Vortex model of the airborne wind energy systems aerodynamic wake

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1. Background
2. Helicoidal vortex filament
3. Near wake
4. Far wake
5. Power equation
6. Power coefficient
7. Conclusions

Trevisi, F., Riboldi, C. E. D., and Croce, A.: Vortex model of the airborne wind energy systems aerodynamic wake, *accepted for publication in Wind Energy Science*, 2023.

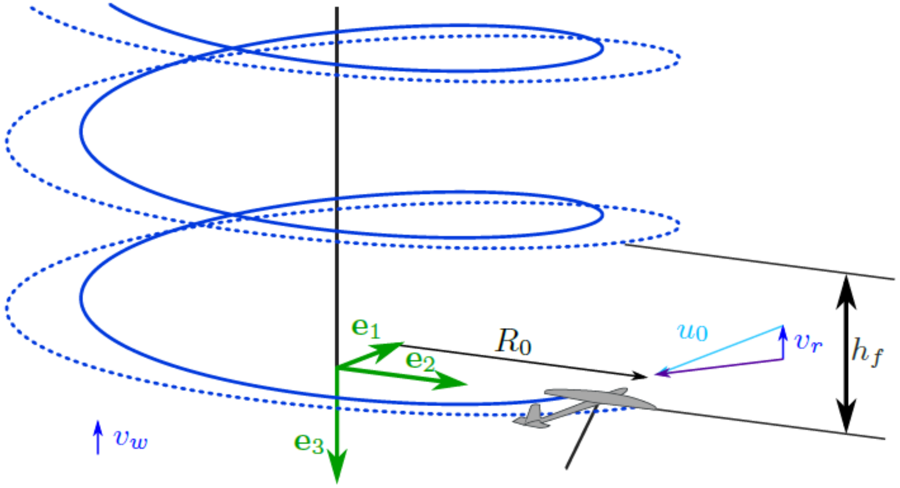
Trevisi, F., Riboldi, C. E. D., and Croce, A.: Refining the airborne wind energy systems power equations with a vortex wake model, *Wind Energy Science Discussions*, 2023.

Optimal control →

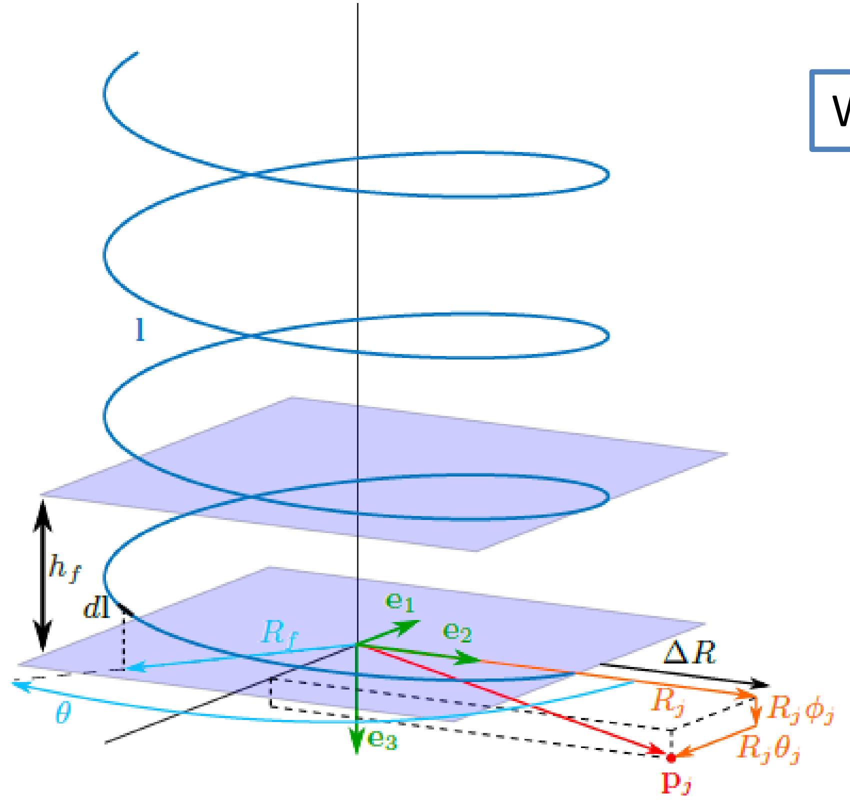
It is beneficial to have low mass and low turning radius R_0 [1]

→ The aerodynamics gets worst at low R_0. Can we model it?

- Momentum methods → They can hardly be used to evaluate the induction at the AWES [2]
- Vortex methods
- CFD →
  - Too expensive
  - Cannot be used in conceptual design



[1] Trevisi, F., Castro-Fernandez, I., Pasquinelli, G., Riboldi, C. E. D., and Croce, A.: Flight trajectory optimization of Fly-Gen airborne wind energy systems through a harmonic balance method, Wind Energy Science, 7, 2039–2058, 2022.
[2] Gaunaa, M., Forsting, A. M., and Trevisi, F.: An engineering model for the induction of crosswind kite power systems, Journal of Physics: Conference Series, 1618, 032 010, 2020.



$$\lambda_0 = \frac{2\pi R_0}{h_0}$$

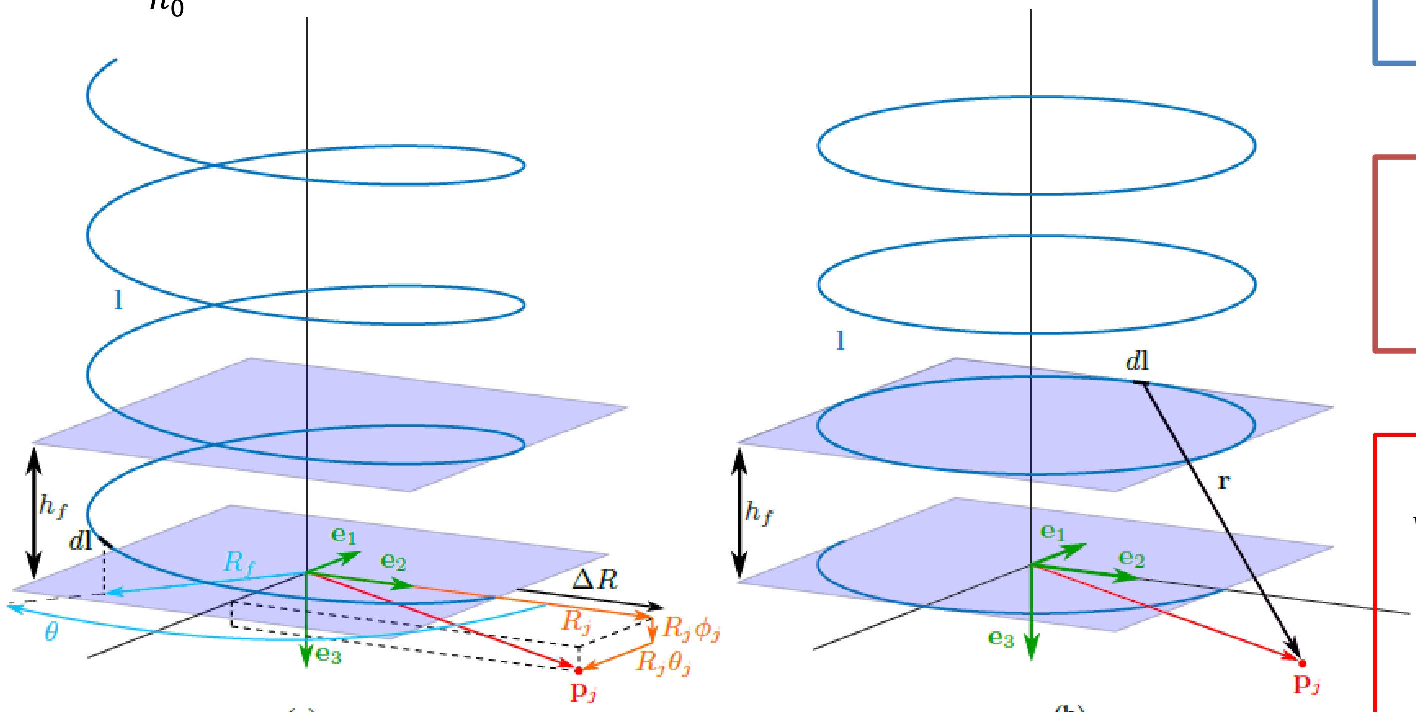
We parametrize the helicoidal vortex filament to write the Biot-Savat law

$$\boldsymbol{w}_{j,f} = \frac{\Gamma}{4\pi}\int_0^{+\infty} \frac{d\boldsymbol{l}\times\boldsymbol{r}}{|\boldsymbol{r}|^3}$$

$$w_{j,f} = -\frac{\Gamma}{4\pi\Delta R}\int_0^{+\infty} \frac{N_\sim(\eta,(\theta-\theta_j))}{\left(D_\sim\left(\eta,(\theta-\theta_j)\right) + D_/\left(\phi_j, \frac{1-\eta}{1-\eta_0}\frac{\theta}{\lambda_0}\right)\right)^{3/2}} d\theta = -\frac{\Gamma}{4\pi\Delta R}\Upsilon$$

How can we solve this integral?

$$\lambda_0 = \frac{2\pi R_0}{h_0}$$



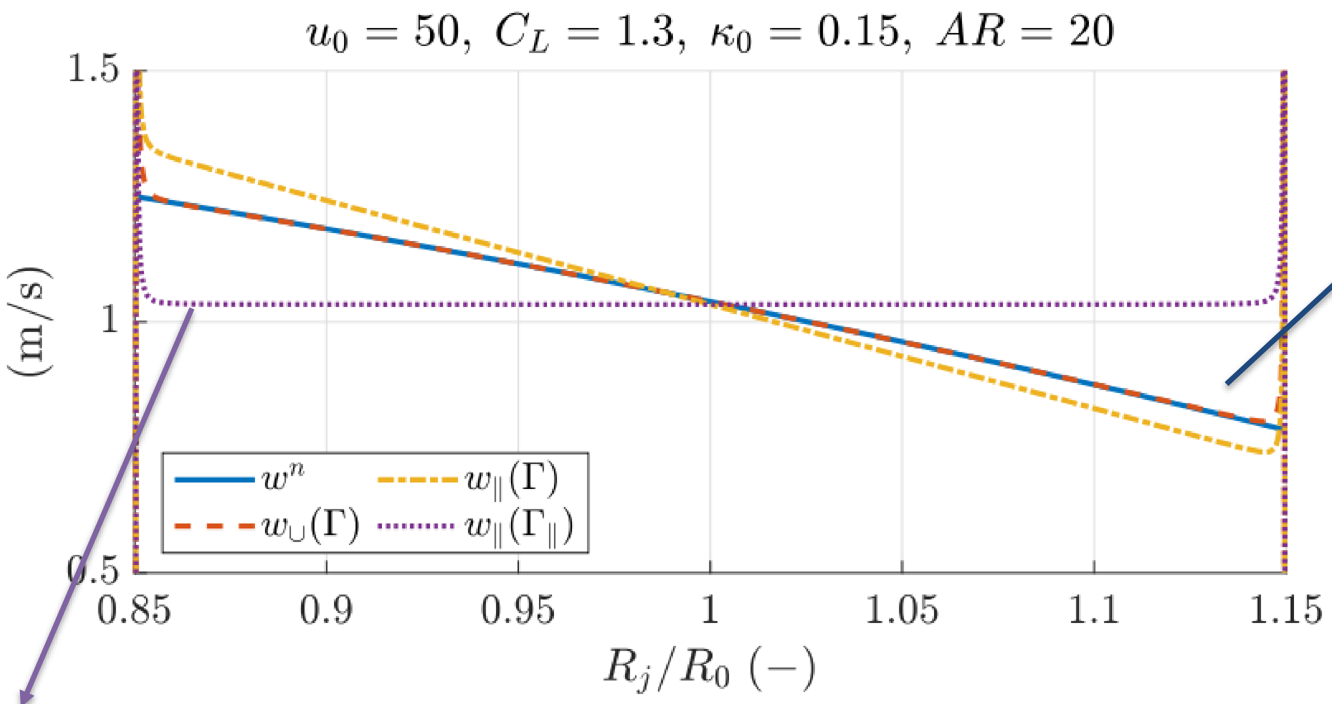
(a) (b)

We split the integration interval and neglect the non periodic term proportional to θ

The helicoidal vortex model reduces to half a ring (near vortex filament) plus an infinite cascade of vortex rings (far vortex filament)

$$w_{j,f} \approx -\frac{\Gamma}{4\pi\Delta R}\left[\int_0^{\pi} \frac{N_{\sim}(\eta,(\theta-\theta_j))}{D_{\sim}\left(\eta,(\theta-\theta_j)\right)^{3/2}} d\theta + \sum_{k=1}^{+\infty}\int_{-\pi}^{\pi} \frac{N_{\sim}(\eta,(\theta-\theta_j))}{\left(D_{\sim}\left(\eta,(\theta-\theta_j)\right) + D_{/}\left(\phi_j, \frac{1-\eta}{1-\eta_0}\frac{2\pi k}{\lambda_0}\right)\right)^{3/2}} d\theta\right]$$

Near wake: induced drag coefficient



Induced velocities the same wing would have in forward flight

$$\kappa_0 = \frac{b/2}{R_0}$$

Induced velocities due to the near wake

Induced change in angle of attack

Induced drag coefficient $C_{Di}^n \approx \frac{C_L^2}{\pi AR}$

Far wake: induced drag coefficient

We assume the wake to be already rolled up into two distinct vortices and we find a fitted solution to the infinite summation of vortex rings

Induced velocities due to the far wake → Induced drag coefficient $C_{Di}^f \approx \frac{1}{4\pi}\frac{C_L^2}{\pi AR}\kappa_0^{\pi/2}\lambda_0^{3/2}$

C_{Di}^f is function of

- Near wake drag coefficient $\frac{C_L^2}{\pi AR}$
- Inverse turning ratio $\kappa_0 = \frac{b/2}{R_0}$
- Normalized torsional parameter $\lambda_0 = \frac{2\pi R_0}{h_0}$ Unknown

The system glide ratio can be written as

$$G = \frac{C_L}{C_D + C_{T,t}} = \frac{C_L}{(C_{D0} + C_{Di}^n + C_{Di}^f)(1+\gamma)} = \frac{1}{\left(\frac{1}{G_0} + \frac{C_L}{\pi AR} + \frac{1}{4\pi}\frac{C_L}{\pi AR}\kappa_0^{\frac{\pi}{2}}\lambda_0^{\frac{3}{2}}\right)(1+\gamma)}$$

$$C_{T,t} = \gamma C_D$$
$$C_D = C_{D0} + C_{Di}^n + C_{Di}^f$$

Far wake: closure model for the normalized torsional parameter λ_0

Assumption: the axial velocity of the far vortex filaments is equal in modulus to the velocity at the wing center $v_w\sqrt{(1-a_z)^2+a_r^2}$

For given: [C_L, C_{D0}, γ, AR, κ_0], the normalized torsional parameter λ_0 (and consequently $\lambda = G$ and C_D) can be found iteratively by setting *h=0*

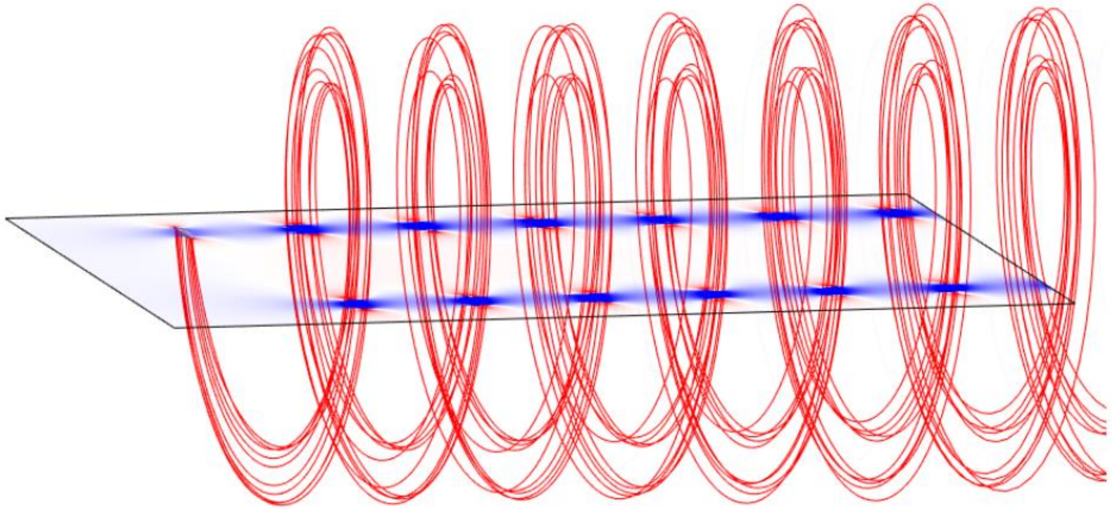
$$h(\lambda_0, C_L, C_{D0}, \gamma, AR, \kappa_0) = \lambda_0 - \frac{\lambda}{\sqrt{(1-a_z)^2+a_r^2}} = 0$$

$$a_r \approx \lambda \frac{2}{9\pi}\frac{C_L}{\pi AR}\kappa_0^{\pi/2}\lambda_0^{1.1}$$

$$a_z \approx \lambda \frac{C_L}{\pi AR}\left(1+\kappa_0^{\pi/2}\lambda_0^{3/2}\right)$$

$$\lambda = G = \frac{1}{\left(\frac{1}{G_0}+\frac{C_L}{\pi AR}+\frac{1}{4\pi}\frac{C_L}{\pi AR}\kappa_0^{\frac{\pi}{2}}\lambda_0^{\frac{3}{2}}\right)(1+\gamma)}$$

The model is validated with the free vortex wake model implemented in QBlade



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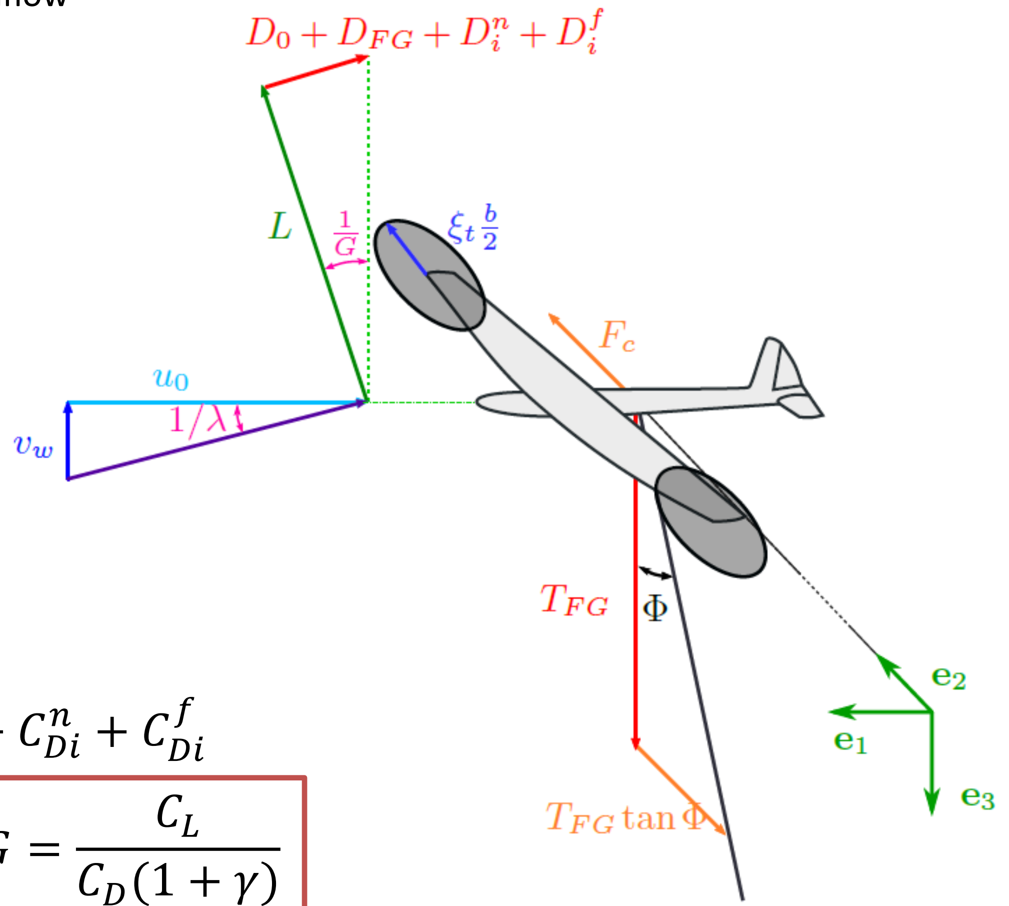
Power equation refinement (of Fly-Gen AWES)

- Given C_L, C_{D0}, γ, AR, $\kappa_0 = \frac{b/2}{R_0}$, ξ_t
- No gravity
- Constant inflow
- $G^2 \gg 1$



$$C_{T,t} = \gamma C_D$$

$$C_D = C_{D0} + C_{Di}^n + C_{Di}^f$$

$$\lambda = \frac{u_0}{v_w} = G = \frac{C_L}{C_D(1+\gamma)}$$

Onboard wind turbine thrust

$$D_{FG} \approx \frac{1}{2} \rho A \gamma C_D G^2 v_w^2$$

Onboard thrust power

$$P_t \approx D_{FG} \cdot u_0 = \frac{1}{2} \rho A \gamma C_D G^3 v_w^3$$

Shaft power

$$P = (1 - a_t) P_t \approx \left(1 - \frac{\gamma C_D}{2\pi AR \xi_t^2}\right) P_t$$

Considering the wake model

$$h(\lambda_0, C_L, C_{D0}, \gamma, AR, \kappa_0) = \lambda_0 - \frac{\lambda}{\sqrt{(1-a_z)^2 + a_r^2}} = 0$$

Power coefficient: choice of reference area

$$C_P = \frac{\frac{1}{2}\,\rho\, A\gamma C_D G^3 v_w^3\left(1-\frac{\gamma C_D}{2\pi AR\xi_t^2}\right)}{\frac{1}{2}\,\rho v_w^3 A_{ref}}$$

Which reference area A_{ref} should we take?

$A_{ref} = 2\pi R_0 b$?

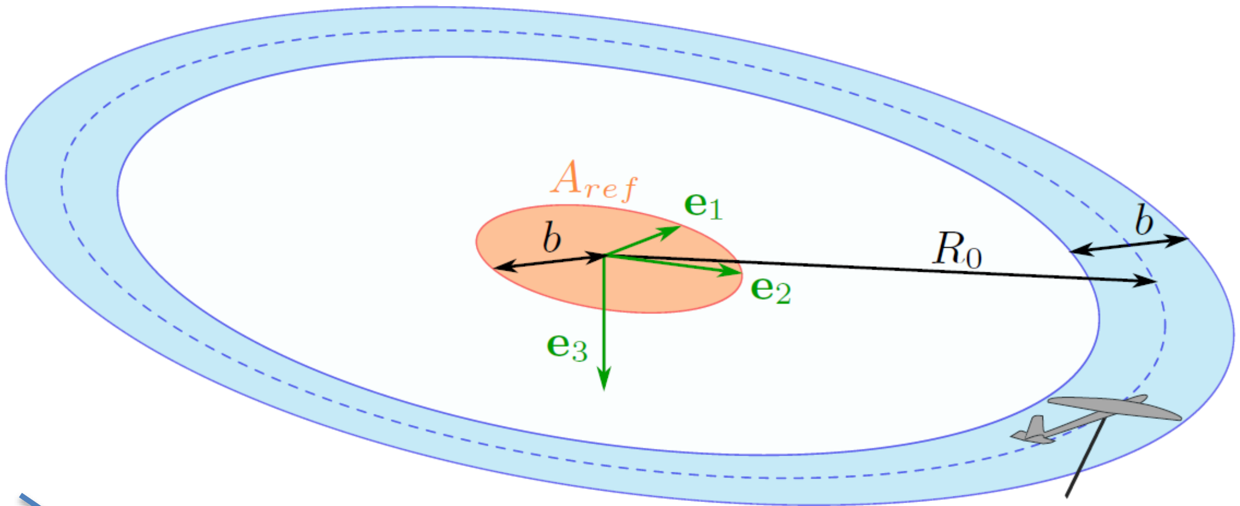
It chances according to the wind speed

$A_{ref} = A$?

Equal to the lifting body area

$A_{ref} = \pi b^2$?

Function of the lifting body span

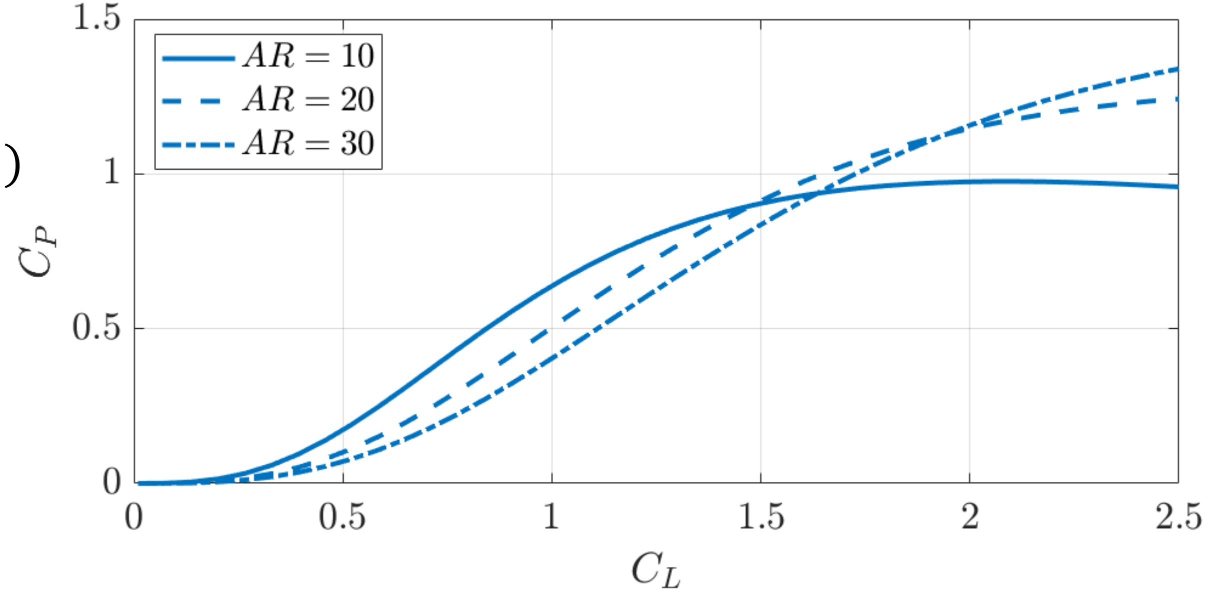


Same reference area of WT with blade length = *b*

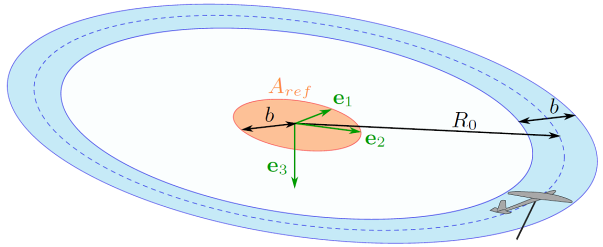
$$C_P = \frac{\frac{1}{2}\,\rho\, A\gamma C_D G^3 v_w^3\left(1-\frac{\gamma C_D}{2\pi AR\xi_t^2}\right)}{\frac{1}{2}\,\rho v_w^3 (\pi b^2)} = \frac{\gamma}{(1+\gamma)^3}\frac{C_L}{\pi AR}\left(\frac{C_L}{C_D}\right)^2\left(1-\frac{\gamma C_D}{2\pi AR\xi_t^2}\right)$$

$$(\gamma, \lambda_0)^* = \arg\left(\max_{(\gamma, \lambda_0)} C_P(\gamma, \lambda_0, C_L, C_{D0}, \text{AR}, \kappa_0, \xi_t) \right)$$

Subject to: $h(\gamma, \lambda_0, C_L, C_{D0}, \gamma, AR, \kappa_0) = 0$



Case with κ_0 = 0.15, C_{D0} = 0.05, ξ_t = 0.15.



- C_P can exceed the Betz limit and 1 because of the reference area definition
- Higher aspect ratio are optimal at higher design lift coefficients

Power coefficient: Given a wingspan *b*, which is the aspect ratio *AR* that maximizes power?

$$\kappa_0 = \frac{b/2}{R_0}$$

Numerical approach:

$$(\gamma, AR, \lambda_0)^* = \arg\left(\max_{(\gamma, AR, \lambda_0)} C_P(\gamma, \lambda_0, C_L, C_{D0}, \text{AR}, \kappa_0, \xi_t)\right)$$

Subject to: $h(\gamma, \lambda_0, C_L, C_{D0}, \gamma, AR, \kappa_0) = 0$

Analytical approach: $C_{Pt}\left(\gamma = \frac{1}{2}, \kappa_0 = 0\right) = \frac{4}{27}\frac{C_L}{\pi AR}\left(\frac{C_L}{C_D}\right)^2$

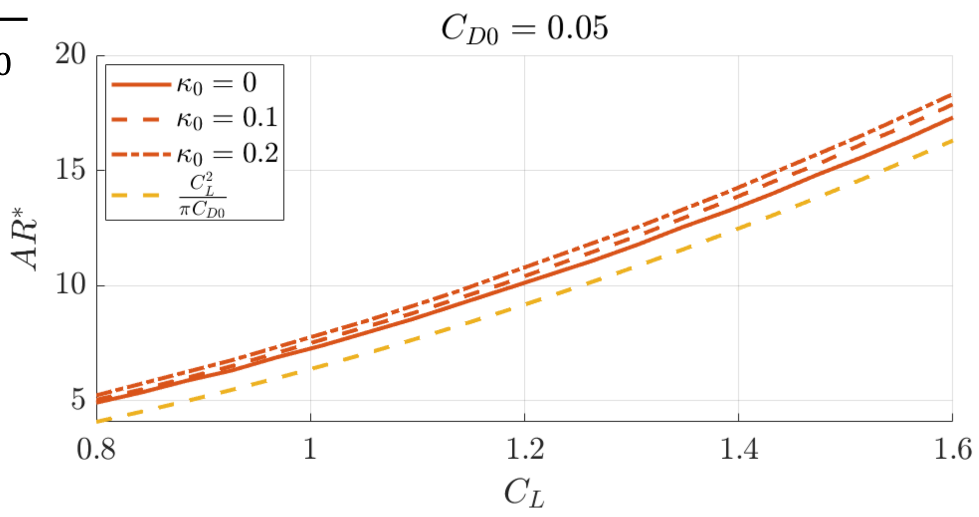
$$\frac{\partial C_{Pt}\left(\gamma = \frac{1}{2}, \kappa_0 = 0\right)}{\partial AR} = \frac{4}{27}\frac{C_L^3}{\pi}\frac{\partial\left(\frac{1}{AR}\frac{1}{C_D^2}\right)}{\partial AR} = 0$$

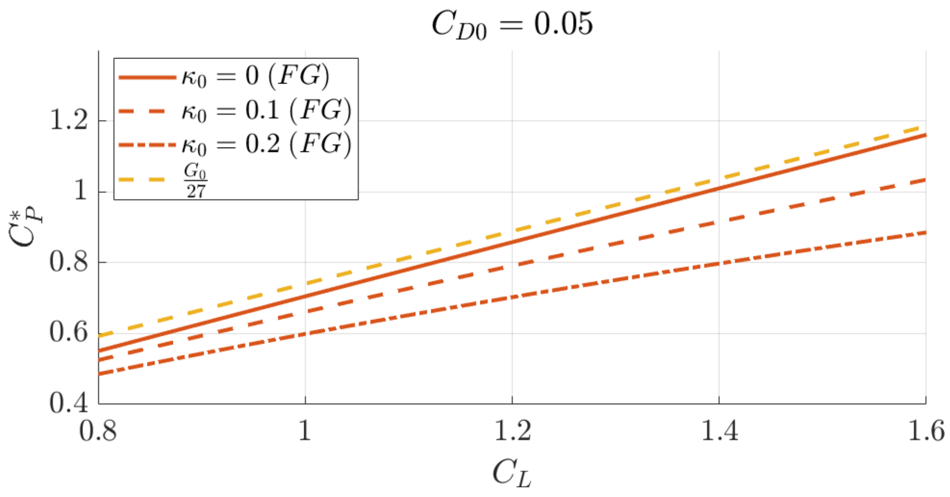
↓

$$AR^{\otimes} = \frac{C_L^2}{\pi C_{D0}}$$

↓

$$C_P^{\otimes} = C_{Pt}\left(\gamma = \frac{1}{2}, \kappa_0 = 0, AR = AR^{\otimes}\right) = \frac{C_L}{27 C_{D0}} = \frac{G_0}{27}$$





Conclusions

- Vortex based aerodynamic model:
    - An induced drag coefficient models the near wake and one the far wake;
    - The model is validated with QBlade
- The power equation is refined by including the wake
- The power coefficient uses the reference area of a disc with radius equal to the AWES wing span.
- For a given wing span:
    - The optimal aspect ratio is finite, and its analytical approximation is found
    - The maximum theoretical power is found

Thanks for the attention!

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Assumption: the axial velocity of the far vortex filaments is equal to the axial velocity at the wing center $v_w(1-a_z)$

$$h_0 = v_w(1-a_z)\frac{2\pi R_0}{u_0}$$

Helix pitch — Far vortex filaments velocity — Revolution period

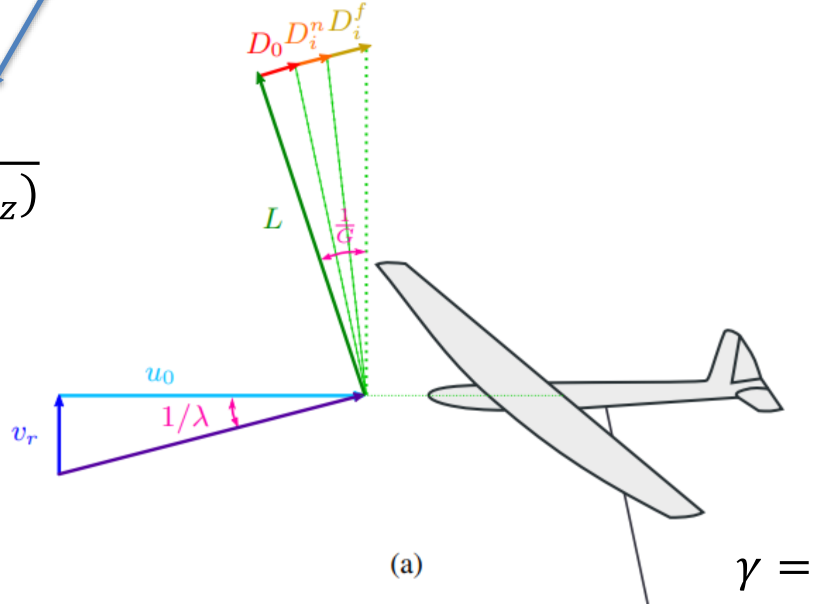
$$\lambda_0 = \frac{\lambda}{(1-a_z)}$$

....

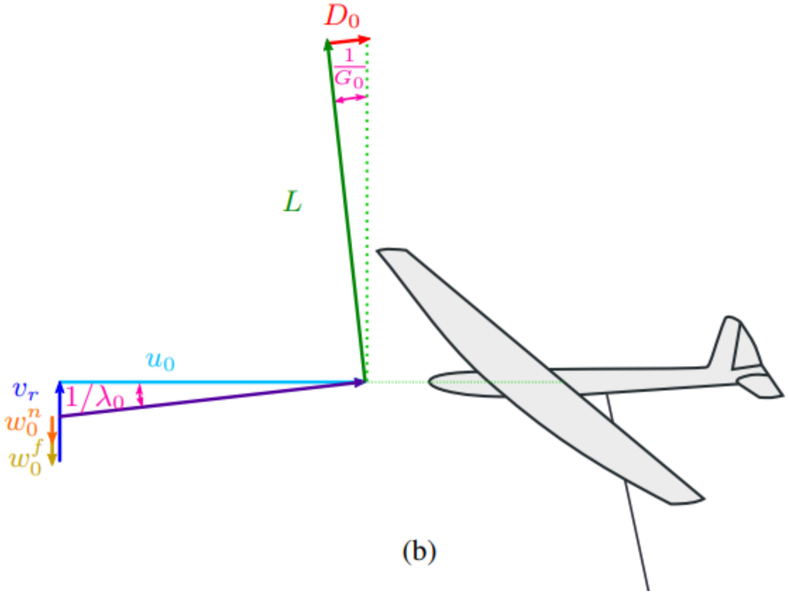
$$\lambda = \frac{u_0}{v_w} = G = \frac{C_L}{C_D}$$

$$\lambda_0 = \frac{2\pi R_0}{h_0} = G_0 = \frac{C_L}{C_{D0}}$$

$$G = \frac{1}{\left(\frac{1}{G_0} + \frac{C_L}{\pi AR} + \frac{1}{4\pi}\frac{C_L}{\pi AR}\kappa_0^{\frac{\pi}{2}} G_0^{\frac{3}{2}}\right)(1+\gamma)}$$



$\gamma = 0$

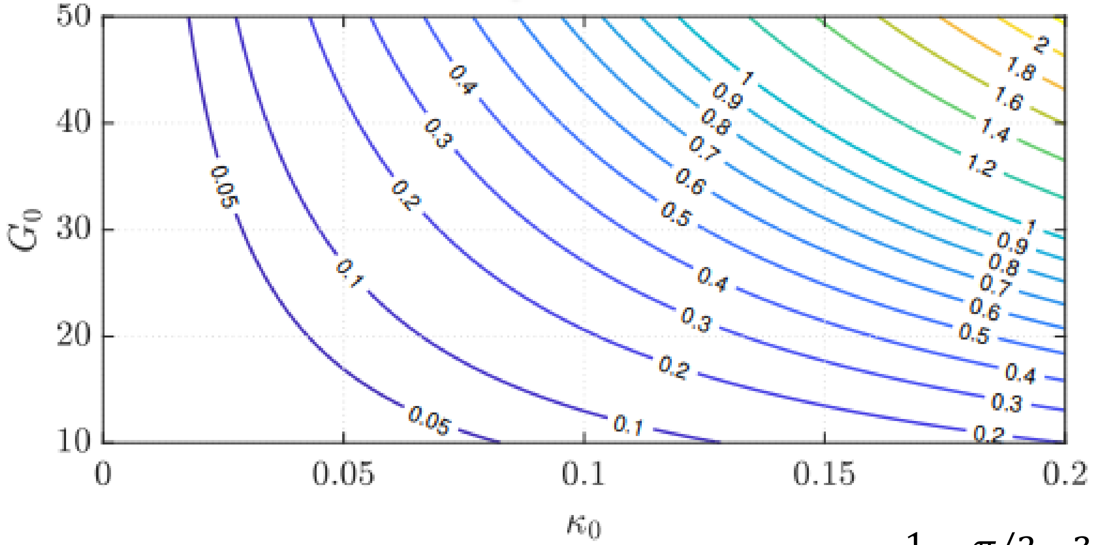


Far wake: explicit closure model for the normalized torsional parameter λ_0

Optimal control

It is beneficial to have low mass m and low turning radius R_0

The aerodynamics gets worst at low R_0. Can we model it?

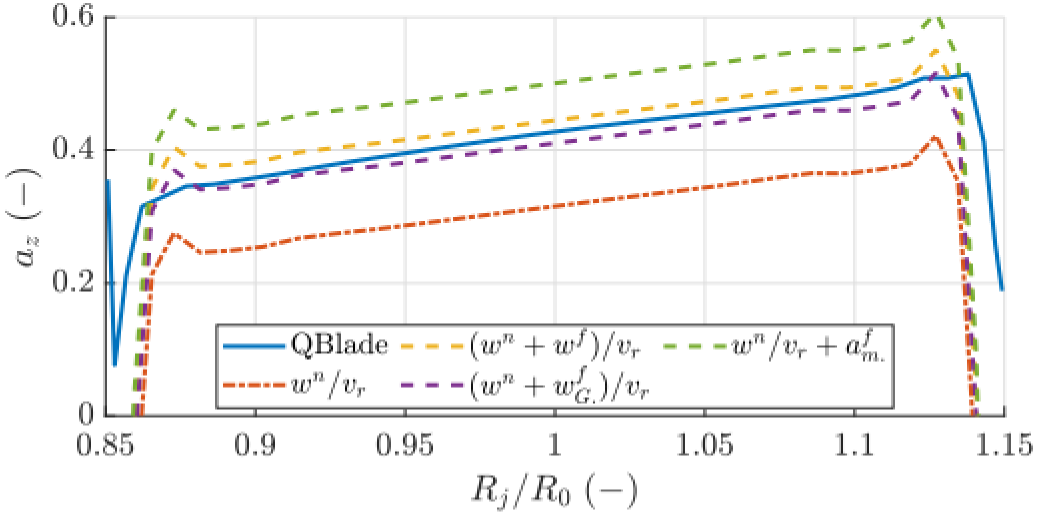


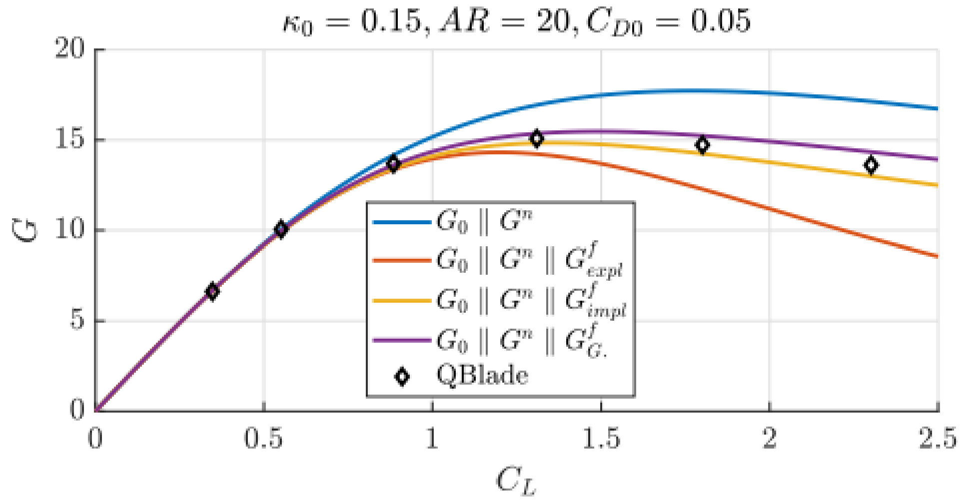
Ratio between the far and near wake induced drag: $\frac{1}{4\pi}\kappa_0^{\pi/2}G_0^{3/2}$

$$G_0 = \frac{C_L}{C_{D0}}$$

$$\kappa_0 = \frac{b/2}{R_0}$$

Validation with QBlade: axial induction and glide ratio





$w^f_{G.}$ and $G^f_{G.}$: induced velocities and glide ratio due to the far wake from Gaunaa et al. (2020)

$a^f_{m.}$: induction based on momentum theory from Kheiri at al. (2018)

Gaunaa, M., Forsting, A. M., and Trevisi, F.: An engineering model for the induction of crosswind kite power systems, Journal of Physics: Conference Series, 1618, 032 010, 2020.

Kheiri, M., Bourgault, F., Saberi Nasrabad, V., and Victor, S.: On the aerodynamic performance of crosswind kite power systems, Journal of 610 Wind Engineering and Industrial Aerodynamics, 181, 1–13, 2018.