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CALCULATION OF THE STATIC CHARACTERISTIC OF A HYBRID STEPPING MOTOR BY MEANS OF AN ENERGETIC METHOD

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1. Introduction.

The present work is concerned with methodologies for the hybrid stepping motor project.

The static characteristic of the torque is an important element in the evaluation: its knowledge makes it possible to determine the static behaviour of the machine and is also indicative of the behaviour at speed. This characteristic can be divided into three contributions: detent torque, due to magnet action; reluctance torque, associated with the stator m.m.f. only; holding torque, caused by interaction between the magnet and the stator m.m.f..

For each of these contributions, which are periodic functions, the amplitude of the higher harmonics is generally small with respect to the fundamental. In practice, the above-mentioned torque components are sinusoidal.

The relationship between torque and angular position can be determined point by point, calculating the derivative of the magnetic energy as a function of the angular stator-rotor position. This method, while complete and rigorous, requires very onerous calculations.

A simplified method of calculation is suggested by the sinusoidal nature of the various contributions to the torque. This method evaluates their maximum values only, giving origin to the proposed approach, which reaches its target using calculations carried out at just a few angular positions. The method makes reference to the energy balance equation, applied to a suitable finite turning of the rotor. It is presented in the general case, in which saturation is taken into account.
For project optimization, an analysis in linearity conditions, i.e. in absence of saturation, involving very simple expressions, is also considered.

2. General considerations on the hybrid stepping motor.

The hybrid stepping motor is a machine capable of high torque and speed, with a large number of steps per turn and with very precise positioning. The holding torque is of average magnitude and the rotor inertia is small. The damping of oscillations and the efficiency are both good.

The structure used most frequently is of the two-phase type. The rotor is uniformly toothed; the stator can be uniformly toothed, with two distributed windings, or it can be supplied with toothed field poles with concentrated windings. In the first case, the rotor teeth pitch is different from that of the stator. In the second case, usually the pitch is the same, but the stator poles are displaced of a fraction of a tooth pitch, plus a distance of one or more pitches.

The study carried out is basically applicable to both types of stator. All the same, for simplicity of description, reference will be made to a machine supplied with stator field poles. In practice, the conceptual difference between the two situations constists of treating as a single reluctance element at the air-gap the reluctance relating to a single stator tooth or that relating to a field pole.

In the case considered, there are eight field poles displaced of a quarter tooth pitch between them. The two rotor casings, toothed and displaced by half a tooth pitch between them, enclose axially a disc-type permanent magnet, made of a material with high residual induction. To limit the losses, the magnetic circuit is laminated. This introduces an equivalent air-gap in the longitudinal direction.

The method of calculation normally used starts from the energy balance equation. Let us call:
i_1 and i_2 the currents in the two phase windings,
Φ_{c1} and Φ_{c2} the corresponding total linkages,
θ the rotor rotation angle, referred to any equilibrium position with unpowered machine,
W_m the total magnetic energy within the system.
For an infinitesimal rotation $d\theta$ we can write:

$$i_1 \cdot d\Phi_{c1} + i_2 \cdot d\Phi_{c2} = T \cdot d\theta + dW_m \quad , \tag{2.1}$$

from which this expression for torque follows:

$$T(\theta) = -(\partial W_m/\partial\theta)|_{\Phi=\mathrm{const}} \ . \tag{2.2}$$

In equation (2.2), the expression $\partial W_m/\partial\theta$ is reduced to the only derivative of the energy W_{mv}, stored in the reluctances at the air-gap, with variable geometry. This is because the energy W_{mf}, stored in the reluctances with fixed geometry is, at constant fluxes, independent of the angle of rotation θ. Considering that the reluctances at the air-gap are linear, W_{mv} can be expressed as:

$$W_{mv} = \sum_1^8{}_k \int_0^{\Phi_k} U_{ok} \cdot d\Phi_{ok} = \sum_1^8{}_k \tfrac{1}{2} \cdot P_{ok} \cdot (U_{ok})^2 \quad , \tag{2.3}$$

where U_{ok}, Φ_{ok}, P_{ok} represent, respectively, the magnetic voltage drop, flux and permeance referring to the reluctance at the air-gap under the K-th stator pole.
From equations (2.2) and (2.3) one obtains:

$$T(\theta) = \tfrac{1}{2} \cdot \sum_1^8{}_k (\partial P_{ok}/\partial\theta) \cdot (U_{ok})^2 \ . \tag{2.4}$$

The advantages of using this relationship are:
- equation (2.4) enables the calculation to be carried out at any position inside the tooth pitch, and hence makes it possible to determine exactly the variation of the torque $T(\theta)$;
- this expression applies even under dynamic or transient conditions;
- it is possible to allow for any asymmetries due to construction defects.

On the other hand, some observations can be made:
- the use of equation (2.4) requires the resolution of as many magnetic circuits as the number of points of $T(\theta)$ to be calculated;
- to calculate the derivatives $(\partial P_{ok}/\partial\theta)$ of the permeances at the air-gap, analytical expressions for the P_{ok} are required;

-for the optimized design of the machine, the method employing (2.4) provides a lot of redundant information;
-on the other hand, the method does not provide final expressions, even in approximate form, for the maximum values of the torque components, which are of interest in guiding the design.

3. The energy variation method.

A method will now be described which permits a simplified evaluation of the maximum values of the torque components. With this in view, it is convenient to split the energy W_{mf} into different components: the energy stored in the magnet (W_{mfm}); the energy in the linear reluctances associated with magnetic circuit lamination and leakage fluxes (W_{mfl}); finally, the energy in saturated reluctances (W_{mfs}). We can thus write:

$$(3.1)\qquad W_m = W_{mv} + W_{mfm} + W_{mfl} + W_{mfs} \quad .$$

-W_{mv} is the energy stored in the air-gap and equals:

$$(3.2)\qquad W_{mv} = \sum_{k=1}^{8} \tfrac{1}{2}\cdot R_{\delta k}(\theta)\cdot(\Phi_{\delta k})^2 \quad ,$$

where $R_{\delta k}(\theta)$ is the reluctance at the k-th pole air-gap, as a function of the rotation angle θ.
-For W_{mfm}, let us consider a magnet of cross-section A_m, length l_m, residual induction B_r: the straight segment of the hysteresis cycle, along which there are the points of regular operation, intersects the H axis in the point $-H_o$ (with $H_o>H_c$=coercitive force). As it is well-known, the following relationship between flux and magnetic voltage applies to it:

$$(3.3)\qquad U_m = R_m\cdot\Phi_m - M_m \quad ,$$
with $R_m=(1/\mu_d)\cdot l_m/A_m$; $\mu_d=B_r/H_o$; $\Phi_m=B_m\cdot A_m$; $M_m=H_o\cdot l_m$.

Hence, the energy can be expressed thus:

$$(3.4)\qquad W_{mfm} = \int_0^{\Phi_m} U_m\cdot d\Phi_m = \tfrac{1}{2}\cdot R_m\cdot(\Phi_m)^2 - M_m\cdot\Phi_m \quad .$$

-The energy W_{mfl} equals:

$$(3.5)\qquad W_{mfl} = \Sigma_h\, \tfrac{1}{2}\cdot R_{lh}\cdot(\Phi_{lh})^2 \quad ,$$

where the generic R_{lh} is represented by the lamination reluctances and by the reluctances associated with the stray fluxes.
-W_{mfs} is the integral, over the volume V occupied by ferromagnetic material, of the energy w_{mfs} per unit volume (energy density):

$$(3.6)\qquad W_{mfs}=\int_V w_{mfs}\cdot dV \quad , \text{ where } w_{mfs}=\int_0^B H_s\cdot dB_s \quad .$$

The terms in the first part of equation (2.1), as a function of the m.m.f. M_k of each winding and of the related fluxes Φ_k, equal:

$$(3.7)\qquad i_1\cdot d\Phi_{c1} + i_2\cdot d\Phi_{c2} = \sum_{k=1}^{8} M_k\cdot d\Phi_k \quad .$$

The M_k can assume only two different values, as they are due to the currents in the two phases.

The energy variation method consists of integrating expression (2.1) over a finite rotation between two suitable positions, θ_1 and θ_2, at constant currents (in (2.2), on the other hand, the calculation is at constant fluxes). Bearing in mind equation (3.7), we obtain:

$$(3.8)\qquad \sum_{k=1}^{8} M_k\cdot\int_{\Phi_{k(\theta 1)}}^{\Phi_{k(\theta 2)}} d\Phi_k = \int_{\theta_1}^{\theta_2} T(\theta)\cdot d\theta + \int_{W_{m(\theta 1)}}^{W_{m(\theta 2)}} dW_m \quad ,$$

from which:

$$(3.9)\qquad \sum_{k=1}^{8} M_k\cdot[\Phi_{k(\theta 2)} - \Phi_{k(\theta 1)}] = \int_{\theta_1}^{\theta_2} T(\theta)\cdot d\theta + [W_{m(\theta 2)} - W_{m(\theta 1)}] \quad .$$

At constant currents, the work of the m.m.f. sources is transformed partly into mechanical work and partly into a finite variation in the total magnetic energy stored.

It is clear that, provided that θ_1 and θ_2 are chosen suitably, equation (3.9) provides the work done by the torque in the rotation as above.
The knowledge of the form of $T(\theta)$ then makes it possible to evaluate the maximum value of the torque component desired.

It is possible to obtain a more explicit formulation of equation (3.9) using equations (3.1)÷(3.6), developing the variation of the stored magnetic energy. In this way, it can be found that the variation relating to the m.m.f. of the magnet (M_m) is of the same type of the variation of the stator m.m.f.'s, and hence can be associated with them. We hence obtain:

$$(3.10)\qquad \Sigma_h\, M_h\cdot\Big[\Phi_{h(\theta)}\Big]_{\theta_1}^{\theta_2} = \int_{\theta_1}^{\theta_2} T(\theta)\cdot d\theta + \sum_{k=1}^{8} \tfrac{1}{2}\cdot\Big[R_{\delta k(\theta)}\cdot(\Phi_{\delta k(\theta)})^2\Big]_{\theta_1}^{\theta_2} + \Sigma_j\, \tfrac{1}{2}\cdot R_j\cdot\Big[(\Phi_{j(\theta)})^2\Big]_{\theta_1}^{\theta_2} + \Big[W_{mfs(\theta)}\Big]_{\theta_1}^{\theta_2}.$$

The calculation of the work done by the torque $T(\theta)$ reduces to the determination of the air-gap reluctances, of the fluxes flowing across the reluctances and the m.m.f.'s, and of the variation of the energy stored in saturated branches, as a function of θ_1 and θ_2.

3.1 Calculation of maximum values of the torque components.

In the following, all the quantities pertinent to the condition of 1 or 2 phases powered are indicated with the subscripts I and II respectively.

The choice of θ_1 and θ_2 is based on the following observation: when only one phase is powered, the function $T_I(\theta)$ is an odd periodic function with respect to any equilibrium position. Assuming that the origin of θ is in such a position, the function can thus be expressed as a series of sine functions only:

$$(3.11)\qquad T_I(\theta) = \sum_{k=1}^{\infty} T_{IMk}\cdot\sin\Big[k\cdot(\pi\cdot\theta)/(2\cdot\theta_s)\Big] \quad ,$$

where θ_s is the step angle; the quantity of interest is the amplitude T_{IM1} of the first harmonic of the torque due to 1 phase only, indicated with T_{IM} for simplicity. If the torque function consisted of a fundamental only, it would be sufficient to integrate over a half-period (for example with θ_1=0 and θ_2=2$\cdot\theta_s$) to calculate its maximum value T_{IM}. The presence of higher harmonics suggests a different approach: let x be a generic relative fraction of the step angle ($0\le|x|\le 1$).
The work L_{Ik} done by the k-th harmonic of the torque for a rotation from $\theta_1=x\cdot\theta_s$ to $\theta_2=(x+2)\cdot\theta_s$ equals:

$$(3.12)\qquad L_{Ik}=\int_{x\cdot\theta_s}^{(x+2)\cdot\theta_s} T_{Ik}(\theta)\cdot d\theta \; \begin{cases} 0 & \text{for k even} \\ T_{IMk}\cdot(4\cdot\theta_s/\pi)\cdot\cos(x\cdot k\cdot\pi/2)/k & \text{for k odd} \end{cases} .$$

This work is inversely proportional to k. In addition, T_{IMk} declines with the increasing order of the harmonic; its amplitude depends mainly on the teeth design and on the saturation condition, and is normally of the order of a few percent of the fundamental for lower-order harmonics.
In order to minimize the influence of the harmonics on the value of T_{IM} to be calculated, it is useful to choose x in such a way as to cancel the work done by the lowest-order odd harmonic, i.e. the 3rd. Equation (3.12) shows that this occurs when x=1/3. This choice also cancels the work done by the multiples of the 3rd harmonic and reduces that done by the 5th and the 7th harmonics to 0.866 times what it would be for x=0. The choice of x=1/3 also reduces the work done by the fundamental, but, in order to calculate T_{IM}, it is sufficient to allow for that by means of a coefficient obtained from (3.12) for k=1.

As far as the holding torque $T_{II}(\theta)$ is concerned, when both phases are fed with equal currents, the torque of one phase is shifted by θ_s relative to that of the other phase. Thus, in the case of a linear magnetic circuit, the resultant torque amplitude of two phases is $\sqrt{2}$ times the corresponding value due to one phase only. This is no longer true under conditions of saturation, while the phase relationships remain valid both for the fundamental and for the harmonics. The torque due to two phases, $T_{xx}(\theta)$, can also be expressed as a series of sine functions only, by means of the following equation:

$$(3.13)\qquad T_{xx}(\theta) = \sum_{1}^{\infty} {}_k\, T_{xxMk}\cdot \sin\left[k\cdot\pi\cdot[\theta/(2\cdot\theta_m)+1/4]\right]\cdot\sqrt{2}\cdot\cos(k\cdot\pi/4)\ .$$

The quantity of interest is T_{xxM1}, indicated as T_{xxM} for simplicity. It can easily be checked that all the harmonics of the order $k=2\cdot(2\cdot h+1)$, where $h=0,1,2,\ldots$, are equal to zero. This applies, in particular, to the 2nd harmonic, which corresponds to the reluctance torque.
The work L_{xxk} done by the k-th harmonic between the positions $\theta_1=x\cdot\theta_m$ and $\theta_2=(2+x)\cdot\theta_m$ is:

$$(3.14)\qquad L_{xxk}=\begin{cases} 0 & \text{for } k \text{ even} \\ T_{xxMk}\cdot(\sqrt{2}\cdot 4\cdot\theta_m/\pi)\cdot\cos(k\cdot\pi/4)\cdot\cos\left[k\cdot(\pi/2)\cdot(x+\frac{1}{2})\right]/k & \text{for } k \text{ odd.}\end{cases}$$

If it is desired to cancel the work done by the 3rd harmonic, it is sufficient to choose $x=-1/6$.

As for the reluctance torque, which has only a modest value in a hybrid stepping motor, it must be calculated, in saturated conditions, as the 2nd harmonic of the main torque with one phase excited. Equation (3.11) applies, but the harmonic components have to be integrated between the positions $\theta_1=x\cdot\theta_m$ and $\theta_2=(1+x)\cdot\theta_m$, from which:

$$(3.15)\qquad L_k = T_{xMk}\cdot(2\cdot\theta_m/\pi)\cdot\left[\cos(k\cdot\pi\cdot x/2)-\cos(k\cdot\pi/2+k\cdot\pi\cdot x/2)\right]\ .$$

Thanks to the fact that T_{xM} is known from the earlier calculations, the reluctance torque is obtained as difference of the work contributions, with a value for x which cancels the 3rd harmonic: bearing in mind that the 4th harmonic makes a zero work contribution, this value is $x=1/6$.

Finally, as for the calculation of the maximum value T_{mM} of the fundamental of the torque due to the magnet only, the same considerations apply as in the case of the main torque with only one phase excited. It should be noted that, in this case, the fundamental has a periodicity four times greater than the main torque. Hence, the work is evaluated for a rotation equal to $\theta_m/2$, under conditions of no power supply to the stator. Also in this case the work done by the even harmonics is zero, and a more accurate calculation of T_{mM} is possible by cancelling out the work done by the 3rd harmonic. This is obtained when $x=1/12$, with the following positions: $\theta_1=\theta_m/12$; $\theta_2=\theta_m\cdot(7/12)$.

4. The study of the hybrid stepping motor under conditions of saturation.

Fig.1 shows the magnetic network used. The pairs of field poles with an equal relative position between the teeth of the stator and of the rotor are connected in parallel and are represented by a single branch.

The symbols shown have the following meaning:

- the fluxes and the reluctances with super-scripts ' and " refer respectively to values on the North and South sides of the magnet;
- the subscripts a, b, c and d identify values of the four different pole pairs in parallel;
- the subscripts ts and tr refer to stator teeth and rotor teeth quantities respectively;
- the subscript δ refers to air-gap quantities;
- the subscripts w and m refer to the reluctances of stator poles on the winding side and on the magnet side respectively;
- the subscripts l and t apply to the laminations and to the longitudinal dispersion reluctance of the stator respectively.

The circuit of fig.1 leads to the writing of 23 equations referring to as many fluxes. All the same, it should be noted that all the reluctances and fluxes marked with superscripts 'and " and having the same subscripts are equal. Later on, by

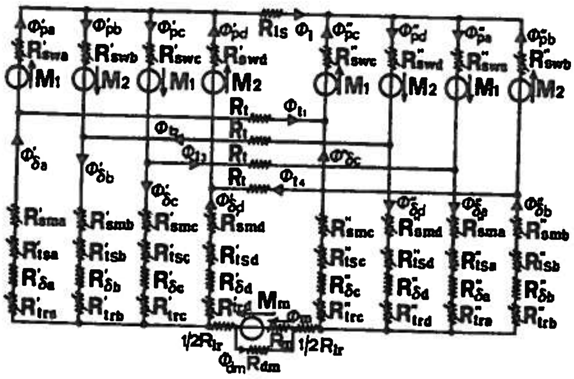


Fig.1 - Magnetic network of a hybrid stepping motor, considering the saturated reluctances.

eliminating from the equations the flux in the longitudinal reluctance and the fluxes in the branches of the magnet, one arrives at a set of 8 equations applying to 4 separate fluxes in the air-gap and a further 4 in the m.m.f. sources. Once these fluxes are known, one can calculate the remaining ones in all the branches by means of simple expressions. The calculation of the value T_M follows from the above by determining all the contributions to energy variation.

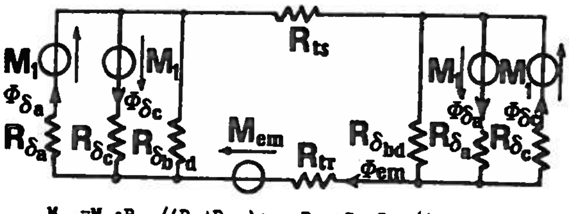
The method is easily applied on a P.C.: the fluxes are calculated using a system of linear equations, in which the saturated reluctances are worked out by iteratively updating the permeability value by means of the material magnetic curve. The fact that the two considered positions are almost coincidental to those corresponding to maximum and minimum reluctance at the air-gap (case established when x=0) makes it possible, in addition, to evaluate the extreme values of induction in the various branches of the circuit, as well as the working conditions of the magnet.

4.1. Study in the absence of saturation.

In this case, in the circuit of fig.1 there remain the reluctances at the air-gap, those of the magnet, longitudinal and dispersion ones. Thanks to linearity, it can easily be shown that eq. (3.9) reduces to the following expression:

$$(4.1)\qquad \int_{\theta_1}^{\theta_2} T(\theta)\cdot d\theta = \tfrac{1}{2}\cdot\Sigma_n\, M_n\cdot\left[\Phi_{n(\theta 2)}-\Phi_{n(\theta 1)}\right]\ .$$

The conclusion is that, in the case of a linear magnetic circuit, half the variation of the magnetic energy of the m.m.f.'s is transformed into mechanical work, while the other half appears as a change in the magnetic energy stored. Eq. (4.1) shows that it is sufficient to determine only the fluxes which flow through the m.m.f. sources. Subject to some circuit transformation, this allows the resolution of the problem in closed form. The circuit reached is shown in fig.2, in which



$M_{em}=M_m\cdot R_{dm}/(R_m+R_{dm})$; $R_{em}=R_m\cdot R_{dm}/(R_m+R_{dm})$;
$R_{\delta bd}=1/(P_{\delta b}+P_{\delta d})$; $R_{ts}=1/(1/R_{ls}+4/R_t)$; $R_{tr}=R_{lr}+R_{em}$

Fig.2 - Magnetic network of a hybrid stepping motor, to be used in non-saturated conditions.

it is sufficient to consider one phase only, thanks to the linearity. The fluxes of interest, for the purpose of equation (4.1), are: $\Phi_{\delta a}$, $\Phi_{\delta c}$, Φ_{em}. It is easily shown that the energy variations in the sources of fig.2 are equal to those in circuit of fig.1 under non-saturated conditions.

The analytical study of the circuit leads to simple expressions for the various components of the torque. These have been determined by putting $\theta_1=0$, in order to ease the calculations.

- Holding torque, with one phase excited.

The maximum value of this torque is determined by observing that:

$$P_{\delta a}(\theta_1)=P_{\delta c}(\theta_2)=P_{\delta max};\quad P_{\delta bd}(\theta_1)=P_{\delta bd}(\theta_2)=P_\delta^*$$

$$\text{(4.2)}\quad P_{\delta c}(\theta_1)=P_{\delta a}(\theta_2)=P_{\delta min},\ \text{with}\ \theta_1=0,\ \theta_2=2\cdot\theta_s\ .$$

The expression obtained is as follows:

$$\text{(4.3)}\quad T_{IM}=\frac{\pi\cdot(P_{\delta max}-P_{\delta min})\cdot M_{em}\cdot M_1}{\theta_s\cdot[(R_{ts}+R_{tr})\cdot(P_{\delta max}+P_{\delta min}+P_\delta^*)+2]}\ .$$

It can be seen that the holding torque is proportional to the stator m.m.f., to the equivalent m.m.f. of the magnet and to the difference between the maximum and minimum permeances at the air-gap. The presence of longitudinal reluctances tends to reduce its value.

- Detent torque, due to the magnet only.

The following relationships are valid in this case, while the four air-gap reluctances in branches a, b, c, d, are kept separate:

$$\text{(4.4)}\quad P_{\delta b}(\theta_1)=P_{\delta d}(\theta_1)\ ,\quad P_{\delta a}(\theta_2)=P_{\delta d}(\theta_2)\ ,$$
$$P_{\delta c}(\theta_2)=P_{\delta b}(\theta_2)\ ,\ \text{with}\ \theta_1=0,\ \theta_2=\theta_s/2\ .$$

Indicating the total reluctance at the air-gap as $R_{\delta t}(\theta)=1/\Sigma_k P_{\delta k}(\theta)$, (k=a,b,c,d), the detent torque due to the magnet only is obtained as follows:

$$\text{(4.5)}\quad T_{mM}=\frac{(\pi/\theta_s)\cdot[R_{\delta t}(\theta_2)-R_{\delta t}(\theta_1)]\cdot(M_{em})^2}{[R_{ts}+R_{tr}+2\cdot R_{\delta t}(\theta_2)]\cdot[R_{ts}+R_{tr}+2\cdot R_{\delta t}(\theta_1)]}\ .$$

One should note the dependence on the square of the equivalent m.m.f. of the magnet and the dependence on the difference between the total reluctances at the air-gap in the two positions.

It is also possible to carry out a calculation, analogous to the above, for the reluctance torque. As its magnitude is small compared with the main torque, the relevant equations are being omitted.

- The influence of the permeance harmonics.

To show the influence of the harmonics of the permeance at the air-gap, it is useful to express this permeance as a series. It is an even function with respect to a maximum position, and hence can be expanded in terms of cosine functions only:

$$\text{(4.6)}\quad P_\delta(\theta)=\sum_{0}^{\infty}{}_k\, P_{\delta(k)}\cdot\cos\left[k\cdot(\theta/\theta_s)\cdot(\pi/2)\right]\ .$$

Using equation (4.6) and neglecting the harmonics higher than those of interest, the earlier expressions change as follows.

Equation (4.3) assumes the following form:

$$\text{(4.7)}\quad T_{IM}\cong\frac{\pi}{\theta_s}\cdot\frac{P_{\delta(1)}}{1+2\cdot P_{\delta(0)}\cdot(R_{ts}+R_{tr})}\cdot M_{em}\cdot M_1\ .$$

Hence, the maximum value of the main torque depends of the first harmonic $P_{\delta(1)}$ of the permeance at the air-gap and on the average value $P_{\delta(0)}$ of this permeance. From equation (4.5) we obtain:

$$\text{(4.8)}\quad T_{mM}\cong\frac{\pi}{\theta_s}\cdot\frac{2\cdot P_{\delta(4)}}{[1+2\cdot P_{\delta(0)}\cdot(R_{ts}+R_{tr})]^2}\cdot(M_{em})^2\ .$$

The maximum value of the torque due to the magnet only is proportional to the 4-th harmonic $P_{\delta(4)}$ of the permeance at the air-gap and is, hence, usually of a modest amplitude. This torque also depends on the average value $P_{\delta(0)}$ of the permeance.

Because of the great importance of the main torque, it is useful to analyse it in more depth:

-for this purpose, it is useful to put $R_{ts}+R_{tr}=R_t+R_{em}$, where R_t is the total lamination reluctance of the stator and the rotor;

-the following relationships apply to the ratio M_{em}/R_{em}: $M_{em}/R_{em}=M_m/R_m=B_r\cdot A_m$;

-the m.m.f. M_1, can be expressed as $M_1=S\cdot A_{cu}$, where S is the current density and A_{cu} is the total copper cross-section of the coil around a stator pole.

Using the above, eq. (4.7) becomes:

$$\text{(4.9)}\quad T_{IM}=\frac{N_s}{2}\cdot\frac{P_{\delta(1)}/P_{\delta(0)}}{P_{em}/P_{\delta(0)}+2\cdot(1+R_t\cdot P_{em})}\cdot A_m\cdot A_{cu}\cdot S\cdot B_r\ ,$$

with N_s number of steps per turn.

In choosing the design parameters, it appears useful to use the following guidelines:

-use permanent magnets of a high cross-section and high residual induction and coercive force;

-use high current densities and high copper cross-sections;

-secure a high ratio between the first harmonic and the mean value of the permeance at the air-gap. At equal tooth configurations, this corresponds to the adoption of narrow air-gaps.

As to the working point of the magnet, for the rare earth magnets the usefulness of working at the point of maximum energy product $(B_r/2,-H_c/2)$ is well known. When the stator windings are not supplied with power, it can be checked that this position is obtained at a value of the internal reluctance of the magnet equal to $R_m=R_{\delta(0)}/2+R_t$. Having chosen A_m, this relationship yields the value of the length l_m, or the other way round. A first approximation to the value of $R_{\delta(0)}$ can be obtained by using the Carter factor for the toothed structures considered together.

The expressions above constitute useful design formulae for getting the project started. In the presence of saturation, the calculation calls for an approach of the numerical type.

4.2. Calculation of permeances at the air-gap.

The evaluation of the permeances at the air-gap between the two toothed structures is one of the most crucial points in the study of stepping motors. This is because the precision with which the performance of the machine can be calculated depends on it. Two methods of approach are normally used for this calculation. One is an analytical method, which uses an approximate description of the magnetic field lines, using straight-line and circular segments. The other is a numerical procedure which determines the field, for example by using the finite element method.

The analytical approach is easy to use, as the expressions are relatively simple and it is easy to calculate their derivatives. This approach, however, is of limited precision; this becomes particularly critical in the case of certain tooth configurations.

The finite element method makes it possible to obtain a high degree of precision, at the cost, however, of a more onerous calculation. The latter may become impracticable, particularly on a P.C, in the case where the classical method is used for the calculation of T(θ). This not only requires many values of permeance, but also does not permit a simple calculation of the derivative of the permeance with respect to position.

Up till now, we have used the energy method with analytical expressions for the permeances. The use of the finite element method can, however, be of some interest, thanks to the fact that the number of positions at which permeances have to be calculated is small and that it is not necessary to find their derivatives.

Once the stator-rotor permeance corresponding to one tooth pitch τ_t is found, the total permeance under a stator pole is the above quantity multiplied by the number of teeth per stator pole, to which has to be added the right part of the permeance due to fluxes between the stator pole sides and the rotor.

The analytical method of calculation employs an approximate representation of the field, with magnetic induction lines following either straight or circular segments (see fig.3a). The assumption is often made that the teeth are sufficiently high to be able to neglect the induction lines at the bottom of the slot. This makes the integrals simple to work out.

For example, considering the position y represented in fig.3a, the expression obtained for the permeance P_t (referred to a width τ_t and to a unit length) is given by 6 contributions, corresponding to 6 different types of flux tubes within 1 tooth pitch. Using the symbols of fig.3a, if we call:

$$K_1=(b_{ts}-b_{tr})/2 \quad , \quad K_2=(\tau_t-b_{ts})/2 \quad ,$$

for every position y in the range:

$$K_1 \le y \le K_2 \quad \text{we can write:}$$

$$(4.10) \qquad P_t(y) = \sum_1^6 {}_n P_{tn}(y) \quad , \text{ with}$$

$$P_{t1} = \frac{\mu_o \cdot y}{\delta+\beta\cdot(K_1-y)+(\alpha+\beta)\cdot K_2} \qquad \text{for } \alpha=\beta$$

$$P_{t1} = \frac{\mu_o}{\beta-\alpha}\cdot\ln\left[1+\frac{(\beta-\alpha)\cdot y}{\delta+\beta\cdot(K_1-y)+(\alpha+\beta)\cdot K_2}\right] \quad \text{for } \alpha\neq\beta$$

$$P_{t2} = \frac{\mu_o}{\alpha+\beta}\cdot\ln\left[1+\frac{(\alpha+\beta)\cdot(K_2-y)}{\delta+\beta\cdot(K_1+y)}\right]$$

$$P_{t3} = \frac{\mu_o}{\beta}\cdot\ln\left[1+\frac{\beta}{\delta}\cdot(K_1+y)\right]$$

$$P_{t4} = \mu_o\cdot\frac{b_{ts}-K_1-y}{\delta} \quad ; \quad P_{t5} = \frac{\mu_o}{\alpha}\cdot\ln\left[1+\frac{\alpha}{\delta}\cdot(y-K_1)\right]$$

$$P_{t6} = \frac{\mu_o}{\alpha+\beta}\cdot\ln\left[1+\frac{(\alpha+\beta)\cdot(K_1+K_2-y)}{\delta+\alpha\cdot(y-K_1)}\right] \quad .$$

The big problem is how to evaluate the error committed in adopting the above field representation instead of the real one: in fig.3b one can see a qualitative indication of the deviation between the real lines (continuous lines) and the fictitious lines, using straight-line and circle approximation (dashed lines). As can be seen, the deviation of some of the lines is considerable. There are some, for example those which spread out near the corners of the teeth, where the lines of the representation are longer than the real ones, while other lines, which can be found in the central part of the slot, have a fictitious length smaller than that of the corresponding real ones. The overall effect is that of partial compensation, thanks to the fact that permeance is an integral type quantity, hence its calculation tends to minimize the effect of local errors.

The direct use of permeance values derived from this treatment of the field has shown up some limits of precision. It has been found that the width of the tooth heads (b_{ts} and b_{tr}) was a particularly critical element, also in relation to the value of the air-gap δ. In the calculations carried out, the greatest error has been found in the maximum value of the detent torque, due to the magnet only. This is easily understood, if it is remembered that this torque depends on the value of the fourth harmonic of the permeance. The latter value can, in certain cases, be of the same order as the error in the calculation of the permeance itself.

The above limitations on precision have led to a search for fictitious lines which would approximate the real ones more closely, both in shape and in length. Let us first consider the situation shown in fig.4, in which a toothed structure is faced with a smoothed one, across an air-gap δ.

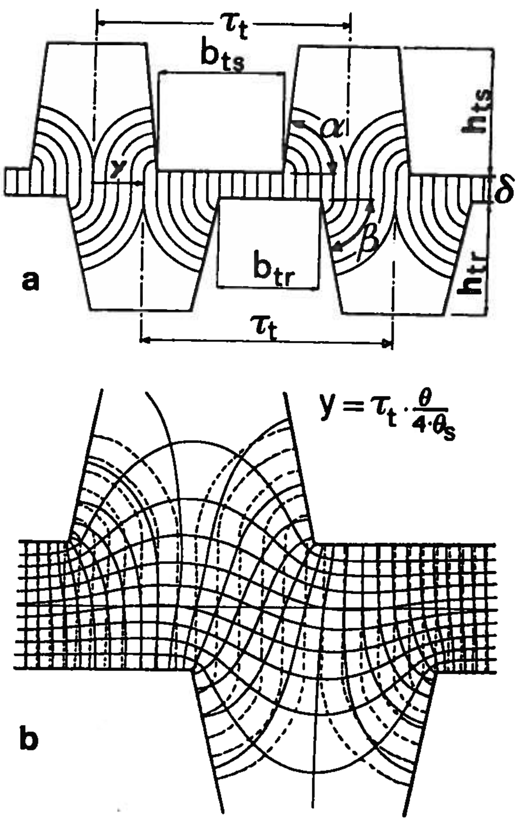


Fig.3 - Toothed structures, faced each other:
a) field approximation with fictitious lines consisting of straight and circular segments;
b) comparison between real and fictitious field lines.

Fictitious field lines are drawn, using straight-line and circular segments. A broken trace shows a modified generic line which, starting from the line joining the tooth heads, diverges from the original arc of the circumference. The problem is how to formulate a law for such a displacement which would, when applied to all the lines, lead to the exact value of the permeance between the two structures. The generic shift d between the circumference and the modified line can, by using a coefficient K_d, be assumed to be proportional to the distance z from the corner of the tooth and to the angle Γ. By a simple integration, the length of the modified arc is found in this way to be equal to $K_m\cdot\alpha\cdot z$, where $K_m=1+K_d/2$, while the length of the original circumferential arc was equal to $\alpha\cdot z$. The comparison between the two

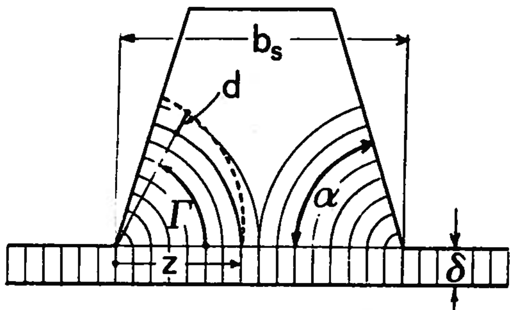


Fig.4 - Toothed and smoothed facing structures, to define the modified fictitious field lines.

lengths suggests the introduction of the modified angle $K_\alpha \cdot \alpha$, in place of the geometrical angle α, in the formula for the calculation of permeance. Remembering the origin of the Carter factor, it is possible to determine K_α. It is enough to stipulate that the ratio between the value of the permeance as calculated with lines modified by the use of K_α and the value of the permeance between smoothed structures (with an equal air-gap and corresponding to a tooth pitch) should be equal to the inverse of the Carter factor K_c evaluated for the structure shown in fig.4. Utilizing the expression for the Carter factor, the iterative calculation formula shown below follows from it:

$$K_\alpha^{(n+1)}=(2/\alpha)\cdot\ln[1+K_\alpha^{(n)}\cdot\alpha\cdot b_s/(2\cdot\delta)]\cdot(1/5+\delta/b_s) \quad (4.11)$$

When considering arrangements where both the stator and rotor structures are toothed, it is understood (by the image principle, applied with respect to the line at the middle of the air-gap) that in the situation where teeth face one another, it is necessary to evaluate two distinct coefficients, relating to an air-gap of half the real width.
While this correction criterion significantly improves the calculation of permeances, it is still subject to two limitations:
- the factors calculated are adequate only for configurations where the shapes of the stator and rotor teeth are not too dissimilar. Otherwise, the intermediate equipotential line, to which refers the image principle, will depart excessively from a straight line;
- factors calculated in the position of teeth facing each other are then assumed to remain constant in any other relative position of stator and rotor.

4.3. Examples of calculations.

Some results of calculations carried out on a hybrid stepping motor of 200 steps per revolution are reported below. The motor characteristics are listed in Tab.I. The motor referred to has already been described in the literature [6] hence useful comparisons can be drawn.

Table I - Principal motor data

Tooth pitch: τ_t=1.184 mm	air-gap: δ=0.1 mm
Stator teeth: width b_{ts}=0.53 mm,	angle $\alpha=\pi/2$ rad
Rotor teeth: width b_{tr}=0.50 mm,	angle β=1.46 rad
N° of teeth/pole: 5	N° of stator poles: 8
N° of rotor teeth: 50	Rotor diameter=18.85 mm
Ext.stator diameter=40.12 mm	Stator length=7.5 mm
Toothed wheels length: 3.0mm	Magnet length=1.5 mm
Magnet diameter=15.5 mm	Magnet material: SmCo
Rated current: I_n=0.185 A	N° of turns/pole=160

Table II shows the values of the maximum torques of the magnet only, with one phase and with two phases supplied with the nominal current.

Table II- Maximum holding and detent torque values

Torque type	T_M [mNm]	(a)	(b)	(c)
Detent	T_{mM}	0.82	0.76	0.52
Holding (1 ph.)	T_{IM}	18.9	18.9	19.1
Holding (2 ph.)	T_{IIM}	27.8	27.8	27.6

(a) measured (b) calculated [6] from eq. (2.4)
(c) calculated from eq. (3.10)

It should be noted that there is an acceptable agreement between the energy variation method (with fictitious field lines modified in accordance with equation (4.11)) on one hand and experimental values and those calculated using equation (2.4) on the other. This applies to the cases of both the torque due to one phase and that due to two phases. As for the detent torque of the magnet, the error is significant in relative values, though in absolute terms it can be compared with the error in the holding torque.

In order to demonstrate the sensitivity to parameters of certain salient quantities in the design, fig.5 shows the variation in the maximum torque (one phase powered) as a function of the width of the rotor teeth, of the air-gap and of the length of the magnet.
All the quantities are normalized with respect to the values corresponding to the data in Tab.I. It can be seen that the results are in agreement with the indications of par.4.1. In particular, the torque increases (other conditions being equal and within certain limits) with diminishing width of the rotor teeth, with a reduction in the air-gap and with the increase in magnet length at constant diameter.

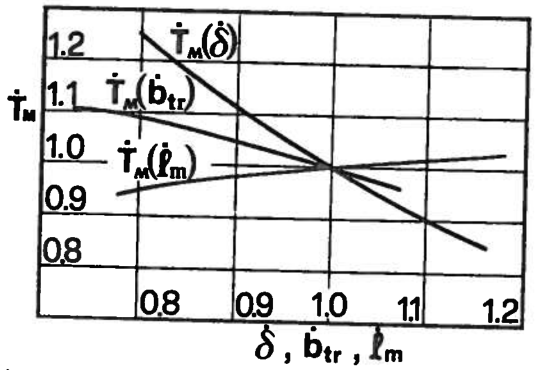


Fig.5 - Variation of the maximum torque due to one phase, as a function of: rotor teeth width, air-gap and magnet length. All the quantities are normalized, referred to those of Tab.I

As for the induction level, it has been found that, if the saturation is pushed up to significant levels, it produces a large reduction in the rice rate of the maximum values of the torques, caused by a changed flux distribution in the various circuit branches. It follows that working at high induction level is of little value, also considering the increased losses caused by it when functioning at speed.

5. Conclusions.

The work has developed a simplified method for the calculation of the static torque characteristic of a hybrid stepping motor.
The method, which is of general applicability, differs from the classical one in that the characteristic is evaluated not on a point-by-point basis, but in global terms. By applying the energy balance equation, at constant currents, to a finite turn of the rotor, the complexity of the calculation is substantially reduced in comparison with the differential formulation carried out at constant fluxes. While the latter is necessary when the study is concerned with the dynamic functioning of the machine, the energy variation method is particularly useful when the aim is to optimize the design.
On the assumption of a linear magnetic circuit, the method leads to simple expressions for the maximum values of the torque components as a function of the construction and operating parameters, useful in guiding the choice of design quantities. By expanding the air-gap permeance into a harmonic series, the dependence of all the quantities on the permeance harmonics is demonstrated.
The point of greatest difficulty is the model of the permeance at the air-gap.
The energy variation method considered in this paper has the property of using permeance values corresponding to only a few stator-rotor positions, without requiring their derivatives.
This property opens the practical suitability of

resolving the field directly, for example by the use of the finite element method, as well as permitting recourse to approximate analytical expressions for such permeances.

The work, which is still in progress, will be directed towards a further elaboration of the above subjects.

Acknowledgement.

The authors wish to thank Prof. I. Vistoli for his guidance and encouragement during the studies and for the fruitful discussions during the drafting of this paper.

List of symbols

α, β [rad]: stator and rotor tooth side angles
b_{ts}, b_{tr} [m]: stator and rotor tooth head width
b_s [m]: slot width, at the air-gap
B_r [T]: residual induction of the magnet material
B_s [T]: induction in the saturated material:
$B_s=B_s(H_s)$: magnetization curve of the ferromagnetic material
Γ [rad]: generic angle for calculating K_d
d [m]: generic shift between fictitious and modified field lines
δ [m]: air-gap
Φ_{cj} [Wb]: flux linkage relative to j-th winding
Φ_{dm} [Wb]: flux in the magnet stray reluctance
$\Phi_{\delta k}$ [Wb]: flux in the air-gap reluctance under the k-th stator pole (k=a,b,c,d)
Φ_{em} [Wb]: flux in the equivalent magnet branch
Φ_l[Wb]: flux in the longitudinal stator reluctance
Φ_{lh} [Wb]: flux in the generic linear reluctance
Φ_m [Wb]: flux in the magnet
Φ_{pk} [Wb]: flux in the stator reluctances, on the magnet side (k=a,b,c,d)
Φ_{tk} [Wb]: flux in the longitudinal stator stray reluctances (k=1,2,3,4)
H_c [A/m]: coercitive force of the magnet material
H_o [A/m]: intersect with H axis of the straight line segment of the magnet hysteresis cycle
H_s [A/m]: magnetic force in the ferromagnetic material
i_j [A] : current in the j-th winding
K_1 [m]: auxiliary variable: $K_1 = (b_{ts}-b_{tr})/2$
K_2 [m]: auxiliary variable: $K_2 = (\tau_t-b_{ts})/2$
K_α [-]: correction factor of the tooth geometric angle
K_c [-]: Carter factor for toothed structures
K_d [-]: modification coefficient for the fictitious field lines
L_{Ik} [J]: work done by the k-th torque harmonic (1 phase powered)
L_{IIk} [J]: work done by the k-th torque harmonic (2 phases powered)
μ_d [H/m]: reversible permeability: $\mu_d= B_r/H_o$
M_j [A]: m.m.f of the j-th phase (j=1,2)
M_m [A]: m.m.f. of the magnet: $M_m=H_o\cdot l_m$
M_{em} [A]: m.m.f. of the equivalent magnet branch
N_s [steps/turn]: number of steps per turn
$P_{\delta k}$ [H]: air-gap permeance under the k-th stator pole (k=a,b,c,d)
$P_{\delta(\nu)}$ [H]: amplitude of the v-th harmonic of air-gap permeance
P_t [H]: air-gap permeance, relative to a tooth pitch τ_t, for a unit longitudinal length
$R_{\delta t}$ [H^{-1}]: total air-gap reluctance
R_{dm},R_m,R_{em} [H^{-1}]: stray magnet reluctance, magnet reluctance, reluctance of the equivalent magnet branch
R_{lr},R_{ls} [H^{-1}]: rotor and stator longitudinal reluctances
R_{swk} [H^{-1}]: stator pole reluctance on the winding side (k=a,b,c,d)
R_{smk} [H^{-1}]: stator pole reluctance on the magnet side (k=a,b,c,d)
R_t [H^{-1}]: stray longitudinal reluctance
R_{tsk},R_{trk} [H^{-1}]: stator and rotor teeth reluctance (k=a,b,c,d)
R_{ts},R_{tr} [H^{-1}]: stator and rotor total longitudinal reluctances
R_t [H^{-1}]: stator-rotor total lamination reluctance
$T_I(\theta)$, $T_{II}(\theta)$ [Nm]: holding torque with 1 or 2 powered phases
$T_m(\theta)$ [Nm]: detent torque, due to the magnet only
T_{IMk}, T_{IIMk}, T_{mMk} [Nm]: amplitude of the k-th torque harmonic (for 1,2,no phases powered, resp.)
τ_t [m]: tooth pitch
θ [rad]: rotor rotation angle, referred to any equilibrium position with unpowered machine
θ_1, θ_2 [rad]: starting and final rotation angles
θ_s [rad]: step angle
W_{mv} [J]: energy in the reluctances having variable geometry
W_{mfm},W_{mfl},W_{mfs} [J]: energies in the reluctances having fixed geometry: in the magnet, in the linear and in the saturated reluctances
$U_{\delta k}$ [A]: magnetic voltage drop over the k-th air-gap reluctance (k=a,b,c,d)
U_m [A]: magnetic voltage drop over the magnet
x [-]: fraction of the step angle (in per unit)
y [m]: stator-rotor linear relative position
z [m]: auxiliary variable distance

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