Design and Optimisation of Weak Stability Boundary Transfers to Unstable Libration-Point Orbits: an Application to the LUMIO CubeSat

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***Abstract* – Due to the recently renewed interest in lunar exploration, two novel approaches in trajectory design are emerging. Firstly, Weak Stability Boundary (WSB) transfers are becoming a cost-effective alternative for lunar missions, utilizing the Sun's gravity to reduce propellant requirements despite longer times of flight. Secondly, libration-point orbits around the Moon are being exploited to further minimise delta-v costs by exploiting their unstable dynamics. These two techniques are particularly advantageous for limited-capability spacecraft like CubeSats, exemplified by the LUnar Meteoroid Impacts Observer (LUMIO) mission, which will observe lunar meteoroid impacts from a quasi-halo orbit around the Earth-Moon L_2 point. To reach its final orbit, LUMIO is expected to exploit a shared launch opportunity on a WSB transfer.**

The focus of this work is on designing and optimizing WSB transfers to LUMIO operative orbit, but envisioning a dedicated launch scenario. The design process starts from a simplified gravitational model, where dynamical system theory can be easily exploited to model the transfers. Then, transfers are optimised in a complete ephemerides-based model of the Solar System. Results show that, by exploiting a dedicated launch, completely ballistic trajectories are possible, and the transfer delta-v for the LUMIO mission can be lowered to less than 0.1 m/s. Comparisons are drawn between these results and those with a shared launch opportunity, highlighting the sensitivity of WSB transfers to launch conditions.

I. INTRODUCTION

In recent times, there has been a resurgence of interest in lunar exploration, as a high number of space missions have flown or are planned to fly to the Moon in this decade. Traditionally, lunar missions have always utilized direct, Hohmann-like transfers to reach the Moon [1]. However, in recent times there has been a paradigm shift in the way to reach the Moon: Weak Stability Boundary (WSB) transfers are becoming a valid alternative as they use less propellant with respect to traditional transfers [2].

The concept of WSB was first introduced by Belbruno [3], as a sort of expansion of the concept of sphere of influence of a body. Indeed, the WSB of a central body is defined as the region in which stable closed motion breaks down, due to the balance between the gravitational attraction of the main body and the perturbations of other bodies. These regions can be exploited to perform an Earth–Moon WSB transfer: the spacecraft is sent to the Earth WSB, where solar gravity acts as a substitute for a manoeuvre by increasing the periapsis radius up to the one of the lunar orbit and by decreasing the characteristic energy value with respect to the Moon [4]. These characteristics help reducing the propellant required for the transfer, at the expense of increasing the time of flight. Two examples of recent missions leveraging on WSB transfers are the KPLO mission [5], which reached a Low Lunar Orbit in December 2022, and the CubeSat CAPSTONE [6], which reached a Near Rectilinear Halo Orbit (NRHO) around the Earth–Moon L_2 point in November 2022.

In addition to the use of WSB, new lunar missions also exploit libration-point orbits around the Moon, such as the one used by the CAPSTONE mission. Indeed, these kinds of orbits offer some advantages, such as continuous communications with Earth, and the possibility of exploiting their unstable dynamics to reduce the Δv of the transfer, as a quasi-asymptotic approach to the target can be achieved [7]. Some missions flying on libration-point orbits are the proposed Lunar Gateway [8], which will fly on the same orbit as CAPSTONE, and the EQUULEUS mission [9], which is targeting a halo orbit around the second lunar Lagrange point.

As WSB transfers and libration-point orbits offer numerous advantages for lunar missions, a great effort in research is devoted to trajectory design which combines these two novelties. In particular, dynamical system theory has been widely used to design WSB transfer to lunar libration-point orbits. A pioneering work in this field is the one of Parker [10], where the simplified dynamical model of the Patched Circular Restricted 3-Body Problem is used. Moreover, other works, such as [11], exploited the dynamical insights of the Bi-Circular Restricted 4-Body Problem to design WSB transfers to both libration-point and conic orbits around the Moon.

Finally, the advantages of these trajectory design techniques become even more relevant when applied to limited-capability spacecraft such as CubeSats. A CubeSat mission capitalizing on these techniques is the LUnar Meteoroid Impacts Observer (LUMIO) [12]. This mission aims at observing and characterizing meteoroid impacts on the lunar far side, enhancing lunar situational awareness and complementing Earth-based observations. To do that, LUMIO will operate on a quasi-halo orbit around the Earth-Moon L_2 point, which will be reached via a WSB transfer. As it is often the case for CubeSats, LUMIO is expected to exploit a shared launch opportunity [13], meaning that it will be inserted in a trans lunar trajectory not optimised for its own target. Then, it will rely on its own propulsion to achieve the operational orbit.

This work, instead, focuses on designing and optimizing WSB transfers for the LUMIO mission by envisioning a dedicated launch scenario. For this reason, new methodologies are required with respect to the previous LUMIO mission analysis [13]. These are developed in this work but are general, so that they can be easily applied to other missions, provided that an unstable libration-point orbit is the final target.

The WSB design process starts from a simplified gravitational model, where dynamical system theory can be easily exploited to model the trajectory. Then, transfers are optimized in a comprehensive ephemerides-based model, accounting for all major bodies in the Solar System and including the effects of the solar radiation pressure.

Regarding the organization of the paper, first the utilized dynamical models are introduced. Then, the methodologies regarding the WSB design and optimization are explained, Next, results of their application to the LUMIO mission are discussed. Finally, these results are compared to the ones of the LUMIO mission analysis, particularly highlighting the differences between the dedicated and shared launch scenarios.

II. DYNAMICAL MODELS

In this section, the dynamical models used in this work will be introduced.

A. Patched Circular Restricted Three-Body Problem

The minimum set of gravitational bodies that must be considered to compute a lunar WSB transfer is composed by the Earth, the Moon, and the Sun. The simplest dynamical model which includes these bodies is the Patched Circular Restricted 3-Body Problem. In this model, the gravitational attractions of the Sun and the Moon are switched on and off depending on the spacecraft distance to the Moon. Indeed, if the spacecraft is inside the Moon 3-Body Sphere of Influence (3BSOI), the Earth-Moon CR3BP is considered, otherwise the Sun-Earth CR3BP is employed. The radius of the Moon 3-Body SOI is defined as [10]:

$$r_{3BSOI} = a\left(\frac{m_M}{m_S}\right)^{\frac{2}{5}} \quad (1)$$

where a is the distance between the Moon and the Sun, which is approximated with the Astronomical Unit and m_M and m_S are the mass of the Moon and the Sun.

Being this model a simple patching of two CR3BPs, no modifications in the usual equations of the CR3BP are needed. Assuming a reference frame centred on the primaries barycentre and rotating with them, a normalization of the units such that the distance between the primaries, their angular speed and sum of their masses are all equal to 1, the equations of motion of the CR3BP read [14]:

$$x'' = 2y' + x - (1-\mu)\frac{x+\mu}{r_1^3} - \mu\frac{x-1+\mu}{r_2^3} \quad (2)$$

$$y'' = -2x' + y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} \quad (3)$$

$$z'' = -(1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3} \quad (4)$$

where r_1 and r_2 are the distances between the spacecraft and the primary bodies and $\mu = m_1/(m_1+m_2)$, m_1 and m_2 being the masses of the primaries.

B. Roto-Pulsating Restricted n-Body Problem

The other dynamical model used in this work is the Roto-Pulsating Restricted n-Body Problem (RPRnBP). This is a complete ephemerides-based gravitational model of the Solar System, where the attractions of all the major bodies are included, together with the perturbation of the solar radiation pressure. Following the derivation in [15], the equations of motion are written as a perturbation of the CR3BP. The reference frame is centred on the primaries barycentre: as the primaries follow the motion dictated by the ephemerides, the frame rotates and pulsates with them. Units are normalized in an analogous way to the CR3BP, so that the equations of motion are [16]:

$$\boldsymbol{r}'' = -\frac{2}{n}\left(\frac{\dot{k}}{k}I + C^T\dot{C}\right)\boldsymbol{r}' - \frac{1}{n^2}\left[\left(\frac{\ddot{k}}{k}I + 2\frac{\dot{k}}{k}C^T\dot{C} + C^T\ddot{C}\right)\boldsymbol{r} + \frac{1}{k}C^T\ddot{\boldsymbol{b}}\right] + \mathrm{G}\frac{m_1+m_2}{n^2k^3}\left[(1-\mu)\frac{\boldsymbol{r}-\boldsymbol{r}_1}{\|\boldsymbol{r}-\boldsymbol{r}_1\|^3} + \mu\frac{\boldsymbol{r}-\boldsymbol{r}_2}{\|\boldsymbol{r}-\boldsymbol{r}_2\|^3} + \sum_i \hat{\mu}_i\frac{\boldsymbol{r}-\boldsymbol{r}_i}{\|\boldsymbol{r}-\boldsymbol{r}_i\|^3}\right] + \boldsymbol{a}_{SRP} \quad (5)$$

where:

- the symbols $'$ and $\dot{}$ are used for a derivative with respect to the non-dimensional and dimensional time, respectively;
- $\boldsymbol{r}$, $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{r}_i$ are the position vectors of the spacecraft, the primaries, and the remaining i-th body, in the roto-pulsating frame;
- n is the mean motion of the primaries;
- k is the distance between the primaries;
- C is the rotation matrix between the inertial frame centred on the Solar System barycentre and the roto-pulsating frame;
- I is the identity matrix;
- $\boldsymbol{b}$ is the position vector of the primaries

barycentre in the inertial frame;

- G is the gravitational constant;
- m_1 and m_2 are the masses of the primaries;
- $\mu = m_2/(m_1 + m_2)$;
- $\hat{\mu}_i = m_i/(m_1 + m_2)$;
- $\boldsymbol{a}_{SRP}$ is the normalized acceleration due to Solar Radiation Pressure (SRP).

The SRP acceleration is modelled by considering the spacecraft as a point mass. Thus:

$$\boldsymbol{a}_{SRP} = \frac{1}{kn^2} \frac{c_r A P_0}{m} \frac{d_0^2}{d^2} \frac{\boldsymbol{r} - \boldsymbol{r}_S}{\|\boldsymbol{r} - \boldsymbol{r}_S\|} \tag{6}$$

where $1/(kn^2)$ is the normalization coefficient, c_r is the coefficient of reflectivity of the spacecraft, A is the illuminated area of the spacecraft, m is its mass, P_0 is the SRP at a distance d_0 of 1 AU, d is the current distance between the spacecraft and the Sun and $\boldsymbol{r}_S$ is the position of Sun in the roto-pulsating frame.

III. METHODOLOGY

In this section, the methodology to design, refine, and optimise WSB transfers to unstable libration-point orbits is explained.

A. WSB Design

The design strategy presented here is similar to the one developed in [10]. The fundamental idea behind the design of this type of transfers is to exploit the unstable dynamics of the target libration-point orbit. This allows to obtain a quasi-asymptotic approach to the orbit, without the need of a relevant orbit insertion manoeuvre. To do that, the simplified dynamical model of the Patched CR3BP is exploited. In this way, the target orbit is computed in the Earth-Moon CR3BP, where its stable invariant manifold can be computed. To do that, stable eigenvectors are evaluated for each point on the orbit. These points are individuated by the parameter τ, which is measured as a fraction of the orbital period. Each state on the orbit is perturbed in the direction of its stable eigenvector, thus giving the final conditions for the transfer. These are propagated backwards through the dynamics of the Patched CR3BP for a fixed interval of time. Since this dynamical model is not autonomous, the backpropagation depends on the starting time of the integration, t_f, which will be the final time of the transfer. This is measured as the number of days after the full Moon condition: in this way t_f measures the advancement of the synodic month, and, therefore, is indicative of the relative configuration between the Earth, the Moon, and the Sun at the arrival on the orbit. Given a certain orbit, a fixed interval of propagation and a certain perturbation size, the complete integrated trajectory is determined only by τ and t_f, which represent the degrees of freedom of the design problem. Therefore, by varying independently these parameters, a 2-dimensional grid can be constructed with the aim of finding integrated trajectories that come close to the Earth. In this way, given a certain perigee radius threshold, WSB transfers from the Earth to the target orbit can be obtained.

A schematic representation of the adopted design strategy is shown in Fig. 1.

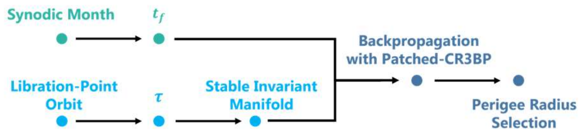

Fig. 1. WSB design strategy.

B. WSB Refinement and Optimisation

The main objective of the refinement step is to transition the trajectories from the simplified dynamical model to the full ephemerides one, while at the same time maintaining as much as possible the ballistic nature of the transfer. This is achieved by simultaneously optimising the sum of the Δv of any possible intermediate manoeuvre, which will be called Deep Space Manoeuvres (DSM), and the final Orbital Insertion Manoeuvre (OIM).

As a first point, it is important to note that this refinement can be achieved only if the relative geometry between the Earth, the Sun, and the Moon is preserved from the simplified to the complete gravitational model. This is guaranteed by selecting the right ephemerides time for the transfer, by considering that t_f measures the number of days after a full Moon configuration.

Secondly, to achieve a realistic trajectory, also the target orbit should be refined in the complete model. Given the complex nature of these interlinked problems, a step-by-step procedure is adopted:

- Problem 1: The transfer is refined in the RPRnBP and the Δv is optimised, but the target is still the CR3BP orbit.
- Problem 2: The target orbit is refined in the RPRnBP.
- Problem 3: Using the solutions of Problem 1 and 2, the transfer is optimised with the final RPRnBP orbit as the target.

A schematic representation of the adopted strategy is shown in Fig. 2.

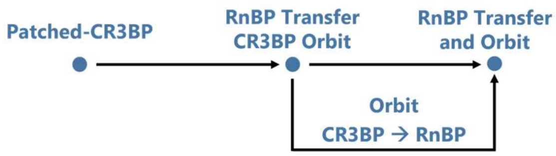

Fig. 2. Refinement and optimisation strategy.

The formulation of the first problem is the following:

Problem 1: Find the initial state of the spacecraft, $\boldsymbol{x}_1 = [\boldsymbol{r}_1, \boldsymbol{v}_1]^T$, the manoeuvres magnitude, direction, and epoch such that

$$J = \sum_{i=1}^{N} \left\| \Delta \boldsymbol{v}_{DSM}^{i} \right\| + \left\| \Delta \boldsymbol{v}_{OIM} \right\| \quad (7)$$

is minimised, while abiding to the constraints:

$$\|\boldsymbol{r}_1\| = r_c \quad (8)$$
$$\boldsymbol{r}_1 \cdot \boldsymbol{v}_1 = 0 \quad (9)$$
$$\boldsymbol{r}_f = \boldsymbol{r}_{TO}(t_f) \quad (10)$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_f$ are the initial and final position of the spacecraft, $\boldsymbol{v}_1$ is the initial velocity, r_c is the given radius of a circular parking orbit and $\boldsymbol{r}_{TO}$ is the final position on the target orbit.

It is important to note that the cost of the Trans–Lunar Injection (TLI) manoeuvre is not accounted for in the objective function J, as this manoeuvre is usually given by the launcher and not the spacecraft itself. Moreover, in this formulation only the initial altitude of the TLI is constrained, so that the optimiser can choose the best launch conditions for the WSB transfer (in terms of magnitude, direction, and epoch of the manoeuvre). Finally, also the final time can be varied during the optimisation, so that the best OIM can be achieved.

Problem 1 is transcribed into a nonlinear programming problem (NLP) and solved with a multiple-burn multiple-shooting technique, following the methodology explained in [17]. This means that the trajectory is discretised in ballistic arcs which are separated by consecutive manoeuvres, while inside each arc a number of multiple-shooting segments is defined. During the optimisation, the continuity of the trajectory must be imposed in position and velocity between each segment, and only in position between arcs.

Regarding the second problem, its formulation can be stated as follows:

Problem 2: Find the best possible dynamical substitute of a 3-body libration-point orbit in a n-body gravitational model.

This problem is solved with the methodology developed in [18]: a multiple shooting algorithm is used to impose the continuity of the whole trajectory, while minimising a scalar function that depends only on the continuity constraints. In addition, the following constraints on the difference between the insertion point of the 3-body and n-body orbits are imposed:

$$\left\| \boldsymbol{r}_{3B}(t_f) - \boldsymbol{r}_{NB}(t_f) \right\| \leq \varepsilon_r \quad (11)$$
$$\left\| \boldsymbol{v}_{3B}(t_f) - \boldsymbol{v}_{NB}(t_f) \right\| \leq \varepsilon_v \quad (12)$$

where ε_r and ε_v represent a certain tolerance on the position and velocity difference.

Finally, **Problem 3**, is stated and solved exactly as Problem 1, with the difference of the refined target orbit.

The solver adopted for all 3 of these optimisation problems is MATLAB *fmincon*, using a combination of the *interior-point* [19] and the *active-set* algorithms, the latter one being a type of Sequential Quadratic Programming algorithm [20]. It is heuristically found that Problem 1 converges more easily to the solution with interior-point, while Problem 2 and 3 prefer the active-set algorithm.

IV. RESULTS

As explained in the introduction, the WSB design and optimisation presented in this work is applied to the LUMIO mission. Hence, the selected target orbit is a southern halo around the Earth-Moon L_2 point, with a Jacobi Constant of 3.09, which is shown in Fig. 3.

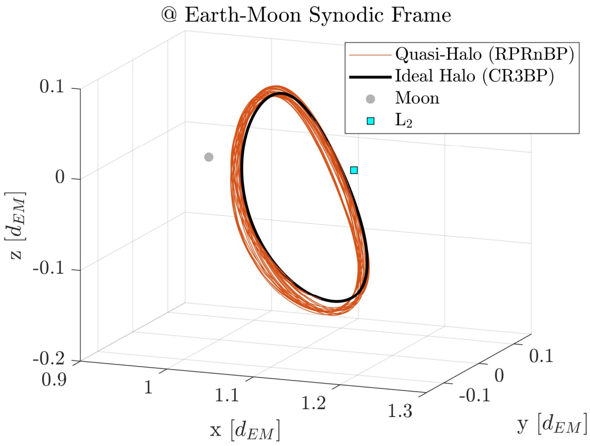

Fig. 3. Ideal halo and quasi-halo for the LUMIO mission.

The results of the WSB design for this orbit are condensed in Fig. 4, where the results of a 200x200 grid search over the variables τ and t_f are shown. Each point of the graph represents an integrated trajectory, and the colour indicates the minimum perigee radius achieved by each trajectory. As it was expected, the majority of the trajectories are not feasible WSB transfers, as their perigee radii are not in the vicinity of the Earth. However, dark blue regions indicate areas where WSB transfers are possible. These areas contain, for example, 59 transfers with a perigee radius between 6000 and 7000 km. Their trajectories are shown in Fig. 5. The shape of the transfers in the Sun–Earth Synodic frame confirms that the trajectories are indeed WSB transfers. In particular, it can be seen that the apogees are placed in the 2nd and 4th quadrant of the frame, which is in accordance with the theoretical findings on this type of transfers [21][21].

Regarding the refinement and optimisation problems, these are solved by starting from selected trajectories on the set that is shown in Fig. 5. In particular, the trajectories are chosen so that a solution is obtained for each week of the 2027, which is the currently selected year of the launch of LUMIO, for a total of 52 computed transfers. A perigee altitude of 200 km is enforced, while the maximum number of DSM is limited to 8.

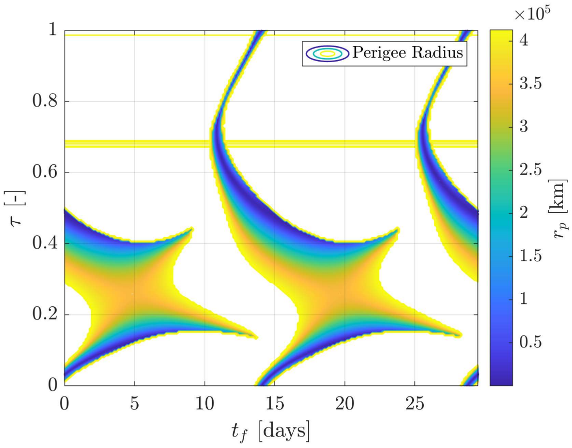

Fig. 4. WSB Design Results.

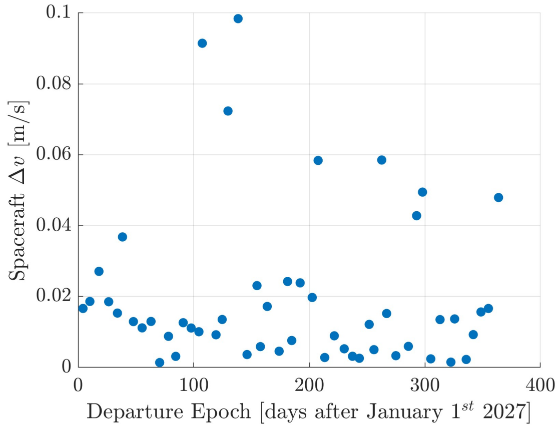

Fig. 6. Δv cost with respect to departure epoch.

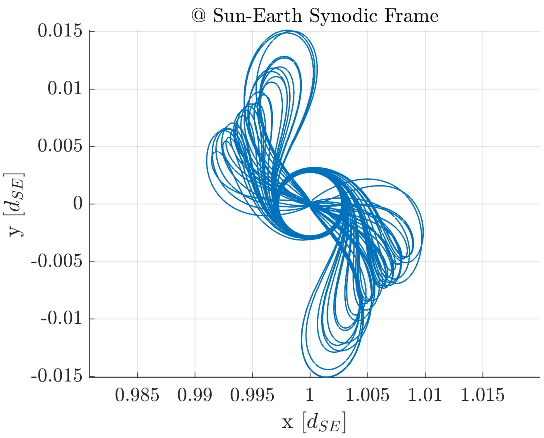

Fig. 5. WSB Transfers in the Sun-Earth Synodic Frame.

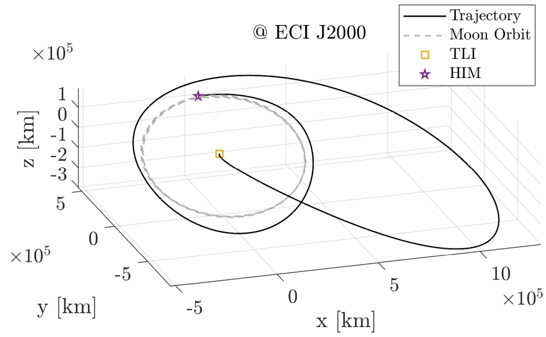

Fig. 7. Example of a WSB transfer to LUMIO orbit.

The major finding is that the adopted procedure is able to preserve the ballistic nature of the trajectories of the simplified gravitational model also in the complete one. Indeed, every intermediate DSM is set to zero, and the final Halo Insertion Manoeuvre (HIM) is always less than 0.1 m/s. This means that a realistic transfer trajectory from the Earth to LUMIO quasi-halo would not need any deterministic manoeuvre, as the transfer would follow the natural dynamics and mimic the stable manifold of the target libration-point orbit. Indeed, the little cost of the HIM, comparable to a station-keeping manoeuvre cost, is indicative of the quasi-asymptotic approach of the transfer to the orbit. The total Δv cost for all the optimised transfers is shown in Fig. 6.

Finally, an example of a WSB trajectory in the Earth-Centred Inertial (ECI) frame is shown in Fig. 7. From this figure both the typical "bielliptic" shape of the WSB transfer, and the final quasi-asymptotic approach to the target orbit can be appreciated.

V. COMPARISON WITH LUMIO MISSION ANALYSIS

In this section, the results obtained in this work will be compared to the results of the LUMIO Phase B Mission Analysis (MA) [13]. Both works consider a WSB transfer from the Earth to the same quasi-halo operational orbit. However, for the LUMIO MA, a shared launch opportunity was considered. In particular, it was foreseen a launch for a main spacecraft that flies on a WSB transfer trajectory towards a Low Lunar Orbit; from this trajectory LUMIO needs to deviate to reach its operational orbit. This means that the launch conditions are not perfectly optimised for LUMIO's target, which is instead the case for the trajectories computed with the methodology presented in this work, where a dedicated launch is considered. The trajectory characteristics shared by these two different approaches are reported in Table 1.

Table 1. Similarities between shared and dedicated launch trajectories

Type of Transfer	WSB
Perigee Altitude	200 km
Departure Epochs	Once per week in 2027
Target	LUMIO's quasi-halo

The relevant differences are, instead, reported in Table 2, where average properties of the transfers along the year 2027 are shown.

Table 2. Average properties of dedicated and shared launch trajectories over the year 2027.

	Shared Launch	Dedicated Launch
Δv [m/s]	38.56	0.02
n° DSM [-]	2	0
Δv_{TLI} [km/s]	3.196	3.201
ToF [days]	117.71	121.84
Apogee [km]	$1.527 \cdot 10^6$	$1.593 \cdot 10^6$

The major difference is in the Δv cost of the two types of transfers: shared launch trajectories have a mean Δv of 38.56 m/s, which is given, on average, with two Deep Space Manoeuvres; on the other hand, trajectories with a dedicated launch are completely ballistic, as no intermediate manoeuvre is required. From a mission analysis point of view, it seems that a dedicated launch would be the obvious best choice for LUMIO, as it greatly reduces the Δv budget. This, however, comes at the expense of higher launch costs, as the launch need to be reserved entirely for LUMIO. In addition, the availability of small launchers capable of performing a TLI manoeuvre is low, whereas rideshare opportunities are more frequent. Therefore, the choice between reserving or sharing a launch should be made at mission management level.

Nonetheless, understanding the causes behind the significant difference between dedicated and shared launch trajectories could greatly help in reducing the Δv budget for LUMIO and similar missions. Therefore, an analysis on the characteristic of the TLI conditions is performed for both classes of launches. As it is shown in Table 2, the average magnitude of the manoeuvre is similar, as both launches insert the spacecraft in a WSB transfer towards the lunar region. The marginally higher average TLI manoeuvre for dedicated launches could be explained considering that LUMIO has on operational orbit which is slightly beyond the Moon orbit (which is the target of the shared launch). As a consequence, also the average apogee radius and Time of Flight (ToF) are slightly higher. However, these small differences do not justify the vast change in the Δv of the transfer. Instead, looking at the angular Keplerian parameters of the TLI state of dedicated launches reveals some peculiarities. In Fig. 8, the Right Ascension of the Ascending Node (RAAN) of the TLI state is shown together with the inclination, with respect to the ECI J2000 frame, and for all computed transfers, which are indicated with dots. It is clear that the disposition of the TLI angular states is not random, as dots are concentrated in a semi-elliptic shaped region. Moreover, the majority of the transfers, 32 out of 52, have an inclination higher than 90 degrees, which means that these trajectories start as retrograde orbit. These peculiarities do not appear in the analogue graph made for shared launch trajectories, which is shown in Fig. 9. In this figure, a colour map based on the transfer Δv is applied. Here, the semi-elliptic shape disappears and the majority of the transfers, 39 out of 52, have an inclination lower than 90 degrees. In particular, there seems to be a higher occurrence for TLI states with low inclination and RAAN close to 0 degrees (or equivalently 360 degrees). This is something that is not found in the dedicated launch case. Therefore, from the comparison of these two figures, it seems that the shared launch conditions do not respect some sort of undefined dynamical constraint that influences the dedicated launches and makes them more optimal for the Δv budget of the mission.

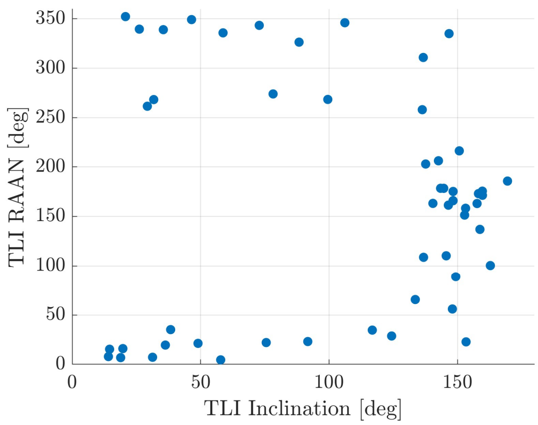

Fig. 8. Dedicated launch trajectories: TLI conditions.

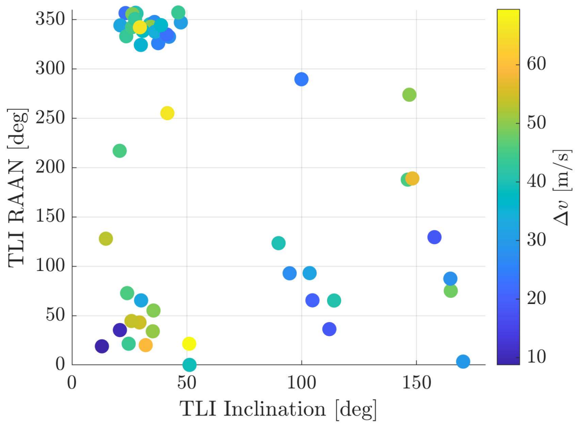

Fig. 9. Shared launch trajectories: TLI conditions.

This consideration is also remarked by the comparison between the final HIM conditions between dedicated and shared launch trajectories, which can be seen in Fig. 10 and Fig. 11. Here, the position of the insertion point along the quasi-halo orbit is measured in terms of the parameters τ and t_f, which come from the design of the WSB transfer. The parameter τ is measured as the number of days after the initial point of the halo orbit,

which coincides with the upper *y-z* plane crossing; t_f is, instead, the number of days after the full Moon configuration. It is important to note that these parameters used to be independent when designing the transfer, as the Halo orbit was computed in the CR3BP, so the position on the orbit, function of τ, was independent from the epoch, which is a function of t_f. Now, instead, in the RPRnBP the position on the orbit depends on the epoch, so τ and t_f are dependent variables. This dependency is clearly visible in the dedicated launch trajectories graph, in Fig. 10, where the polynomial fit reveals a sinusoidal relation. Moreover, the period of this sinusoidal function matches the 14 days period of the quasi-halo orbit, suggesting that the best insertion point moves periodically along the orbit as the synodic month advances, which is measured by t_f.
It is important to stress that, in this case, the HIM conditions are a function of the flow of the TLI initial conditions, since the dedicated launch transfers are ballistic trajectories.
On the other hand, considering Fig. 11, no clear dynamical relation is evident between τ and t_f, as the disposition of the dots seems to be random. This, again, is interpreted as the inability of shared launch transfers to follow the best dynamical evolution of the trajectory which is, instead, achieved with dedicated launches, where the TLI manoeuvre states can be chosen freely and optimised for.
These considerations highlight the sensitivity of this kind of WSB transfers on the launch conditions, which should be optimised to obtain the minimum Δv cost for the transfer.

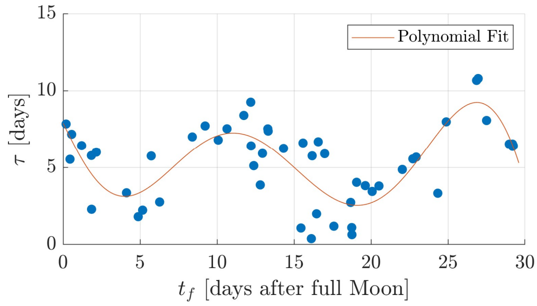

Fig. 10. Dedicated launch trajectories: HIM conditions.

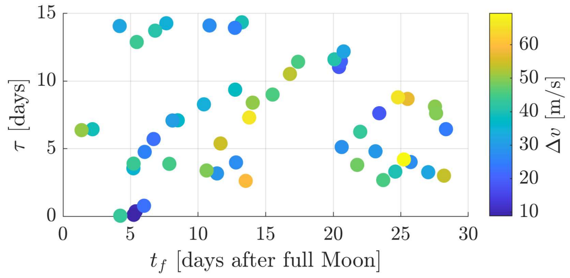

Fig. 11. Shared launch trajectories: HIM conditions.

VI. CONCLUSIONS

The work presented in this paper is focused on designing and optimising Weak Stability Boundary transfers to unstable libration-point orbits in the Earth–Moon system. A design methodology able to exploit the dynamical theory around 3-body orbits is employed. In particular, the backpropagation of the stable invariant manifold through the Patched Circular Restricted 3-Body Problem is the fundamental design technique used to obtain WSB transfers. The refinement and optimisation of the transfers in the Roto-Pulsating Restricted n-Body Problem is performed with a step-by-step procedure, where both the transfers and the target orbits are refined under the complete ephemerides model of the Solar System.
These methodologies are applied to the LUMIO mission, where a WSB transfer to an Earth-Moon quasi-halo orbit around the L_2 point is needed. In particular, 52 transfers are computed with a weekly frequency along the year 2027. In contrast with previous results for the LUMIO mission analysis, where a shared launch opportunity was envisioned, in this work a dedicated launch scenario is considered. By doing so, the launch conditions can be optimised for the trajectory of LUMIO and its final target.
The results show that the adopted procedure is able to obtain ballistic transfers, where no intermediate manoeuvres are needed. Moreover, the halo insertion manoeuvre is minimised to less than 0.1 m/s for each transfer. This represents a great improvement for LUMIO transfer Δv budget, as shared launch transfer trajectories cost on average 38.56 m/s and need on average 2 Deep Space Manoeuvrers (DSM).
An analysis is performed to understand the dynamical reasons behind this relevant difference. It is found that, although the Trans Lunar Injection (TLI) manoeuvre magnitude is similar between the two approaches, the angular states differ considerably. Consequently, also the final conditions on the halo insertion point are quite different, with the ones of the dedicated launch trajectories showing a clear dynamical relation between the position of the insertion point and the epoch of the insertion, with respect to the Earth–Moon–Sun configuration. This is, instead, not found for the shared launch trajectories. Therefore, this proves that with a dedicated launch scenario, the best possible evolution of the trajectory is achieved up to the final libration point target, which, instead, is something not possible with a shared-launch opportunity, where costly DSMs need to be performed to reach the target.
In conclusion, this study reveals that an optimisation of the launch conditions is of great advantage for WSB missions towards libration point orbits, highlighting the sensitivity of this kind of transfers to initial conditions. These results may be used by mission designers to justify the need of a more expensive dedicated launch for CubeSats flying on WSB transfers, as the propellant

savings may be relevant. Moreover, even if a shared launch opportunity is to be used, future studies may build upon the analysis performed in this work to understand which are the TLI states that produce a low Δv cost for the transfer. Thus, the possibility of predicting the Δv of a WSB transfer, in function of the TLI states and final target, may be an interesting and relevant research topic for future works.

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