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Enhancing Incremental NDI Control with $\mathcal{L}_1$ Adaptive Augmentation in a Longitudinal Autopilot

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Abstract. In this work the effectiveness of an adaptive control augmentation for high-performance fixed-wing aircraft is investigated. An Incremental Nonlinear Dynamic Inversion (INDI) pitch rate control loop is designed within the cascade of a longitudinal autopilot. A minimally invasive $\mathcal{L}_1$-adaptive controller is then designed to counteract system uncertainties and variations, with an hedging procedure that ensures bounded adaptation during saturation events. A time-domain performance analysis is performed, showing that the baseline INDI controller draws a clear benefit from the $\mathcal{L}_1$-adaptive augmentation.

Introduction

Most modern fixed-wing aircraft achieve stability through a Flight Control System (FCS) that relies on Gain-Scheduling (GS). In this method, the varying system dynamics is taken into account by switching the controller parameters in real time, based on some scheduling rules. Although there are techniques that can speed up the scheduling process [1], a significant effort in design and analysis is often required, and the full capabilities of the aircraft may not be exploited. In Incremental Nonlinear Dynamic Inversion (INDI) [2] architectures, a feedback linearization of the system dynamics allows for the desired response to be enforced in the complete flight envelope without the need for scheduling, in a modular and easily interpretable structure. However, the model-based implementations of INDI control laws suffer from model mismatches, whose effect on the response can be mitigated using an adaptive control augmentation. $\mathcal{L}_1$ adaptive control theory [3] offers a promising alternative to comparable architectures, as it avoids inducing the system into dangerous oscillatory behaviors resulting from fast adaptation. In this work we consider a nonlinear model for the longitudinal dynamics of an airframe, where a cascade autopilot controlling the climb angle is designed with a model-based INDI pitch rate control loop. Then, a minimally invasive $\mathcal{L}_1$ Piecewise Constant (PwC) adaptive [3] augmentation is designed to compensate for any uncertainty in the open-loop pitch rate dynamics. With a proposed hedging [4] method for the predictor in the adaptive controller, safe adaptation during actuator saturation events is ensured, which represents a step forward when compared to similar works [5].

Longitudinal Autopilot Baseline Design

Control Structure. A cascade architecture, visible in Fig. 1, is adopted to control a model for the longitudinal dynamics of the airframe shown in Fig. 2, using a tail elevator deflection δ, whose actuator is modeled through a second-order transfer function $G_{\mathrm{a}}(s)$ with position and rate limits $\delta_{\mathrm{M}}, \dot{\delta}_{\mathrm{M}}$. The two outer loops are closed with proportional gains K_γ and K_{az}, controlling respectively the climb angle γ and the vertical acceleration $a_z = \dot{w}$. The inner loop contains a model-based INDI controller, designed so that the pitch rate tracking error $e_q = q - q_{ref}$ remains small.

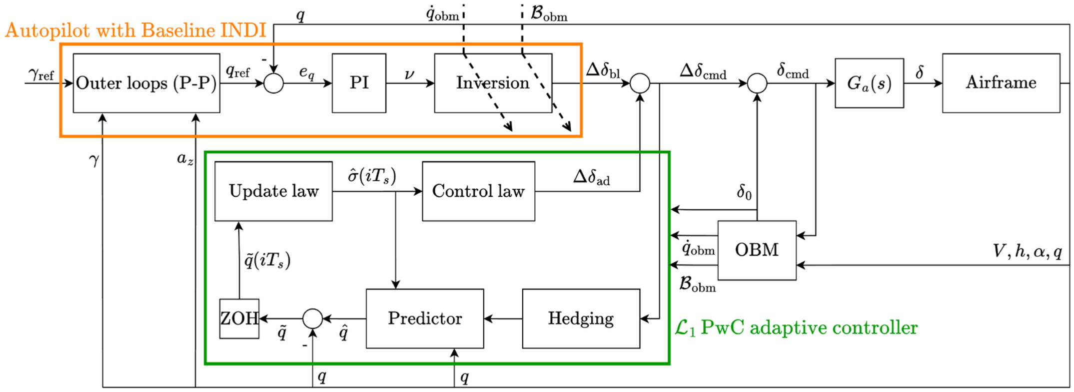

Fig. 1. Full autopilot structure with baseline INDI and $\mathcal{L}_1$-adaptive controllers.

Pitch dynamics. The second body-frame component of the Euler equations with the aerodynamic model adopted from [1] reads

$$\dot{q} = \frac{1}{J} q_\infty S_{\text{ref}} d_{\text{ref}} \left(C_{\mathcal{M}/q} q + C_{\mathcal{M},\alpha}(\alpha, V, M) + C_{\mathcal{M}/\delta} \delta \right), \tag{1}$$

where J is the moment of inertia of the airframe, q_∞ is the asymptotic dynamic pressure, S_{ref} and d_{ref} are the reference surface and length, $C_{\mathcal{M}/q}$ is the damping-in-pitch derivative, $C_{\mathcal{M},\alpha}(\cdot)$ is a nonlinear function depending on the angle of attack α, the airspeed V, and the Mach number M, while $C_{\mathcal{M}/\delta}$ is the elevator control derivative.

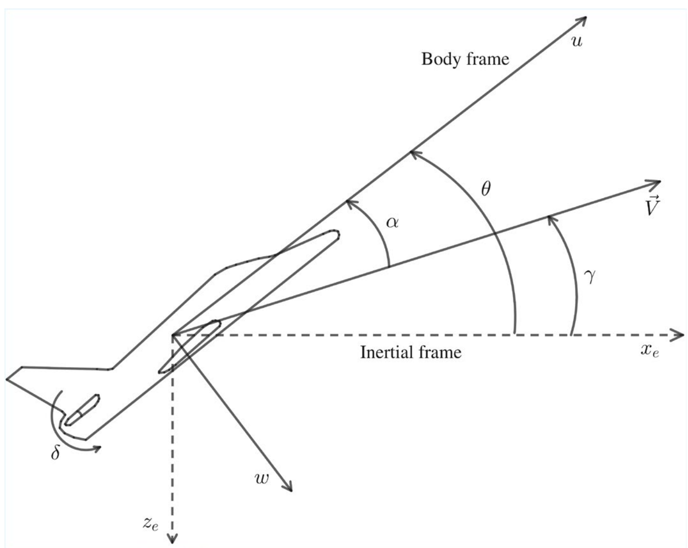

Fig. 2. The motion of the longitudinal airframe in the body and inertial frames.

INDI Baseline Controller. The first-order Taylor expansion of Eq. (1) is computed at each time step around the current condition, denoted by the subscript 0. The assumption of a separation in the timescales between (slow) rigid-body dynamics and (fast) actuator dynamics is adopted to simplify the obtained incremental dynamics [2] down to

$$\dot{y} \approx \dot{y}_0 + \mathcal{B}(\delta - \delta_0) \approx \dot{y}_0 + \mathcal{B}\Delta\delta_{cmd} \tag{2}$$

where $\mathcal{B} = \frac{1}{J} q_{\infty,0} C_{\mathcal{M}/\delta} S_{ref} d_{ref}$ is the control effectiveness and $\Delta\delta_{\text{cmd}}$ is the total incremental command. The baseline INDI incremental command $\Delta\delta_{\text{bl}}$ is then computed as

$$\Delta\delta_{\text{bl}} = \mathcal{B}_{\text{obm}}^{-1}(\nu - \dot{q}_{\text{obm}}) \tag{3}$$

with $\dot{q}_{obm}$ and $\mathcal{B}_{obm}$ representing estimates of $\mathcal{B}$ and $\dot{q}_0$, obtained from an On-Board Model (OBM) using nominal aerodynamics $\bar{C}_{\mathcal{M}/q}$, $\bar{C}_{\mathcal{M},\alpha}$, $\bar{C}_{\mathcal{M}/\delta}$. The total commanded input coming from

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the baseline INDI controller is $\delta_{cmd} = \Delta\delta_{\mathrm{bl}} + \delta_0$, with δ_0 estimated with a model of the actuator $\bar{G}_{\mathrm{a}}(\mathrm{s})$. The pseudo control $\nu = K_P(q_{ref} - q) + K_I \int (q_{ref} - q)$, with K_P, K_I positive constants, relates to the desired second-order pitch rate dynamics, achieved almost exactly within the whole flight envelope as long as the OBM estimates are sufficiently accurate.

$\mathcal{L}_1$-Adaptive Control Augmentation Design

In a practical scenario, model uncertainty and exogenous disturbances will affect the pitch rate dynamics of Eq. 1. These effects can be considered through a lumped uncertainty σ_m, matched inside the control channel of the open-loop pitch rate dynamics as

$$\dot{q} = \dot{q}_{\mathrm{obm}} + \mathcal{B}_{\mathrm{obm}}(\Delta\delta + \sigma_m). \tag{4}$$

Predictor. Within the $\mathcal{L}_1$-adaptive controller, the predictor is a dynamical system which computes an estimate of the pitch rate $\hat{q}$ according to:

$$\dot{\hat{q}} = \dot{q}_{\mathrm{obm}} + \mathcal{B}_{\mathrm{obm}}(\Delta\delta_{\mathrm{a}} + \hat{\sigma}_m) + L(q - \hat{q}), \qquad L > 0 \tag{5}$$

Hedging. In Eq. (4), $\Delta\delta_{\mathrm{a}}$ is obtained by estimating the incremental elevator deflection according to

$$\Delta\delta_{\mathrm{a}} = \bar{G}_{\mathrm{a}}(t) * \mathrm{sat}_{\delta_{\mathrm{M}}}\left\{\int_0^t \mathrm{sat}_{\dot{\delta}_{\mathrm{M}}}\left[\frac{d}{dt}(\delta_{\mathrm{cmd}}(\tau))\right] d\tau\right\} - \delta_0, \quad \delta_{\mathrm{cmd}} = \Delta\delta_{\mathrm{ad}} + \Delta\delta_{\mathrm{bl}} + \delta_0 \tag{6}$$

where $*$ is the convolution operator and $\mathrm{sat}_\delta(\cdot)$ is the symmetric saturation function. This modification guaranties that adaptation is not mistakenly driven during maneuvers or actuator saturation, avoiding potentially catastrophic events.

Update Law. The prediction error $\tilde{q} = \hat{q} - q$ is used to drive the parameter update law that estimates the uncertainty using a piecewise constant approximation according to

$$\hat{\sigma}_m(t) = K_{\mathcal{L}_1}(T_s)\tilde{q}(iT_s), \quad \forall\, t \in [iT_s, (i+1)T_s), \tag{7}$$

$$K_{\mathcal{L}_1}(T_s) = -\mathcal{B}_{\mathrm{obm}}^{-1}\left(1 - e^{-LT_s}\right)^{-1} L e^{-LT_s}, \tag{8}$$

where the adaptation speed is regulated through L and the sampling time of the controller T_s.

Control Law. The adaptive control law that compensates for the uncertainty is

$$\Delta\delta_{\mathrm{ad}} = -C(s)\,\hat{\sigma}_m(s), \qquad C(s) = \frac{\omega_{\mathrm{f}}}{s + \omega_{\mathrm{f}}}, \tag{9}$$

where s is the Laplace variable and $\omega_{\mathrm{f}} > 0$ adjusts the filtering action that prevents exciting the system at high frequencies.

Tuning remarks. The sampling period T_s should be selected as high as the available CPU allows. With $T_s \to 0$, and $F_\sigma(s)$ denoting the transfer function from σ_m to $\hat{\sigma}_m$ obtained by closing the loop in the prediction dynamics with Eq. 7 and Eq. 8, one gets that

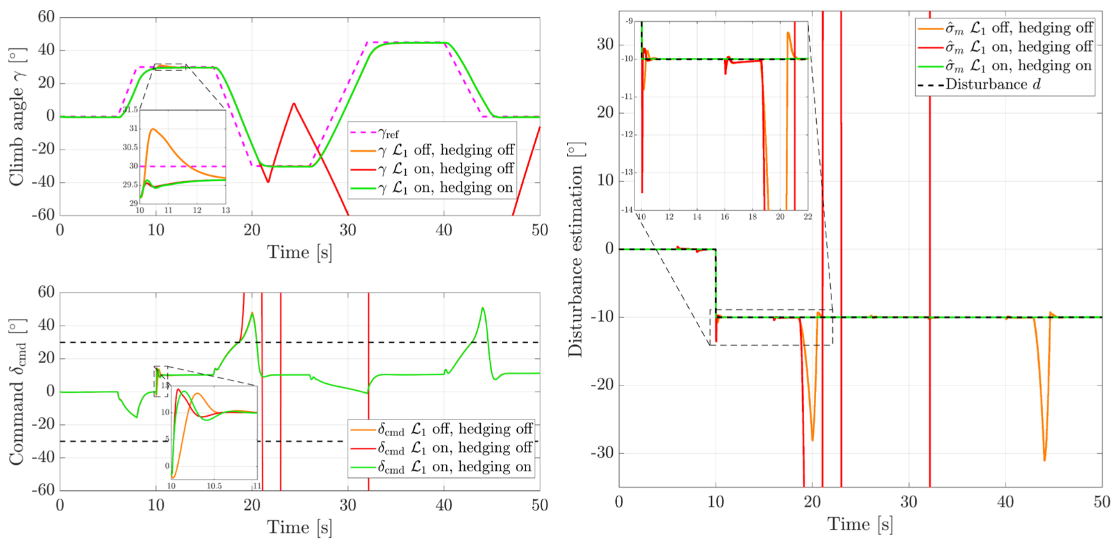
$$F_\sigma(s) = -K_{\mathcal{L}_1}\left(s - (-L + \mathcal{B}_{\mathrm{obm}} K_{\mathcal{L}_1})\right)^{-1} \mathcal{B}_{\mathrm{obm}} = \frac{e^{-LT_s}}{1 + s(1 - e^{-LT_s}/L)}, \tag{10}$$

so that $\lim_{t\to\infty}[\hat{\sigma}_m(t) - \sigma_m(t)] \overset{L\to 0}{\rightarrow} 0$. The bandwidth of the filter ω_{f} should be as high as possible without exceeding the available bandwidth of the system.

Performance Analysis
The controllers and the airframe model are implemented in MATLAB/Simulink®. The adopted controller parameters are shown in Table 1. The gains of the baseline controller were tuned with the MATLAB® function systune, while the adaptive controller was tuned conservatively using the guidelines described in the previous section.

Table 1. Tuned controller parameters

Parameter	Value
K_γ	-1324
K_{az}	-0.0093
K_P	18.55
K_I	67.58
T_s	0.01s
L	0.1
ω_f	50 rad/s



(a) Climb angle γ and commanded elevator deflection δ_{cmd}.

(b) Estimation performance in the adaptive controller with respect to disturbance d.

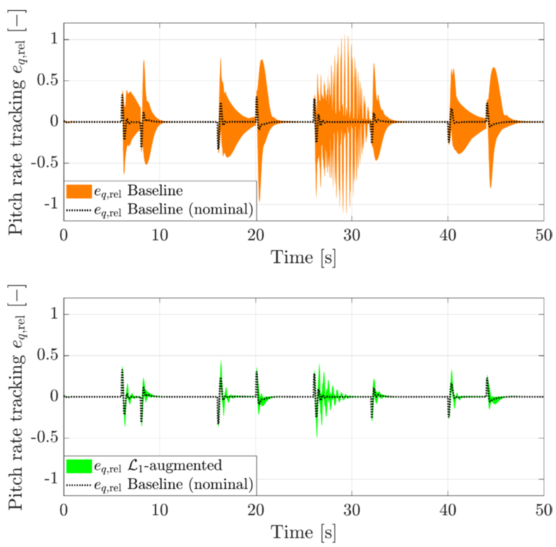
Fig. 3. Results of nonlinear simulations with injected disturbance d, comparing the baseline and $\mathcal{L}_1$-augmented controllers with and without predictor hedging.

Disturbance Rejection. With a reference signal γ_{ref} that makes the system cross its envelope, a disturbance $d(t) = -10\text{step}(t - 10)$ deg is injected in the control channel, with the objective of proving the effectiveness of adaptation and the importance of hedging. Fig. 3a shows the climb angle response and the corresponding total command, while Fig. 3b highlights the estimation performance of the adaptive controller. The $\mathcal{L}_1$ controller slightly increases the system responsiveness. However, with active adaptation, the proposed hedging methodology proves to be fundamental in avoiding instability and providing correct disturbance estimation. The baseline INDI controller also achieves an acceptable performance degradation, thanks to the integral action tuned by the gain K_I.

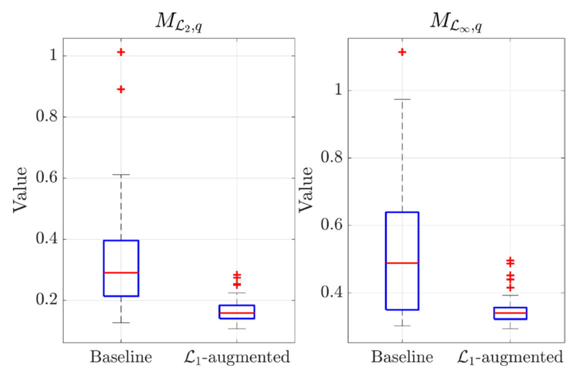
Monte Carlo Analysis. The following quantities are defined:

$$e_{q,\mathrm{rel}} = \frac{e_q}{\| q_{\mathrm{ref}} \|_{\mathcal{L}_\infty}}, \quad M_{\mathcal{L}_2,q} = \frac{\| e_q \|_{\mathcal{L}_2}}{\| q_{\mathrm{ref}} \|_{\mathcal{L}_2}}, \quad M_{\mathcal{L}_\infty,q} = \frac{\| e_q \|_{\mathcal{L}_\infty}}{\| q_{\mathrm{ref}} \|_{\mathcal{L}_\infty}}; \tag{11}$$

and a Monte Carlo study through nonlinear simulations is performed, considering parametric uncertainties. The relative pitch rate tracking error $e_{q,rel}$ is shown in Fig. 4a, comparing the baseline and the $\mathcal{L}_1$-augmented controllers to a nominal simulation. Undesirable pitch oscillations are greatly reduced by the adaptive augmentation when compared to the responses obtained with the baseline controller. The box plots in Fig. 4b highlight an improved consistency of performance under the varying conditions resulting from uncertainty.



(a) Time histories of the relative pitch rate tracking error



(b) Values for the pitch rate tracking performance metrics $M_{\mathcal{L}_2,q}$, $M_{\mathcal{L}_\infty,q}$.

Fig.4. Results of the Monte Carlo analysis comparing pitch rate tracking performance for baseline and $\mathcal{L}_1$-augmented controllers

Conclusions

The $\mathcal{L}_1$-adaptive augmentation of the INDI controller presented in this paper proved to be beneficial to the dynamic response of the airframe. Moreover, the proposed hedging method is fundamental to avoid inducing the system into unstable behavior in the case of actuator saturation events. Although the observations from time-domain simulations show an increase in tracking and disturbance rejection capabilities, a strict stability analysis would complete the evaluation of the architecture, which can potentially be extended to a complete aircraft model with multiple control channels.

References

[1] P. Gahinet, P. Apkarian, Automated tuning of gain-scheduled control systems, 52nd IEEE Conference on Decision and Control (2013) pp. 2740-2745. https://doi.org/10.1109/CDC.2013.6760297

[2] S. Sieberling, Q. P. Chu, J. A. Mulder, Robust flight control using incremental nonlinear dynamic inversion and angular acceleration prediction, Journal of guidance, control, and dynamics (2010) 33(6) pp. 1732-1742. https://doi.org/10.2514/1.49978

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[3] N. Hovakimyan, C. Cao, L1 adaptive control theory: Guaranteed robustness with fast adaptation. Society for Industrial and Applied Mathematics, Philadelphia, 2010. https://doi.org/10.1137/1.9780898719376

[4] E. N. Johnson, A. J. Calise. Pseudo-control hedging: A new method for adaptive control. Advances in navigation guidance and control technology workshop (2000), Alabama, USA.

[5] P. Bhardwaj, V. S. Akkinapalli, J. Zhang, S. Saboo, F. Holzapfel, Adaptive augmentation of incremental nonlinear dynamic inversion controller for an extended f-16 model. AIAA Scitech 2019 Forum (2019) p. 1923. https://doi.org/10.2514/6.2019-1923