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INVESTIGATION OF DEBRIS CLOUD EXPANSION USING METRIC-BASED ANALYSIS

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The continuous growth of the space debris population around the Earth is undermining the use of space as a resource, which is critical for several space-based services used in everyday life. The sustainability of space activities and the safety of operational satellites is threatened by the increasing number of in-orbit breakups occurring each year, caused by collisions and explosions of inactive spacecraft. Therefore, a fast assessment of the debris cloud produced after catastrophic events is crucial to provide information for tasks such as collision avoidance maneuvers, observation planning and short-term risk investigations. Moreover, long-term investigations can provide insights into the consequences of such events.
In this work, novel metrics for the characterization of the expansion of a debris cloud throughout time are proposed. In the first phase, the formulation exploits the synthetic generation of fragmentations through the application of the NASA Standard Breakup Model and the propagation of the cloud for a given time interval using a density-based approach. The latter treats the debris cloud as a continuum, rather than employing a piece-by-piece formulation, speeding up the propagation process. The typical timescales of the evolution of the cloud are investigated, leveraging Ashenberg's formulation for the time of band formation and comparing it with numerical goodness-of-fit tests to assess its accuracy. The aim of the metrics is to compute at each snapshot the spread of the fragments, which enables to capture the phase of the cloud evolution and the dispersion of the cloud itself. This method allows to detach the characterization of the cloud from the density-based model, which yields a complete description of the evolution of the cloud at the expense of higher complexity.
The metric was devised in the context of the Orbital Fragmentation rEconstruction tool for forensics Analysis (OFELIA), an ESA-funded project, where the characterization of the debris cloud through the metric is integrated into the full analysis of the breakup event to aid several operations such as sensor pointing for observation, fragmentation reconstruction and collision risk assessments.

Keywords: space debris, fragmentation, cloud expansion, pinch points

1. Introduction

The proliferation of space has become a critical challenge in the sustainable use of outer space. The growth in the number of these objects is leading to an increase in the frequency of in-orbit breakups, which in turn threatens the long-term stability of the space environment. Routine in-orbit operations are put at risk by the generation of new debris following a fragmentation event, either a collision or an explosion, further contributing to the congestion of the orbital environment around the Earth.
Consequently, the early characterization of debris clouds is essential to provide insights on the occurred breakup, to be able to assess the hazards that the event poses for active neighboring satellites and schedule ad-hoc operations such as sensor tasking and collision avoidance maneuvers. A timely analysis of the event in its early stages is therefore crucial to mitigate the associated threats from an operational point of view and to quantify the characteristic timescales connected to its evolution.
In the past, several works focused on determining the relevant transition times between evolution phases of a debris cloud. Chobotov [1] stated that the last phase of cloud evolution is reached when the fastest fragment encounters the slowest one, making the apsidal and nodal dispersion complete. McKnight [2] instead considers that each phase of cloud evolution is defined when the relevant Keplerian element (namely true anomaly for the first phase, argument of pericenter for the second and right ascension of the ascending node for the third) catches up with the center of mass of the cloud. Moreover, he assesses the hazard posed by each phase of the cloud to active satellites by developing the SCREEN model, based on the computation of the collision risk. Ashenberg [3] famously developed a formulation to describe the time required for the cloud to become uniform, i.e. to complete the last phase of cloud evolution. More recently, Letizia et al. [4] conducted an investigation of the times required for the randomization of the angles typical of each phase, finding that the time required for the final evolution phase to be completed was underestimated by previous formulations. In addition to the analysis of the relevant times of cloud evolution, the work in [4] develops the debris Cloud Evolution in Low Orbits (CiELO) method to model a debris cloud and its evolution, though an in-depth analysis of the cloud in the first phases upon its formation is lacking. The dynamics of the first phases of cloud evolution were studied in [5], where an orbital density map was defined to describe the structure and evolution of the early cloud.
In this context, this work focused on the characterization of debris clouds in the first stages of its evolution. Particularly, the typical timescales of cloud dynamics in its first three phases were reassessed with respect to previous formulations and two metric formulations were devised for the analysis of the first stage concerning the randomization of the mean anomaly.
The paper is organized as follows: Section 2 delves into the explanation of the phases of cloud evolution and the investigation of the typical timescales associated with them, by assessing the accuracy of previous formulations. Section 3 presents the metrics for the characterization of the first phase of cloud expansion, until the

randomization of mean anomaly, by focusing on the clustering of fragments and on the pinch points of the cloud. Section 4 draws the conclusions and outlines future work.

2. Phases of cloud evolution and typical timescales

The evolution of a debris cloud following a fragmentation event was extensively investigated in the literature. A broad consensus exists that its dynamical evolution is split into three main phases, each characterized by distinct dominant perturbations and spatial geometries [2]. Jehn [6] classifies the first phase as the time period necessary to go from an ellipsoidal cloud, close to the trajectory of the parent, to a ring. At the fragmentation epoch, all fragments share the same position but have different velocities imparted by the breakup, hence leading to different orbits and eventually to a stretch of the cloud into the ring around the Earth [4]. In this phase, which is the fastest, the mean anomaly M becomes uniformly distributed, effectively being randomized. This evolution stage is characterized by the presence of a pinch point, which corresponds to the breakup location, where each fragment orbit meets [1,4]. The following phases are slower, and account for the effect of orbital perturbations, particularly the effect of Earth's oblateness. In the second phase, the shape of the cloud evolves from a ring to a flat torus around the Earth due to the spreading of the argument of pericenter ω, leading to the dispersion of the pinch point(s) as well. Lastly, the right ascension of the ascending node Ω becomes randomized, consequently making the torus into a band, limited in latitude by the inclination of the parent [4]. Phases 2 and 3 are sometimes overlapped or swapped, depending on the inclination of the parent orbit, however once the band has formed, all three angles (M, Ω, ω) have a uniform distribution, i.e. they are random, and for the purpose of this work they will still be called phase 2 and 3.

The typical timescales associated with this process were extensively investigated in [1,2,3], providing *a priori* criteria to predict when the angles distributions would be uniform and therefore when the band could be considered formed. Among them, Ashenberg's expression for the time required to reach band formation is particularly appealing, as it links the band formation time directly to the orbital characteristics of the parent orbit.
These analytical formulations were tested against goodness-of-fit statistical tests to assess their accuracy in estimating the uniformity of distributions. To this aim, Letizia et al. [4] used the Kolmogorov-Smirnov (KS) test to provide a numerical estimation of band formation, by comparing the empirical cumulative distribution function (CDF) and a reference cumulative distribution function (the uniform distribution). However, as pointed out by Stephens [7], the KS statistic is not invariant under cyclic transformations. Indeed, the KS statistic depends on the maximum absolute difference between the two CDFs, yet for circular data the CDF is not uniquely defined as it depends on the choice of origin. Therefore, by rotating the same circular data, the KS statistic may yield different results, making it unsuitable for circular data and sensitive to the end points of the interval. To solve the issue, Kuiper's test [8] is generally considered a better alternative when analyzing periodical data as it is

invariant under cyclic transformations. This test was developed to be a rotation-invariant version of the KS test, therefore it is a preferable approach when dealing with angles. For this reason, it was deemed as a better test for the typical timescales of debris cloud evolution, to assess when the distributions of angles become uniform.

2.1 Kuiper's test

Kuiper's test compares the empirical cumulative distribution function $F_n(x)$ with a reference one, usually the uniform distribution, to test whether a data sample comes from the given distribution $F(x)$ [7]. The empirical CDF is defined as:

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} 1_{\{x_i<x\}}$$

Where $x \in [0,2\pi)$ and n is the number of samples.
The Kuiper statistic, V, is defined as follows:

$$V = D^+ + D^-$$

where D^+ is a measure of the maximum positive difference between the empirical and the theoretical distributions and D^- is the maximum negative distance between them. Differently from KS test, which takes the maximum of the two differences, the Kuiper formulation allows to have equal sensitivity on the full domain. The intuitive interpretation of Kuiper's statistic considers that the larger V, the more discrepancy between the two distributions, i.e. the empirical distribution is not uniform. Moreover, the significance of the Kuiper statistic must be assessed through its associated p-value.

2.2 Application of Kuiper's test for debris cloud phases

The Kuiper statistic was applied to each of the three phases of early cloud evolution, to numerically define the time required for the relevant angle distribution to become uniform and compare it with the time computed with the KS test. To this aim, breakup events were simulated with the NASA Standard Breakup Model [8] and propagated with a density-based approach (the Starling suite) developed at Politecnico di Milano [10, 16].

2.2.1 First phase

For the first phase, immediately following the fragmentation event, the time required for the mean anomaly M to randomize was assessed. Figure 1 shows the distribution of M with respect to the semi-major axis for an explosion occurred at 600 km altitude (with zero eccentricity and zero inclination of the parent orbit), at three different snapshots. The progressive homogenization of the distribution over time is evident, and the Kuiper test is applied to quantify the moment at which M can be considered as statistically uniform.

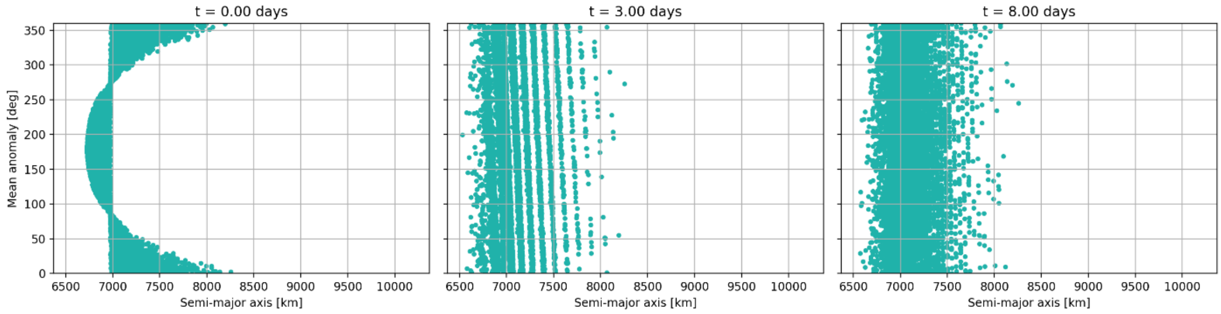

Figure 1. Mean anomaly distribution for a debris cloud generated by an explosion at 600 km at the time of the Breakup (left), 3 days after (center) and 8 days after (right).

Each snapshot contains about 10000 fragments, considering fragments down to 10 cm. This, paired with the very fast dynamics of the first phase, makes classical goodness-of-fit p-values for Kuiper and KS overly sensitive. They remain very small even after the distribution appears uniform by eye, detecting each small change as a deviation from a uniform distribution and therefore never confirming the randomization of the mean anomaly. For this reason, the adopted criterion for this phase looks at the value of the Kuiper and KS statistics, aiming at assessing when the distribution becomes "uniform enough" and determining this condition is met when the statistics are below 0.025. This threshold is about 3 times the expected statistic value for a truly uniform distribution of comparable sample size, making it a conservative and physically meaningful threshold.
The test was repeated on explosions occurred between 400 km and 1100 km with a step of 50 km, preserving the ΔV and area-to-mass distributions of the fragments while changing the altitude of the breakup. All parent objects orbits were considered circular and equatorial. The time for phase 1 estimated with the Kuiper test and the KS test is reported in Figure 2.

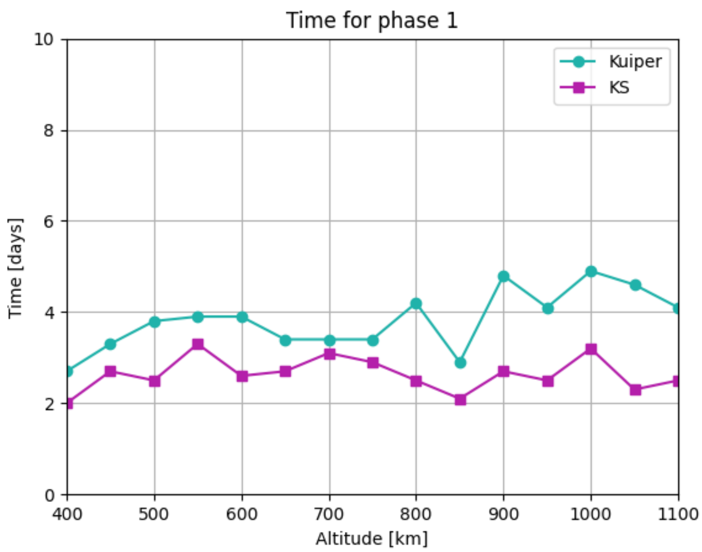

Figure 2. Time for randomization of mean anomaly, i.e. time for phase 1, computed with Kuiper's test and KS test for explosions at altitudes between 400 km and 1100 km.

Across all altitudes, the KS test estimates a lower time for the full randomization of the mean anomaly with respect to the Kuiper test, with the highest difference amounting to 2.3 days. Although small, such differences are not negligible on the timescale of phase 1, therefore showing that the KS test may lead to underestimation in the typical time associated with cloud evolution. Moreover, the Kuiper test shows a mild dependency with respect to the altitude of the breakup itself. This is coherent, as the time required for the spreading of M is driven by the differences in mean motion, and consequently in semi-major axis among the fragments, due to the ΔV imparted to the fragments at breakup. Indeed, over the short timescales considered here (days), perturbations are negligible, and each fragment's mean anomaly evolves approximately linearly [13]:

$$\boldsymbol{M_i(t) = M_{i0} + n_i t} \qquad [\mathbf{1}]$$

Hence, the relative phase between any pair of fragments i, j is given by:

$$\Delta M_{i_j}(t) = M_i(t) - M_j(t) = \Delta n_{i_j} t \qquad [2]$$

The fragments reach a uniform distribution in mean anomaly when relative phases span 2π, implying $|\Delta M_{ij}(t)| = 2\pi$, leading to:

$$t = \frac{2\pi}{|\Delta n_{ij}|} \qquad [3]$$

Therefore, by deriving Δn_{ij} for small spreads with respect to the parent orbit, one obtains:

$$\Delta n_{ij} \approx \frac{dn}{da} \Delta a_{ij} = -\frac{3}{2} \frac{\Delta a_{ij}}{a} n \qquad [4]$$

Hence:

$$t = -\frac{4}{3} \frac{\pi a}{n|\Delta a_{ij}|} \qquad [5]$$

This expression shows that the time for the full randomization of M does not increase monotonically with semi-major axis (i.e., altitude for circular parent orbits). As altitude increases, a increases as well, leading to an increase in time (t depends on $a^{\frac{5}{2}}$). However, the distribution of the fragments Δa_{ij} also tends to increase with altitude, leading to a decrease in time. The two effects coexist leading to a mild and non-monotonic dependence on altitude, consistent with the Kuiper statistic results.

2.2.2 Second phase

The same tests were conducted on the timescale for the randomization of the argument of pericenter ω, i.e. typically the second phase of cloud evolution. For the argument of pericenter and RAAN values randomization, whose J_2-driven precession mixes over much longer timescales, p-value thresholds remain informative, therefore a p-value of 0.05 was defined to assess statistical significance. Explosion events at increasing altitudes were used for testing, propagating the breakups for 500 days to ensure that the argument of pericenter distribution reaches uniformity. Figure 3 reports the results of the analysis.

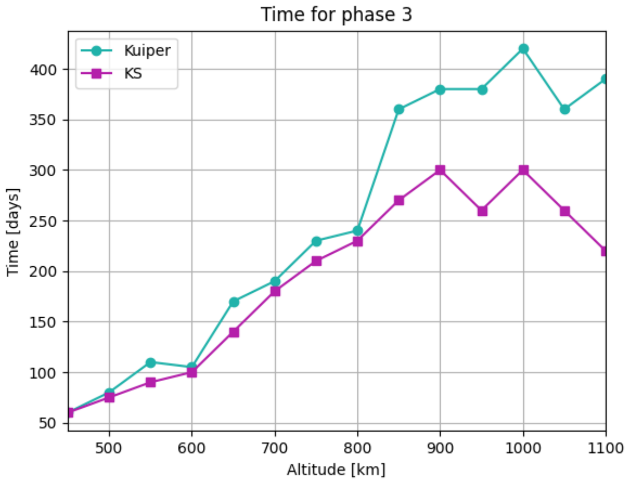

Figure 3 Time for randomization of argument of pericenter, i.e. time for phase 2, computed with Kuiper's test and KS test for explosions at altitudes between 450 km and 1100 km.

The results show that, similarly to phase 1, the KS test underestimates the time required for the angle distribution to become uniform. The discrepancy between the two goodness-of-fit tests grows with increasing altitudes, leading to significant and non-negligible differences in the time estimation. Moreover, for this phase of cloud evolution, a clear, altitude-dependent trend is observable: the time required for the randomization of the argument of pericenter increases approximately linearly with the breakup altitude. This is consistent with the fact that this phase of cloud expansion, as well as the third phase, is driven by the secular perturbation due to Earth's oblateness (J_2). Consequently, higher altitude breakups experience slower differential precession, causing a slower spreading of ω.

2.2.3 Third phase

Lastly, the same checks on the time required for the completion of the last phase of band formation were carried out with respect to the randomization of the RAAN (Ω), on the same explosions. Figure 4 shows the results.

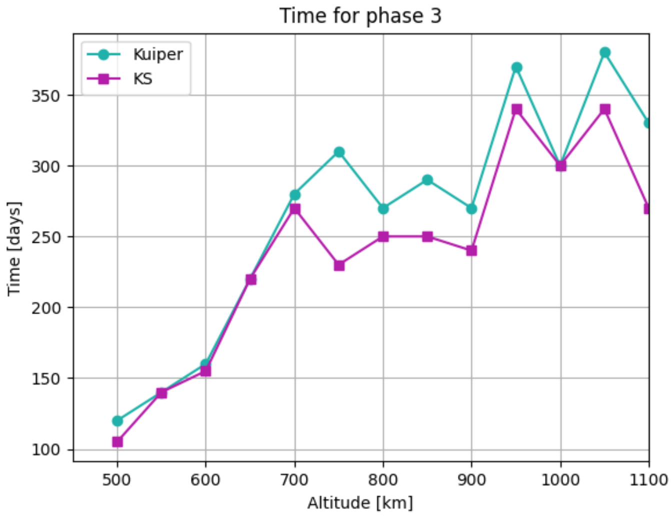

Figure 4 Time for randomization of right ascension of ascending node, i.e. time for phase 3, computed with Kuiper's test and KS test for explosions at altitudes between 500 km and 1100 km.

The goodness-of-fit tests evidence that, as for ω, the time required for the randomization of Ω tends to increase with breakup altitude, as the J_2-driven secular precession is the main mechanism dominating this phase of cloud evolution. The systematic underestimation of the randomization time produced by the KS test relative to the Kuiper test is still observed, although to a lower degree. Furthermore, the time necessary for the randomization of ω and of Ω are comparable. This is expected because the secular precession rates of both angles are determined by J_2 and differ only through their inclination-dependent coefficients. Consequently, the second and third phase can be overlapped depending on the inclination of the parent orbit. For this reason, Ashenberg's formulation for the estimation of band formation time adopts the maximum time between the randomization times of ω and Ω as the time of band formation [3].

2.2.4 Comparison with Ashenberg's time

Ashenberg's formulation [3] is widely employed for estimating the characteristic timescale associated with band formation during the early evolution of a debris cloud. Previous analyses [4] found, however, that the times predicted by this method tend to be underestimated, as the formulation measures the moment when the fastest and slowest fragments meet. For this reason, Letizia et al. [4] adopted a safety factor of 3 when computing this time with Ashenberg's formulation, found by carrying out multiple simulations. These prior works based their numerical validation on the KS test. For the reasons discussed in the previous sections, the same comparisons were carried out with the Kuiper test as well.
Considering a circular and equatorial parent orbit for an explosion at 800 km, Figure 5 reports the Kuiper and KS statistics concerning the randomization of the RAAN, together with the times computed with Ashenberg's formulation, the corrected value with the safety factor and the numerical times computed by the KS and Kuiper tests.

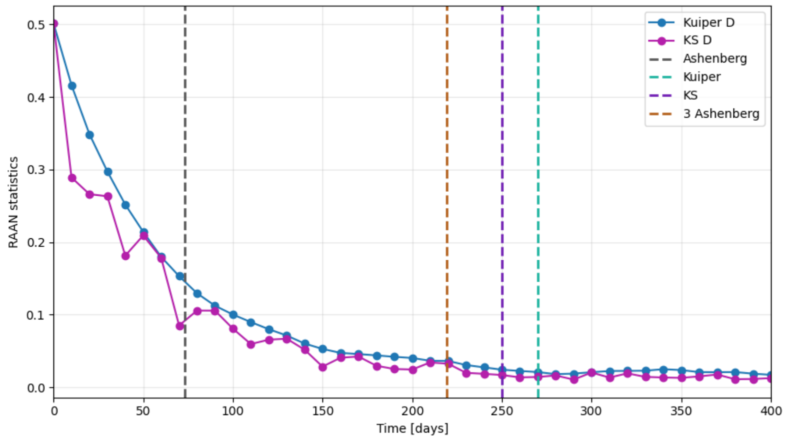

Figure 5. Kuiper and KS statistics applied to an explosion at 800 km. The time of randomization of Ω computed with Ashenberg's formulation, the safety factor, the KS test and the Kuiper test are also reported.

The results clearly show that the original formulation by Ashenberg significantly underestimates the time of band formation, which occurs at 73 days with this approach. Even the modified estimate, obtained by applying a factor of 3, appears insufficient. The time of randomization numerically computed by the Kuiper test is 270 days, which leads to a non-negligible difference of 51 days with respect to the previously accepted time with the safety factor.
An analogous analysis was conducted on the distribution of the argument of pericenter, as Ashenberg's formulation provides analytical predictions for the randomization of ω as well. The results, referred to the same 800 km explosion, are reported in Figure 6 and demonstrate again an underestimation of the time required for the randomization by Ashenberg's formulation, which leads to an underestimation in the time computed with the safety factor as well, particularly if compared with the time found by the Kuiper test.
Overall, this analysis underscores the limitations of the classical Ashenberg approach. The systematic underestimation observed, even after applying the conventional safety factor, highlights the need for new formulations to compute the time of band formations, more accurate with respect to the underlying physics of the evolution of the cloud of fragments. More analyses should be conducted with a Monte Carlo approach, to test a wider set of fragmentation events and assess the average behavior over multiple test cases, however this preliminary analysis shows the need for a re-evaluation of the typical expressions to evaluate the timescales of cloud expansion.

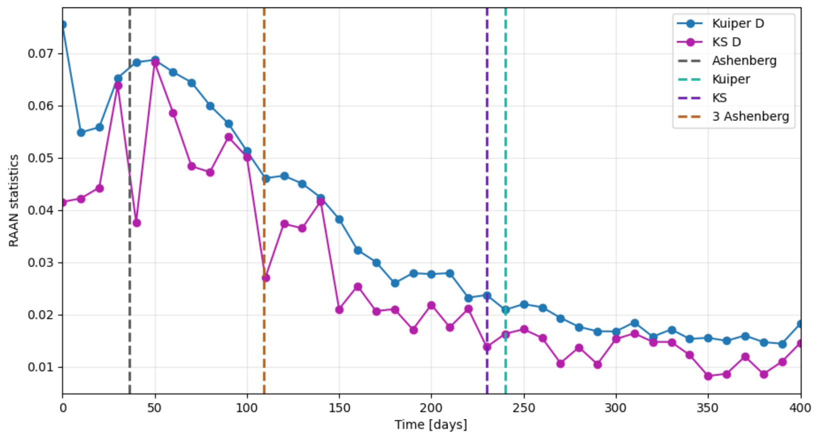

Figure 6. Kuiper and KS statistics applied to an explosion at 800 km. The time of randomization of ω computed

with Ashenberg's formulation, the safety factor, the KS test and the Kuiper test are also reported.

3. Metrics for phase 1

To fully characterize a debris cloud and its evolution phases, it is crucial to capture the features of each evolution stage. This enables better-informed mitigation strategies tailored to the debris cloud current state, supporting operational activities such as sensor tasking for observation and collision avoidance maneuvers for neighboring satellites which must be performed in a timely manner.

The work in this paper focuses on the initial phase of cloud evolution, i.e. the first days following the event until the cloud forms a ring around the Earth and the mean anomaly is randomized. Two different metrics were devised for the analysis of this phase of the cloud, leveraging the dynamics of the cloud orbital dispersion process. The first metric is based on the mean resultant length, which is a common statistical indicator for circular data. The second metric aims at capturing the pinch point(s) behavior, typical of this stage.

3.1 Mean Resultant Length metric

When dealing with circular data, such as the mean motion distribution, a common descriptive statistic is the mean resultant length (R). This quantity ranges from 0 to 1, reflecting how tightly the data cluster on a circle [11], hence providing a measure of their concentration or dispersion. Given a sample of angles $\{\theta_1, \ldots, \theta_n\}$, each observation is mapped to the unit circle through its complex representation $e^{i\theta_k}$[12]. The sample mean resultant vector is defined as:

$$R = \frac{1}{n}\left|\sum_{i=1}^{n} e^{i\theta_k}\right| \qquad [6]$$

If applied to the mean motion evolution, this is particularly appropriate to describe the first phase of cloud evolution, as the main dynamics to capture is represented by the spread of M in time, until the full randomization. The interpretation of R is such that when its value approaches 1, it means that all data are concentrated at a single value, while when it approaches 0 the spread is large. Therefore, it is expected that the value of R will have a peak right after the fragmentation event and will decrease afterwards, providing a measure of the rate of spreading of the mean anomaly distribution.

This metric was applied to explosion events simulated with the NASA Standard Breakup Model [9] and propagated with Starling [10, 16]. The synthetic breakups were generated from circular and equatorial orbits at altitudes ranging from 400 km to 1000 km, propagating them for 10 days and taking snapshots every quarter of a period for each event. The results of the metric are reported in Figure 7.

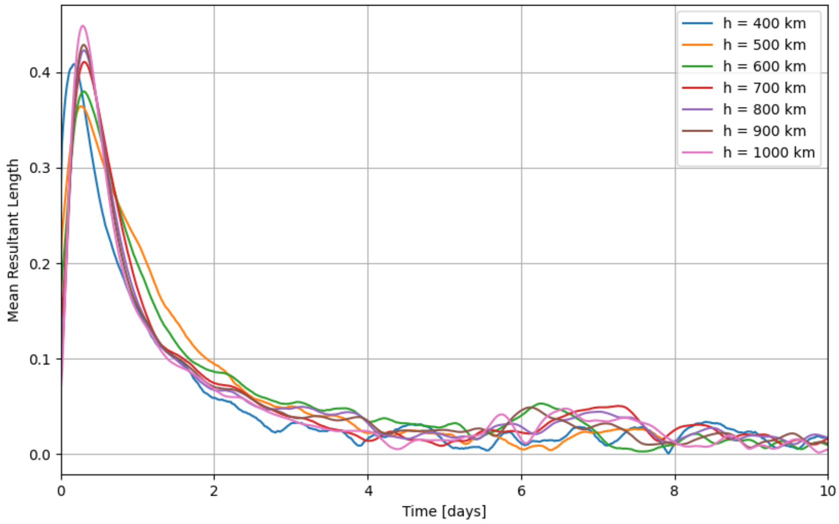

Figure 7 Mean resultant length metric applied at simulated explosions at altitudes from 400 km to 1000 km.

Several conclusions can be drawn from Figure 7. First, the behavior of the mean resultant length appears to be largely independent of the breakup altitude. All curves exhibit a similar trend: an initial phase of strong clustering at the beginning, followed by a quick decay within 2-3 days. This indicates that the cloud undergoes a rapid early spreading as it transitions to a ring-like shape, leading to a significant stretch in mean anomaly and a quasi-ring shape within the first few days, although the distribution cannot be considered uniform yet. The transition towards a uniform distribution occurs on a much slower timescale, as the value of R remains confined between 0.02 and 0.04 for longer times. An additional and unexpected behavior is also observed: contrary to the anticipated peak at the initial time instant (breakup epoch) followed by a monotonic decrease of the metric, the mean resultant length value exhibits a peak occurring some time after the fragmentation event. Figure 8 and Figure 9 report this effect for a breakup at 600 km, with respect to the parent orbit orbital periods and zooming on the first ten periods. The peak is observed at approximately four orbital periods. This value appears to be essentially invariant across all investigated breakup altitudes. As shown in Figure 3, the temporal position of the peak is nearly identical for all curves, and since the parent orbit periods differ only by a few minutes, the peak systematically occurs close to the fourth orbital period.

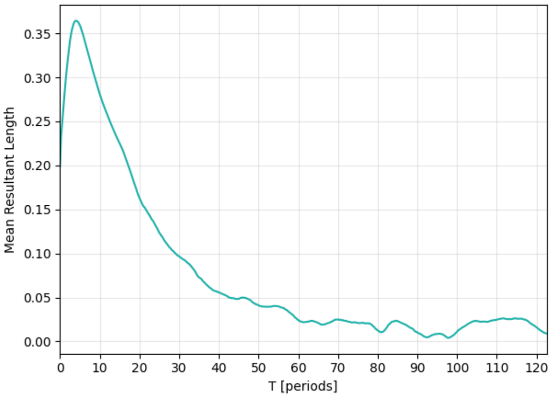

Figure 8 Mean resultant length metric applied at explosion occurred at 600 km.

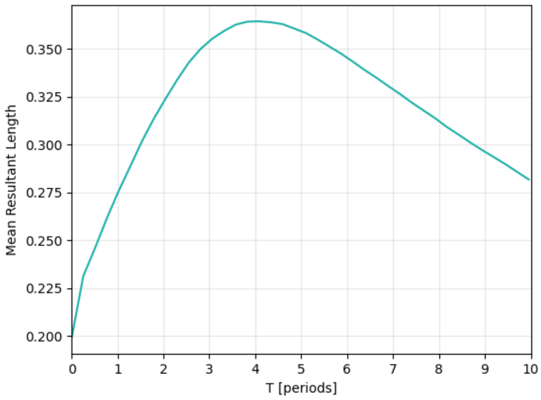

Figure 9 Mean resultant length metric applied at explosion occurred at 600 km, for the first 10 periods.

This behavior actually represents a limitation to the metric approach, as the tightest clustering of the data should be observed at the fragmentation epoch, however it is useful to understand the dynamics of the cloud and the timescales associated with the ring-like spreading. Indeed, the mean resultant length is a measure of concentration intended for circular data, but in the first few periods after fragmentation the mean anomaly distribution is too irregular to be considered as such. For this reason, the metric acquires validity once the fragments have spread, in terms of M, along the orbit enough to assume circular data. This sub-phase of phase 1 is driven by the ΔV imparted to the fragments at breakup, rather than by the effect of orbital perturbations, causing differences in mean motion n among the generated objects. Over these short times, the expression for the time required to achieve a span of 2π among fragments mean anomalies is reported in Equation [3]. Since $|\Delta n_{ij}|$ varies across fragments pairs, its behavior can be characterized with a percentile (quantile) of the corresponding distribution. This allows to estimate that for all simulated breakups four orbital periods correspond to roughly 0.1% of fragments achieving a full 2π separation. Consequently, this value marks the beginning of the validity of this metric formulation and coincides with the start of the transition of the cloud to a ring-like shape.

3.1 Pinch point metric

One of the distinctive features of the first phase of cloud expansion is the presence of pinch points in the cloud distribution, where the dispersion of fragments disappears in one or two directions [14]. These pinch points appear every half orbit, and progressively disappear throughout the first phase of cloud expansion, hence a metric aimed at characterizing this stage of cloud evolution should capture the behavior of the pinch points. Figure 10 represents the cloud evolution of three subsequent snapshots for an explosion at 600 km. It is possible to notice that the cross-sectional thickness of the cloud is reduced in the two pinch points, hence this metric should characterize such thickness in time.

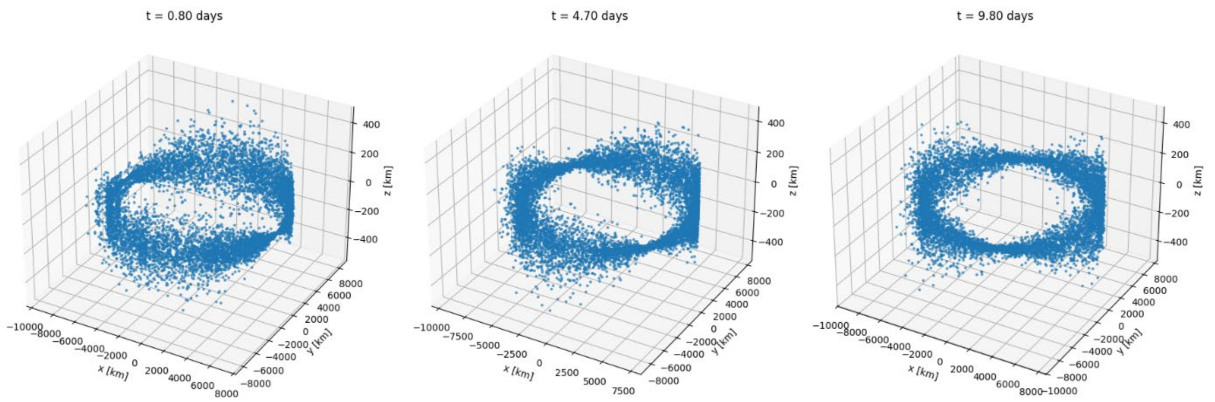

Figure 10 3D cloud evolution for an explosion occurred at 600 km.

To this aim, the Principal Component Analysis (PCA) was adopted. PCA is a variable reduction method, particularly useful with large datasets, whose purpose is to identify a small set of features representing the original data [15] and finding directions in which the dataset varies the most. When applied to a debris cloud, represented by the position of all its fragments, PCA returns three orthogonal principal components corresponding to the directions of maximum, intermediate, and minimum variance, denoted as λ_1, λ_2, and λ_3, respectively. The third component, i.e. the direction of minimum spread, typically represents the out of plane dispersion, orthogonal with respect to the other two principal components which span the best-fit plane of the cloud. Therefore, the third principal component can be used to measure the cross-sectional thickness of the cloud. The principal components are obtained by computing the eigenvalues and eigenvectors of the covariance matrix of the data [15], in this case the fragment positions. This approach is particularly suited for this application because it is invariant under rotations of the reference frame, ensuring that the direction of minimum spread is consistently identified regardless of the coordinate system used.
For these reasons, the second metric formulation proposed in this work applies PCA to the debris cloud at several snapshots during its evolution, then evaluates the local vertical dispersion of the cloud in mean anomaly bins as:

$$d = \sqrt{\lambda_3} \quad [7]$$

In this way, the metric represents the thickness of the cloud.
The analysis of the behavior of d in time gives insights on the phase of the debris cloud. Indeed, in time, the cross-sectional thickness should become uniform i.e. fragments become more dispersed.

This metric was applied to an explosion occurring at 600 km, from a circular and equatorial parent orbit. The cloud was propagated for 10 days, with 10 snapshots per day. Figure 11 reports the results of the metric in time.

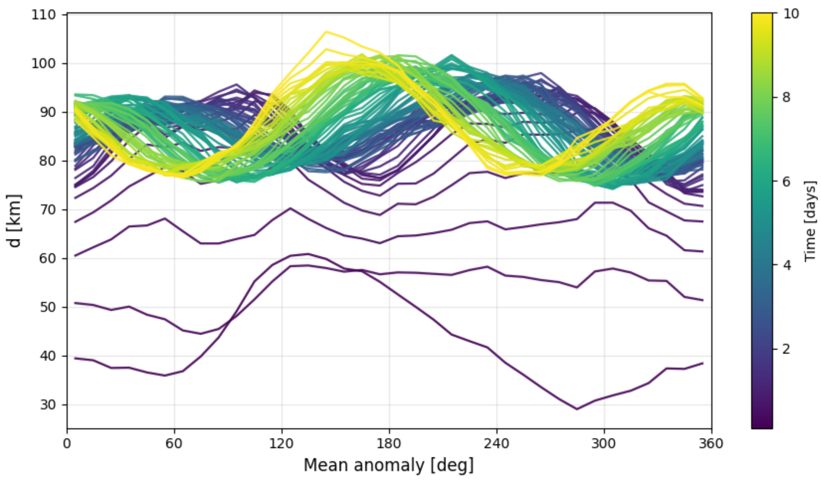

Figure 11 Pinch point metric applied to an explosion event at 600 km.

To reduce numerical noise, the thickness values were smoothed both along mean anomaly (using a circular moving average) and across consecutive snapshots in time (using a light Gaussian temporal filter), without altering the large-scale physical trends.

The plot highlights how the metric correctly interprets and characterizes the physical evolution of the cloud in this first stage. The early curves show strong oscillations with mean anomaly, reflecting the anisotropic structure derived from the parent orbit. As time progresses, the oscillations diminish and curves become flatter, indicating that the cross-sectional thickness becomes more uniform around the orbit, increasing the overall dispersion and losing the trace of the parent orbit. Moreover, the curves corresponding to later snapshots have higher values of d, as the cross-sectional spread is growing in time while the cloud evolves to a torus-like configuration.

These trends demonstrate that this formulation effectively captures the characteristic features of the early evolutionary regime of debris clouds, therefore providing a robust metric to interpret their early geometry right after the fragmentation event.

4. Conclusions

The increase in the frequency of breakup events, due to the growing population of space debris, is threatening daily in-orbit operations in terms of safety and sustainability. The early characterization of debris clouds upon their formation is crucial to predict the evolution of the cloud and to inform mitigation strategies. The purpose is the reduction of the risk posed by fragmentation events to neighboring satellites, by carrying out operations as sensor tasking for observations and collision avoidance maneuvers.

The first step to the complete analysis of a fragment cloud in its early phase is the identification of the typical timescales associated with it. Several formulations were proposed in the past to do this, however they were often underestimating the time required for the cloud to reach a band configuration. Moreover, the timescales associated with the very first phase of cloud expansion, i.e. the randomization of true anomaly, were not investigated. This work proposed a different statistical goodness-of-fit test, i.e. the Kuiper test, to numerically verify when the distributions of the angles typical of each phase of cloud formation become uniform. This test is more suited to circular data with respect to previously employed statistical measures. Several preliminary analyses applied to each initial phase of cloud evolution proved that the usual formulations yielded an underestimation of the times of cloud evolution. Despite the need for more analyses, this preliminary result highlights the inaccuracies in previously employed methods. Moreover, two metric formulations were proposed for the characterization of the first phase of the cloud expansion, until mean anomaly randomization. The first metric uses the mean resultant length, a statistical indicator of the clustering of circular data, to investigate how fragmentations at different altitudes expand, until they reach a ring-like formulation. The second metric instead characterizes the behavior of the pinch points of the cloud in time, which are features of the first few days of cloud evolution, via Principal Component Analysis. Future work will be dedicated to finding formulations for metrics to characterize the subsequent phases of cloud evolution as well, to offer a complete analysis of the first stages of cloud expansion and to support mitigation strategies.

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