Periodic Disturbance Learning Model Predictive Control

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***Abstract*—A novel Model Predictive Control (MPC) framework called disturbance-learning MPC (DL-MPC) for constrained LTI systems subject to bounded disturbances is proposed. The primary objective is to improve the disturbance rejection performance of the tube-based MPC (tube-MPC) law, especially focusing on periodic disturbance signals. Based on convex optimization, the method uses real-time measurements to learn a model of the disturbance, to predict its future behavior. By including this model in the MPC, the latter can proactively counteract the disturbance, significantly improving closed-loop performance. The presented technique includes the disturbance model while preserving robust recursive feasibility and constraint satisfaction. The effectiveness of DL-MPC is demonstrated through simulation of a multivariable nonlinear system, a Continuous-flow Stirred Tank Reactor, subject to periodic disturbances. The results clearly show enhanced tracking accuracy compared to nominal MPC and tube-MPC methods.**

***Index Terms*—Constrained control, estimation, predictive control for linear systems, robust control.**

I. Introduction

INDUSTRIAL systems commonly encounter periodic disturbances arising from factors like machinery vibrations or environmental conditions. Such disturbances adversely affect the performance and can lead to safety risks. Hence, developing control methods capable of mitigating these effects is crucial. Model Predictive Control (MPC) is widely recognized for optimizing control actions over a finite prediction horizon, inherently providing disturbance rejection within the closed-loop bandwidth. MPC strategies for handling constant disturbances have been extensively studied. Early methods like dynamic matrix control (see, e.g., [1]) introduced correction terms for steady-state errors. Later techniques proposed the use of augmented models to estimate the constant disturbance and compensate it, or using velocity-form approaches to include an integral action [2], [3] Conversely, the compensation of non-constant disturbance in MPC remains less explored. Generally, disturbances comprise periodic and non-periodic components; we focus here on predominantly periodic disturbances with potentially time-varying frequency spectra. Existing MPC methods for periodic disturbances include repetitive internal-model MPC (RC-MPC, [4]), disturbance-observer MPC (DOB-MPC, [5], [6]), and frequency-aware observer–MPC hybrids [7]. These strategies either augment the plant model with fixed disturbance models or employ dedicated observers, requiring offline specification of disturbance spectra. Consequently, these methods increase state dimensions or controller complexity, limiting the capability to handle a large number of frequencies in the disturbance signal. Moreover, robust constraint satisfaction using techniques like tube-MPC [8] remains unclear in these frameworks.

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To address these limitations, the main contribution of this letter is to introduce a new approach, called disturbance learning MPC (DL-MPC). Our method augments a tube-MPC structure with a real-time disturbance learning capability. The disturbance is modeled as a linear combination of spectral components selected via discrete Fourier transform (DFT), with coefficients identified through convex optimization. By integrating this model into predictions, DL-MPC proactively rejects the disturbances, resulting in better close-loop performance. Our formulation does not rely on predefined augmented models, thus it can manage efficiently numerous and time-varying frequency components. By means of a multi-trajectory concept, our approach adds the disturbance model in the control law, while ensuring robust recursive feasibility and constraint satisfaction identical to standard tube-MPC. The effectiveness of DL-MPC is validated via simulations on a nonlinear Continuous-flow Stirred Tank Reactor (CSTR), demonstrating substantial performance improvements over standard and tube-based MPC.

Notation: In the remainder, given two sets W, Z we denote $W \oplus Z = \{w + z : w \in W, z \in Z\}$ and $W \ominus Z = \{x : x + z \in W, \forall z \in Z\}$. For a vector $z \in \mathbb{R}^n$ and a symmetric positive semi-definite matrix $Q \in \mathbb{R}^{n\times n}$, $\|z\|_Q^2 \doteq z^T Q z$.

This letter is structured as follows: the problem formulation is described in Section II and the proposed approach in Section III; Section IV presents the simulation results and Section V the conclusions.

II. Problem Formulation

We consider a discrete time, linear time invariant system:

$$x(k+1) = Ax(k) + Bu(k) + w(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ are the system states and control inputs at time step k, respectively, A, B are constant matrices and $w(k) \in \mathbb{R}^n$ is the vector of exogenous disturbances.

Assumption 1: The disturbance $w(k)$, $\forall k$, belongs to a convex and compact polytopic set $W \subset \mathbb{R}^n$.

We consider a tube-MPC approach as in [8], briefly recalled for completeness. The state and input trajectories are split into $x(k) = z(k) + e(k)$, $u(k) = v(k) + K\,e(k)$, where z is the nominal state, v the nominal input, e the disturbance-driven state deviation and K a pre-computed stabilizing gain. The error dynamics follows $e(k+1) = (A+BK)\,e(k) + w(k)$; a robust positively invariant (RPI) polytopic set Z is computed such that $(A+BK)Z \oplus W \subseteq Z$. Methods to compute the RPI set are given in, e.g., [9], [10].

We consider convex state and input constraint sets, denoted with $\mathbb{X} \subseteq \mathbb{R}^n$, $\mathbb{U} \subseteq \mathbb{R}^m$ respectively, containing the origin in their interior, and assume that the tightened constraint sets $X_t = \mathbb{X} \ominus Z$, $U_t = \mathbb{U} \ominus KZ$ are non-empty. We denote the nominal state and input predictions pertaining to time $k+i$, computed at time k, with $z(i|k)$, $v(i|k)$, and the vector of stacked predicted nominal inputs as:

$$V = \left[v(0|k)^T, v(1|k)^T, \ldots, v(N-1|k)^T\right]^T. \tag{2}$$

where N is the prediction horizon and $\cdot^T$ is the transpose operation. The considered tube-MPC formulation features the following Finite-Horizon Optimal Control Problem (FHOCP) at time step k, denoted with $\mathscr{P}_t(x(k))$:

$$\min_{V,\, z(0|k)} \sum_{i=0}^{N-1} \left(\|z(i|k)\|_Q^2 + \|v(i|k)\|_R^2 \right) + \|z(N|k)\|_P^2 \tag{3a}$$

subject to:

$$z(i+1|k) = Az(i|k) + Bv(i|k),\, i = 0, \ldots, N-1 \tag{3b}$$

$$z(0|k) - x(k) \in Z \tag{3c}$$

$$z(i|k) \in X_t,\; i = 1, \ldots, N \tag{3d}$$

$$v(i|k) \in U_t,\; i = 0, \ldots, N-1 \tag{3e}$$

$$z(N|k) \in Z_T \tag{3f}$$

where Q, R, P are positive definite symmetric matrices. As usual, Q and R are weighting matrices that balance responsiveness and control effort, while P defines the terminal cost and Z_T is a terminal polytopic set containing the origin in its interior and such that $(A+BK)Z_T \subseteq Z_T$ and $KZ_T \subseteq U_t$. Let us denote the set

$$\mathbb{X}_t \doteq \{x : \mathscr{P}_t(x) \text{ is feasible}\} \tag{4}$$

and consider the following assumption.

Assumption 2: $\mathbb{X}_t \neq \emptyset$.

Assumption 2 holds under the conditions considered in the tube-MPC theory [8]. Since $\mathscr{P}_t(x)$ is a convex parametric QP with x as parameter, the feasibility set $\mathbb{X}_t$ is a convex polytope. Moreover, $\mathbb{X}_t$ is RPI under the tube-MPC law for all disturbance signals satisfying Assumption 1, thanks to the recursive feasibility property. The problem that we address in this letter is how to augment the tube-MPC law with the capability to learn over time possible periodic patterns in the disturbance, in order to compensate them.

III. Proposed Approach

A. Disturbance Model Identification

At each time k, we use the current and past measurements of state and input to infer the disturbance that affected the system:

$$\tilde{w}(k-1) = x(k) - (Ax(k-1) + Bu(k-1))$$

For simplicity, in this letter we are neglecting the presence of noise affecting the state measurements. This is a rather commonly adopted assumption in the MPC literature. It can be justified if such noise is zero-mean and its frequency content lies mostly in an interval whose lower bound is much higher than the close-loop bandwidth, following common practices in control system design. The close-loop bandwidth under the MPC law can be tuned by selecting the Q, R matrices; a systematic way to estimate it (when no constraints are active and the prediction horizon is long enough) is to compute the eigenvalues of matrix $A + BK_{lqr}$, where K_{lqr} is the matrix of feedback gains of the linear quadratic regulator obtained with cost matrices Q and R.

Let us now focus on a generic component j of the disturbance vector, the approach being the same for all $j = 1, \ldots, n$. We store the measurements, collected over a window of $M > 0$ past time steps, in vector $\tilde{W}_j(k-1)$:

$$\tilde{W}_j(k-1) = [\tilde{w}_j(k-1), \tilde{w}_j(k-2), \ldots, \tilde{w}_j(k-M)]^T \tag{5}$$

We consider a model of the disturbance with the following form:

$$\hat{w}_j(k-1) = \sum_{g=1}^{L} \phi_g(k-1)\theta_{j,g} \tag{6}$$

where $\phi_g(k)$, $g = 1\ldots, L$ are suitably chosen basis functions, and $\theta_{j,g}$ the corresponding parameters to be identified from data. In our case, since we deal with periodic disturbances, it's sensible to choose sine and cosine functions with a finite number N_f of user-selected frequencies:

$$\begin{array}{r} \phi_{2\ell-1}(k) = \sin(\omega_\ell\, k) \\ \phi_{2\ell}(k) = \cos(\omega_\ell\, k) \end{array},\; \ell = 1, 2, \ldots, N_f$$

obtaining, if a constant term $\phi_0 = 1$ is also included, $L = 2N_f + 1$ trigonometric basis functions in the model (6). The choice of N_f and of the specific frequencies to be included in the model is important: selecting too many frequencies can lead to overfitting, while selecting too few may result in a model that fails to accurately capture the disturbance signal. As presented later on, we propose to analyze the disturbance data via a discrete Fourier transform (DFT) over a moving window of r_{DFT} past samples (where $r_{DFT} \geq 2N_f$ is a user-defined parameter) and select the N_f most relevant frequency components in the obtained spectrum. The value of M in (5) is the other important tuning parameter in our approach. Generally, the larger it is, the better, since with more data one can obtain a more accurate disturbance model, at the cost of higher computational time to estimate the model parameters.

We collect the latter in vector $\Theta_j = [\theta_{j,1}, \dots, \theta_{j,L}]^T$ and build the regressors' matrix $\Phi(k-1)$, computed at time k, as:

$$\Phi(k-1) = \begin{bmatrix} \phi_1(k-1) & \phi_2(k-1) & \dots & \phi_L(k-1) \\ \phi_1(k-2) & \phi_2(k-2) & \dots & \phi_L(k-2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1(k-M) & \phi_2(k-M) & \dots & \phi_L(k-M) \end{bmatrix}$$

i.e., with each row containing the basis functions' values pertaining to a different time step. We can then write compactly $\hat{W}_j(k-1) = [\hat{w}_j(k-1), \hat{w}_j(k-2), \dots, \hat{w}_j(k-M)]^T = \Phi(k-1)\Theta_j$. The problem of estimating the parameters Θ_j of the linear model (6) is a standard one and can be approached with different possible methods, here we consider (and compare in the example in Section IV) the following two:

a) Least Squares (LS):

$$\begin{aligned} \Theta_j &= \arg\min_{\Theta \in \mathbb{R}^L} \|\tilde{W}_j(k-1) - \Phi(k-1)\Theta\|_2^2 \\ &= \left(\Phi(k-1)^T\Phi(k-1)\right)^{-1}\Phi(k-1)^T\tilde{W}_j(k-1) \end{aligned} \tag{7}$$

This approach is computationally efficient and can be implemented recursively. However, it risks overfitting if too many frequency components are included or if irrelevant frequencies are chosen.

b) Least Absolute Shrinkage and Selection Operator (LASSO, [11]):

$$\begin{aligned} \Theta_j = &\arg\min_{\Theta \in \mathbb{R}^L} \|\Theta\|_1 \\ &\text{s.t} \quad \|\tilde{W}_j(k-1) - \Phi(k-1)\Theta\|_\infty < \varepsilon \end{aligned} \tag{8}$$

Here, ε is user-defined, balancing model accuracy and complexity. LASSO typically yields sparse solutions, setting many parameters to zero, and is less prone to overfitting. A practical choice is to initially set ε to the maximum fitting error from LS, increasing it to achieve further sparsity. Problem (8) is efficiently solvable as a Linear Program (LP), though it demands slightly higher computational resources compared to LS.

Please note that a different set of basis functions $\phi_{j,g}$ can be also used in (6) for each state component j to capture channel-specific spectra, with a negligible increase in memory usage. Here, we employ the same basis functions for simplicity.

B. Proposed Control Law

After obtaining the values $\Theta_j(k-1)$, $j = 1 \dots, n$ from data, we can predict the present and future disturbance vectors as:

$$\hat{w}(i|k) = \begin{bmatrix} \hat{w}_1(i|k) \\ \vdots \\ \hat{w}_n(i|k) \end{bmatrix}; \; \hat{w}_j(i|k) = \sum_{g=1}^{L} \phi_g(i+k)\theta_{j,g} \tag{9}$$

Algorithm 1 DL-MPC for Periodic Disturbances

1: At time k, for each $j = 1, \dots, n$,
   a. Update the vector of disturbance measurements $\tilde{W}_j(k-1)$ with the new value $\tilde{w}_j(k-1) = x_j(k) - \hat{x}_j(k)$, where $\hat{x}(k) = Ax(k-1) + Bu(k-1)$;
   b. Compute the DFT of the measured disturbance time series over the last r_{DFT} samples, sort frequencies w.r.t the amplitude of the corresponding DFT component and select the first N_f frequencies in the sorted vector;
   c. Compute the parameter estimate Θ_j, for example via LS (7) or LASSO (8);
2: Build the disturbance prediction model (9);
3: Solve the FHOCP (10), let $z^*(0|k)$, $v^*(0|k)$ be the optimal nominal state and input values pertaining to the current time step;
4: Apply the control input $u(k) = v^*(0|k) + K(x(k) - z^*(0|k))$ to the system;
5: Go to 1 at $k \leftarrow k+1$.

for $i \geq 0$. These estimates are integrated into the optimal control problem of the DL-MPC approach. To do so, while still retaining the robust constraint satisfaction and recursive feasibility guarantees of tube-MPC, we resort to a multi-trajectory approach, conceptually similar to that of [12]. Let us introduce two input trajectories to be optimized

$$\begin{aligned} V^I &= \left[v^I(1|k)^T, \dots, v^I(N-1|k)^T\right]^T \\ V^{II} &= \left[v^{II}(1|k)^T, \dots, v^{II}(N-1|k)^T\right]^T. \end{aligned}$$

Then, we formulate the following FHOCP, denoted with $\mathscr{P}_{DL}(x(k))$:

$$\min_{\substack{v(0|k),\, z(0|k) \\ V^I,\, V^{II}}} \sum_{i=0}^{N-1}\left(\|z^I(i|k)\|_Q^2 + \|v^I(i|k)\|_R^2\right) + \|z^I(N|k)\|_P^2 \tag{10a}$$

subject to:

$$z^I(1|k) = Az(0|k) + Bv(0|k) + \hat{w}(0|k) \tag{10b}$$
$$z^I(i+1|k) = Az^I(i|k) + Bv^I(i|k) + \hat{w}(i|k), \quad i = 1, \dots, N-1 \tag{10c}$$
$$z^{II}(1|k) = Az(0|k) + Bv(0|k) \tag{10d}$$
$$z^{II}(i+1|k) = Az^{II}(i|k) + Bv^{II}(i|k), \quad i = 1, \dots, N-1 \tag{10e}$$
$$z(0|k) - x(k) \in Z \tag{10f}$$
$$z^{II}(i|k) \in X_t, \; i = 1, \dots, N \tag{10g}$$
$$v(0|k) \in U_t \tag{10h}$$
$$v^I(i|k) \in U_t, \; i = 1, \dots, N-1 \tag{10i}$$
$$v^{II}(i|k) \in U_t, \; i = 1, \dots, N-1 \tag{10j}$$
$$z^{II}(N|k) \in Z_T \tag{10k}$$

The FHOCP (10) is embedded in the DL-MPC approach, given by Algorithm 1.

Namely, in (10) we consider two input trajectories to be optimized, $v^I(i|t)$ and $v^{II}(i|t)$, and consequently two state trajectories, $z^I(i|t)$ and $z^{II}(i|t)$, that share the initial state $z(0|t)$

and input $v(0|t)$ but are otherwise generally different: the state trajectory $z^I(i|t)$ ("performance trajectory") is predicted using the disturbance model, it is used in the cost function and it is not subject to the tightened state constraints, while the state trajectory $z^{II}(i|t)$ ("safe trajectory") is predicted without the disturbance model, it is not used in the cost function and it is subject to the tightened state constraints. With this approach, the optimizer takes into account the disturbance prediction in the cost function, resulting practically in better performance (if the prediction is accurate enough, see Remark 2). At the same time, recursive feasibility and constraint satisfaction are guaranteed no matter how (in)accurate the disturbance prediction is, as stated by the next result.

Proposition 1: Let Assumptions 1-2 hold and assume that the plant is controlled by DL-MPC as given in Algorithm 1. Then, problem $\mathscr{P}_{DL}(x(\underline{k}))$ is feasible $\forall x(\underline{k}) \in \mathbb{X}_t$, it is recursively feasible, and the state and input constraints are robustly satisfied for any $k > \underline{k}$.

Proof: Constraints (10d)-(10h) and (10j)-(10k) are the same as in tube-MPC, they therefore define the same feasibility set $\mathbb{X}_t$. Constraints (10i) involve only the decision vector V^I and since $U_t \neq \emptyset$, they are always satisfied. Equalities (10b)-(10c) hold by construction because the performance trajectory $z^I(i|t)$ is not subject to state constraints. Hence $\mathscr{P}_{DL}(x)$ is feasible for all $x \in \mathbb{X}_t$.

Recursive feasibility is ensured by shifting the previously optimal "safe" trajectory: $v(0|k) = v^{II*}(1|k-1)$, $v^{II}(i|k) = v^{II*}(i+1|k-1)$, for $i = 1, \ldots, N-2$ (padded with the terminal control law), and $z(0|k) = Az^*(0|k-1) + Bv^*(0|k-1)$. All constraints remain satisfied, while V^{I} again fulfils (10i).

Finally, the applied control input $u(k) = v^*(0|k) + K(x(k) - z^*(0|k))$ follows a nominal trajectory that respects the tightened state and input constraints, guaranteeing robust constraint satisfaction. ■

Remark 1: The FHOCP $\mathscr{P}_{DL}(x(k))$ (10) is a convex QP and it can be solved efficiently for a global minimizer. It features a higher number of decision variables and constraints than problem $\mathscr{P}_t$ (3), however with a structure that can be exploited, since the newly added variables, i.e., V^I, enter the cost function but are subject to input constraints only, while variables V^{II} are subject to both input and state constraints but do not enter the cost function. An alternative is to consider one predicted input sequence, but still two trajectories: one for the cost function, accounting for the disturbance model, and the other one for the state and terminal constraints, without the disturbance model. This can be easily done by using the same sequence V in (10b)-(10e), instead of two different ones, and eliminating one of (10i)-(10j), which would become redundant. The resulting QP would have the same complexity as the standard tube-MPC one, at the cost of lower degrees of freedom in the optimization.

Remark 2: The proposed approach does not rely on any additional assumption on the disturbance other than boundedness, thus its applicability is the same as that of most robust MPC techniques. On the other hand, in this way it is hard to derive theoretical results on the accuracy of the disturbance model and its effects on the close-loop performance. More assumptions on the frequency content of the disturbance and/or its time-invariance would be required to provide further guarantees. A formal investigation of these aspects is subject of future research. On the other hand, the results of Section IV show that the approach copes well with abruptly changing frequency content and delivers significantly better performance than both nominal and tube-MPC. Moreover, one can adopt practical techniques to cope with situations where the data are scarce and/or the frequency content changes. One idea is to employ a residual-based detector to monitor the goodness of the disturbance model over the recent past, to decide whether it shall be used or not in the control computation. Finally, we note that step 1.b. of Algorithm 1 can be carried out at a lower frequency than each time step, especially if the spectrum of the process disturbance does not change over time, and that smoothing approaches can be used to avoid abrupt changes in either the model frequencies or the estimated parameters.

TABLE I
CSTR MODEL PARAMETERS

Parameter	Description	Value
r	Reactor base radius	$0.219\ m$
R	Gas constant	$8.314\ \frac{J}{mol}$
k_0	Reaction frequency factor	$7.2 \cdot 10^{10}\ \frac{1}{min}$
E	Reaction activation energy	$72748\ \frac{J}{mol}$
ΔH	Reaction enthalpy	$-5 \cdot 10^4\ \frac{J}{mol}$
ρ	Solution density	$1\ \frac{Kg}{L}$
C_p	Solution specific heat capacity	$239\ \frac{J}{Kg\,K}$
U	Reactor surface heat conduction	$54936\ \frac{J}{min\,m^2 K}$

IV. SIMULATION RESULTS

We tested the approach on a nonlinear continuous-flow stirred tank reactor (CSTR) model [13], a commonly used chemical system where maintaining steady operating conditions is crucial, despite disturbances. The model in continuous time reads:

$$\begin{aligned}
\frac{dC_A(t)}{dt} &= \frac{F_{\text{in}}(t)C_{A0} - F_{\text{out}}(t)C_A(t)}{\pi r^2 h(t)} - k_0 C_A(t) e^{-\frac{E}{RT(t)}},\\
\frac{dC_B(t)}{dt} &= -\frac{F_{\text{out}}(t)C_B(t)}{\pi r^2 h(t)} + k_0 C_A(t) e^{-\frac{E}{RT(t)}},\\
\frac{dT(t)}{dt} &= \frac{F_{\text{in}}(t)T_0 - F_{\text{out}}(t)T(t)}{\pi r^2 h(t)} - \frac{\Delta H}{\rho C_p} k_0 C_A(t) e^{-\frac{E}{RT(t)}}\\
&\quad + \frac{2U}{r\rho C_p}(T_C - T(t)),\\
\frac{dh(t)}{dt} &= \frac{F_{\text{in}}(t) - F_{\text{out}}(t)}{\pi r^2},
\end{aligned}$$

where t is the continuous time in minutes, C_A, C_B the concentrations of reactant and product, T the reactor temperature, h the fluid level, F_{in} and F_{out} the inflow and outflow rates, T_C the coolant temperature, and k_0, E, ΔH, ρ, C_p, U, r are parameters defining reaction kinetics and heat-transfer dynamics. F_{out} and T_C are the control inputs, while F_{in} is the process disturbance. For simplicity we omit the time dependency of the variables in the remainder. The model parameters are listed in Table I.

The steady-state to be regulated has been computed by solving a constrained optimization problem minimizing the following economic cost:

$$J_{ss}(z, u) = -k_e F_{out} C_B + 0.1(T_C - 293)^2 \quad (11)$$

where the parameter k_e quantifies the economic worth of the output product B from the reactor. The found equilibrium is $[C_A \; C_B \; T \; h \; T_C \; F_{out}] = [0.5806 \; 0.4194 \; 340 \; 1 \; 309.1004 \; 100.0000]$. The sets $\mathbb{X}$, $\mathbb{U}$ are boxes with upper and lower limits $[1 \; 1 \; 350 \; 1 \; 350 \; 120]$ and $[0.1 \; 0.1 \; 290 \; 0.1 \; 280 \; 80]$ respectively. The plant is subjected to a bounded additive disturbance in the set $W = \{w = B_d \Delta F_{in} : \|\Delta F_{in}\|_\infty \leq 15\}$, where B_d is the Jacobian of the CSTR model equations w.r.t F_{in}, evaluated at the considered equilibrium, and $\Delta F_{in} = F_{in} - F_{\text{in}_{\text{mean}}}$. To design the MPC law, we linearized the model at the computed equilibrium and discretized it with sampling period $T_s = 0.1$ min. We chose K as the LQR gain with $Q_k = \text{diag}([1, 1, 1, 20])$ and $R_k = \text{diag}([3.3, 0.43])$. The RPI set Z for the linearized model is contained in a box with upper and lower limits $[0.1390 \; 0.1316 \; 8.7252 \; 0.0466]$ and $[-0.1390 \; -0.1316 \; -8.7252 \; -0.0466]$ respectively, computed with a box-propagation approach. Regarding the process disturbance, we built a periodic signal with three main frequency components plus filtered random noise and, after $t = 30$ min, we injected two additional frequency components, to observe the adaptation of the disturbance learning algorithm to changes in the spectrum of the disturbance. In particular, the signal $F_{\text{in}}(t)$ is generated as $F_{\text{in}}(t) = \eta_{\text{white noise}}(t) + F_{\text{in}_{\text{mean}}} + 3\sin(2\pi f_1 t) + 3\cos(2\pi f_2 t) + 3\sin(2\pi f_3 t)$ for $t < 30$ min, and $F_{\text{in}}(t) = \eta_{\text{white noise}}(t) + F_{\text{in}_{\text{mean}}} + 3\sin(2\pi f_1 t) + 3\cos(2\pi f_2 t) + 3\sin(2\pi f_3 t) + 3\sin(2\pi f_4 t) + 3\sin(2\pi f_5 t)$ afterwards, where:

- $\eta_{\text{white noise}}(t)$ is zero-mean, band-pass filtered white noise with a frequency range of 0.1 min^{-1} to 3 min^{-1};
- $f_1 = 0.08\,\text{min}^{-1}$, $f_2 = 1.1\,\text{min}^{-1}$, $f_3 = 2.2\,\text{min}^{-1}$, $f_4 = 0.65\,\text{min}^{-1}$, $f_5 = 1.6\,\text{min}^{-1}$
- $F_{\text{in}_{\text{mean}}} = 100$ is the signal's mean component.

The resulting signal is presented in Fig. 1; it has predominant periodic components plus non-periodic ones due to the white noise. We designed a nominal MPC, a tube-MPC, and a DL-MPC to regulate the steady-state. The MPC parameters are $N = 21$, $Q = \text{diag}([10^8, 10^8, 10^4, 10^8])$ and $R = \text{diag}([1, 1])$, and in addition we used $r_{DFT} = 160$, $N_f = 60$ (i.e., $L = 121$ including the constant term) and $M = 8$ in the DL-MPC. For performance analysis, we consider the Mean Absolute Percentage Error (MAPE). The results are summarized in Table II. It can be noted that DL-MPC was able to significantly reduce the tracking error compared to standard MPC and tube-MPC. Fig. 2 presents the results in the time domain, while Fig. 3 in the frequency domain. The latter clearly shows how the disturbance frequency components have been suppressed in close-loop by DL-MPC.

Table III reports the average CPU times obtained on a single core of a 13^{th}-generation Intel i7-13650HX (2.60 GHz). Even with a 60-term Fourier basis, the disturbance-learning stage raises the per-step cost by a very small fraction, while, as expected, the DL-MPC has higher computational cost for the QP solution due to the additional optimization variables, but still well below the sampling period used in many industrial applications.

Note that these results have been obtained by simulating the continuous-time nonlinear CSTR model, thus also model mismatch and discretization errors are present. As a more theoretical analysis, in Fig. 4 we present a comparison between the magnitude of the close-loop sensitivity function with the standard MPC and the one pertaining to DL-MPC, both evaluated assuming an LTI system, no active input nor state constraints, perfect knowledge of the disturbance spectrum, and purely periodic disturbance. The reduction of the sensitivity to the disturbance achieved by DL-MPC is clear and of several orders of magnitude at low frequency, up to about a factor 0.5 around the close-loop bandwidth. A comparative test of DL-MPC with a single input sequence (see Remark 1) delivered practically identical performance; this result suggests that, in this example, either configuration can be adopted in practice without loss of effectiveness, with the latter having the same computational complexity as standard MPC.

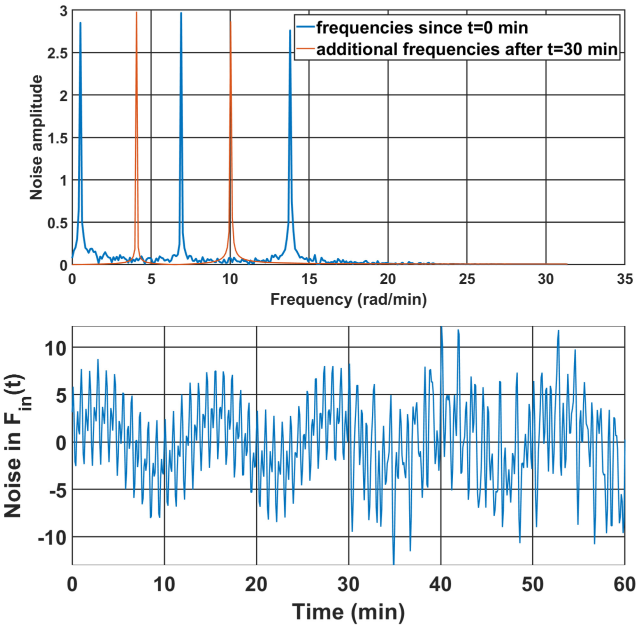

Fig. 1. Top: frequency spectrum of the disturbance $F_{\text{in}}(t)$ used in the simulation: at $t = 30$ min, two new frequencies depicted in orange are injected; bottom: disturbance $F_{\text{in}}(t)$ in the time domain.

TABLE II
SIMULATION RESULTS: ERROR METRICS

Tracking error (MAPE):	C_A	C_B	T	h
Standard MPC	0.8469%	1.3069%	0.3256%	0.3399%
tube-MPC	0.3497%	0.5251%	0.2222%	0.2321%
DL-MPC	**0.1335%**	**0.2086%**	**0.1415%**	**0.1455%**

TABLE III
AVERAGE CPU TIME PER CONTROL STEP ON A 13^{TH}-GEN INTEL I7-13650HX @ 2.60 GHZ (SINGLE CORE)

Controller	Total time (ms)	QP solve (ms)	FFT (ms)
DL-MPC	**3.8**	1.6	0.08
MPC	3.4	0.754	—
tube-MPC	3.0	0.773	—

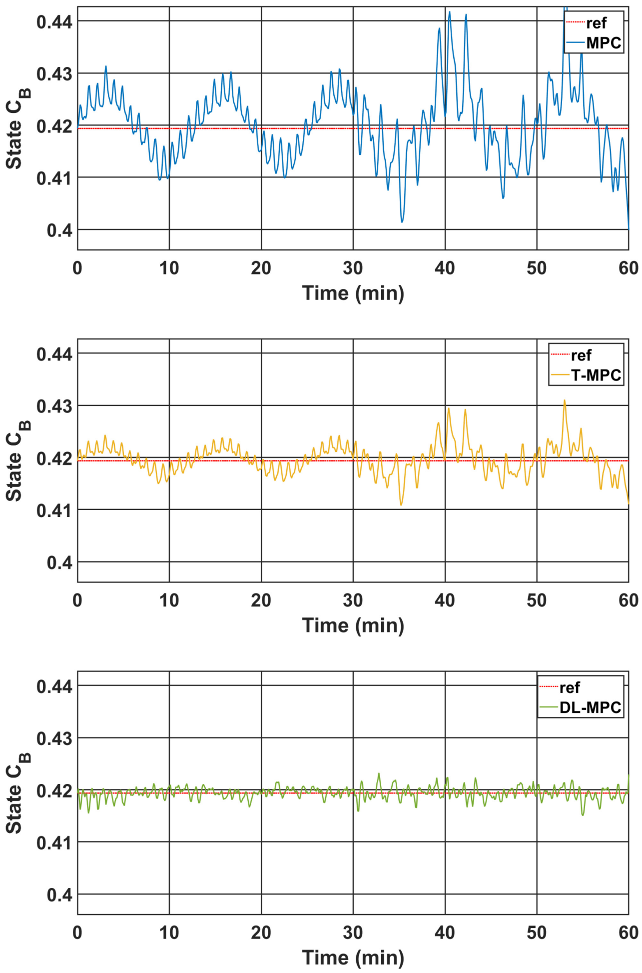

Fig. 2. Simulation results, from top to bottom: reference tracking of state C_B with MPC, tube-MPC and DL-MPC.

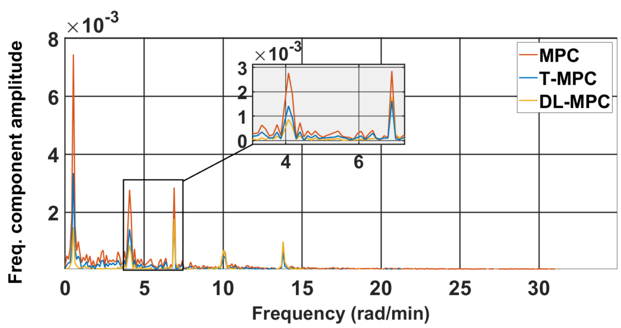

Fig. 3. Spectrum of the tracking error of state C_B with traditional MPC, tube-MPC and DL-MPC.

V. Conclusion and Future Developments

This letter proposed a novel disturbance-learning model predictive control method. The main idea is to repeatedly identify a disturbance model from real-time system measurements, and use it in the MPC law to predict future disturbance values. The effectiveness of this approach has been demonstrated through simulations on a continuous stirred-tank reactor subjected to periodic disturbances.

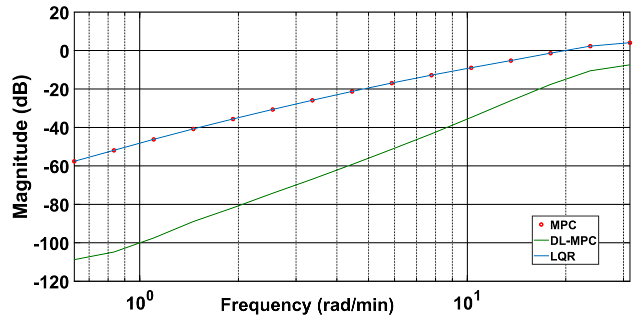

Fig. 4. Close-loop sensitivity magnitude between the disturbance and the error in state C_B.

Future research will explore the theoretical aspects of the estimation error and its effects on the close-loop performance, the effects of measurement noise, the use of different disturbance models, also including deep learning approaches, the extension to output-feedback MPC, and applying DL-MPC to practical industrial systems.

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