycle, making visible which modeling steps were supported and which were largely absent. This analytical work was conceived as a necessary preparatory phase for T3, in which teachers were asked to redesign the task by constructing an authentic real-life situation aligned with Blum’s model, using ChatGPT-4o as a design support. In this way, reflection on skills

development and modeling structure (T1–T2) provided the conceptual foundations for the subsequent ChatGPT-4o-assisted instructional design activity (T3).

As mentioned earlier, the course was carried out online. Participants interacted with each other within each group via Google Meet and queried ChatGPT-4o individually (using separate devices and accounts). However, the prompts were formulated jointly through discussion within the group.

#### 4.2. Participants and Case Study

Eleven teachers from high schools and technical institutes in various regions of Italy (North, Center, South) participated in the PROMOTED training program. Nine teachers had over 10 years of teaching experience, all teachers had previously taken general teaching courses, and nine teachers had taken courses in mathematics education. Most of them frequently use online platforms, educational software, and instructional videos produced by others; only 2 out of 11 state that they often use GenAI in their teaching practices, while the majority (7 out of 11) report having used it occasionally. None of the participants had any prior knowledge of Blum's modeling cycle.

From all the participants, we selected a group of two mathematics teachers, Silvia and Paolo (fictitious names), as a case study. The case study was selected based on criteria of heterogeneity, namely experience, technological expertise, and familiarity with modeling. The analysis was deliberately limited to two teachers within a case study in order to enable an in-depth and fine-grained examination of teachers' interactions with ChatGPT-4o and their instructional design processes. The choice of the case study method is motivated by the fact that we are interested in a complex phenomenon that requires a broad and in-depth description, namely the discovery of details within a collaborative experience of teachers relating to instructional design following a critical analysis of the available teaching resources. From a methodological perspective, this choice is consistent with a qualitative case study approach (Yin, 2009), which prioritizes analytical depth and contextualized understanding over breadth, and is particularly suited to exploring complex and emerging phenomena such as ChatGPT-4o-supported instructional design in mathematics education. Furthermore, the case study offers the opportunity to analyze episodes of interaction with generative AI in a qualitative and situated manner, capturing the dynamics of appropriation, negotiation, and reflection that would be difficult to discern using quantitative approaches (Yin, 2009).

Silvia is a teacher at a classical high school in Southern Italy, with more than 10 years of teaching experience, but with limited prior training in mathematics education (according to her, she had never participated in training courses in mathematics education). She enrolled in the course because she was attracted by the "course presentation" and her desire to learn "new methods of teaching". In her teaching practice, she has "often" used self-produced slides and educational software such as "GeoGebra and PhET Colorado, with guided worksheets" prepared by her. Only occasionally has she used Generative Artificial Intelligence (ChatGPT-4o) "to create questions from a text".

Paolo is a teacher at a scientific high school in Central Italy, also with more than 10 years of teaching experience, who has participated in several training courses in general education and mathematics education. He decided to participate in the course "to deepen his understanding of problem solving" and because he was attracted by the "digital approach". In his teaching practice, he has "often" used videos produced by others "to introduce new topics during explanations" and educational software such as "Geogebra to visualize or investigate problem solving". He has sometimes used online platforms such as "Graspable Math to introduce a topic or invite students to collaborate in problem solving". He has never used GenAI environments.

#### 4.3. Data Collection and Analysis Criteria

The data collected during the PROMOTED training program includes:

- teachers' responses to tasks T1, T2, and T3;
- interactions with ChatGPT-4o: prompts from teachers, model responses, collaborative revisions, reflective comments.

All interactions and materials were anonymized and organized in an internal database.

This exploratory work focuses on Silvia and Paolo's responses to tasks T1, T2, and T3 and their prompts during the interactions with ChatGPT-4o. According to the case study methods, we employed a qualitative holistic analysis of all the data. More precisely, for the purposes of this study, the analysis focused on a clearly delimited corpus of written data produced by the two selected teachers, including their written responses to tasks T1, T2, and T3, the final version of the instructional activity they designed, and the complete set of prompts and textual exchanges with ChatGPT-4o generated during the design process. The analytical keywords were defined deductively on the basis of the modeling steps and related sub-competencies presented in Table 1, which provided a theoretically grounded coding framework. These keywords were used to identify explicit references to modeling actions (e.g., identifying variables, mathematizing relationships, validating results). In cases where such references did not appear explicitly in the text, excerpts were analyzed interpretively, focusing on the functional role they played in the design process and their alignment with specific modeling sub-competencies. To ensure the reliability and consistency of the analysis, all materials were independently coded by the authors and subsequently compared and discussed; discrepancies were resolved through iterative discussion until a shared interpretation and full agreement were reached.

### 5. Data Analysis and Results

In the following, we present the analysis of the data sources, beginning with the teachers' responses to Tasks T1, T2, and T3 and concluding with the analysis of the prompts.

#### 5.1. Tasks T1 and T2

We recall that tasks T1 and T2 concerned the analysis of the "Dadi" problem without the use of ChatGPT-4o. More precisely, T1 said: "Investigate the characteristics of the "Dadi" problem from the point of view of both the type of problem and the skills that students need to tackle it and that it aims to develop in students.", while T2 says: "Analyze the problem taking into account the steps of the modeling cycle."

In responding to task T1, Silvia and Paolo considered the "Dadi" problem to be "too simple" for a second-year high school class. They recognized critical issues in the question, identifying it as a question of "an observational nature rather than a modeling problem". They identified the skills students need to tackle this question as mainly "reading and interpreting" the question (text and figure), and the difficulties students face as "understanding data and requests" and "constructing a model". These statements refer to the management of different semiotic registers (propositional and graphic) and to actions by students relating to the first two steps of the modeling cycle (understanding the task, simplifying and structuring) according to the sub-competencies S1-2.2 and S1-2.4 (see Table 1).

The analysis of the "Dadi" problem was getting deeper when the two teachers (Silvia and Paolo) addressed Task T2. They confirm the problem is easy with the following comment:

"The question [i.e., the task of the "Dadi" problem] is quite simple, interpretation is aided by an image, the mathematical procedure consists of simple subtractions, validation of the results does not require particular attention (if mistakes are made in the operations and the score is negative or higher than 6, it is not difficult

to notice that it does not fit the context), and presentation/communication of the results consists only of filling in the table”.

We note that the two teachers seem to refer properly to the steps of the modeling cycle: they said that “mathematical procedure” (namely, S4 “working mathematically”) is limited to “simple subtractions”, the step S6 “validating” does not require particular attention because it is sufficient to check whether the number is negative or positive, and step S7 “presenting” is aided by the existence of a pre-constructed table. In addition, they highlighted critical issues in the “Dadi” problem that allow students to work adequately only on the first two steps of the modeling cycle (Understanding the task and Simplifying/structuring), and neglect the other steps of modeling. It seems that the two teachers are becoming acquainted with Blum’s cycle, which they had not previously encountered.

### 5.2. Task T3

After critically analyzing the question, the teachers worked on task T3, which involves redesigning the question to transform it into a modeling problem. We recall that task T3 required participants to create a real-life situation based on the original problem in accordance with Blum’s model, highlighting the model situation and doing so with the support of ChatGPT-4o. More specifically, task T3 was as follows: “With the support of ChatGPT-4o and starting from the original problem, create a real-life situation in accordance with Blum’s model, bringing out the model situation”. In tackling the task, as specified in the assignment, Silvia and Paolo relied on ChatGPT-4o. The initial prompt from the teachers was as follows: “Come up with a real-world problem for a first-year high school class that uses dice”. This formulation seems to indicate the teachers’ desire to recognize a real-world context (the use of “dice”) but does not yet identify an authentic mathematical need because it lacks a substantial reason. ChatGPT-4o responds with a playful scenario, namely a dice tournament:

*Problem: The dice tournament*

A group of friends organizes a dice tournament. Each player rolls two six-sided dice at the same time.

Base score: The score for a round is given by the sum of the two numbers rolled on the dice.

Bonus: If the two dice show the same number (for example, 4 and 4), you get a bonus of 5 points.

Penalty: If the sum of the two dice is an odd number, you lose 2 points.

*Questions:*

What is the probability of obtaining a base score greater than 10?

What is the probability of getting a bonus?

What is the probability of incurring a penalty?

Calculate the average score for a round.

*Practical extension:*

Organize a small tournament in class, actually rolling the dice and scoring according to the rules above. Then compare the results obtained with the calculated theoretical probabilities.

This scenario may stimulate curiosity, but it does not justify the need for a model. The initial situation is therefore in an embryonic stage: mathematics is evoked, but not yet necessary. Starting from this response, a conversation begins, consisting of six increasingly detailed and specific iterations from the teachers (Table 3). Below, we analyze the conversation between the teachers and ChatGPT-4o, focusing on the teachers’ prompts. In particular, in light of the seven steps of modeling and the related sub-competencies, we analyze each prompt, showing how teachers are modeling even when they are designing the activity.

**Table 3.** The analysis of the prompts provided by Silvia and Paolo.

| #  | Prompt  | Sub-Competencies                 | Step      |
|----|---|----------------------------------|-----------|
| P1 | Two teams of six must be formed using the roll of two dice as the selection criterion.  | S1–2.1<br>S1–2.2<br>S1–2.3       | 1–2       |
| P2 | Come up with a real-world problem for a 9th grade class that uses dice.   | S1–2.1<br>S1–2.4                 | 1         |
| P3 | Two teams of six must be formed using the result of the roll of two dice, other than odd and even, as the selection criterion.  | S1–2.2<br>S1–2.3<br>S3.1         | 1–2, 3    |
| P4 | Two teams of six must be formed using the result of rolling two dice, other than even and odd, as the selection criterion. What is the probability of being assigned to one of the two teams?   | S3.1<br>S3.3<br>S4.2             | 3, 4      |
| P5 | Two teams must be formed to play 5-a-side soccer, starting with 12 players; each team will have one reserve, the last player drawn. The selection criterion will be the result of two dice being thrown by each player, other than the odd or even choice. What is the probability of being assigned to one of the two teams? | S1–2.2<br>S1–2.3<br>S3.1<br>S5.1 | 1–2, 3, 5 |
| P6 | Propose a criterion different from the one used so far.   | S6.1<br>S6.3<br>S1–2.3           | 1–2, 6    |

The sub-competencies involved in prompt P1 are recognizing the problem (S1–2.1), identifying variables (S1–2.2), and establishing relationships between them (S1–2.3). In fact, the prompt sets a situated goal (“Two teams of six must be formed”) and specifies a decision-making mechanism (“the roll of two dice as the selection criterion”). This defines a real-world problem, thus activating step 1. It also identifies variables (players, teams, dice) and the causal relationship between the outcome of the roll and team assignment, i.e., it already constructs a first structure of the real model, thus activating step 2.

As for the sub-competencies activated in prompt P2, they are the identification of a possible problem to be modeled (S1–2.1) and the selection of the relevant context (S1–2.4) because the teachers indicate the constraints of realism (school environment, first year of high school) but does not yet define assignment structures or rules. The prompt requires the creation of a “real” situation, i.e., one that is contextual and plausible for “a 9th grade class”, in which dice appear as an object of interest. This corresponds to step 1 because it aims to identify a non-mathematical situation from which a modellable problem can arise. No variables or operational relationships are specified yet, so we do not enter step 2.

This occurs with prompt P3, which defines a real model structure (“Two teams of six must be formed using the result of the roll of two dice”). The sub-competencies activated are: identifying variables and relationships in the system (S1–2.2; S1–2.3) and initiating the translation of these relationships into mathematical terms (S3.1), because the distinction between teams is required to depend on a non-trivial numerical property of the dice outcomes. Then, this prompt activates step 2. In addition, teachers explicitly ask ChatGPT-4o for “other than odd and even, as the selection criterion”, i.e., they request the choice/creation of an assignment function based on the numerical outcomes of the dice. This request for an alternative criterion implies a first mathematical formalization of the assignment rule, thus triggering step 3.

The first part of prompt P4 (“Two teams of six must be formed using the result of rolling two dice, other than even and odd, as the selection criterion”) requires the definition of a formal assignment rule based on the results of two dice. The second part (“What is

the probability. . .”) asks to calculate the probability of assignment to teams according to that rule. The sub-competencies activated are, therefore, the translation of the real situation into a probabilistic model (S3.1), its formal representation (S3.3), and the request to analyze the impact of the choice of model (S4.2), since knowing the probability is equivalent to assessing its fairness. The shift towards calculating the probability of outcomes corresponds to step 3 because it explicitly requires the construction of a mathematical model of the outcomes of two dice, and to step 4 because it requires the mathematical elaboration of the model (counts, distributions, probabilities).

Prompt P5 introduces a real and socially recognizable context (“to play 5-a-side soccer”), specifies structural constraints (“starting with 12 players; each team will have one reserve, the last player drawn”), and defines the assignment mechanism (“the result of two dice being thrown by each player, other than the odd or even choice”). The sub-competencies involved are the definition of the variables and constraints of the problem (S1–2.2, S1–2.3), the probabilistic formalization of the situation (S3.1), and the connection of the results to the real classroom situation (S5.1). This corresponds to step 2, because it precisely constructs the real model with concrete data and organizational constraints. Next, the question “What is the probability. . .” refers to step 3, i.e., mathematizing of the assignment process in terms of probability. Furthermore, since the task is formulated with a real-world situation in mind (dividing people into game teams), there is an implicit need to give meaning to the calculated probability in terms of the ‘fairness’ of the division, which is typical of step 5 (interpretation of results in the real world).

Prompt P6 explicitly asks for “a criterion different from the one used so far”, i.e., it prompts the generation of an alternative to the model already discussed. The sub-competencies activated are those of plausibility checking and model comparison (S6.1, S6.3) and, implicitly, the reformulation of the relationships between variables for a new version of the model (S1–2.3). This corresponds to step 6 of the Blum cycle, because it involves critically validating the current model and comparing it with alternative models in order to find a more suitable solution. The request for a new criterion also implies the potential start of a new construction of the real model (step 2), since an alternative rule redefines the link between dice results and team assignments.

At the end of this conversation with ChatGPT-4o, the two teachers produce the following instructional design (GD) as the result of the two teachers’ interactions with ChatGPT-4o:

#### *Game Designers (GD) Activity*

The activity is aimed at a class of twenty students in their first or second year of high school. The estimated time is two hours, including the return of the results found.

The class is divided into groups of five, and each group is given two dice.

#### Prerequisite

Knowledge of the basic elements of statistics and probability

#### Delivery

You are designing a new game and before you start, you need to divide the 12 players into two teams of six at random using the two dice contained in the box. Suggest a strategy for doing this.

#### Work phases

1. Discussion phase: 20 min. After reviewing the assignment, propose a solution strategy, test it, and collect data using the dice provided at the beginning.
2. Discussion phase between the various groups 20 min Each group presents its strategy and the results obtained.

3. Reflection phase 20 min of discussion by the other groups and the teacher of the proposed solution, considering possible simplifications and alternative strategies
4. Could you achieve the same result with just one die? What if you used three dice? Could you use a coin or another tool?
5. Implementation and execution phase 20 min construction of a theoretical model to support the chosen strategy
6. Validation phase 20 min preparation of an instruction booklet to be attached to the game, explaining how to divide it up
7. Final presentation of the finished product 20 min

#### Competencies

Communicating in the presentation of work; interacting in group work; using probabilistic tools to assess the feasibility of the chosen strategy; constructing and analyzing models of extra-mathematical contexts and situations; evaluating and producing justifications for statements; manipulating material aids and tools for mathematical activities.

The activity produced by Silvia and Paolo is very different from the “dice tournament” problem created by ChatGPT-4o and it is notably more open problem. The analysis of its excerpts, reported in Table 4, seems to show that the GD activity allows students to go through almost all the steps of the modeling cycle.

**Table 4.** The analysis of the GD activity designed by Silvia and Paolo.

| #  | Excerpt from the Activity  | Sub-Competencies | Step |
|----|--|------------------|------|
| E1 | You are designing a new game and before you start, you need to divide the 12 players into two teams of six at random using the two dice contained in the box. Suggest a strategy for doing this. | S1–2.1<br>S1–2.4 | 1–2  |
| E2 | Propose a solution strategy, test it, and collect data using the dice provided at the beginning.   | S1–2.2<br>S1–2.3 | 1–2  |
| E3 | Construction of a theoretical model to support the chosen strategy   | S3.1<br>S3.3     | 3    |
| E5 | Could you achieve the same result with just one die? What if you used three dice? Could you use a coin or another tool?  | S5.1<br>S6.3     | 5, 6 |
| E6 | Preparation of an instruction booklet to be attached to the game, explaining how to divide it up   | S6.1<br>S6.2     | 6    |
| E7 | Each group presents its strategy and the results obtained.   | S7.1<br>S7.2     | 7    |

Excerpt E1 seems to encourage the recognition of a problem that can be modeled (S1–2.1) and the identification of the context and real constraints (S1–2.4), i.e., the steps of understanding and structuring (steps 1–2). Students are required to interpret the situation as an authentic probabilistic challenge, in which the construction of the model emerges from concrete needs. The request in E2 seems to facilitate the identification of variables (S1–2.2) and the definition of the relationships between them (S1–2.3), laying the empirical foundations for the construction of the real model (step 2). The transition to mathematizing step (step 3) is explicit in E3, as can be seen from the translation of the situation into mathematical terms (S3.1) and the formalization of relationships (S3.3).

The interpreting and validating steps (steps 5 and 6) seem to emerge in requests E5 for comparison and variation of the model (“Could you achieve the same result with just one die? What if you used three dice? Could you use a coin or another tool?”) and E6 for

“Preparation of an instruction booklet”. These actions involve recontextualizing the results (S5.1) and critically comparing alternative models (S6.3).

Finally, the requirements in E7 refer to the sub-competencies of communication (S7.1) and reflection on the modeling process (S7.2), i.e., step 7. The reference to this step is reinforced in the section on “Competencies” that the two teachers include at the end of their design, when they indicate the competence “Communicating in the presentation of work; interacting in group work”. Moreover, it is worth noting that there is no explicit reference to step 4 in the phases of the activity planned by Silvia and Paolo. However, a reference to this step can be found in the section on competencies mentioned above. In fact, the request to “use probabilistic tools to assess the feasibility of the chosen strategy” refers to the sub-competencies of applying mathematical procedures (S4.1) and justifying results (S4.2), which refer to the step of working mathematically, i.e., step 4.

Considering the absence of prior formal training in mathematical modeling, we can say that the increasing specificity and modeling orientation of the teachers’ instructions appear to be fostered by the professional development program (along with the use of ChatGPT-4o) in which they participated, which oriented their design reasoning toward the progressive structuring and mathematization of the task.

## 6. Discussion and Conclusions

The rapid diffusion of Generative Artificial Intelligence (GenAI) in education has opened new opportunities and challenges for teachers’ instructional design. While AI tools such as ChatGPT-4o can provide immediate access to ideas, texts, and ready-made lesson materials, their pedagogical effectiveness depends on teachers’ ability to interpret, adapt, and critically validate the generated content. Within mathematics education, this issue becomes particularly relevant, since meaningful learning requires activities that connect real-world contexts with formal mathematical reasoning (Blum & Borromeo Ferri, 2009; Niss & Højgaard, 2019). Against this background, we wonder if the modeling cycle can offer a powerful theoretical and practical framework to guide teachers in navigating GenAI-assisted design processes. More precisely, in this study, we addressed the following research question:

(RQ) How can the modeling cycle serve as a reference framework for GenAI-assisted instructional design of learning activities by teachers?

To address such a question, the present study investigated how upper secondary school mathematics teachers engage with ChatGPT-4o in the design of modeling-based activities. We focused on the case of Silvia and Paolo within the PROMOTED training program. Data were collected from multiple sources and analyzed qualitatively according to Blum’s (2015) modeling cycle and its sub-competencies (Maass, 2006; Bassi & Brunetto, 2023). The analysis aimed to identify explicit and implicit references to the modeling steps (understanding, structuring, mathematizing, working mathematically, interpreting, validating, presenting) and to map how these were enacted in the teachers’ dialogue with GenAI.

The case of Silvia and Paolo seems to show that the understanding and structuring steps of the modeling cycle can guide teachers in formulating prompts that orient the GenAI toward meaningful real-world contexts. By framing their design questions through the initial modeling steps, the teachers were able to shift from ChatGPT-4o’s generic “dice tournament” scenario to a problem involving decision-making under uncertainty—a situation requiring probabilistic reasoning. Thus, the modeling cycle operated as a scaffold that allowed teachers to recognize and redefine authenticity in AI-generated contexts.

During the mathematizing and working mathematically steps, the cycle provided a structure for validating and refining AI proposals. The teachers’ iterative prompts seem

to show that the modeling framework encouraged them to interrogate ChatGPT-4o's suggestions critically, ensuring that the generated activities included clear mathematical relationships and calculations. This aligns with findings by [Gurl et al. \(2025\)](#) and [Noster et al. \(2024\)](#), who argue that GenAI can stimulate teachers' creativity but also introduces the need for rigorous mathematical supervision. Within the modeling framework, interaction between teachers and AI becomes a process in which mathematical activity is produced.

The interpreting and validating steps could reveal the metacognitive function of the modeling cycle in ChatGPT-4o-assisted design. In asking for alternative criteria, the teachers activated reflection on the plausibility, fairness, and generalizability of the model—activities that correspond to validation and interpretation steps ([Blum & Borromeo Ferri, 2009](#)). These findings can support the argument of [Drijvers \(2024\)](#) and [Giannakos \(2025\)](#) that frameworks rooted in mathematical practice are essential to transform GenAI from a generator of content into a trigger for professional reflection.

Their final design seems to highlight how the modeling cycle promotes not only task production but also professional development. Following [Ratnayake et al. \(2020\)](#) and [Bassi et al. \(2024\)](#), this process can enable teachers to articulate the rationale of their design choices and to reflect collaboratively on their pedagogical implications. In this sense, the modeling cycle seems to operate as both a cognitive and professional mediation tool between human expertise and generative technology.

We stress that the two teachers (Silvia and Paolo), like all participants in the professional development program, were expert teachers. However, they had no prior knowledge of Blum's modeling cycle. Thus, the professional development program, in combination with the use of ChatGPT-4o, appears to have fostered greater specificity and a stronger modeling orientation in the teachers' instructions, shaping their design reasoning toward the progressive structuring and mathematization of the task.

Moving back to the RQ, the findings of this study seem to show that [Blum's \(2015\)](#) modeling cycle can serve as a framework for teachers' instructional design processes when interacting with Generative Artificial Intelligence (GenAI) tools such as ChatGPT-4o. The cycle can provide a cognitive and operational structure that allows teachers to anchor the otherwise fluid and open-ended generative outputs of AI to specific modeling sub-competencies and pedagogical purposes. The modeling cycle can be reinterpreted as a design framework for orchestrating ChatGPT-4o interactions in teacher education. It may help teachers navigate between exploration and formalization, creativity and validation, human agency and algorithmic generation. Consistent with the literature on TPACK ([Mishra & Koehler, 2006](#)), this integration positions ChatGPT-4o not as a substitute for teachers' professional knowledge but as a catalyst for its activation and reconfiguration within a structured theoretical model.

While the findings provide insights into the potential of the modeling cycle as a framework for ChatGPT-4o-assisted instructional design, this study has some limitations that should be acknowledged. The research was based on a small-scale case study involving two teachers within a specific professional development context; consequently, the results cannot be generalized but rather serve as exploratory evidence of how modeling and AI interaction can co-evolve in teacher reasoning. Moreover, the data were limited to written prompts and textual exchanges, without direct observation of classroom enactments or follow-up interviews that could reveal teachers' evolving beliefs and professional trajectories.

Future research could address these limitations by analyzing all the data collected, such as the whole interaction with ChatGPT-4o, the video recording of the training session and final feedback questionnaire, and examining how teachers appropriate the modeling framework over time and across diverse contexts.

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