

Resonant meta-lattices for wave confinement and energy harvesting

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Precise control of elastic waves is a challenge for many applications in the field of mechanical vibrations, ultrasonic inspection, and energy harvesting. Inspired by the advanced wave manipulation performances of elastic substrates with resonators, we design an elastic meta-lattice with local resonators capable of redirecting and confining elastic energy. The proposed structure is made of square unit cells with a chiral structure, and local resonators at the corners. The interplay between chirality and local resonance allows to redirect elastic waves within the unit cell, amplifying the wavefield in the resonators. By implementing piezoelectric coupling in the resonators, the meta-lattice can be tailored for energy harvesting applications, minimising energy losses due to wave scattering.

Metamaterials, Meta-lattices, Wave confinement, Energy Harvesting

INTRODUCTION

The emergence of mechanisms to manipulate the propagation of waves has attracted growing interest across different realms of physics, with multiple realizations in electromagnetic (Pendry et al., 1999, Pendry et al., 2004), acoustic (Craster and Guenneau, 2012, Liu et al., 2000), and elastic systems (Craster and Guenneau, 2017). In the context of elastic waves, numerous wave mechanisms have been investigated to focus or confine elastic energy, such as the creation of lenses (Tol et al., 2016, Tol et al., 2017, Zareei et al. 2018, Allam et al. 2021), cavities (Wu et al., 2009, Qi et al. 2016), mirrors (Carrara et al., 2013) or topological modes (Darabi et al. 2020) for wave transport with minimal energy dissipation. Metamaterials based on local resonators have been recently employed to confine elastic energy for energy harvesting applications. Arrays of resonators allow to simultaneously filter elastic waves and store the elastic energy (Gonella et al, 2009, Sugino and Erturk 2018). Moreover, broadband characteristics can be achieved leveraging graded metamaterials, i.e. arrays of resonators with spatially varying resonant frequency (Colombi et al., 2016, Colombi et al., 2017). These systems are based on gradually varying arrays of resonators to take advantage of local band-gaps to control wave propagation. The underlying physics, capable of inducing spatial segregation of frequency components, relies on the ability to locally decrease the propagation speed along the array. Such graded line arrays of resonators have been theorised, designed and manufactured also for energy harvesting applications (De Ponti et al, 2020, De Ponti et al. 2021, De Ponti et al., 2022). Inspired by the advanced performances of chiral lattice materials (Bacigalupo et al., 2016, Bacigalupo et al., 2017, Bacigalupo et al., 2019) for elastic wave control and auxetic properties, we decide to combine chirality with local resonance. The design strategy is based on the creation of square unit cells, spatially rotated, with the addition of a resonator on each corner. This allows to redirect elastic waves within the unit cell, amplifying the wavefield in the resonators. By implementing piezoelectric coupling in the resonators, the meta-lattice can be tailored for energy harvesting applications, minimising energy losses due to wave scattering.

ANALYSES AND RESULTS

A study of wave manipulation in lattices through local resonance effects is reported. The aim is to develop a new generation of metalattices that employ local resonance effects in specific position of the structures to better redirect elastic energy flow. The goal is to enhance energy localisation and/or energy flow through predetermined preferential paths to enhance energy harvesting and vibration isolation efficiency.

Elastic wave interaction with single resonator

The first analysis proposed is the comparison between the voltage generated by a lone cantilever resonator attached to an infinite waveguide with the same resonator on the same infinite waveguide, but where a second perpendicular waveguide is attached in the same position as the resonator. The goal is to evaluate, for travelling flexural waves, the impact of the presence of a perpendicular waveguide on the voltage output (and so the interaction efficiency with the travelling wave) of the cantilever. The evaluation of the open circuit voltage generated by a piezoelectric patch of PZT-5H positioned on top of the resonator ($E_p = 61 \text{ GPa}$, $\nu_p = 0.31$, $\rho_p = 7800 \text{ Kg/m}^3$ with dielectric constant $\epsilon_{33}^T/\epsilon_0 = 3500$ and

piezoelectric coefficient $\epsilon_{31} = 9.2C/m^2$) is reported in Fig.1. The resonator and the waveguides are made of aluminum. The resonator has a length of 13 mm, width of 2.5 mm and thickness of 1.5 mm, while the piezoelectric patch has the same length and width of the resonator, but the thickness is 0.3 mm.

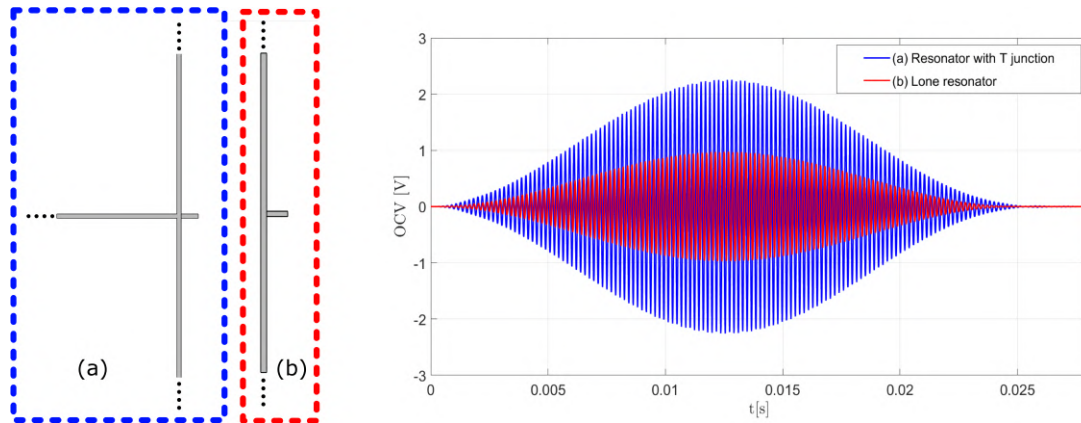


Figure 1 – On the left the two compared geometries are reported. On the right the open circuit voltage is evaluated for the same two cantilevers.

The numerical results reported are obtained by means of time domain implicit analyses in COMSOL Multiphysics. The input is a narrow-band signal centered at the resonance frequency of the composite cantilever (cantilever and piezoelectric patch on top). Furthermore the input is always a vertical excitation on the main waveguide beam. At the waveguide boundaries, ALID boundary conditions (BCs) are implemented to model an infinite domain, which avoids spurious edge reflections. Further damping mechanisms are not contemplated nor implemented in the reported analyses, because of the high quality factor of aluminium structures at low frequency.

The results show a 131.16% increase of the maximum voltage developed when the perpendicular waveguide is connected. Not only there is an increase of the maximum voltage generated but also the overall electrical power generated during the interaction time is enhanced. The enhancement effect can not be attributed simply to a shift in the resonance frequency of the cantilever due to the new boundary conditions. This is because the evaluation is done at the resonance frequency of the single cantilever on the waveguide. The enhancement can be attributed to the interaction between the traversing wave on the beam and the perpendicular waveguide. This connection is able to slow down the wave and this in turn grants a better interaction between the resonator and the passing wave. The perpendicular waveguide also offers a scattering point for the traversing wave that helps the interaction with the lateral resonator. This result in of itself can show that, for a complex 2D structured lattice with perpendicular connections, the positioning of the resonators is key from an energy harvesting point of view.

Wave redirection effect

Given these first results the idea of evaluating redirection of the wave thanks to this mechanism is discussed. The hypothesis that is studied in this paper is that the presence of the resonator is capable of enhancing perpendicular redirection of the wave. This is done thanks to quasi-radial anchor losses of the vibrating resonator. This brings to an even redistribution of the wave field over the waveguides.

To show this effect Fig.2 reports the vertical displacement field of the mid point on the perpendicular waveguide far enough from the junction with the main waveguide. The comparison is between the case of just the simple perpendicular junction between the two waveguides and the same case, but with also the resonator present at the junction point. The presence of the resonator shows that, for a wave which is at the resonance frequency of the resonator, the maximum displacement field in the perpendicular waveguide is enhanced by 75.5%. This opens new design ideas for wave manipulation in 2D and even 3D lattices. Even if the wave is not completely stopped by the junction, partial redirection can be achieved, this could limit reflection losses typical of locally resonance materials or even rainbow reflection or rainbow trapping arrays.

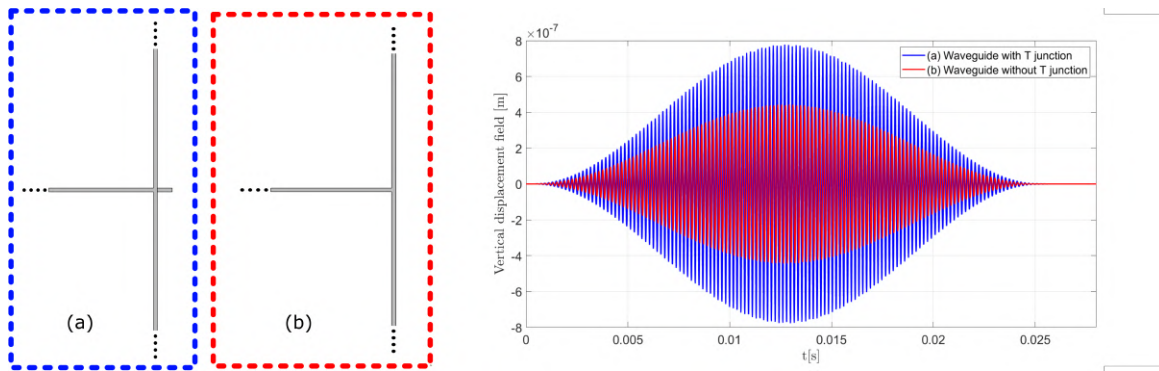


Figure 2 – On the left the two compared geometries are reported. Both structures consider an elastic beam with one resonator attached, while in (a) a perpendicular waveguide is added with respect to (b). On the right the vertical displacement field of the mid-point of the perpendicular waveguide is reported for the two structures.

Tetrachiral locally resonant lattices

To fully exploit this kind of redirection effect in a closed geometry, a fundamental lattice cell that creates a partially closed energy loop is studied. The idea is to use the redirection effect seen above to create closed fluxes of elastic energy to localise energy inside a cell. This way energy can be stored in specific structure points for prolonged periods of time. The simplest cell that can create a closed loop in a squared lattice is the one shown in Fig.3. In the figure the open circuit voltage developed by the four resonators is also reported. It is lower with respect to the previous cases, given the different BCs of the finite cell, nonetheless there is still a good interaction with the travelling wave. The lattice is able to partially redirect the wave by implementing the resonators at the intersections of the perpendicular beams. The wave is partially forced to move clockwise inside the cell, enhancing interaction and limiting reflection losses compared to common locally resonant lattices.

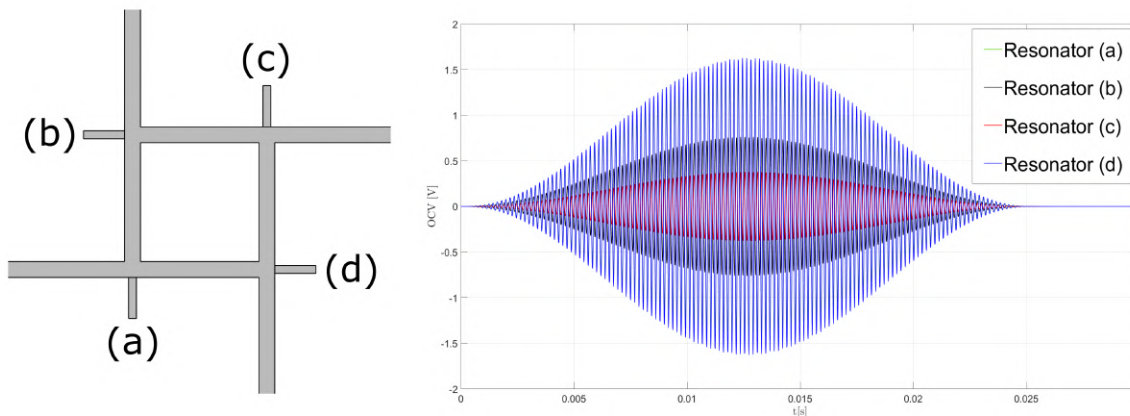


Figure 3 – On the left the complete cell of the tetrachiral locally resonant lattice. On the right, the open circuit voltage developed by the lateral resonators is shown.

The lattice obtained by repeating this cell in the 2D space, without the lateral resonators, is called tetrachiral and possesses peculiar mechanical properties in and of itself. A dispersion analysis is conducted on the cell to see how the behavior of the periodic system changes by implementing the lateral resonators analysed above. Fig.4 shows the dispersion relation of the tetrachiral lattice both with and without the resonators. The implementation of the lateral resonators generates new dispersion branches that are able to deeply localise the wave. Moreover the specific branch that is analysed is found inside a band gap intrinsically generated by the pristine tetrachiral structure. So the presence of the resonators fill a band gap that would be otherwise inaccessible by the travelling flexural wave. This is then a locally resonant tetrachiral lattice, that influences the direction of wave propagation thanks to its micro-structure.

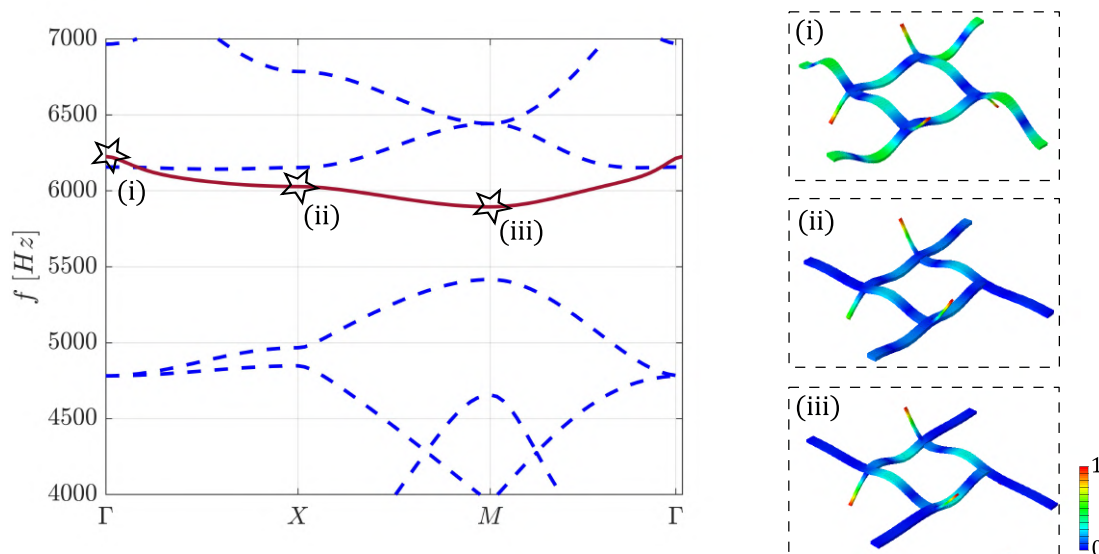


Figure 4 – On the left, the dispersion relation of the pristine tetrachiral lattice is reported with dashed blue lines. The implementation of the lateral resonators gives rise to the red line in the middle (that is not present in the pristine tetrachiral structure). On the right the mode shapes of the branch at the three main symmetry points of the reciprocal space are reported.

CONCLUSIONS

The results of multiphysical analyses on elastic waves interacting with local resonance elements in different waveguide configurations were reported. From them it has been highlighted the higher interaction of the lateral resonator with the traversing wave when a second perpendicular waveguide is attached in the same position as the resonator. The configuration greatly improves the voltage generated by the cantilever, resulting in better wave localisation. This result can be an interesting development for energy harvesting purposes, given the importance of localising passing elastic energy in specific space positions for prolonged periods of time. Furthermore it has been shown that the wavefield is enhanced in the perpendicular waveguide if the resonator is attached at the intersection. The results showed that the better cantilever interaction grants also better wave redirection effects by means of radial anchor losses of the resonator. The presence of the cantilever is capable of enhancing the displacement field of the perpendicular waveguide if the wave traversing the structure is at the resonance frequency of the cantilever. Finally, these two effects were combined and employed into a tetrachiral lattice to create closed energy loops for localising in space a travelling wave. The lattice can be used for energy harvesting or vibration isolation purposes thanks to its partial localisation effect. A future development of the lattice would be to increase redirection efficiency given by the lateral resonator to better localise waves inside the lattice.

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