Multi-parametric programming with constraint relaxation for the optimal operation of micro-grids integrating renewables

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Abstract. Motivated by the integration of distributed renewable energy sources in the electrical grid, we address the optimal operation of a micro-grid that has to adhere to a daily power exchange profile agreed in advance with the grid, by compensating local deviations that may occur along the day using dispatchable generators, loads, and storage units. We propose a solution that is based on explicit Model Predictive Control (eMPC) with constraint relaxation. More specifically, we formulate the problem as a multi-parametric quadratic program (mp-QP) whose solution is the optimal power to be dispatched expressed as a function of a parameter vector representing the actual power consumption/generation profile, which can then be assessed during the day via reliable predictions. Preferred operating regions for the involved dispatchable units are encoded in the mp-QP by introducing soft constraints, which are relaxed using an exact penalty approach when determining the eMPC solution so that they are violated but only when needed to recover feasibility. Performance of the proposed approach is shown on a self-consumption numerical example.

Keywords: micro-grids, renewables integration, multi-parametric programming, explicit MPC, exact penalty

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1 Introduction

The massive diffusion of renewable energy sources (RESs) is significantly reshaping the electrical system which was initially conceived as a centralized network model with unidirectional power flows from large-scale production systems to consumption units. Indeed, the decentralized nature of RESs makes traditional centralized electrical plants less adept at ensuring the secure operation of the electrical system in this evolving model, prompting a policy shift advocating for the adoption of distributed energy resources (DERs). This shift in policy reflects an acknowledgment of the potential of DERs to enhance grid resilience, promote energy independence, and facilitate the integration of RESs into the system. This necessitates defining favorable configurations or architectures of distributed energy resources (DERs), such as microgrids (MGs), for effective management and control. Essentially, a MG is a single organized power subsystem comprising multiple distributed generation systems, both renewable (such as photovoltaic, wind power devices), and/or conventional generation (such as internal combustion engines, micro-turbines and diesel generators), loads, and storages, [1], [2]. From a main grid point of view, a MG acts as a single controlled load within the power system, and, also, having the generation situated in close proximity to the load, significantly reduces transmission losses. However, uncertainties related to RESs availability, and load fluctuations can pose challenges to the secure and economical operation of MGs. This asks for suitable daily optimal MG operation scheme, as well as coordination and interaction mechanisms with the main grid, [3]. Typically, the MG and the main grid coordinate in advance to establish a daily exchange profile. As a result, each day, the MG strives to maintain adherence to this scheduled profile, in order to avoid incurring imbalance charges from the main grid operator. Within a MG, dispatchable generators and loads, and energy storages can be employed as flexible resources to pursue the fulfilment of scheduled programs, thus coping with the uncertainties and fluctuations of the source and load.
To solve this exchange profile enforcement problem, several control scheme based on Model Predictive Control (MPC) applied at MG level have been proposed. In [4], a hierarchical control scheme is implemented to manage the exchange of flexibility within a hybrid AC/DC grid consisting of multiple MGs. Specifically, an MPC strategy is initially applied to each MG to locally address power management. Then, a coordination mechanism among MGs facilitates flexibility exchange in situations where local power sources cannot adequately compensate for variability. In [5], a two-layer control scheme based on MPC operating at two different timescales is proposed. The high-level optimizer operates at a slow timescale and is responsible for calculating the nominal operating conditions for each component over a long time horizon. The low-level controller operates at a higher frequency and fine-tunes the MG operation to minimize the discrepancy between the planned energy exchange and the actual one. Similarly, in [6] a multi-time scale (day-ahead and intra-day) economic scheduling method combining robust optimization and MPC has been proposed.

In this paper, we firstly frame the problem of adhering to the exchange profile agreed with the grid as an optimization program with quadratic cost and linear constraints where the power to be dispatched is determined by minimizing the operating costs subject to both hard and soft constraints on the dispatchable units. Whilst hard constraints must be enforced to have a solution that is applicable in practice, soft constraints represent preferred operating conditions and can be relaxed if needed to enforce the pre-defined exchange profile with the grid. Based on the evidence that the quadratic program problem depends on several quantities that needs to be updated at each time slot t, such as the DERs production, loads consumption, and the state of flexible units, we opt for computing offline the optimal map which provides the power to dispatch as a function of these quantities that are treated as a parameter vector according to the explicit MPC (eMPC) approach. This allows to determine the power to be dispatched by simply evaluating the optimal map for the value taken by the uncertain quantities and without solving online the optimization problem. Differently from standard eMPC (see e.g. [7]), however, we adopt the approach in [8] where violation of the soft constraints is allowed but subject to a penalization of its 1-norm in the cost through an additive term that is weighted by some coefficient.

Paper [8] fully characterizes the connection between the original constrained problem and its relaxed version as the penalty coefficient varies, proving effective in providing an optimal solution for the original constrained optimization problem within its feasible region, while returning a solution with minimal 1-norm violation of the soft constraints otherwise. Since hard constraints that cannot be relaxed are still enforced in [8], the obtained eMPC solution with relaxed soft constraints is particularly suited for the considered MG operation problem where operative non relaxable constraints and performance/comfort relaxable requirements co-exist. Performance of the proposed approach is shown on a numerical example where a MG is operating in island mode, i.e., with zero net power exchange with the main grid. The eMPC policy with constraint relaxation derived according to [8] is applied during the reference one-day time window according to the receding-horizon strategy, using a control horizon of 1-hour over which predictions of DERs production and loads consumption are quite reliable.

The paper is organized as follows. In Section 2, the MG optimal operation problem is presented together with the proposed solution based on multiparametric programming with constraint relaxation. Section 3 provides a model for dispatchable units. Section 4 describes the considered case study and the obtained results. Finally, concluding remarks are drawn in Section 5.

Notation We denote with $\mathbb{R}$ the set of real numbers, and with $\mathbb{R}_+$ the set of non-negative real numbers. The transpose of a matrix $M \in \mathbb{R}^{n_r,n_c}$ is denoted with $M^\top$. Inequalities between two vectors u and v, $u \leq v$, have to be intended as component-wise. For a symmetric matrix $M = M^\top$, $M \succ 0$ ($M \succeq 0$) denotes that M is positive (semi-)definite. For a vector $v \in \mathbb{R}^n$, $\|v\|_1$ denotes its 1-norm and $[v]_+ = \max\{v, 0_n\}$, where $0_n \in \mathbb{R}^n$ is the zero vector, has to be intended as the component-wise maximum between the two vectors.

2 Optimal MG operation problem and proposed solution

We consider the operation of an MG over a one-day horizon divided in time slots of a certain duration Δ. The MG is connected with the grid and is equipped with both dispatchable (e.g., a battery, a thermostatically controlled load or a generator like a diesel internal combustion engine/microturbine) and non-dispatchable (e.g., a wind turbine, a solar photovoltaic panel installation, a non-flexible load) units. The two sets of units are respectively denoted as $\mathcal{D}$ and $\mathcal{N}$. Accordingly, the (average) power $p_{net}(t)$ exchanged by the MG with the grid in time slot t is given by:

$$p_{net}(t) = \sum_{i \in \mathcal{D}} p_i(t) + \sum_{i \in \mathcal{N}} p_i(t), \tag{1}$$

where $p_i(t)$ denotes the power output of the i-th unit (see Figure 1).

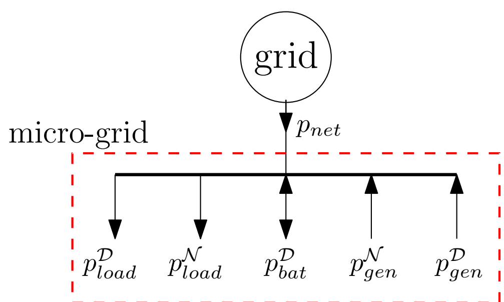

Fig. 1: Schematic view of the exchange flows between a micro-grid and the grid.

We suppose that the MG and the main grid agreed a daily power exchange profile $\bar{p}(t)$, $t = 1, \ldots, N$, with N denoting the number of time slots in a day. The goal of the MG is to enforce the scheduled profile, i.e., $p_{net}(t) = \bar{p}(t)$, $t = 1, \ldots, N$, by compensating the possible deviations of the power exchanged by the non-dispatchable units in $\mathcal{N}$ with respect to the profile that was predicted earlier. This entails optimally dispatching power among the flexible units $\mathcal{D}$ within the limits dictated by their operative constraints.

To this purpose, one could adopt the Model Predictive Control (MPC) approach and compute at each time slot t the power of the dispatchable units over some prediction horizon $H_t = \{t, t+1, \ldots, t+T-1\}$ of length $T > 0$ as the solution $p_i^\star(t+k)$, $i \in \mathcal{D}$, $k \in \mathcal{T} = \{0, 1, \ldots, T-1\}$, to the following optimization problem:

$$\min_{\{p_i(t+k)\}_{i \in \mathcal{D}, k \in \mathcal{T}}} \quad \mathcal{J}^{\mathcal{D}} = \sum_{i \in \mathcal{D}} \sum_{k \in \mathcal{T}} \mathcal{J}_i^{\mathcal{D}}(p_i(t+k)) \tag{2a}$$

$$\text{s.t:} \quad \sum_{i \in \mathcal{D}} p_i(t+k) + \sum_{i \in \mathcal{N}} p_i(t+k) = \bar{p}(t+k), \quad k \in \mathcal{T} \tag{2b}$$

$$\sum_{k \in \mathcal{T}} a_{j,k} p_i(t+k) \leq d_{j,i}(t), \quad j = 1, \ldots, n_{c,i}, i \in \mathcal{D} \tag{2c}$$

where $\mathcal{J}^{\mathcal{D}}$ in (2a) is the sum of operating costs of the dispatchable units $\mathcal{J}_i^{\mathcal{D}}$ which are typically quadratic or linear functions of the power, (2b) is the tracking constraint, and (2c) collects all $n_{c,i}$ operative constraints (including those that can be relaxed) per dispatchable unit $i \in \mathcal{D}$ that are assumed to be linear. The bounds $d_{j,i}(t)$, $j = 1, \ldots, n_{c,i}$, in (2c) may depend on the time slot t as it is indeed the case for the capacity constraint of a battery that depends on its initial status of charge or for the ramp-rate constraint of a generator that depends on the power production in the previous time slot. According to the receding-horizon strategy adopted in MPC, one then dispatches only the power $p_i(t) = p_i^\star(t)$, $i \in \mathcal{D}$, solve again the problem at $t+1$, and so on. This allows to select at each t the best $p_i^\star(t)$, $i \in \mathcal{D}$, according to the most updated information on the power exchanged by the non-dispatchable units $\sum_{i \in \mathcal{N}} p_i(k)$ over H_t. However, this basic MPC approach has two main drawbacks: i) it requires to repeatedly solve problem (2) online, and ii) it does not allow to distinguish between hard constraints representing e.g., actuation or safety limits, and soft constraints representing e.g., desired operating regions.

In this paper we propose an alternative approach which overcomes these two drawbacks by collecting in a parameter vector ϑ all the quantities appearing in (2) that depends on t except for the decision variables (i.e., $\sum_{i \in \mathcal{N}} p_i(t+k)$ and $\bar{p}(t+k)$, $k \in \mathcal{T}$, and $d_{j,i}(t)$, $j = 1, \ldots, n_{c,i}$, $i \in \mathcal{D}$), and solving offline a variant of the resulting multi-parametric Quadratic Program (mp-QP) where the 1-norm violation of the soft constraints is penalized in the cost. Based on the results in [8], we can derive an optimal map that provides the power values $p_i^\star(t+k)$, $i \in \mathcal{D}$, $k \in \mathcal{T}$, to dispatch as a function of ϑ and has the following properties: for those values of ϑ such that the associated original problem (2) is feasible, then, the map returns the optimal solution to (2), otherwise it provides a solution that minimizes the 1-norm of the relaxed constraints violation.
At each time slot t, one can then determine the power dispatch $p_i^\star(t)$, $i \in \mathcal{D}$, by simply evaluating the map at the value of the parameter vector ϑ corresponding to $\sum_{i \in \mathcal{N}} p_i(t+k)$ and $\bar{p}(t+k)$, $k \in \mathcal{T}$, and $d_{j,i}(t)$, $j = 1, \ldots, n_{c,i}$, $i \in \mathcal{D}$, where the contribution of non-dispatchable units over H_t can be assessed based on reliable predictors.

We next reformulate problem (2) according to the notations in the reference work [8] and recall its main results that are instrumental to the MG optimal operation problem solution.

Under the assumption that the operating cost $\mathcal{J}^{\mathcal{D}}$ is quadratic or linear and due to the linearity of all constraints, problem (2) results in a mp-QP in the form:

$$\begin{aligned} \min_z \quad & \tfrac{1}{2} z^\top Q z + c^\top z \\ \text{s.t:} \quad & G_h z \leq b_h + S_h \vartheta \end{aligned} \tag{3}$$

$$G_s z \leq b_s + S_s \vartheta$$

where $z \in \mathbb{R}^{n_z}$ is the decision vector collecting $p_i(t+k)$, $i \in \mathcal{D}$, $k \in \mathcal{T}$; $\vartheta \in \mathbb{R}^{n_\vartheta}$ is the parameter vector collecting $\sum_{i \in \mathcal{N}} p_i(t+k)$, $k \in \mathcal{T}$, $\bar{p}(t+k)$, $k \in \mathcal{T}$, and $d_{j,i}(t)$, $j = 1, \ldots, n_{c,i}$, $i \in \mathcal{D}$; $0 \preceq Q = Q^\top \in \mathbb{R}^{n_z, n_z}$ and $c \in \mathbb{R}^{n_z}$ define the cost function; $G_h \in \mathbb{R}^{n_h, n_z}$, $b_h \in \mathbb{R}^{n_h}$, and $S_h \in \mathbb{R}^{n_h, n_\vartheta}$ define the n_h hard constraints (including the tracking constraint (2b) that is converted into double-sided inequalities); and $G_s \in \mathbb{R}^{n_s, n_z}$, $b_s \in \mathbb{R}^{n_s}$, and $S_s \in \mathbb{R}^{n_s, n_\vartheta}$ define the n_s soft constraints.

Assumption 1 (Well-posedness, [8]) *The set $\Theta_f \subseteq \mathbb{R}^{n_\vartheta}$ of ϑ for which* (3) *is feasible is full dimensional. Moreover, the constraint set $\mathcal{Z}_\vartheta = \{z \in \mathbb{R}^{n_z} : G_h z \leq b_h + S_h \vartheta\}$ is bounded for some $\vartheta \in \mathbb{R}^{n_\vartheta}$.*

Under the quite standard well-posedness Assumption 1, problem (3) admits as a solution an optimal map $z^\star : \Theta_f \to \mathcal{Z}$ taking values in $\mathcal{Z} = \bigcup_{\vartheta \in \Theta_f} \mathcal{Z}_\vartheta$ which is a PieceWise Affine (PWA) function given by

$$z^\star(\vartheta) = \Gamma_z^j \vartheta + \gamma_z^j, \; \vartheta \in \Theta_j, \; j = 1, \ldots, r, \tag{4}$$

where $\{\Theta_j\}_{j=1}^r$ is a polyhedral partition of Θ_f, which is also polyhedral, [9].

There might exist values of $\vartheta \notin \Theta_f$ such that problem (3) without the soft constraints would be feasible. In the considered MG optimal operation problem, infeasibility may for instance arise due to violations of comfort/performance requirements rather than hard actuation/safety constraints. By admitting violations of soft constraints, one can enlarge the set of parameter values for which the problem admits a solution from Θ_f to $\Theta = \{\vartheta \in \mathbb{R}^{n_\vartheta} : \mathcal{Z}_\vartheta \neq \emptyset\} \supseteq \Theta_f$ while still enforcing the hard constraints. To this purpose, one can consider the following relaxed counterpart of problem (3):

$$\begin{aligned} \min_z \quad & \tfrac{1}{2} z^\top Q z + c^\top z + \mu \|[G_s z - b_s - S_s \vartheta]_+\|_1 \\ \text{s.t:} \quad & G_h z \leq b_h + S_h \vartheta, \end{aligned} \tag{5}$$

where $\mu > 0$ is a penalty coefficient that can be added to ϑ and interpreted as a further parameter of the resulting multi-parametric optimization problem.

In [8], the relation between problem (3) and its relaxed counterpart in (5) is fully characterized by means of the following two theorems.

Theorem 1 ([8]). *Under Assumption 1, any PWA optimal map $z : \Theta \times \mathbb{R}_+ \to \mathcal{Z}$ of the multi-parametric problem* (5) *satisfies*

$$\lim_{\mu \to \infty} z(\vartheta, \mu) = \bar{z}(\vartheta), \tag{6}$$

where $\bar{z} : \Theta \to \mathcal{Z}$ is a PWA function defined over a polyhedral partition $\{\Theta_j\}_{j=1}^q$ of Θ, which is continuous whenever $z(\vartheta, \mu)$ is continuous.
In particular, for each $j = 1, \ldots, q$, Θ_j is related to the i-th region $\mathcal{R}_j = \{(\vartheta, \mu) \in \Theta \times \mathbb{R}_+ : F_j \vartheta + f_j \mu \leq g_j\}$ of $z(\vartheta, \mu)$ receding along the μ-axis (i.e., with $f_j \leq 0$)

and is given by $\Theta_j = \{\vartheta \in \Theta : \ F_j^0 \vartheta \leq g_j^0\}$*, with* F_j^0 *and* g_j^0 *containing the rows of* F_j *and* g_j *corresponding to zero elements of* f_j*. Also, for each* $j = 1, \ldots, q$*,*

$$\bar{z}(\vartheta) = \Gamma_z^j \vartheta + \gamma_z^j, \quad \vartheta \in \Theta_j, \tag{7}$$

is derived from the expression of $z(\vartheta, \mu)$ *over* $\mathcal{R}_j$ *which is independent of* μ *and given by* $z(\vartheta, \mu) = \Gamma_z^j \vartheta + \gamma_z^j$*,* $(\vartheta, \mu) \in \mathcal{R}_j$*.*

Theorem 2 ([8]). *Under Assumption 1, for all* $\vartheta \in \Theta$*, any limit map* $\bar{z}(\vartheta)$ *obtained via* (6) *satisfies*

$$\begin{aligned} \bar{z}(\vartheta) \in \arg\min_{z} \quad & \tfrac{1}{2} z^\top Q z + c^\top z \\ \textit{s.t:} \quad & G_h z \leq b_h + S_h \vartheta \\ & \|[G_s z - b_s - S_s \vartheta]_+\|_1 \leq \bar{v}(\vartheta), \end{aligned} \tag{8}$$

with

$$\begin{aligned} \bar{v}(\vartheta) = \min_{z} \quad & \|[G_s z - b_s - S_s \vartheta]_+\|_1 \\ \textit{s.t:} \quad & G_h z \leq b_h + S_h \vartheta. \end{aligned} \tag{9}$$

According to Theorem 1 any PWA optimal map $z(\vartheta, \mu)$ of (5) approaches a PWA limit map $\bar{z}(\vartheta)$ as μ grows to infinity, whereas Theorem 2 states that, in the limit as $\mu \to \infty$, solving (5) for a given ϑ is equivalent to first finding all the $z \in \mathcal{Z}_\vartheta$ that minimize the 1-norm of the soft constraint violation and then, among these ones, those that minimize the original cost function. This in turns entails that $\bar{z}(\vartheta)$ is identical to the optimal map $z^\star(\vartheta)$ in (4) for any value of the parameter $\vartheta \in \Theta_f$ for which (3) is feasible. A procedure to compute the limit map $\bar{z}(\vartheta)$ without taking any limit is presented in [8, Algorithm 1] by exploiting the constructive proof of Theorem 1.

Note that the dimensionality of the parameter space increases with the time horizon length T. This could, in principle, create a scalability issue when solving the resulting mp-QP. However, in the considered context, T is typically small since the controlled units are operated on time slots of a quarter of an hour and accurate predictions of DERs production and loads consumption are typically available over a few hours only.

We now present a possible modelling of flexible resources within the MG that fits the presented problem formulation.

3 Resource modelling

As discussed in [10], the set of admissible power profiles offered by many different types of dispatchable units can be modelled via a polyhedron set $\mathcal{P}_i$, possibly accounting for both operating constraints and comfort/performance conditions, expressed in terms of power and energy limits. We next focus on those dispactchable units that are more relevant to our MG application.

3.1 Dispatchable generator

The power output $p_i(t+k)$, $k \in \mathcal{T}$, is constrained by

$$\ell_h^{(p,i)} \leq \ell_s^{(p,i)} \leq p_i(t+k) \leq u_s^{(p,i)} \leq u_h^{(p,i)}, \quad k \in \mathcal{T}, \tag{10}$$

to remain between a minimum and a maximum value. Values $\ell_h^{(p,i)}$ and $u_h^{(p,i)}$ represent safe operating limits (hard constraints), whereas $\ell_s^{(p,i)}$ and $u_s^{(p,i)}$ represent desired performance conditions (soft constraints), e.g., generators are rarely operated at their maximum/minimum safety limits, but conservative operative ranges are defined so as to maintain a margin for further increasing or decreasing the power in response to specific requests from the grid.

We can also consider ramp-rate constraints

$$\ell_h^{(r,i)} \leq \ell_s^{(r,i)} \leq p_i(t+k) - p_i(t+k-1) \leq u_s^{(r,i)} \leq u_h^{(r,i)}, \quad k \in \mathcal{T}, \tag{11}$$

if the rate at which the generator power can vary is limited. Note that for $k = 0$, $p(t-1)$ appears in (11) should be included in ϑ as a parameter.

Operating cost For a small-scale fuel generator within a MG, the predominant operating cost typically stems from fuel expenses, which can be defined as:

$$\begin{aligned} \mathcal{J}_i^{\mathcal{D}}(p_i(t+k)) &= c_g^{(i)} \mathcal{F}_g^{(i)}(p_i(t+k)) \\ &= c_g^{(i)} \left[\alpha_g^{(i)} (p_i(t+k))^2 + \beta_g^{(i)} p_i(t+k) + \gamma_g^{(i)} \right], \end{aligned} \tag{12}$$

where $c_g^{(i)}$ is the fuel price [€/p.u.], $\mathcal{F}_g^{(i)}(p_i(t+k))$ denotes the fuel consumption [p.u./h] which is modelled as a quadratic cost, with $\alpha_g^{(i)}$, $\beta_g^{(i)}$, and $\gamma_g^{(i)}$ being comprehensive cost coefficients of the generator output, [13]. Cost $c_g^{(i)}$ accounts for the cost for fuel and other costs, such as transportation and on-site storage facility costs. The operation cost of fuel cells and micro-turbines can also be captured by (12), albeit with different cost coefficients.

3.2 Battery

In case of a battery, beside having power and (possibly) rate constraints like (10) and (11) respectively, we have to account also for constraints on the minimum/maximum amount of energy $e_i(t+k+1)$ that can be stored at the beginning of time slot $t+k+1$, where $e_i(t+k+1)$ is described by the following (recursive) equation:

$$e_i(t+k+1) = \zeta_i e_i(t+k) + \Delta p_i(t+k), \tag{13}$$

with $\zeta_i \in (0,1]$ being the self-discharge coefficient[3] and Δ the discrete time slot duration. Generally, batteries are never operated at their physical limits due to

[3] Most lithium-ion batteries have a self-discharge rate $(1-\zeta_i)$ of between $0.5 \div 3\%$ per month.

degradation phenomena, which are more emphasized at very high and low state of charge, and also to prevent over/under voltage issues and battery stability problems (for very low charge levels), [11]. This motivates the presence of the following hard and soft limits:

$$\ell_h^{(e,i)} \leq \ell_s^{(e,i)} \leq e_i(t+1+k) \leq u_s^{(e,i)} \leq u_h^{(e,i)}, \quad k \in \mathcal{T}. \tag{14}$$

By unrolling the dynamics of e_i in (13), one obtains:

$$e_i(t+1+k) = \zeta_i^{k+1} e_i(t) + \Delta \sum_{s=t}^{t+k} \zeta_i^{t+k-s} p_i(s), \quad k \in \mathcal{T},$$

with $e_i(t)$ being the energy content at the beginning of time slot t. Accordingly, constraint (14) can be equivalently expressed in terms of power as:

$$\tilde{\ell}_\star^{(e,i)}(t) \leq \Delta \sum_{s=0}^{k} \zeta_i^{k-s} p_i(t+s) \leq \tilde{u}_\star^{(e,i)}(t), \quad k \in \mathcal{T}, \tag{15}$$

with $\tilde{\ell}_\star^{(e,i)}(t) = \ell_\star^{(e,i)} - \zeta_i^{k+1} e_i(t)$ and $\tilde{u}_\star^{(e,i)}(t) = u_\star^{(e,i)} - \zeta_i^{k+1} e_i(t)$, where $\star$ denotes hard (h) or soft (s) limits. Notice that (15) depends on the state at the beginning of the time slot t, to be included in ϑ.

Operating cost The cost of operating a battery can be calculated as proposed in [6]:

$$\mathcal{J}_i^{\mathcal{D}}(p_i(t+k)) = c_b^{(i)} (p_i(t+k))^2, \tag{16}$$

where the cost coefficient $c_b^{(i)}$ accounts for the kWh price cost, heat power losses during charging and discharging, which in turn depends on specific battery design parameters, and maintenance costs, [13]. The latter can be thought of as the cost incurred for having 1 kWh of storage capacity available, which depends on the replacement cost and the total lifetime cycling capacity of the battery[4].

Notice that, thermostatically controlled loads such as residential air conditioners, heat pumps, space heaters, refrigerators and water heaters can be viewed as general energy storage devices hence modelled in the form of (13), [12]. In this case, power and energy limits are related to temperature limits and the operating cost can be expressed as a linear function of the absorbed power. This cost represents, for example, the expense incurred to maintain home temperature within the desired comfort range.

[4] The total lifetime cycling capacity depends on the rated depth of discharge, capacity and lifetime.

4 Numerical example

We consider an MG that consists of five distributed units, with a total generation capacity of 20 kW, 20 kW of load and 20 kWh of storage. Our goal is letting the MG work in island-mode (net zero exchange with the main grid) over a one-day horizon divided in 96 time slots of duration $\Delta = 15$ minutes. To this purpose, we set $\bar{p}(t) = 0$, $\forall t$, and consider an optimization problem in the form of (2) with a finite horizon of 1-hour length (i.e., $T = 4$), parametric in $\{\sum_{i\in\mathcal{N}} p_i(t+k)\}_{k\in\mathcal{T}}$ and the initial battery energy content, with power and energy constraints as defined in (10) and (14) (see Table 1, where values between brackets denote (tighter) soft operational limits imposed on both the generator power and the battery energy content over time).
Values for $\sum_{i\in\mathcal{N}} p_i(t)$ over the one-day horizon are taken from [14] and reported in Figure 2. Table 2 presents coefficients for determining operational costs for both the battery, [6], and the generator, [13].

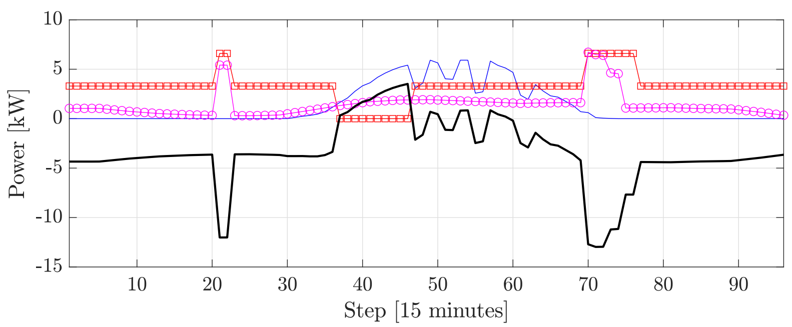

Fig. 2: Daily profiles of loads and PV generation, [14]. Non-dispatchable net power (black solid line); PV generation (blu solid line); resistive load (red squared markers); electronic load (magenta circle markers).

Table 1: Parameters of the resources in the micro-grid. Within brackets the desired soft operative limits.

	Resource	$P_{min} \div P_{max}$ [kW]	E_n [kWh]	$E_{min} \div E_{max}$ [kWh]
$\mathcal{N}$	Resistive Load	$0 \div 10$	-	-
	Electronic Load	$0 \div 10$	-	-
	PV	$0 \div 10$	-	-
$\mathcal{D}$	Generator	$0 \div 10(7)$	-	-
	Battery	$-20 \div 20$	20	$[0.02(0.2) \div 0.98(0.8)]E_n$

Table 2: Operational cost coefficients for dispatchable resources (equations (12) and (16)).

	Resource	Cost coefficients
$\mathcal{D}$	Generator	$c_g = 1.5$, $\alpha_g = 3.1e-4$, $\beta_g = 0.052$, $\gamma_g = 0.8$
	Battery	$c_b = 0.02$

To investigate the (positive) impact of the violation of performance limits imposed on power and energy limits (soft constraints) on the fulfillment of the scheduled power exchange enforcement problem, we run Algorithm 1 in [8] (Step 1 was performed using the MPT3 toolbox) to solve offline the relaxed mp-QP problem (5) and obtain the limit map $\bar{z}(\vartheta)$ (6) providing the optimal PWA power dispatch $\{p_i^\star(t+k)\}_{i\in\mathcal{D},k\in\mathcal{T}}$ as a function of the parameter vector containing $\{\sum_{i\in\mathcal{N}} p_i(t+k)\}_{k\in\mathcal{T}}$ and the battery energy content at the beginning of the time slot t. Such an optimal map is then applied in a receding-horizon fashion. That is, at any given time slot t, we evaluate the map at the current parameter value, apply only $\{p_i^\star(t)\}_{i\in\mathcal{D}}$, move to the next time slot $t+1$, and repeat the process.

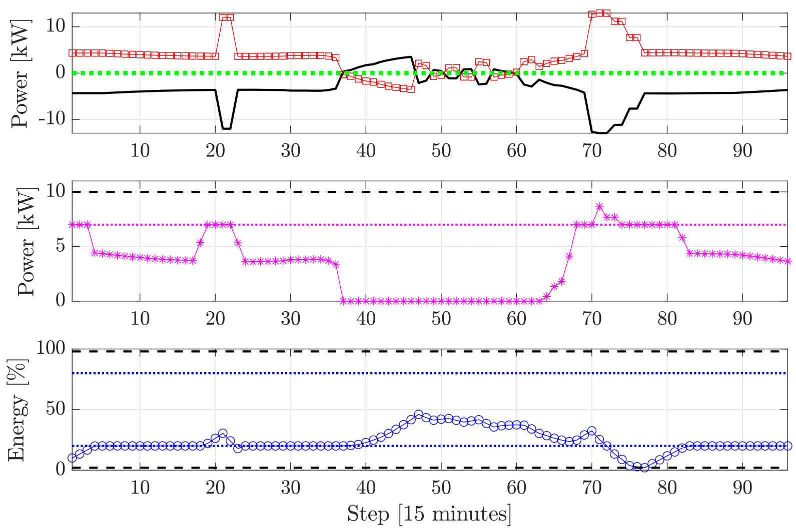

Fig. 3: Results for the case when the initial battery energy content is 10%. Top: non-dispatchable net power (black solid line); dispatchable net power (red square markers); resulting exchange program (green dotted line). Middle: generator dispatched power (magenta star markers). Bottom: battery dispatched energy (blue circle markers). Dotted and dashed lines denote soft and hard limits, respectively.

We report in Figure 3 the resulting optimal power dispatch when the initial battery energy content is set to 10%, hence below the lower operative (soft) limit of 20%. Firstly, observe that we nonetheless succeeded in obtaining a net-zero power exchange with the main grid (green dotted line). Secondly, the performed run confirms the nature of the adopted softening approach in achieving the minimization of the 1-norm of the soft constraint violation as a primary objective and then the minimization of the cost function of (3), as a secondary objective. In fact, in the early stages, the generator is operated at its upper operative (soft) limit so as to charge the battery in order to minimize the energy battery soft limit violation, and meanwhile satisfying the non-dispatchable load demand. Another interesting aspect is the optimal dispatch policy just before the first demand peak at step 21. Indeed, according to control horizon length ($T = 4$), from step 18 to 20, the generator is operated to charge the battery, which will subsequently provide the required power to meet the peak demand without causing the generator to exceed its upper power limit (soft constraint). The second peak demand, around step 70, is managed similarly, but in this case the generator is asked to violate its upper power limit so as to satisfy the demand.

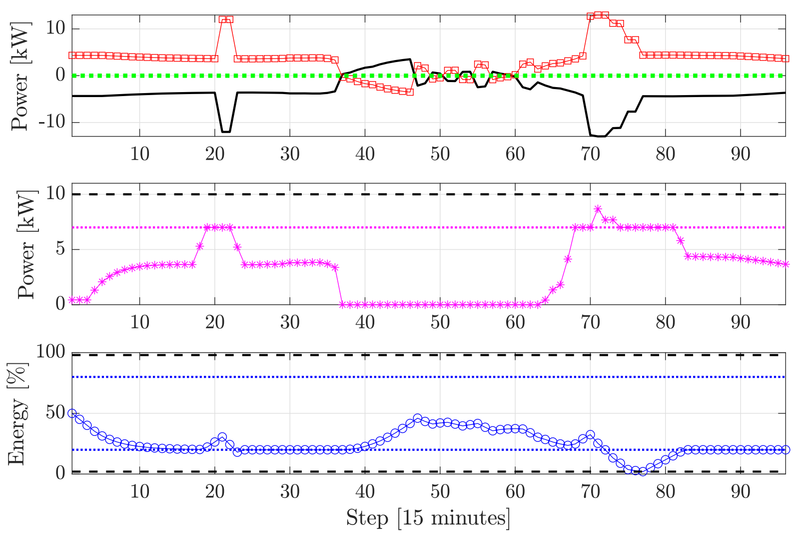

Fig. 4: Results for the case when the initial battery energy content is 50%. Top: non-dispatchable net power (black solid line); dispatchable net power (red square markers); resulting exchange program (green dotted line). Middle: generator dispatched power (magenta star markers). Bottom: battery dispatched energy (blue circle markers). Dotted and dashed lines denote soft and hard limits, respectively.

Figure 4 reports the optimal dispatch of a second run carried out considering an initial battery energy content equal to 50%, thus within the set operative (soft) limit. Due to the battery operational cost which is consistently lower than that of the generator for power values below 10 kW, the battery is primarily designated to meet the initial net load demand (approximately equal to 5 kW) thus discharging rapidly. Gradually, the generator's contribution increases, enabling the battery to smoothly reduce its injection and reach its lower (soft) energy limit just before step 18. Following this, similar to the previously illustrated case, the generator starts charging the battery to address the peak demand at step 21. Subsequently, the optimal dispatch corresponds to that obtained for an initial battery energy content of 10%.

5 Conclusion

This paper focuses on optimally controlling the operation of a microgrid to ensure that effective power exchanges with the main grid adhere to the scheduled ones, thereby responding to fluctuations in DERs production and loads consumption. Specifically, we frame this task as a multi-parametric quadratic programming problem and we leverage recent advancements in soft-constrained optimization for this class of problem to handle infeasibility issues resulting from violations of performance and operational constraints on resources. The proposed approach is applied to MGs operating in island mode, with results demonstrating effective alignment with zero net power exchange programs. Also, results give evidence that the adopted relaxation approach indeed minimizes constraint violation in the region of infeasibility.

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