Possible strategies to enhance students' learning and achievement in mathematical modelling teaching experiences

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In the present work preliminary results about strategies to enhance students' learning and achievement in mathematical modelling teaching experiences are presented. The results confirm the importance of employing real world problems which are perceived as important by students. Moreover, they also suggest that the design of teaching experiences in which the initial steps of the modelling cycle can be carried out with a few or even any mathematical language could enhance students' perception of self-efficacy, thus providing a motivational push also for the subsequent steps, where specific disciplinary language must be employed. Finally, also the use of ICT and mathematical tools which are perceived by the students as belonging to the university and employment world can increase students' perception of value.

Keywords: Real-world problem, enjoyment, self-efficacy, modelling cycle.

Introduction and theoretical background

In the last decades mathematical modelling (MM) has been recognized as relevant both by scholars and educational policies for teaching mathematics (Barquero et al., 2019). However, bringing MM within the mathematical curriculum is demanding both for teachers and students, so that in everyday mathematics teaching practice, there is still relatively few genuine modelling (Blum, 2015). Indeed, MM requires mathematical and extra-mathematical knowledge, as well as appropriate beliefs and attitudes, especially for more complex modelling activities (Blum, 2015; Niss, 2003). We agree with the scholars that claim the relevance of affect factors for teaching and learning mathematics exploiting MM. In particular, we agree with the fact that using MM to deal with real world situations increases students' enjoyment and interest, motivates them and makes them retain longer what they learned. The research *problematique* of the present paper frames within two of the five leading themes emerged during the discussion of the thematic working group TWG6 at CERME 11 (Barquero et al., 2019), namely *T1. Analysis of modelling process when solving modeling problems* and *T3. Strategies to support design and implementation of modelling.* To that end, in this work we present the findings of a preliminary study about MM in high school.

Mathematical modelling and modelling cycle

The term *mathematical modelling* indicates the process of translating between the real world and mathematics in both directions (Blum & Borromeo-Ferri, 2009). More precisely, *a mathematical model* is a "deliberately simplified and formalized image of some part of the real world" (Blum, 2015, p. 77), such as nature, society, everyday life and other disciplines. The purpose of building and making use of a model is to understand how to tackle problems related to the real world (Niss et al., 2007). For our purpose, we focus on the modelling cycle (Blum, 2015; Greefrath, 2011), which is composed of seven steps within and between reality and mathematics (see Figure 1).

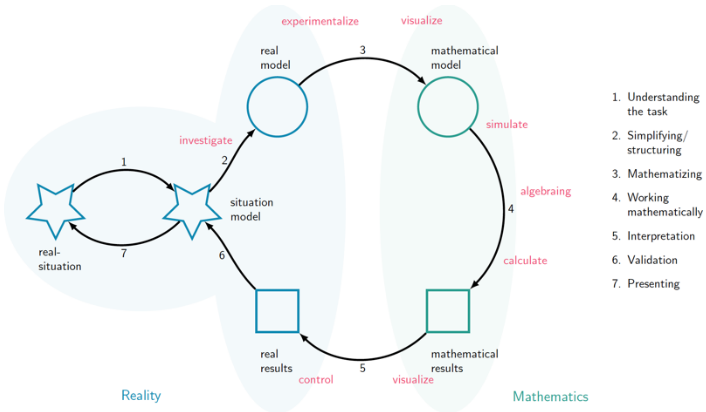

Figure 1: Blum's modelling cycle with added influence of digital tools (red terms)

The first two steps are within the reality and allow to move from the real situation to situation model (understanding) and to the real model (simplifying); the third step (mathematizing) bring to the mathematical model within the mathematics domain, where it is possible to work mathematically (step 4) reaching a mathematical result. The further two steps allow to go back to reality interpreting (step 5) and validating (step 6) the results. In the last step, the results (if any) are presented in the real situation. According to Greefrath (2011), we share the importance of digital tools along such a cycle. For instance, a digital tool may support the learner to calculate in step 4. Teaching mathematics using MM means to carry out all steps of the modelling cycle and drive students through them. To achieve that, certain competencies or sub-competencies are required (Niss, 2003), for instance understanding a given real world situation or interpreting mathematical results in relation to a situation (Blum, 2015). Modelling competency means the ability to construct, to use/apply mathematical models by carrying out appropriate steps as well as to analyze or to compare given models (Blum, 2015). Moreover, non-mathematical and extra-mathematical knowledge are required for MM. As a result this activity is highly demanding: several studies (see Blum (2015) and the references within) have shown that each step can be a cognitive barrier for students, even the first one "understanding the situation and constructing a situation model".

Affect framework

The role of affect factors, such as "emotions", "beliefs" and "attitudes", is pivotal in learning mathematics, also when students deal with problems (Schukajlow et al., 2012; Hannula, 2012). The characterization of affective factors is challenging for the researchers, because affective variables tend to form a cluster (Liljedhal, 2018). But in one of the possible classifications, Hannula (2012) identifies three types of affects: i) *cognitive* (e.g., beliefs), ii) *motivational* (e.g., values), and iii) *emotional* (e.g., feelings). *Cognitive* type refers to the affect variables that concern the beliefs towards the learning process and the achievement. We dwell on the self-efficacy belief (Bandura, 1977), which is the engine for mathematical thinking and doing (Andrà et al., 2020). More precisely, self-efficacy is a major factor in whether students will attempt a given task, how much effort they will put on it, and how resilient they will be when difficulties arise. Self-efficacy beliefs can be represented with the variable "I can", that range continuously from zero (lack of) to a maximum (strong) (Andrà et al., 2020). *Motivational* type concerns value and interest. The value is the perceived importance attributed to something (Eccles & Wigfield, 2002), while the interest concerns a specific relationship between a person and something in one's "life-space" (Krapp, 2000, p.111). Value and interest are nuances of motivational factors, for instance one can recognize that mathematical skills are valuable for obtaining a specific job but has not any interest in developing mathematical skills because that job is not part of one's "life-space". Finally, *emotional* types are all those feelings which do not belong to the previous ones. Emotions prepare our actions, accompany these actions, and influence reflection about their outcomes (Schukajlow et al., 2012). Among them, we distinguish positive and negative emotions (Pekrun et al., 2002), that can be task- and self-related or social related. For instance, *hope*, *boredom* and *anxiety* are task- and self-related emotions, while *sympathy* and *antipathy* are social related. In this paper we strongly focus on the emotion *enjoyment*, since it has been proven that there is a positive correlation between students' enjoyment and academic achievement (Pekrun et al., 2002). The levels of enjoyment can be seen as nuances of the variable "like" (Di Martino & Zan, 2015), that can range continuously from zero (boredom) to a maximum.

Research questions

We recall that the rationale behind the present study is the problematique of finding possible strategies which can be employed in the design of mathematical modelling teaching experiences in order to enhance students' learning and achievement. In particular, due to the strong connection between learning process and affective factors, we formulate the following specific questions: RQ1. Which activity carried out during the project the students enjoy the most? And RQ2. Which affective factors, cognitive and motivational, linked to the emotion enjoyment, can be associated to the different activities of the project?

Methodology

The research context

The present study, which is part of a wider MM project, involved two researchers (the authors), 18 students (grade-11) and their teacher. The data analyzed in this work has a peculiarity because the didactical situation stems from a real-world problem posed by a stakeholder. The alderman to the culture of a small town wanted to know how young people use and what they would like to find in the municipal library. The stakeholder involved the local high school for answering her question. The project was carried out through eight meetings of two hours each. The first three meetings, delivered as frontal lessons by statistics experts, aim at providing some fundamental statistical instruments. The remaining five meetings, delivered by the authors of the paper, had a student-centered approach such as group work and classroom discussion and focused on the mathematical modelling process. Such methodologies have been used since they have proven to be particularly suited for modelling activity: they are able to activate students both cognitively and metacognitively (Blum, 2015) and they have a positive impact in terms of enjoyment, interest and self-efficacy (Schukajlow et al., 2012). Since the question posed by the stakeholder is open-ended, as highlighted in (Blomhøj et al., 2003), this could generate in the students a feeling of "perplexity due to too many roads to take and no compass given". For this reason, the students have been guided through the modelling process by means of the activities described in the following subsection.

The mathematical modelling activities

The question posed by the stakeholder (how young people use and what they would like to find in the municipal library) has been investigated by means of a survey which has been built and successively analyzed by the students using statistical and ICT instruments. In the last part of the experience the students wrote a scientific report which has been delivered to the stakeholder. The activities were designed to allow students to pass through all the steps of the Blum's modelling cycle, but the ones considered in this work are related to steps 2, 3, 4 and 5. More precisely, during activity A1 (*Proposing the more relevant questions*) students were asked to work in groups and propose questions for the survey, suggesting them to take inspiration from surveys to which they answered in the past and from the internet. Then, they were asked to identify which dimensions (e.g., frequency of attendance, reason for attendance) of the phenomenon "library" their questions aim at exploring. Finally, the questions were classified according to such a criterium. Therefore, students discarded the questions which were not relevant with respect to the stakeholder's request. Thus, A1 is related to the "step 2. simplifying/structuring" of the Blum's modelling cycle. While activities A2 (*Formulating the questions for the survey*) and A3 (*Formulating the answer options for the survey*) are linked to "step 3, mathematizing". Indeed, during such activities, the students were asked to work in groups on the formulation of questions and answers to make them as clear as possible for the participants to the survey, to reduce possible ambiguities in their answers and to answer effectively to the initial stakeholder's question. The main part of such activities concerns the characterization of variables beyond the identified dimensions. Roughly speaking, the students decided which form to give to a particular question (e.g., Likert scale, open question, multiple choice). Even if it isn't written in mathematical language, the survey produced by the students is the mathematical model of the initial problem posed by the stakeholder. Activities A4 (*Using the statistics software R*) and A5 (A*nalyzing the results of the survey*) are strongly intertwined. In activity A4, a lesson-tutorial about the software R has been delivered to the students and the students were asked to practice with the software, with given datasets and with some preliminary data coming from the first draft of the survey. While during activity A5, the students were asked to work in groups with the software R to analyze and to interpret the results of the survey. We can then relate both activities A4 and A5 to step 4 ("working mathematically") and 5 ("interpreting"). In particular, the software R support the students in "calculate" and "visualize" (Greefrath, 2011).

Data gathering and method of analysis

The participants of this study are 18 high school students (grade-11), 4 male and 14 female. The average grade of the class in maths is good (6.9 over 10), according to their last report card. To address the above research questions, one survey has been developed and administered online by google form the day after the last meeting. The survey is composed of a set of Likert (5 levels) questions and a set of open questions, according to the narrative analysis (Di Martino & Zan, 2015). Question Q1 and Q2 investigate the "like" and "can" variables, respectively. They ask students to rank each of five activities according to their level of "like" and "can", more precisely: *Q1. Consider the following activities you took during the project and indicate how much you liked them from 1 (I didn't like it at all) to 5 (I liked it very much). Q2. Consider the following activities you took during the project and indicate how much you perceived it as easy from 1 (Not easy at all) to 5 (Very easy).*

For questions Q1 and Q2 and for each activity the mean of the answers has been computed. Cognitive, motivational and emotional factors related to the emotion enjoyment have been investigated more in detail by means of the following two open questions: *Q3. Choose the activity that you liked MOST among the options of the previous question and describe what you liked of that activity and why. Q4. Choose the activity that you liked LESS among the options of the previous question and describe what you DIDN'T like of that activity and why.*

The answers to both questions have been qualitatively analyzed based on the affective types which can be retrieved in them. These types have been identified in students' answers by means of keywords and sentences. For example, the answer *"Formulating the questions for the survey because we had interesting discussions and it was also easy"* is labelled as both motivational type (interest) and cognitive type (self-efficacy). Finally for each activity, the frequency of a particular type has been computed.

Data analysis and findings

Figure 2 (left) summarizes the analysis of the responses to question Q1 and Q2. For each activity, we employed the mean of the score assigned for the level "like" and "can". In this way each activity can be seen as a point in the cartesian plane. We can notice that the activities which have been enjoyed the most were activities A2 (Formulating the questions for the survey) and A3 (Formulating the answer options for the survey), closely followed by activity A1 (Proposing the most relevant questions). We then find at an intermediate level activity A5 (Analyzing the results of the survey), while activity A4 (Using the software R) has been considered as the less enjoyable one. Figure 2 is also a first indication about the fact that, as expected, high levels of enjoyment (I like) correspond to high levels of perceived control and self-efficacy (I can), even if for all the activities the "I like" level is greater than the corresponding "I can" level.

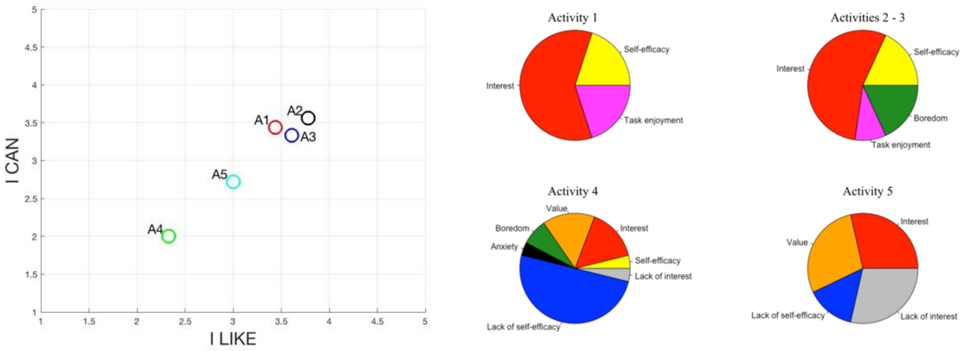

Figure 2: Left: Graphical representation of the dimensions I can – I like. Right: Frequencies of emotional, motivational and cognitive types for the different activities

In order to identify which cognitive, motivational and emotional factors, related to the emotion enjoyment, are reported by the students when carrying out different activities (RQ3), we analyze the answers to the open questions Q3 and Q4. Figure 2 (right) shows the overview of such an analysis, and in the following we report detailed examples of the students' responses.

Considering activity A1, 5 students have indicated it as the preferred one. Analyzing their answers, we notice that the motivational factor (interest) is the more frequent justification for their enjoyment, as can be seen from the following answer: *"thinking about the questions because they are tightly bound to reality"*. In other answers we can retrieve the emotion of task related enjoyment and the self-efficacy: *"because in general I like sharing my point of view with other people and discussing points of view different from mine [...]"* and *"We have been challenged in finding questions inherent to the project which had to be carried out"*. Activities A2 and A3 have been indicated as the most enjoyable ones by 6 out of 18 students. Also for these activities interest was the most popular justification. Moreover, confirming the trend highlighted in Figure 2, also the self-efficacy and perception of control emerge: *"[...] because we had interesting discussions and it was also easy"*. As we can notice in Figure 2, even if activities A1, A2 and A3 have been appreciated by the majority of students, their score in terms of "I like" is not the maximum possible. Analyzing the open question Q4, we can infer that these activities have sometimes triggered negative emotions, like boredom: *"[...] because, for what I am, I didn't find this activity exciting and sometimes boring"*. We now consider activity A4, using the statistics software R, which has been chosen as the less enjoyable activity by 15 out of 18 students. Confirming the trend reported in Figure 2, we can notice that the lack of control and self-efficacy is the most frequent reason for their choice, as can be seen from the following answer: *"I wasn't able to use the software as I would have to"*. In other cases, the activity also triggered negative emotions, as anxiety and boredom: *"Using R needs precise codes and high attention in every little detail of them otherwise an error is computed"* and *"The initial approach was difficult and, since we spent many hours using R, the activity seemed quite repetitive to me. Overall, it was however interesting learning this new language"*. Notice that in the last few lines of the previous answer also the interest factor comes out. Even if activity A4 was the less preferred activity for the majority of students, 3 students out of 18 have indicated this activity as the preferred one. Also in their answers we can recognize the types of interest and value: *"Using the statistics software R is the activity I enjoyed the most because it was a new experience and mostly because it can be useful for my future studies"*. Activity A5 was preferred by only 3 out of 18 students, due to value and interest: *"To me it seemed the most useful and interesting thing among the different activities"*. Summing up, for activities A1, A2 and A3, which have been considered as the most enjoyable, enjoyment is mainly associated to positive types such as interest and, to a lesser extent, task related enjoyment and self-efficacy, even if also the negative emotion of boredom emerges. Activity A4, which has been evaluated as the less enjoyable, triggers mainly negative types. The most frequent one is the low self-efficacy. However also positive motivational types, such interest and value, come out. Finally, in activity A5, which has received an intermediate score with respect to enjoyment, positive and negative types are more balanced: we find in similar proportions value, interest and lack of self-efficacy and lack of interest.

Discussion and conclusions

This study focuses on the relation between the emotion enjoyment and mathematical modelling activities. More precisely, we wonder which activity the students enjoy the most and which cognitive and motivational types can be associated to the different activities. Briefly, we were able to identify self-efficacy and lack of self-efficacy, concerning the cognitive factor; value, interest and lack of interest for the motivational factor; task enjoyment, boredom and anxiety for the emotional factor. Back to the MM, in Section 4 we argue the correspondence between activities and steps of the Blum's modelling cycle. Focusing on "Simplifying" step (activity A1), Blum (2015) argues that this step is usually challenging for the students since they are not used to making assumptions by themselves. However, in our case students show a high sense of self-efficacy. An explanation could be the strong link with reality of activity A1, which could have enhanced students' interest. This is a confirmation of the fact that the employment of real-world problems felt as important by students is fundamental in order to make students interested. Also for the "Mathematising" step, a potential cognitive barrier, students' answers show high levels of self-efficacy. In this case an aspect which could have impacted is the fact that this step has been realized without employing explicitly specific mathematical language. In fact, in activities A2 and A3 only common language was employed. The design of teaching experiences in which a few specific mathematical language is employed in the "Mathematising" step, could increase students' perception of self-efficacy, thus providing them a motivational push also for the following steps of the modelling cycle. On the contrary, in activities A4 and A5, associated to the Blum's step of "Working mathematically" and "Interpreting", the students' difficulty in dealing with mathematical language and problems linked to instrumental genesis strongly come out: the lack of self-efficacy is the cognitive factor more frequently reported in their answers. However, also for these steps, which have been perceived as difficult, the students reported positive motivational types such as interest and value. In particular, value was instead absent in the students' answers referring to the "Simplifying" and "Mathematising" steps. An explanation for this could be that students perceived the statistical and ICT tools as close to the university and employment world, thus contributing to raising the perceived value of the "Working mathematically" and "Interpreting" steps. The use of such tools could be important to enhance the value students attribute to the modelling experience. In order to verify the hypotheses just exposed we are planning to extend the present study by involving more students and by designing additional modelling teaching experiences based on different real-world scenarios.

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