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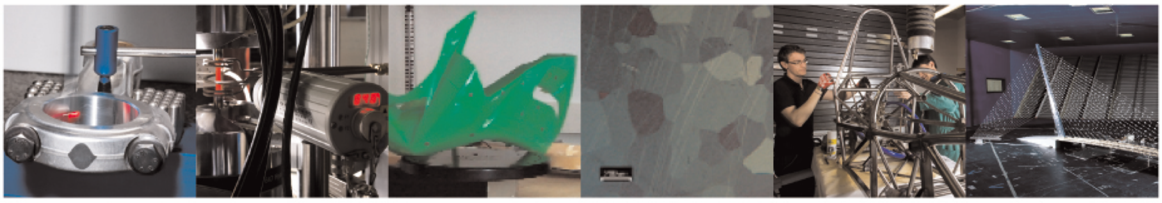

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This is a post-peer-review, pre-copyedit version of an article published in journal title. The final authenticated version is available online at: https://dx.doi.org/10.1016/j.ymssp.2021.108511

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Rev. 0

Human-structure interaction: convolution-based estimation of human-induced vibrations using experimental data

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Abstract: This article addresses the estimation of the effects of human-structure interaction on the dynamic behaviour of slender structures. The paper proposes a method able to provide predictions of human-structure interaction effects in cases in which no models of human dynamic behaviour can be used/are available and only experimental data have to be used. The proposed approach describes the coupled system composed by the structure and the people on it, splitting the action of people into two components: passive and active forces. The sum of the two provides the global action of each person on the dynamics of the structure. Furthermore, an analytical treatment based on modal coordinates and convolution operations is proposed in order to use experimental data, related to the dynamic behaviour of humans, for predicting the vibration level of the occupied structure. An experimental campaign on a slender staircase provided data employed to validate the approach discussed in the article.

Keywords: human-induced vibrations; human-structure interaction; convolution; ground reaction force; vibration; measured apparent mass

1. Introduction

In the last decades many efforts have been spent to study and analyse human-structure interaction (HSI) [1–4]. Different research works investigated experimental cases to evidence the effects of humans on the dynamic behaviour of structures (e.g. [5–8]). Other works proposed models to describe HSI (e.g. [9–18]), as well as control methods for suppressing human-induced vibrations (e.g. [19]). Moreover, some standards (e.g. [20]) proposed guidelines to take into account the influence of people on the dynamic behaviour of structures at the design stage [21].

The structures which have been extensively studied are footbridges [22–29], staircases [30,31], grandstands [6,9] and, more generally, pedestrian structures [32–36], also considering the influence of the occupation density [37]. Indeed, such structures have features which make their dynamic behaviour strongly dependent on the presence of people.

HSI is a complex problem and many aspects must be considered when a model is developed. Furthermore, the problem of modelling HSI gets harder and harder when lateral vibrations [38–40] are taken into account. Indeed, several effects, which have lower relevance for vertical vibrations, become evident for lateral vibrations (e.g. lock-in phenomena [41]). Despite the case of vertical vibrations is the simplest situation, the topic is of great relevance and needs to be further developed. Indeed, many researchers are still trying to propose reliable models able to properly predict the effects of HSI and the consequent change of the dynamic behaviour of the occupied structures.

Among the different methods aimed at describing the effect of people on the structures, an approach proposed in different recent works (e.g. [31,42–45]) seems very promising. The idea behind these works is to decouple the effects of people on the vertical vibrations of structures in two different factors: the passive ground reaction force (PGRF) and the active ground reaction force (AGRF). The PGRF is due to the dynamic characteristics of each person and the structure, while the AGRF is the force generated by the person's active movement. The sum of PGRF and AGRF constitutes the total ground reaction force (GRF). These models demonstrated to be reliable and accurate and, at the same time, are much simpler than using bio-mechanical models.

According to [31], the PGRF is a force generated as a response to the structural movement: if an external force acts on the structure, this vibrates and excites the person. If a person is considered as a dynamic system, he/she starts to vibrate as well. The consequence is that a force (i.e. PGRF) is exerted by the person excited by the structure on the structure itself. Conversely, the AGRFs are not generated by the vibration of the structure and are only due to the active movement of the person. The AGRF can be intended as the force exerted by a moving person on a structure with an infinite stiffness. Thanks to this approach, the GRF can be seen as the sum of PGRF and AGRF.

In the mentioned works, the features of humans (e.g. PGRFs, AGRFs) are often obtained by statistical studies in order to take into account many different scenarios (e.g. [46] for the loads provided by jumping people).

The aim of this paper is to propose an approach to foresee the dynamic response of structures in case of unavailability of statistical models to describe features of humans who occupy the structure (e.g. lack of models in the literature for the considered postures), or in case specific people will occupy the structure and accurate HSI estimations are required. In this latter case, indeed, statistical models are useless and the specific data related to the considered people must be used. These points will be addressed again, in more detail, in Section 2. Moreover, the proposed approach is general, meaning that it can take into account possible changes of both posture and position of people along the structure. This means that the proposed model is expected to accurately describe moving people on the structure; this is made possible by taking advantage of the superimposition of PGRFs and AGRFs.

Furthermore, the mathematical treatment behind the proposed approach is suitable in complex cases (either using statistical models or not), where many people occupy the structure, also having different behaviours (e.g. someone is jumping, someone is walking, etc.), because it allows for a reduction of possible numerical problems and computational burden when foreseeing the structural vibration compared to other approaches available in the literature (see Section 2 for more details).

Despite the proposed method is of general validity, it will be explained focusing on the dynamics of staircases occupied by people. Indeed, this is a very complex case when compared to the case of walking on flat ground because the posture of people changes significantly time instant by time instant when ascending or descending.

The structure of the paper is as follows: Section 2 describes the approach proposed to simulate HSI, Section 3 discusses the inputs required by the developed numerical model, and Section 4 presents an experimental campaign and compare its results to those of the numerical simulations.

2. The approach to predict HSI

As mentioned, the approach proposed in this paper is specifically devoted to two situations: cases in which specific people occupy the structure and thus statistical models of the dynamic behaviour of humans cannot be used, and cases where the postures assumed by people have been never characterised in terms of dynamic behaviour, and thus no statistical models are available. The approach is based on the characterisation of the dynamic behaviour of people by means of experiments, without the need of modelling it (which also means that there is no need of using bio-mechanical models, which often require to be tuned through several tests). This means that few tests with given people are enough to properly estimate HSI in the cases taken into account. This allows to avoid carrying out hundreds or thousands of experiments to find models based on statistical considerations, and this is useful when specific non-general applications are analysed.

Nevertheless, the approach is useful also for another reason: most HSI models proposed in the literature require to solve the dynamics of the coupled system by numerical integration of differential equations in the time domain, which can imply numerical problems or ill-conditioning. Conversely, the approach proposed here does not need to perform numerical integration of differential equations and only requires to calculate convolutions, in turn implying significantly lower numerical problems, and also a significantly lower computational burden.

As mentioned, the method describes the forces exerted by people on the structure (i.e. GRFs) as the sum of PGRFs and AGRFs (see Section 1). The structure is seen as discretised in *n* degrees of freedom (DOFs). The displacement and velocity of the *b*-th DOF are x_b and $\dot{x}_b$, respectively (the dot indicates the derivative with respect to time). A generic linear time-invariant system can be modelled by a state space approach:

$$\dot{\mathbf{r}}(t) = \mathbf{A}\,\mathbf{r}(t) + \mathbf{B}\,\mathbf{u}(t) \qquad (1)$$

$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{r}(t) \qquad (2)$$

where **A**, **B** and **C** are the matrices of the state space representation. Moreover, **r** is the column vector containing the state variables (i.e. displacement $\mathbf{x} = [x_1, \dots, x_b, \dots, x_n]^{\mathrm{T}}$ and velocity $\dot{\mathbf{x}} = [\dot{x}_1, \dots, \dot{x}_b, \dots, \dot{x}_n]^{\mathrm{T}}$ of the DOFs of the system, $\mathbf{r} = [\dot{\mathbf{x}}^{\mathrm{T}} \;\; \mathbf{x}^{\mathrm{T}}]^{\mathrm{T}}$; the superscript T indicates the transposed matrix), *t* is the time, **y** is the output of the system (e.g. displacement of certain DOFs of the system), and **u** is the vector containing the forces acting on the structure (in the case treated in this paper they are intended as the AGRFs; however, the method can be easily extended to take into account other external forces).

The solution of the dynamic system (see Eqs. (1) and (2)) can be expressed as [47]:

$$\mathbf{r}(t) = \boldsymbol{\varphi}(t, t_0)\mathbf{r}_0 + \int_{t_0}^{t} \boldsymbol{\varphi}(t, \tau)\mathbf{B}\,\mathbf{u}(\tau)d\tau \tag{3}$$

$$\mathbf{y}(t) = \mathbf{C}\,\boldsymbol{\varphi}(t, t_0)\mathbf{r}_0 + \int_{t_0}^{t} \boldsymbol{\Omega}(t, \tau)\mathbf{u}(\tau)d\tau \tag{4}$$

where $\mathbf{r}_0 = \mathbf{r}(t = t_0)$ is the initial condition at the initial time t_0, $\boldsymbol{\Omega}$ is a matrix containing the impulse response functions (IRFs) of the system and $\boldsymbol{\varphi}$ is the transition matrix:

$$\boldsymbol{\varphi}(t, t_0) = \exp\left(\int_{t_0}^{t} \mathbf{A}\,d\eta\right) \tag{5}$$

The transition matrix describes the free response of the state at time t due to a unity initial condition (at time t_0) for a system that is otherwise quiescent and unforced [47]. The following relation applies:

$$\boldsymbol{\Omega}(t, \tau) = \mathbf{C}\,\boldsymbol{\varphi}(t, \tau)\mathbf{B} \tag{6}$$

Therefore, the first terms of the right-hand side of Eqs. (3) and (4) are related to the free response of the system and the second terms to the forced response.

Any time-variant system can be approximated as a sequence of time-invariant systems in each time-interval in which the system does not change. In the case of a structure occupied by people, the coupled system (structure plus people) can be considered as time-invariant in the time intervals in which the posture and position of people along the structure do not change (see also Sections 2.1 and 3 for a deeper insight). Furthermore, the values of the system states at the end of the considered time-interval are the initial conditions to be employed for solving the dynamics in the subsequent time-interval. To do this, it is necessary to know $\boldsymbol{\Omega}$ and $\boldsymbol{\varphi}$ of every time-invariant system.

The next subsections explain how it is possible to approximate the matrix $\boldsymbol{\varphi}$ for the case of a structure occupied by people, once the locations of people on the structure and their postures are fixed.

2.1 Modal identification of the occupied structure

The Frequency Response Functions (FRFs) of the empty staircase can be estimated for the nodes *n* in which the structure is discretized [48,49], and stored in the following matrix $\mathbf{G}$:

$$\mathbf{G}(\mathrm{j}\omega) = \sum_{i=1}^{N} \frac{\boldsymbol{\Phi}_i \boldsymbol{\Phi}_i^{\mathrm{T}}}{-\omega^2 + \mathrm{j}2\xi_i\omega_i\omega + \omega_i^2} \tag{7}$$

where $\boldsymbol{\Phi}_i$ is the *i*-th mode shape column vector normalized to the unit modal mass of the empty structure, ω_i is the *i*-th eigenfrequency of the empty structure, ξ_i is the associated *i*-th non-dimensional damping ratio, N is the number of considered modes, j is the imaginary unit, and ω is the angular frequency.

Adopting the approach introduced in [30], it is possible to account for the passive contribution of a generic number of pedestrians placed at any point of the discretized structure. This requires measuring the apparent mass (AM) curves of each person in the considered postures. The AM curve of a person is defined as the FRF between the input acceleration to the person and the consequent force exerted by the person on the vibrating structure (i.e. force over acceleration FRF). The equation to obtain the new FRFs of the occupied structure (i.e. structure under HSI), stored in the $\mathbf{G}_{\mathrm{H}}(\mathrm{j}\omega)$ matrix, is the following [30]:

$$\mathbf{G}_{\mathrm{H}} = [\mathbf{G}^{-1} + \mathbf{WHW}^{\mathrm{T}}]^{-1} = \mathbf{G} - \mathbf{GW}[\mathbf{H}^{-1} + \mathbf{W}^{\mathrm{T}}\mathbf{GW}]^{-1}\mathbf{W}^{\mathrm{T}}\mathbf{G} \tag{8}$$

where **W** is a matrix defining the locations of people on the structure. If *m* subjects are considered on the structure, **u** is a vector *m* x 1, and thus $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_m]$ represents the connection of these m subjects with the nodes of the structure. Indeed, the generic $\mathbf{w}_k$ vector (with $1 \leq k \leq m$) contains as many zeros as the nodes of the structure, but a unit value in correspondence of the node of the structure where the *k*-th pedestrian is located, that is $\mathbf{w}_k = [0, \dots, 1, \dots, 0]^{\mathrm{T}}$. Furthermore, $\mathbf{H}(\mathrm{j}\omega)$ is a diagonal matrix containing the m dynamic stiffness FRFs $H_k(\mathrm{j}\omega)$ (i.e. force over displacement FRFs) of the subjects, with $H_k(\mathrm{j}\omega) = -\omega^2 M_{\mathrm{a},k}(\mathrm{j}\omega)$. $M_{\mathrm{a},k}(\mathrm{j}\omega)$ is the AM of the *k*-th subject in a given posture. The right-hand term of Eq.(8) is derived by means of the Woodbury identity [50].

Therefore, as soon as the configuration of the coupled system is known (position of the pedestrians over the structure and their postures), the matrix $\mathbf{G}_{\mathrm{H}}$ can be calculated starting from the modal parameters of the empty structure (needed to reconstruct **G** and estimated either experimentally or numerically) and the experimental AM curves of the pedestrians in the given postures.

The FRFs gathered in $\mathbf{G}_{\mathrm{H}}$ can be used as inputs to carry out a modal identification on the occupied structure. In this work, the algorithm used for the modal extraction was the polyreference least squares frequency domain method [51]. This allows to estimate the values of $\boldsymbol{\Phi}_i^{\mathrm{oc}}$, ω_i^{oc} and ξ_i^{oc} (the superscript ‘oc’ indicates that the modal parameters are referred to the occupied structure; $\boldsymbol{\Phi}_i^{\mathrm{oc}}$ is scaled to the unit modal mass).

Once the modal parameters of the coupled system are known, it is possible to introduce the modal coordinates q_i:

$$\mathbf{x}(t) = \sum_{i=1}^{N} \boldsymbol{\Phi}_i^{\mathrm{oc}} q_i(t) = \boldsymbol{\Phi}^{\mathrm{oc}} \mathbf{q}(t) \tag{9}$$

where $\mathbf{q} = [q_1 \quad \cdots \quad q_N]^{\mathrm{T}}$ and $\boldsymbol{\Phi}^{\mathrm{oc}} = [\boldsymbol{\Phi}_1^{\mathrm{oc}} \quad \cdots \quad \boldsymbol{\Phi}_N^{\mathrm{oc}}]$. The next subsection explains why modal coordinates are useful.

2.2 Estimation of the structural response

The dynamics of the whole system is now approximated and assumed as a series of decoupled differential equations, one for each mode, by employing the modal coordinates (and assuming proportional damping). Therefore, a sequence of single-degree-of-freedom (SDOF) systems is obtained, each described by an equation of motion having the following form:

$$\ddot{q}_i(t) = -2\omega_i^{\mathrm{oc}} \xi_i^{\mathrm{oc}} \dot{q}_i(t) - (\omega_i^{\mathrm{oc}})^2 q_i(t) + u_i(t) \qquad \forall\, i = 1, \dots, N \tag{10}$$

Where $u_i(t)$ is the modal force: $u_i(t) = -(\boldsymbol{\Phi}_i^{\mathrm{oc}})^{\mathrm{T}} \mathbf{W}\, \mathbf{u}(t)$. Refer to Figure 1 for the convention of sign for the AGRF and thus the modal force.

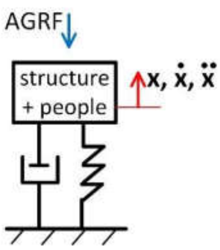

Figure 1: Convention of signs for the AGRFs. The coupled system is represented as a single DOF system for the sake of clearness of the figure, but a generic multi DOFs system can be used.

Each of these SDOF systems can be modelled through the state space approach [52] as:

$$\dot{\mathbf{z}}(t) = \mathbf{E}\,\mathbf{z}(t) + \mathbf{L}\,\mathbf{u}(t) \tag{11}$$

where $\mathbf{z} = [\dot{q}_i\ q_i]^{\mathrm{T}}$ and:

$$\mathbf{L} = \begin{bmatrix} -(\boldsymbol{\phi}_i^{\mathrm{oc}})^{\mathrm{T}}\mathbf{W} \\ 0, \dots, 0 \end{bmatrix} \tag{12}$$

$$\mathbf{E} = \begin{bmatrix} -2\omega_i^{\mathrm{oc}}\xi_i^{\mathrm{oc}} & -(\omega_i^{\mathrm{oc}})^2 \\ 1 & 0 \end{bmatrix} \tag{13}$$

Recalling Eq. (3), the solution of each of these equations can be written as:

$$\begin{bmatrix} \dot{q}_i(t) \\ q_i(t) \end{bmatrix} = \boldsymbol{\varphi}_i^{\mathrm{oc}}(t, t_0) \begin{bmatrix} \dot{q}_i(t = t_0) \\ q_i(t = t_0) \end{bmatrix} + \int_{t_0}^{t} \boldsymbol{\varphi}_i^{\mathrm{oc}}(t, \tau)\,\mathbf{L}\,\mathbf{u}(\tau) d\tau \quad \forall\, i = 1, \dots, N \tag{14}$$

Defining $\omega_{\mathrm{d},i}^{\mathrm{oc}} = \omega_i^{\mathrm{oc}}\sqrt{1 - \left(\xi_i^{\mathrm{oc}}\right)^2}$ and $\delta_i^{\mathrm{oc}} = \xi_i^{\mathrm{oc}}\omega_i^{\mathrm{oc}}$, the transition matrix $\boldsymbol{\varphi}_i^{\mathrm{oc}}$ of these SDOF systems is:

$$\boldsymbol{\varphi}_i^{\mathrm{oc}}(t, t_0) = \exp[-\delta_i^{\mathrm{oc}}(t - t_0)]\ \mathbf{P}_{\boldsymbol{i}}(t, t_0) \tag{15}$$

with:

$$\mathbf{P}_{\boldsymbol{i}}(t, t_0) = \begin{bmatrix} p_{11}(t, t_0) & p_{12}(t, t_0) \\ p_{21}(t, t_0) & p_{22}(t, t_0) \end{bmatrix} \tag{16}$$

and

$$p_{11}(t, t_0) = \cos\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t - t_0)\right] - \frac{\delta_i^{\mathrm{oc}}}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t - t_0)] \tag{17}$$

$$p_{12}(t, t_0) = -\omega_{\mathrm{d},i}^{\mathrm{oc}}\sin\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t - t_0)\right] - \frac{(\delta_i^{\mathrm{oc}})^2}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t - t_0)] \tag{18}$$

$$p_{21}(t, t_0) = \frac{1}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t - t_0)] \tag{19}$$

$$p_{22}(t,t_0) = \cos\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right] + \frac{\delta_i^{\mathrm{oc}}}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right] \tag{20}$$

The integral of Eq. (14) is a convolution (represented here by the symbol $\otimes$). Therefore, Eq. (14) can be written as:

$$\dot{q}_i(t) = \exp\left[-\delta_i^{\mathrm{oc}}(t-t_0)\right] \left(\left\{\cos\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right] - \frac{\delta_i^{\mathrm{oc}}}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right]\right\}\dot{q}_i(t=t_0) + \left\{-\omega_{\mathrm{d},i}^{\mathrm{oc}}\sin\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right] - \frac{\left(\delta_i^{\mathrm{oc}}\right)^2}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right]\right\}q_i(t=t_0)\right) + \left(\exp\left[-\delta_i^{\mathrm{oc}}(t-t_0)\right]\left\{\cos\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right] - \frac{\delta_i^{\mathrm{oc}}}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right]\right\}\right) \otimes u_i(t) \qquad \forall\, i = 1, \dots, N \tag{21}$$

$$q_i(t) = \exp\left[-\delta_i^{\mathrm{oc}}(t-t_0)\right] \left(\left\{\frac{1}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right]\right\}\dot{q}_i(t=t_0) + \left\{\cos\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right] + \frac{\delta_i^{\mathrm{oc}}}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right]\right\}q_i(t=t_0)\right) + \left(\exp\left[-\delta_i^{\mathrm{oc}}(t-t_0)\right]\left\{\frac{1}{\omega_{\mathrm{d},i}^{\mathrm{oc}}}\sin\left[\omega_{\mathrm{d},i}^{\mathrm{oc}}\,(t-t_0)\right]\right\}\right) \otimes u_i(t) \qquad \forall\, i = 1, \dots, N \tag{22}$$

Finally, the structural displacements and velocities (i.e. **r**) can be estimated as:

$$\mathbf{r}(t) = \begin{bmatrix} \mathbf{\Phi}^{\mathrm{oc}} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}^{\mathrm{oc}} \end{bmatrix} \mathbf{Q} \tag{23}$$

where **0** is a null matrix of size equal to that of $\mathbf{\Phi}^{\mathrm{oc}}$ and $\mathbf{Q}$ is defined as:

$$\mathbf{Q}(t) = \begin{bmatrix} \dot{q}_1 & \dots & \dot{q}_N & q_1 & \dots & q_N \end{bmatrix}^{\mathrm{T}} \tag{24}$$

If accelerations are needed, they can be easily estimated by numerical derivation of velocities. It is noticed that the proposed method is a multi-degrees-of-freedom method able to account for many structural modes together. This allows running the model on any desired frequency range.

There are specific cases in which the people change place over the structure and/or posture (e.g. in case of people walking along a structure, cases in which people is jumping and the contact between the structure and the people is sometimes missing). In these cases, every time the configuration of the whole system changes, Eqs. (21) and (22) can be still used. However, the coefficients of these equations must be recalculated at every change of the system (thus a modal extraction is required every time people change posture/location and thus the system changes), and the motion of the whole system in the new configuration can be solved using the last state of the whole system in the previous configuration as initial conditions.

When there is a continuous change of posture of moving people, it is possible to discretise the motion of people by modelling the movement as a sequence of postures for which the AM curves have been collected in advance, as already demonstrated in different works (e.g. [31,45]). This allows using Eqs. (21) and (22) to describe the structural dynamics even in case of continuous motion of people over the structure.

A remarkable feature of Eqs. (21) and (22) is that there is no need to integrate differential equations, with a consequent reduction of possible numerical problems and computational burden. The reason why the use of convolution allows reducing the computational burden for the simulation, compared to integration, is not only related to the lower amount of time needed by a common computer to perform the convolution, but also to a significant simplification of the whole procedure. Indeed, when time integration is used, each measured AM curve is used to fit a lumped parameter model (made from masses, springs and dampers), which is then employed to simulate the effect of a given person in the corresponding posture. When the trend of the AM curve is not trivial (e.g. due to the presence of more than one resonance), the fit can become difficult and time-consuming, also implying the need of checks from the user for the quality of the fit, thus making difficult to have an automatic identification procedure. Conversely, this additional fit is not needed for the convolution approach because the experimental AM curve is directly employed in Eq. (8).

3 Inputs to the model and procedure to be used

To estimate the structural response for a given HSI problem, the following data are needed as inputs to Eqs. (21) and (22):

1. AM curves of all the people expected to occupy the structure (in all the considered postures). These curves are estimated experimentally. The set-ups used to estimate them are usually based on shakers to provide the input vibration to people, and on dynamometric plates to measure the force exerted by people in response to the input vibration. Different papers in the literature propose experimental set-ups to measure the AM curves (e.g. [30,53–55]). Figure 2a shows the experimental set-up used to the purpose of this work. The person stands over a plate that is connected to the movable part of an electro-dynamic shaker through three load cells

measuring the force exerted by the person while the shaker provides a random disturbance in the frequency range of interest (see further in the paper). The acceleration provided by the shaker is measured by means of accelerometers and the plate is stiff enough to have the first eigenfrequency above the interesting frequency range. The acceleration and force signals are then divided into sub-records and used to estimate the AM curves by employing the H1 estimator [49]. Therefore, each resulting AM curve is an average curve and it represents a reliable estimate of the actual AM curve of the subject in a given posture;

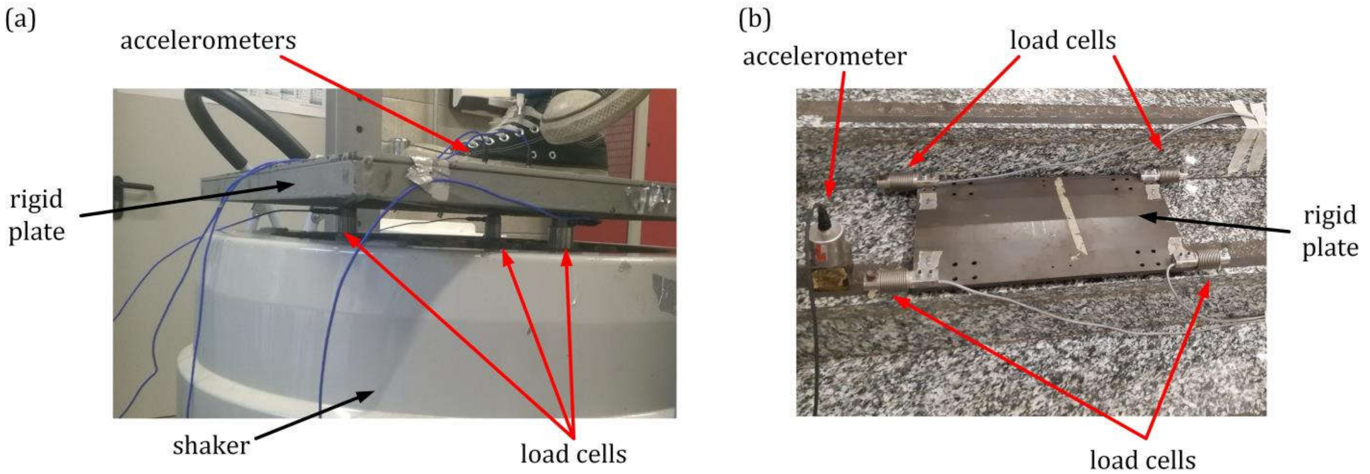

Figure 2: Experimental set-ups for measuring AM curves (a) and AGRFs (b).

2.AGRF time-histories to estimate the modal force terms $u_i(t)$. Even if there are different models in the literature proposing approaches for modelling these forces (e.g. [15,56]), they are related just to specific cases (e.g. people jumping [46], people walking on flat ground [1]). A straightforward approach when models are not available is to build a database of AGRFs measured by means of tailored set-ups (e.g. see the work of Cappellini et al. [31] for people ascending and descending staircases). Figure 2b presents the set-up used to the purpose of this work in order to collect the AGRFs. It is based on a rigid plate laid down on a step of a stiff staircase. The force exerted by the pedestrian is measured by using four load cells. Furthermore, an accelerometer is placed close to the plate to detect the time instant in which the pedestrian starts and stops to be in contact with the dynamometric plate. It is noticed that the plate and the staircase considered for this test are stiff enough to have their first eigenfrequency at a frequency value much higher than the interesting frequency range;

3. user defined parameters of the simulation: number of pedestrians, their trajectory, etc.;

4. modal data of the empty structure, which can be estimated carrying out an initial modal analysis (or even with numerical models).

When all these data are available, the approach described in the previous section can be used to estimate the structural response in case of HSI. The following easy-to-apply procedure is employed to perform the simulation:

1. measure the AM curves for one (or more) people in the postures which are relevant for the specific studied application;
2. measure the AGRFs (if present for the specific application) by means of a tailored set-up;
3. estimate the modal parameters of the empty structure by means of an experimental modal analysis or using models;
4. set the configuration of people on the structure (i.e. number of people, locations, and postures);
5. use Eq. (8) to estimate the FRFs of the occupied structure and then estimate its modal parameters;
6. Eqs. (21), (22) and (23) are used to estimate the structural response **r**(t) under HSI;
7. if the configuration of people on the structure (i.e. number of people, locations, and postures) changes, go at point 4 of this list and start again, using as initial condition (i.e. $\dot{q}_i(t=t_0)$ and $q_i(t=t_0)$) the last of the previous step of simulation.

4 Experiments and validation of the approach

The case considered for the validation of the approach proposed herein is related to pedestrians ascending and descending a staircase. This case is considered a good test-case because it is characterised by many changes of configurations related to both the posture and the location of pedestrians. Indeed, each pedestrian continuously changes posture and location in time.

The structure used for the experiments is a staircase made from steel with steps covered by marble. The number of steps is thirty, with the addition of two plateaus at approximately one third and two thirds of the staircase length. The structure is presented in Figure 3. Its length is 12.03 m, its width is 1.80 m and its height is 5.22 m. The vertical response of the structure was measured by means of 25 seismic piezoelectric accelerometers both during HSI tests and during experimental modal analysis of the empty structure carried out using a small inertial shaker. The accelerometers were placed every three/four steps and in different points of the two plateaus present along the structure (see Figure 3), at both sides of the staircase as well as in the central part. The first two modes of the structure were considered for the tests and their modal data are gathered in Table 1. These two modes are those in the frequency range mostly interesting in HSI problems (i.e. below approximately 20 Hz) and, thus, attention is focused on them.

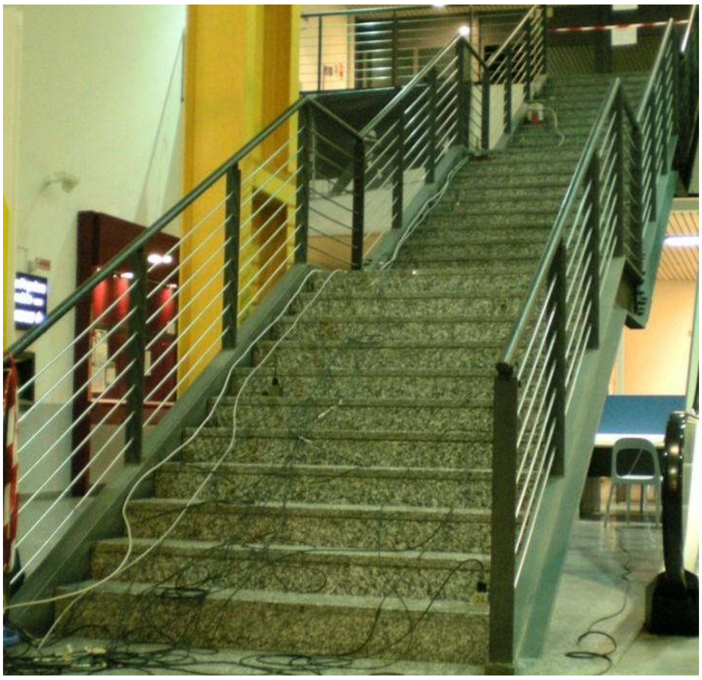

Figure 3: Staircase employed for the experimental tests.

Mode	$\omega_i/2\pi$ [Hz]	ξ_i [%]
1	7.81	0.38
2	8.87	0.35

Table 1: Identified modal parameters of the empty staircase.

The eigenvector components (scaled to the unit modal mass) of the empty structure were estimated in the DOFs where accelerometers were placed, while the mode shape components in the other DOFs of the structure were estimated by interpolation [31]. Different interpolation algorithms were tested and no significant differences were noticed among the results. Finally, a bi-harmonic spline interpolation was used and the number of DOFs where the eigenvector components were estimated was equal to 2500. The two considered mode shapes were close to those presented in [31] for the same structure: the first mode has a bending shape, while the second shows bending coupled to torsion.

Five people were involved in the tests. Their main features are provided in Table 2.

Subject ID	mass [kg]	height [cm]	gender
1	85	175	male
2	90	185	male
3	55	165	female
4	80	185	male
5	70	180	male

Table 2: Subjects involved in the experimental tests.

For each of them, the AM curves were measured in different postures. The motion (both ascending and descending) was discretized in three postures (see Figure 4): one for the first third of the step, one for the second third, and one for the last third. Figure 5 shows some of the AM curves as examples. Twelve AM curves were collected for each pedestrian: three postures for the ascending direction for both the feet, and three postures for the descending direction for both the feet. Figure 6 shows one of the subjects during the tests.

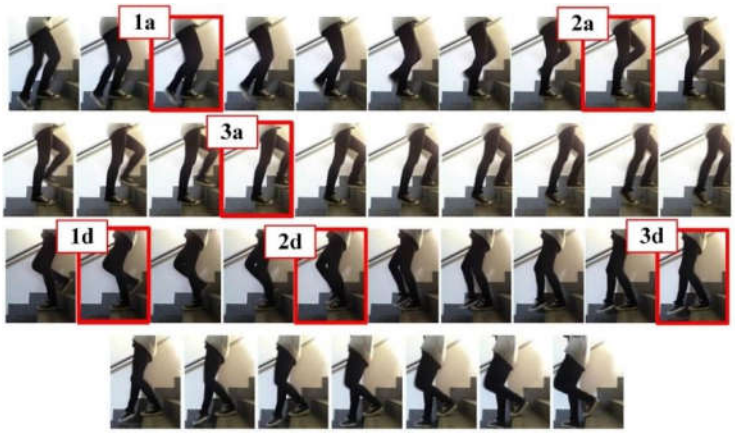

Figure 4: Postures chosen for measuring the apparent mass: 1a, 2a and 3a for the ascending direction and 1d, 2d and 3d for the descending direction. In all cases the pedestrian stands on just one foot. As for the ascending positions, just the foot tip is laid down on the step; as for the descending positions, the foot is completely laid down on the step.

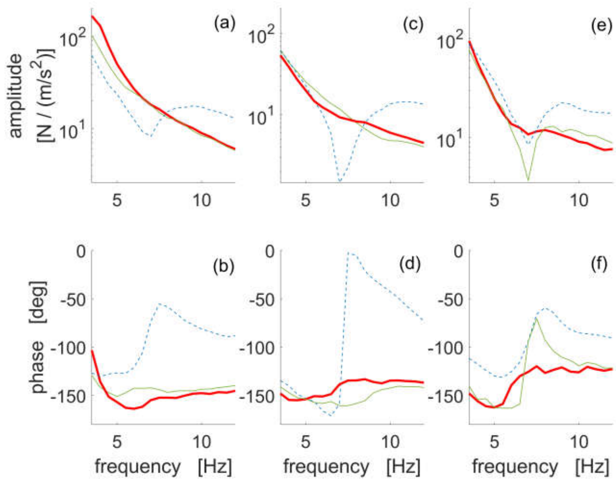

Figure 5: Measured AM curves for the right foot. Subject 1: amplitude (a) and phase (b); subject 5: amplitude (c) and phase (d); subject 2: amplitude (e) and phase (f). Red thick solid lines for posture 1a, green thin solid lines for posture 2a and blue thin dashed lines for posture 3d.

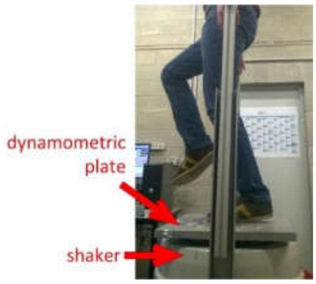

Figure 6: Test in posture 3a for subject 5.

Figure 5 shows the AM curves for different subjects in different postures in the frequency range around the modes considered here (i.e. the first two modes of the structure at 7.81 and 8.87 Hz, see Table 1). It is noticed that the pedestrians showed different AM curves for the same posture (compare the same line type for the different plots in Figure 5), and also that the same pedestrian has different AM curves in different postures (compare the different line types in the same plot of Figure 5).

Furthermore, a database of AGRFs was collected for the five people described in Table 2. The use of a database of measured AGRFs allowed to take into account the great variability of the AGRFs between one pedestrian and the others, between the ascending and descending AGRFs, and also among different steps of the same pedestrian. Furthermore, no models are available in the literature for modelling AGRFs in case of motion along a staircase. Each of the five people was asked to walk forty times on the instrumented step: ten times ascending with the right foot, ten times ascending with the left foot, ten times descending with the right foot, ten times descending with the left foot. Finally, a database of 200 time-histories of AGRFs was stored. Some of the stored AGRFs are depicted as examples in Figure 7.

The experimental tests related to HSI were carried out by asking pedestrians to go upward and downward along predefined paths, on the right and left side of the staircase, respectively. The pedestrians crossed the staircase just once (starting from either the ground floor or the first floor), and then they left the structure. Therefore, the time length of each test was of few tens of seconds, which is the time needed to cross the staircase.

Different types of tests were carried out, where the number of pedestrians involved in the tests changed test by test (see Table 3). Here, just three types of tests are discussed for the sake of conciseness and because the conclusions related to these tests are close to those of all the others. Each test was repeated different times for increasing the statistical reliability of the experimental results.

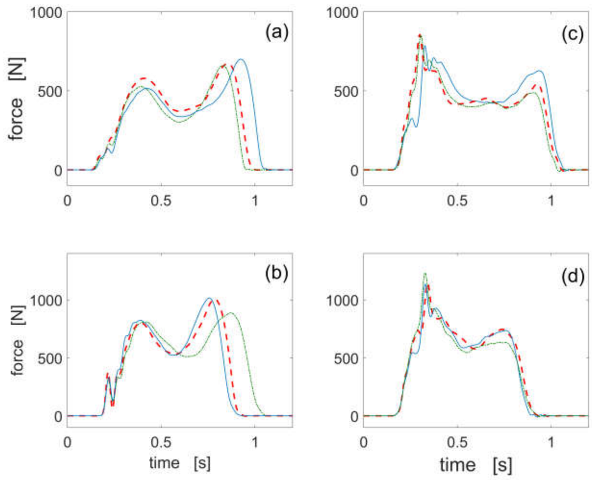

Figure 7: Measured AGRFs: subject 3 - ascending (a), subject 4 - ascending (b), subject 3 - descending (c), and subject 4 - descending (d). The sampling frequency is 256 Hz.

Before ending this subsection, an additional aspect is worth being mentioned. The aim of the experimental campaign is to validate the proposed approach through experimental data. For this reason, only the subjects involved in the experimental tests on the staircase have been characterised in terms of AM curves and only their AM curves are used in the validation process, thus allowing to reproduce exactly the same experimental conditions in the simulations. Furthermore, the use of few people (i.e., five subjects) for the tests allows avoiding any averaging effect that could hide any possible bias effect of the proposed estimation approach. Therefore, considering few people per test enables to have accurate comparisons between experimental and predicted results.

When the proposed approach is instead used for prediciting the human-induced vibrations of a given structure supposing that a variable and undefined number of people will occupy it, it is of course not possible to measure the specific AM curve of all the subjects. Thus, a reasonable approach is to measure the AM curve of several people (for all the needed postures) and then, after determining the mean AM curve together with its scatter, a statistical model of the AM curve can be used to simulate many different subjects.

4.1 Description of the results

The results were expressed as root mean square (RMS) values of the acceleration of the structure in the instrumented DOF showing the highest acceleration values. It is noticed that both the modes considered (see Table 1) have their largest eigenmode components in the same area of the structure (close to the upper plateau, see Figure 3). Therefore, the RMS values, used to compare experiments and simulations, depend on the structural response of both the modes.

The acceleration RMS values were calculated following two different approaches. The first was related to the calculation of the RMS of the whole acceleration time history (WRMS). However, another interesting result which can be extracted from the accelerometer time signals is the moving RMS, which is able to properly take into account the transients. In this case the RMS was calculated on time windows of 3 s (also 1 s was tested without any significant change in terms of result accuracy). Therefore, the accelerometer signal was used to produce a time series of RMS values. The maximum of these values (MRMS) was chosen as another way to express test results. The MRMS can properly take into account the transient in which the vibration is at its maximum level.

All the signals were low-pass filtered in order to compare numerical and experimental signals only in the frequency band related to the first two modes of the structure (which are those considered in the model). Therefore, the cut-off frequency of the filter was set to 12 Hz.

Test ID	number of pedestrians involved	number of repetitions
1	5 (2 ascending and 3 descending)	6
2	3 (2 ascending and 1 descending)	12
3	2 (1 ascending and 1 descending)	10

Table 3: Description of the tests.

4.2 Dispersion associated to numerical results

The aim of the tests described herein is to compare the results of the model with the experimental ones in terms of structural vibration. Obviously, the experimental results are affected by a natural scatter. This subsection explains how to have result scatter also for the numerical simulations in order to have a reliable comparison.

The simulations must be able to reproduce the natural scatter of experimental results due to factors changing from one test repetition to another test repetition (e.g. AGRFs). Therefore, the simulation of each test configuration (see Table 3) was repeated 100 times changing the AGRFs along the steps in order to take into account these random effects also in the simulations. Indeed, there are two main factors able to change the results associated to a given test and which must be taken into account also in the simulation process:

1. the AGRFs exerted on the structure by each pedestrian can be different from one test repetition to another one and from one step to the following one. For this reason, 20 AGRFs for the ascending direction and 20 for the descending direction were stored for each pedestrian involved in the tests (see previously). Therefore, at each new step of a pedestrian, the AGRF is randomly extracted from the database, according to the direction of the motion. The extraction was carried out using a uniform distribution. Even if each pedestrian involved in the tests had his/her own AGRF database, all the AGRFs were put in two common databases (i.e. one for the ascending direction and one for the descending direction) and thus an AGRF of a pedestrian could have been associated to another pedestrian at a given time instant of the simulations. This allowed to have a greater intra-subject variability [30] in the simulations. Indeed, even if 40 time-histories were acquired per person, they were acquired in a short time interval, and it is evident that two AGRFs of the same person measured at short time distance are reasonably close while two AGRFs measured at different times of the day can slightly differ, e.g. due to fatigue;

2. the distances among the pedestrians on the staircase change in time during the experiments. Indeed, the pedestrians were asked to try to keep constant the distance from the preceding pedestrian but without imposing a given rhythm of walk (e.g. using a metronome) in order to make the walking action as natural as possible for each pedestrian. Obviously, this resulted in changes of the distances among pedestrians during

the tests. This effect was reproduced in the simulations thanks to the fact that the AGRFs stored in the database are of different time length (e.g. see Figure 7). Indeed, the time spent by a pedestrian on a given step is imposed in the model by the time length of the AGRF extracted for the same step (see more details below). Therefore, the random extraction of the exerted AGRFs led also to the change in time of the distances among the various pedestrians simulated on the structure.

All these aspects were considered in the simulations in order to reproduce the experimental tests in the most reliable way. Hence, to account for the result dispersion, 100 values of WRMS and MRMS were calculated for each test configuration (one for each simulation repetition). The results of the numerical simulations, being almost Gaussian, are described here with an interval characterised by a central value equal to the mean value μ of the 100 results and bounds equal to $\mu \pm 2\sigma$ (where σ is the standard deviation of the 100 results) [57] (confidence level of approximately 95%).

Finally, it is evidenced that, in the simulations, each pedestrian was considered in contact with a given step of the staircase for the time length of the extracted AGRF (indicated as T_{st}). During this time, the AM curve used is the first one (i.e. 1a and 1d for the ascending and descending direction, respectively; refer to Figure 4) for the first third of T_{st}, the second one (either 2a or 2d) for the second third of T_{st} and the third (either 3a or 3d) for the last third of T_{st}.

4.3 Comparison of results

Figure 8 shows the results of experiments and simulations for the three tests of Table 3. The values of μ are indicated by squares (□), the values of $\mu - 2\sigma$ are indicated by triangles with the tip towards left (◀), and the values of $\mu + 2\sigma$ are indicated by triangles with the tip towards right (▶). As for the experimental results, they are indicated by circles (o). The circles are six for test 1, twelve for test 2 and ten for test 3, being equal to the test repetitions (see Table 3).

It is evident that MRMSs are higher than WRMSs for both experiments and simulations, as expected. Furthermore, the higher the number of pedestrians is, the higher the RMSs are (on the average). This tendency is well predicted by the simulations for both the WRMS and the MRMS. Moreover, also the dispersion of the experimental results is not far from the predicted one.

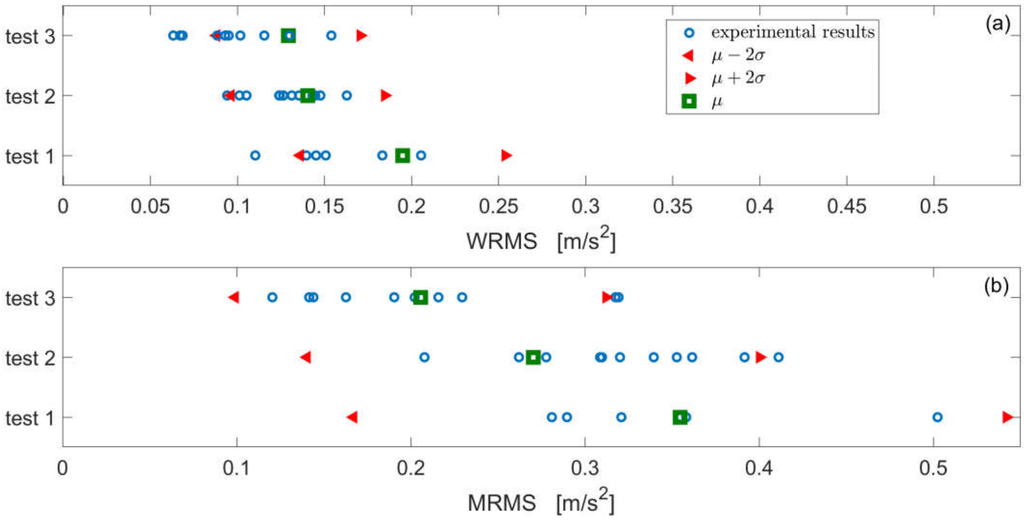

Figure 8: Numerical and experimental results for the tests of Table 3: WRMS (a), and MRMS (b).

In almost all the cases the prediction is reliable. Indeed, few experimental results fall outside the corresponding numerical intervals $\mu \pm 2\sigma$. This occurs especially in the case of WRMS, where there are some experimental results slightly lower than the intervals predicted by the simulations (see tests 1 and 3 in Figure 8a). This is expected to be related to the variability of the AGRFs. However, the model never significantly underestimates the experimental results. This means that no risks of underestimation of the operating vibration levels are found, which is a positive fact under the safety point of view.

A remarkable result is related to MRMSs. They are well predicted by the numerical results (see Figure 8b), which means that the method proposed in this paper is able to accurately describe the transient of a structure under HSI effects. Indeed, the few experimental results outside from the bounds predicted by the numerical model are very close to the bounds. Therefore, the proposed model is able to accurately predict the largest vibration values shown by a vibrating structure under HSI.

Finally, it is underlined that further simulations (not shown here for the sake of conciseness), where the PGRFs were not considered in the numerical model and only AGRFs were employed to predict the structural vibration, gave results significantly overestimating the experimental ones, indicating the need of considering the PGRFs for a proper prediction of the HSI effects.

The proposed approach is expected to work properly in all those cases where there are no evident non-linear behaviours (as in the presented experimental activity). As an example, if soft parts (e.g. made from rubber) showing non-negligible non-linear behaviour were added between the hosting structure and the people, the model would require an update in order to properly describe these additional phenomena and provide reliable estimates of the structural vibrations.

5 Conclusions

This article has addressed the problem of estimating HSI effects on the vertical dynamic behaviour of slender structures. Particularly, the target was to develop a method able to provide predictions of HSI effects when no models of human dynamic behaviours can be used/are available and only experimental data have to be employed. The proposed approach relies on the assumption of considering the effect of each person as the superimposition of two actions: PGRFs and AGRFs. The sum of the two provides the global action of each person on the dynamics of the structure. Furthermore, an analytical treatment based on modal coordinates and convolution operations is proposed and it is shown to allow for the use of experimental data related to the dynamic behaviour of humans. The use of convolution operations in place of integration of differential equations allows for a reduction of possible numerical problems and the computational burden, making the modelling of complex situations where many people occupy a structure, also with different behaviours (e.g. walking, running, jumping), feasible.

The method has been validated by means of an experimental campaign carried out on a slender staircase. It showed to be able to provide satisfactory estimations of the structural response. More in detail, the method showed high accuracy in predicting the transients of the structural dynamics and the largest vibration values due to HSI.

All tests were carried out in accordance with The Code of Ethics of the World Medical Association (Declaration of Helsinki) for experiments involving humans.

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