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ANALYTICAL OPTIMIZATION OF POST MISSION DISPOSAL MANEUVERS TOWARDS AN EARTH RE-ENTRY WITH AVERAGED DYNAMICS MODELS

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This paper develops a triply-averaged dynamics model for an Earth satellite under the perturbation of Earth's oblateness and gravitational attraction from the Moon and the Sun. The dynamics is averaged over one orbital period of a satellite, one orbital period of a third body, and lastly one variation period of right ascension of the ascending node (RAAN) of a satellite, the so-called elimination of the node. The developed model is validated by comparing with a high-fidelity model and then it is integrated in the Hamiltonian formulation of the dynamics. The simplified model is used in developing a post mission disposal design technique for Earth satellites targeting an Earth re-entry. Exploiting the averaged-model significantly facilitates the maneuver optimization procedure and saves much computational time. The proposed technique is applied to a Highly Elliptical Orbit (HEO) satellite and the obtained results are validated through a high-fidelity model.

INTRODUCTION

The space debris problem due to increasing man-made space objects has been of interest by the space community for decades. Several national or international organizations published space mitigation guidelines, such as the Federal Communications Commission (FCC),[1] the Inter-Agency Space Debris Coordination Committee (IADC),[2] and the United Nations (UN),[3] preventing prolonged stay in geostationary orbit (GEO) and limiting passage in low Earth orbit (LEO). Among all the mitigation measures for space debris, post mission disposal is of importance due to its effectiveness in de-orbiting satellites at the end of their mission and large contributions to mitigation of space debris.

Traditionally, disposal maneuvers are computed by global optimization involving numerical orbit propagation,[4–8] which is computationally expensive as propagation of orbits for decades need to be carried out many times during optimization. The heavy computational burden discourages operators to design and implement post mission disposal for their spacecrafts because the required ground resources increase as number of spacecrafts increases. Therefore, it is desired to develop analytical methods for optimizing disposal maneuvers to reduce computation time and even to enable onboard autonomous disposal maneuvers design.

This research focuses on design of post mission disposal of an Earth satellite in HEO through an Earth re-entry. Natural perturbations could be enhanced by impulsive maneuvers, which moves a

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spacecraft to a trajectory naturally evolving towards an Earth re-entry. The remainder of this paper are organized as follows. Section II presents the semianalytical model for orbital perturbations used in the following sections. Section III discusses the process of the elimination of the node. Section IV develops the design technique of post mission disposal. The last section gives the conclusion and remarks on future work.

SIMPLIFIED DYNAMICS MODELS OF SATELLITE ORBITS

Perturbed two-body problem

The dynamics of an orbit of an Earth satellite is generally modeled as a perturbed two-body problem, and the equations of motion are given by

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} + \mathbf{f} \tag{1}$$

where $\mathbf{r}$ is the radial vector of a satellite with respect to the Earth center whose magnitude is r, μ is the gravitational parameter of the Earth, and $\mathbf{f}$ is total acceleration caused by forces other than the central gravitational attraction of the Earth.

The equations of motion Eq. (1) are straightforward to establish but suffer from flaws in numerical computation since all three Cartesian coordinates experience large changing as a satellite evolve around the Earth. One could transform Eq. (1) to another set of equations, the well-known Lagrange planetary equations,[9] by exploiting the method of variation of parameters,

$$\begin{aligned}
\frac{\mathrm{d}a}{\mathrm{d}t} &= \frac{2}{na}\frac{\partial R}{\partial M} \\
\frac{\mathrm{d}e}{\mathrm{d}t} &= \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial M} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega} \\
\frac{\mathrm{d}i}{\mathrm{d}t} &= \frac{1}{na^2\sqrt{1-e^2}\sin i}\left(\cos i\frac{\partial R}{\partial \omega} - \frac{\partial R}{\partial \Omega}\right) \\
\frac{\mathrm{d}\Omega}{\mathrm{d}t} &= \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} \\
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} - \cos i\frac{\mathrm{d}\Omega}{\mathrm{d}t} \\
\frac{\mathrm{d}M}{\mathrm{d}t} &= n - \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial e} - \frac{2}{na}\frac{\partial R}{\partial a}
\end{aligned} \tag{2}$$

where $a, e, i, \Omega, \omega, M$ are classical Keplerian elements, $n = \sqrt{\mu/a^3}$, and R is a disturbing function depending on the perturbations of interest.

The orbit of a satellite in HEO, which is of interest in this research, is mainly affected by perturbations due to the Earth's oblateness, and the gravitational attraction by the Moon and the Sun. The disturbing function of perturbation due to the Earth's oblateness is given by[10]

$$R_{J_2} = -\frac{\mu}{r}J_2\left(\frac{R_\oplus}{r}\right)^2\frac{1}{2}\left[3\sin^2(\omega+f)\sin^2 i - 1\right] \tag{3}$$

where J_2 is the second zonal harmonics, $R_\oplus$ is the equatorial radius of the Earth, and r is given by

$$r = \frac{a(1-e^2)}{1+e\cos f}, \tag{4}$$

where f is true anomaly of an orbit.

The disturbing function of perturbation by a third-body gravitational attraction is given by[11,12]

$$R_{3b} = \frac{\mu_3}{r_3} \sum_{l=2}^{\infty} \left(\frac{r}{r_3}\right)^l P_l(\cos S), \tag{5}$$

where μ_3, r_3 are the gravitational parameter of the third body and the radial distance between the third body and the Earth, respectively, $P_l(\cdot)$ is the l-th order Legendre polynomial, reported in Table 1, and S is the angle between the position vectors of a satellite and a third body as viewed from the Earth, which is given by

$$\cos S = \hat{\mathbf{r}} \cdot \hat{\mathbf{r}}_3 = (\hat{\mathbf{p}} \cos f + \hat{\mathbf{q}} \sin f) \cdot \hat{\mathbf{r}}_3 = A \cos f + B \sin f \tag{6}$$

where $\hat{\mathbf{r}}, \hat{\mathbf{r}}_3$ represent the directions of a satellite and a third body, respectively, $\hat{\mathbf{p}}$ is the unit vector pointing the perigee of a satellite's orbit and $\hat{\mathbf{q}}$ is orthogonal to $\hat{\mathbf{p}}$ in the orbital plane.

Table 1. Legendre polynomials up to 4th order

l	$P_l(x)$
0	1
1	x
2	$\frac{1}{2}(3x^2 - 1)$
3	$\frac{1}{2}(5x^3 - 3x)$
4	$\frac{1}{8}(35x^4 - 30x^2 + 3)$

The formulation of $\hat{\mathbf{r}}_3$, $\hat{\mathbf{p}}$, and $\hat{\mathbf{q}}$ are reported as follows,

$$\hat{\mathbf{r}}_3 = \begin{bmatrix} \cos(\omega_3 + f_3)\cos\Omega_3 - \cos i_3 \sin(\omega_3 + f_3)\sin\Omega_3 \\ \cos(\omega_3 + f_3)\sin\Omega_3 + \cos i_3 \sin(\omega_3 + f_3)\cos\Omega_3 \\ \sin i_3 \sin(\omega_3 + f_3) \end{bmatrix}, \tag{7}$$

$$\hat{\mathbf{p}} = \begin{bmatrix} \cos\omega\cos\Omega - \cos i \sin\omega\sin\Omega \\ \cos\omega\sin\Omega + \cos i \sin\omega\cos\Omega \\ \sin i \sin\omega \end{bmatrix}, \tag{8}$$

$$\hat{\mathbf{q}} = \begin{bmatrix} -\sin\omega\cos\Omega - \cos i \cos\omega\sin\Omega \\ -\sin\omega\sin\Omega + \cos i \cos\omega\cos\Omega \\ \sin i \cos\omega \end{bmatrix}. \tag{9}$$

Partial derivatives of a disturbing function with respect to Keplerian elements can be computed and evolution of a satellite orbit is obtained by numerically integrating Eq. (2). However, numerical integration leads to slow computation, especially when the dynamics is included in the loop of maneuver optimization. On the other hand, short periodic variations of Keplerian elements are of no interest for many applications, for instance in this research, post mission disposal design.

Averaging techniques and averaged models

While numerical methods are witnessed ever-increasing popularity in solving Eq. (1) as computers become faster and faster nowadays, analytical and semi-analytical methods are in no way obsolete. The semianalytical models based on averaging techniques can simplify the dynamics a lot. The core idea is to average the disturbing function over fast angles, to eliminate the short periodic terms in the disturbing function. A disturbing function is averaged over one orbital period of a satellite, as demonstrated in Eq. (10),

$$\overline{R} = \frac{1}{T}\int_{t_0}^{t_0+T} R\,\mathrm{d}t = \frac{1}{2\pi}\int_0^{2\pi} R\,\mathrm{d}M \tag{10}$$

since

$$M = \frac{2\pi}{T}(t - t_0). \tag{11}$$

All elements except mean anomaly are considered constant within one orbital period. Depending upon the formulation of a disturbing function, the variable of integration is changed from mean anomaly M to true anomaly f or eccentric anomaly E for computational convenience through the following relations,

$$\mathrm{d}M = \frac{r^2}{a^2\eta}\mathrm{d}f = \frac{r}{a}\mathrm{d}E. \tag{12}$$

The resulting single-averaged disturbing potential from Eq. (10) is averaged again over one orbital period of a third body in case of third-body perturbation,

$$\overline{\overline{R}} = \frac{1}{T_3}\int_{t_0}^{t_0+T_3} \overline{R}\,\mathrm{d}t = \frac{1}{2\pi}\int_0^{2\pi} \overline{R}\,\mathrm{d}M_3 = \frac{1}{2\pi}\int_0^{2\pi} \overline{R}\frac{r_3^2}{a_3^2\eta_3}\,\mathrm{d}f_3 \tag{13}$$

The disturbing function of the J_2 perturbation is averaged over one orbital period of a satellite,

$$\overline{R}_{J_2} = \frac{\mu J_2 R_\oplus^2}{4a^3\eta^3}\left(2 - 3\sin^2 i\right), \tag{14}$$

where $\eta = \sqrt{1-e^2}$ is defined for computational convenience. In the same manner, the disturbing function of third-body perturbation is averaged as

$$\overline{R}_{3b} = \frac{\mu_3}{r_3}\sum_{l=2}^{\infty}\left(\frac{a}{r_3}\right)^l F_l(A,B,e), \tag{15}$$

in which second to fourth order of F_l are as follows,

$$\begin{aligned}
F_2 &= \frac{1}{4}\left[-2 - 3e^2 + 3A^2\left(1+4e^2\right) + 3B^2\left(1-e^2\right)\right] \\
F_3 &= \frac{5}{16}\left[Ae\left(12+9e^2\right) - 5A^3e\left(3+4e^2\right) - 15AB^2e\left(1-e^2\right)\right] \\
F_4 &= \frac{3}{64}\Big[8 + 40e^2 + 15e^4 + 35B^4\left(1-e^2\right)^2 + 10B^2\left(-4+e^2+3e^4\right) \\
&\quad +35A^4(1+12e^2+8e^4) - 10A^2\left(4+41e^2+18e^4\right) \\
&\quad +70A^2B^2\left(1+5e^2-6e^4\right)\Big]
\end{aligned} \tag{16}$$

As mentioned before, the single-averaged disturbing function is averaged again over one orbital period of the perturbation body,

$$\overline{\overline{R}}_{3b} = \frac{\mu_3}{a_3}\sum_{l=2}^{\infty}\left(\frac{a}{a_3}\right)^l F_l(\alpha_A,\beta_A,\alpha_B,\beta_B,e), \tag{17}$$

where $\alpha_A, \beta_A, \alpha_B, \beta_B$ are defined as

$$\alpha_A = \hat{\mathbf{p}} \cdot \hat{\mathbf{p}}_3, \quad \beta_A = \hat{\mathbf{p}} \cdot \hat{\mathbf{q}}_3, \quad \alpha_B = \hat{\mathbf{q}} \cdot \hat{\mathbf{p}}_3, \quad \alpha_B = \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}_3, \tag{18}$$

in which $\hat{\mathbf{p}}_3, \hat{\mathbf{q}}_3$ are defined in the same manner as $\hat{\mathbf{p}}, \hat{\mathbf{q}}$. The second to fourth order of F_l in Eq. (17) are as follows,

$$\begin{aligned}
F_2 =& \frac{1}{8\eta_3^3}\left[-4 + 3\alpha_A^2 + 3\beta_A^2 + 6e^2(-1 + 2\alpha_A^2 + 2\beta_A^2) + 3\eta^2(\alpha_B^2 + \beta_B^2)\right] \\
F_3 =& -\frac{15ee_3}{64\eta_3^5}\left[(15 + 20e^2)\alpha_A^3 + 10\alpha_B\beta_A\beta_B\eta^2 + \alpha_A(-16 + 15\beta_A^2 + 4e^2(-3 + 5\beta_A^2)\right. \\
& \left.+15\alpha_B^2\eta^2 + 5\beta_B^2\eta^2)\right] \\
F_4 =& \frac{3}{1024\eta_3^7}\left[210\alpha_A^4 + 20\alpha_A^2\left(21\alpha_B^2\eta^2 + 21\beta_A^2 + 7\beta_B^2\eta^2 - 16\right) + 560\alpha_A\alpha_B\beta_A\beta_B\eta^2\right. \\
& + 210\alpha_B^4\eta^4 + 20\beta_A^2\left(7\alpha_B^2\eta^2 + 21\beta_B^2\eta^2 - 16\right) + 420\alpha_B^2\beta_B^2\eta^4 - 320\alpha_B^2\eta^2 + 210\beta_A^4 \\
& + 210\beta_B^4\eta^4 - 320\beta_B^2\eta^2 + 120e^4\left(2\left(7\alpha_A^4 + 2\alpha_A^2\left(7\beta_A^2 - 3\right) + 7\beta_A^4 - 6\beta_A^2 + 1\right)\right. \\
& \left.+e_3^2\left(35\alpha_A^4 + 3\alpha_A^2\left(14\beta_A^2 - 9\right) + 7\beta_A^4 - 9\beta_A^2 + 3\right)\right) \\
& + 20e^2\left(2\left(63\alpha_A^4 + \alpha_A^2\left(63\alpha_B^2\eta^2 + 126\beta_A^2 + 21\beta_B^2\eta^2 - 82\right) + 84\alpha_A\alpha_B\beta_A\beta_B\eta^2\right.\right. \\
& \left.+\beta_A^2\left(21\alpha_B^2\eta^2 + 63\beta_B^2\eta^2 - 82\right) - 6\alpha_B^2\eta^2 + 63\beta_A^4 - 6\beta_B^2\eta^2 + 16\right) \\
& + 3e_3^2\left(105\alpha_A^4 + 3\alpha_A^2\left(35\alpha_B^2\eta^2 + 42\beta_A^2 + 7\beta_B^2\eta^2 - 41\right) + 84\alpha_A\alpha_B\beta_A\beta_B\eta^2\right. \\
& \left.\left.+\beta_A^2\left(21\alpha_B^2\eta^2 + 21\beta_B^2\eta^2 - 41\right) - 9\alpha_B^2\eta^2 + 21\beta_A^4 - 3\beta_B^2\eta^2 + 16\right)\right) \\
& + 3e_3^2\left(175\alpha_A^4 + 10\alpha_A^2\left(35\alpha_B^2\eta^2 + 21\beta_A^2 + 7\beta_B^2\eta^2 - 24\right) + 280\alpha_A\alpha_B\beta_A\beta_B\eta^2\right. \\
& + 175\alpha_B^4\eta^4 + 10\beta_A^2\left(7\alpha_B^2\eta^2 + 7\beta_B^2\eta^2 - 8\right) + 210\alpha_B^2\beta_B^2\eta^4 - 240\alpha_B^2\eta^2 + 35\beta_A^4 \\
& \left.\left.+35\beta_B^4\eta^4 - 80\beta_B^2\eta^2 + 64\right) + 128\right]
\end{aligned} \tag{19}$$

The total single- and double-averaged disturbing functions are following,

$$\begin{aligned}
\overline{R} &= \overline{R}_{J_2} + \overline{R}_{Sun} + \overline{R}_{Moon}, \\
\overline{\overline{R}} &= \overline{\overline{R}}_{J_2} + \overline{\overline{R}}_{Sun} + \overline{\overline{R}}_{Moon}.
\end{aligned} \tag{20}$$

Validation of the Model

The averaging technique allows one to eliminate fast angles in the disturbing function, hence separating long-periodic, and secular effects from short-periodic ones. The procedure is of importance since it simplifies the maneuver optimization. The simplified model is validated against the high-fidelity model described by Gauss' variational equations. The models are propagated for 30 years with the initial Keplerian elements on 22/03/2013 in Table 2. The results are showed in Figure 1, which shows that the single- and double-averaged models coincide well with the high-fidelity model.

Table 2. Keplerian elements of a HEO satellite

a (km)	e $(-)$	i (deg)	Ω (deg)	ω (deg)	M (deg)
87720	0.8766	61.8081	266.4100	253.1972	237.9140

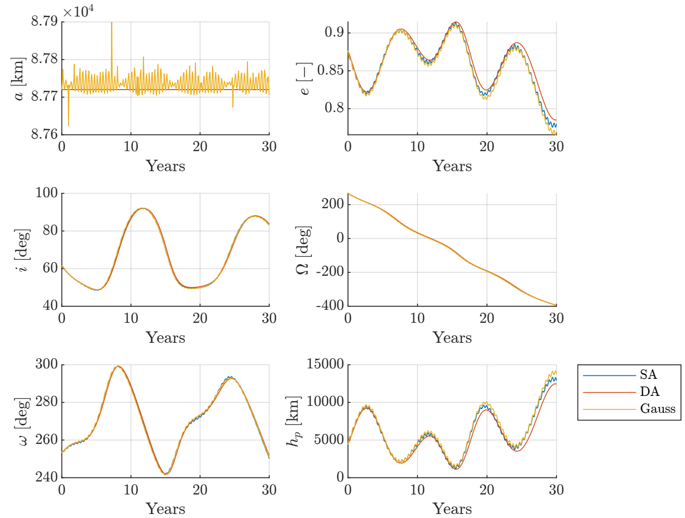

Figure 1. Validation of the averaged models, SA: Single-averaged, DA: Double-averaged, Gauss: High-fidelity

ELIMINATION OF THE NODE

To further simplify the dynamics model, one can average the third body disturbing function over period of variation of Ω, also known as elimination of the node, by

$$\overline{\overline{\overline{R}}}_{3b} = \frac{1}{2\pi} \int_0^{2\pi} \overline{\overline{R}}_{3b} \, \mathrm{d}\Omega = \frac{\mu_3}{a_3} \sum_{l=2}^{\infty} \left(\frac{a}{a_3} \right)^l F_l(e, i, \omega, e_3, i_3, \omega_3), \tag{21}$$

where the node of the third body's orbit Ω_3 is also eliminated since it is coupled with node of a satellite orbit.

DESIGN OF POST MISSION DISPOSAL

The averaged model described in previous sections is now applied to post mission disposal maneuver computation targeting an Earth re-entry. The condition for re-entry is formulated by

$$h_{p,min} = \min h_p(t) < h_{p,target} \tag{22}$$

where

$$h_p = a(1 - e) - R_{\oplus} \tag{23}$$

is perigee height of a satellite orbit, $h_{p,min}$ is the minimal value of perigee height, and $h_{p,target}$ is the target perigee height that we set before the optimization.

Since J_2 and lunisolar perturbation do not affect the value of semimajor axis in long-term, the re-entry condition formulated in Eq. (22) is transformed to

$$e_{max} > e_{crit} \tag{24}$$

where e_{max} is the maximal value of eccentricity and e_{crit} is the critical eccentricity defined by

$$e_{crit} = 1 - \frac{h_{p,target} + R_{\oplus}}{a} \tag{25}$$

since

$$h_{p,min} = a(1 - e_{max}) - R_{\oplus}. \tag{26}$$

The impulsive maneuver is modeled in the common (T, N, H) reference frame as

$$\mathbf{\Delta v} = \begin{bmatrix} \Delta v_T \\ \Delta v_N \\ \Delta v_H \end{bmatrix} = \Delta v \begin{bmatrix} \cos\alpha\cos\beta \\ \sin\alpha\cos\beta \\ \sin beta \end{bmatrix}, \tag{27}$$

where T is the axis tangential to the orbit and pointing to the velocity direction, N axis is normal to T in the orbital plane, and H axis is normal to the orbital plane, $\Delta v, \alpha, \beta$ are the magnitude, in-plane, and out-of-plane angle of the maneuver, respectively.

The Keplerian elements right after the maneuver kep_{post} is computed by adding the Keplerian elements before the maneuver and the variations, as follows,

$$kep_{post} = kep_{pre} + \Delta kep, \tag{28}$$

where Δkep is the combination of variations of Keplerian elements computed by Gauss' variational equations,[13,14]

$$\begin{aligned}
\Delta a &= \frac{2}{n\sqrt{1-e^2}}\sqrt{1+2e\cos f_m + e^2}\Delta v_T \\
\Delta e &= \frac{\sqrt{1-e^2}}{na\sqrt{1+2e\cos f_m + e^2}}\left[2(\cos f_m + e)\Delta v_T - \sqrt{1-e^2}\sin E_m \Delta v_N\right] \\
\Delta i &= \frac{r\cos u_m}{na^2\sqrt{1-e^2}}\Delta v_H \\
\Delta\Omega &= \frac{r\sin u_m}{na^2\sqrt{1-e^2}\sin i}\Delta v_H \\
\Delta\omega &= \frac{\sqrt{1-e^2}}{nae\sqrt{1+2e\cos f_m + e^2}}\left[2\sin f_m \Delta v_T + (\cos E_m + e)\Delta v_N\right] - \cos i\Delta\Omega \\
\Delta M &= -\frac{1-e^2}{nae\sqrt{1+2e\cos f_m + e^2}}\left[\left(2\sin f_m + \frac{2e^2}{\sqrt{1-e^2}}\sin E_m\right)\Delta v_T\right. \\
&\left. +(\cos E_m - e)\Delta v_N\right]
\end{aligned} \tag{29}$$

where f_m is the true anomaly where the maneuver is applied, E_m is the corresponding eccentric anomaly given by

$$\tan\frac{E_m}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{f_m}{2}, \tag{30}$$

and $u_m = \omega + f_m$.

The cost function of optimisation is defined by a weighted sum of the terminal error and magnitude of the maneuver,

$$J = \max\left(\frac{h_{p,min} - h_{p,target}}{h_{p,target}}, 0\right) + w\Delta v \tag{31}$$

where w is weight based on mission scenarios.

CONCLUSION

The paper proposed a post mission disposal manoeuvre design technique for Earth satellites in HEO targeting an Earth re-entry based on semi-analytical models of orbital perturbations, in which disturbing functions and hence the Hamiltonian are averaged three times over one orbital period of a satellite orbit, one orbital period of a third body, and one period of the RAAN variation. The triple averaged model simplifies the maneuver optimization process and considerably reduces the computational burden of maneuver optimization process.

Although the model has relatively less accuracy, the results obtained from the triple averaged model could still be used as a preliminary investigation and first guess for optimisation using the more accurate double averaged model or high-fidelity models to refine the solution.

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