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Between Safety and Conservation—Procedure for the Assessment of Heritage Buildings Based on Historic Research

Antonella Saisi *, **Paolo Borlenghi** and **Carmelo Gentile**

Department of Architecture, Built Environment and Construction Engineering (DABC), Politecnico di Milano, 20133 Milano, Italy; paolo.borlenghi@polimi.it (P.B.); carmelo.gentile@polimi.it (C.G.)
* Correspondence: antonella.saisi@polimi.it

Abstract: Correct approaches to the assessment of historic structures involve the collection of data related to materials, building technology, eventual damage and decay, and transformation that has occurred over time. The procedure proposed by the authors is based on multidisciplinary research, merging data ranging from documentary and archive research to structural modeling. In the developed procedure, the minimization of the costs and timing of the structural assessment were the main requirements. The procedure, implemented on the Arengario, the 13th-century Town Hall of Monza, focuses on the key role of historic and documentary research in order to highlight the difference in the building technology. The overall research program involves the following steps: (i) historical analysis and documentary research; (ii) visual inspections, geometric survey, and decay/damage identification and mapping; and (iii) dynamic testing and modal identification, with these steps driving the choices involved in the subsequent step: (iv) FE modeling and updating.

Keywords: diagnosis; historic buildings; historic research; structural assessment

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1. Introduction

A correct conservation process of a heritage structure—based on a reliable assessment—entails the collection of data on the construction technology, the materials properties, the geometry, the eventual damage and decay, and the evolution in time. The merging of these data drives conservation strategies that prioritize safety, are compatible for re-use with the building properties, and are fully respectful of the heritage values, in agreement with Restoration Principles and Recommendations for the Analysis and Restoration of Architectural Heritage (ICOMOS/ISCARSAH, 2005) [1].

According to this methodology, reliable assessment based on research and inspection is the factor that discriminates the effectiveness and the success of the conservation process [2–8]. Knowledge is the base of planned conservation actions dealing with vulnerability mitigation and risk management [3,8]. In the long term, knowledge involves controls and monitoring guaranteeing the suitable maintenance planned within a specific conservation strategy, which can be considered the key points of effective seismic hazard mitigation, even at the territorial scale [4].

Planned conservation requires the definition of the structural conditions at the starting point of the process. Nevertheless, the assessment usually involves long and extensive procedures, which are difficult to apply to a large set of buildings as risk mitigation plans are needed at the urban scale. Such plans require the development of prompt procedures for the initial assessment to supplement in subsequent times.

In the state-of-the-art literature, several studies are focused on the structural assessment of historic structures. The first reported activities in the technical literature were planned to understand reasons for collapse or when there was concern about the condition of the structure and to support the intervention choice [7,9–11]. The investigation activities often involve extensive tests on-site and on sampled materials, as well as geometric and crack-pattern surveys integrated with visual inspections [2,10]. The analysis of the materials is addressed to recognize the binder, the aggregate origin, the grain size and distribution, the porosity of the blocks (stones, bricks) and the mechanic strength, and the stone characterization [10–12]. On-site investigation is addressed to explore local problems or to collect general input data about the material for the structural model calibration [13–15], generally considered uniform in the overall structure. Flat-jack tests were applied to measure local state of stress and to directly evaluate the elastic properties of the masonry [2,10] to use in the model. Linear elastic models were often developed considering the general geometry and the elastic properties measured in few points with flat-jack tests. These approaches are generally time-consuming, in some case lasting years.

The development of the theoretical and experimental framework of the ambient vibration test (AVT) initially improved the investigation of slender structures through the comparison of frequencies and modal shapes—reliable representative parameters of the overall structural behavior [16–21]—and successively the calibration of structural models [16,17,20–25]. As is known, the availability of updated models allows for behavior control in several structural scenarios.

A further evolution of the assessment based on the AVT led to the development of continue monitoring systems, which are often integrated by static monitoring on the main cracks [26–33].

Local tests, generally non-destructive, are often applied to explore specific problems. Pulse sonic, radar or thermovision tests are often carried out to collect information about the masonry morphology or to control the interventions, progressively substituting destructive actions like coring [2,34–41]. The general trend is to develop fast procedures for material evaluation or to define parameters directly in the codes. Qualitative methods related to the Masonry Quality Index and local research in specific territorial contexts are addressed to define reference values concerning masonry strength [42].

Nevertheless, an open question remains concerning the rationalization of the historic research contribution within structural assessment and model calibration. Despite historic and documentary research often being carried out, their role in the structural knowledge process seems not fully active [43–45].

The most innovative research highlights the complexity of architectural heritage assessment, as well as of data merging without specific guidelines and procedures [2–4,7,8]; the process requires multidisciplinary investigation ranging from historic research to geometric and direct surveys, on site tests and numerical analysis [10,11,17,21–23,34–40].

The collection of technical information from surveys and investigation is generally agreed upon, but the exploitation of documentary and archive research for structural assessment appears less widely shared. Nevertheless, the advanced technical literature [43–45] highlights its contribution for structural analysis, and the Italian Seismic Code [46] requires historic research. Post-seismic surveys highlight frequent damage caused by structural discontinuities due to building evolution. Weak constrains between walls, discontinuous masonry panels, or the alteration of the construction technique involve the high vulnerability of the structure and brittle behavior during earthquakes [4,45,47].

This paper describes a multidisciplinary procedure aimed at the structural assessment of heritage buildings that merges documentary and archive information for structural modelling. Minimizing the costs and timing of the structural assessment were the main requirements of the developed procedure.

More specifically, the overall research program aimed at the evaluation of the state of preservation of the monument, involving the following steps: (i) historical analysis and documentary research; (ii) visual inspections, stratigraphic survey, geometric survey and decay/damage identification and mapping; (iii) dynamic testing and modal identification; (iv) FE modelling and updating [2,10,16,19,25].

The procedure is implemented on the Arengario (Figures 1 and 2), the 13th-century Town Hall of Monza, representing the main symbol of the Commune Age of the city. Over the centuries, the building has been transformed several times due to the changes in its use: Town Hall, Palace of Justice, theatre, museum, and, nowadays, an exhibition space. The changes in use and the different construction stages result in a complex structural arrangement that should be carefully investigated in a multidisciplinary investigation.

(a) (b)

Figure 1. The Arengario. View from the northwest (**a**) and from the southwest with the balcony called "Parlera" (**b**).

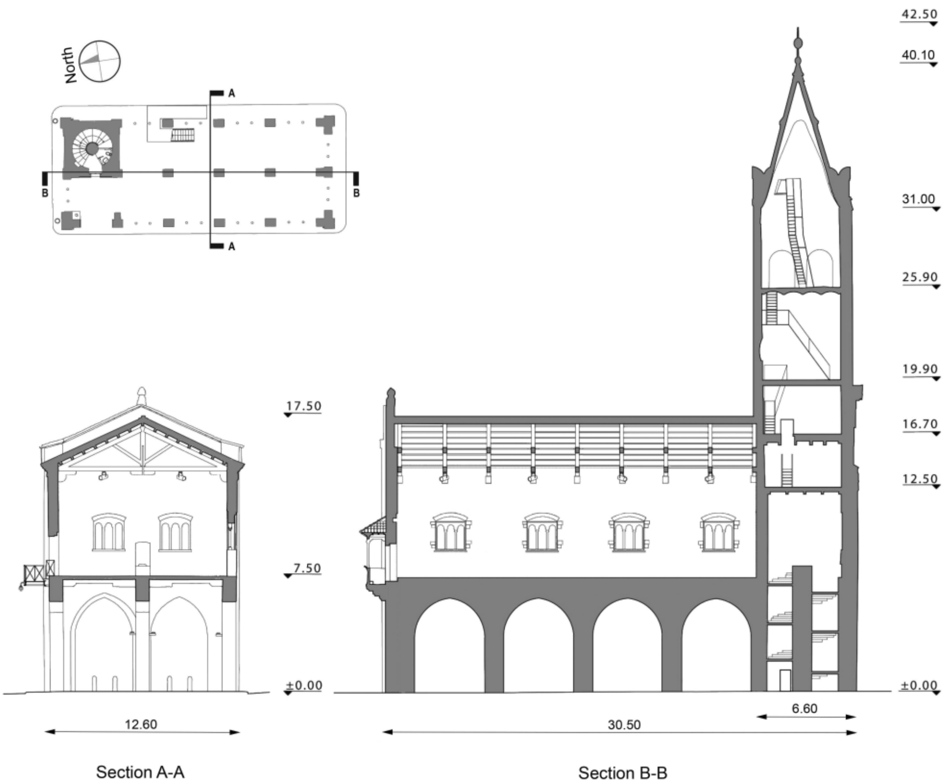


Figure 2. Transversal and longitudinal section of the Arengario.

The paper summarizes some preliminary results of the research program, focusing mainly on the documentary research based on bibliography and archive available sources. Documentary research—twined with geomatic survey, direct inspection and dynamic tests—turned out to provide important data for implementing the structural model.

The developed procedure should represent a robust framework for a fast structural assessment to be extensively applied for establishing priorities at an urban scale.

2. The Process of Knowledge

2.1. The Arengario

The Arengario—i.e., the Middle Ages Town Hall of Monza—is composed of an open arcade on the ground floor, a large hall on the first floor, and a tower of about 40 m on the northeast side (built subsequently to the main building) (Figures 1 and 2). The arcade is composed of 18 stone-masonry columns supporting the wooden floor of the first floor; 3 squared columns are in the transversal direction and 5 squared columns are in the longitudinal direction. Arches and walls are built with brick masonry. At present, the entrance is from the north side through a staircase located in the tower.

The Town Hall was built in the center of the city near the Cathedral (Figure 3), re-proposing the dual relationship between secular and religious power on the basis of many conflicts in the historical period of the Communes in Italy. Monza is a city in north Italy, Lombardia region, about 20 km northeast of Milan, and it had a political key role in the past—it kept the Iron Crown (the reliquary votive crown), deemed one of the most ancient emblems of Christendom and used for centuries to crown the Holy Roman Emperors. The importance of the city in the past is also demonstrated by the installation in 1347 of the third oldest clock in Italy—after those of Padua and Milan—on the Arengario's Tower.

Figure 3. The historic center of Monza on the Theresian Cadastre map (1721) (**a**,**c**) and on a recent map (**b**,**d**). The Arengario is in red and the Cathedral is in light blue. The Palace on the east side of the Arengario was demolished in 1903.

2.2. Historic Information and the Building Transformation

The Arengario construction probably dates back to the second half of the 13th century [48], when it was built as the new Monza's Town Hall; the construction was certainly completed before 1297. Figure 4 resumes the timeline of the main interventions and of the uses of the building.

The original access to the first floor of the Arengario was ensured by an overpass (demolished in 1808) from a nearby building. The tower was built later than the main building, most likely during the first half of the 14th century. In 1380, a balcony (called "Parlera") was added to the northern front, from which the decrees of the Commune were read (Figure 1b). During the 14th century, a double-covered staircase was built on the east front, leading to a door in the center of the façade.

At the beginning of the 19th century, the Municipality evaluated a proposal to demolish the Arengario to obtain more free space in the city center. However, the monument was kept with the idea of transforming it into a shopping center. Luckily, the plan was never realized, though in 1854, the first floor was adapted to host the offices of the Magistrate's Court according to the project of the Engineer Villa [48]. The new use involved a series of changes to the interiors and the façades; the first floor was divided with longitudinal and transversal walls to obtain a series of offices and a wooden floor was installed to separate the volume in height. Furthermore, a new overpass was built over the first arch (counting from north to north) of the east front, and all the openings of the east and east fronts were modified, obtaining tall double-lancet windows. Specifically, the height was increased and, on the east front, the position was changed to obtain the regular distribution of the openings. On the contrary, the north and north facades were not altered.

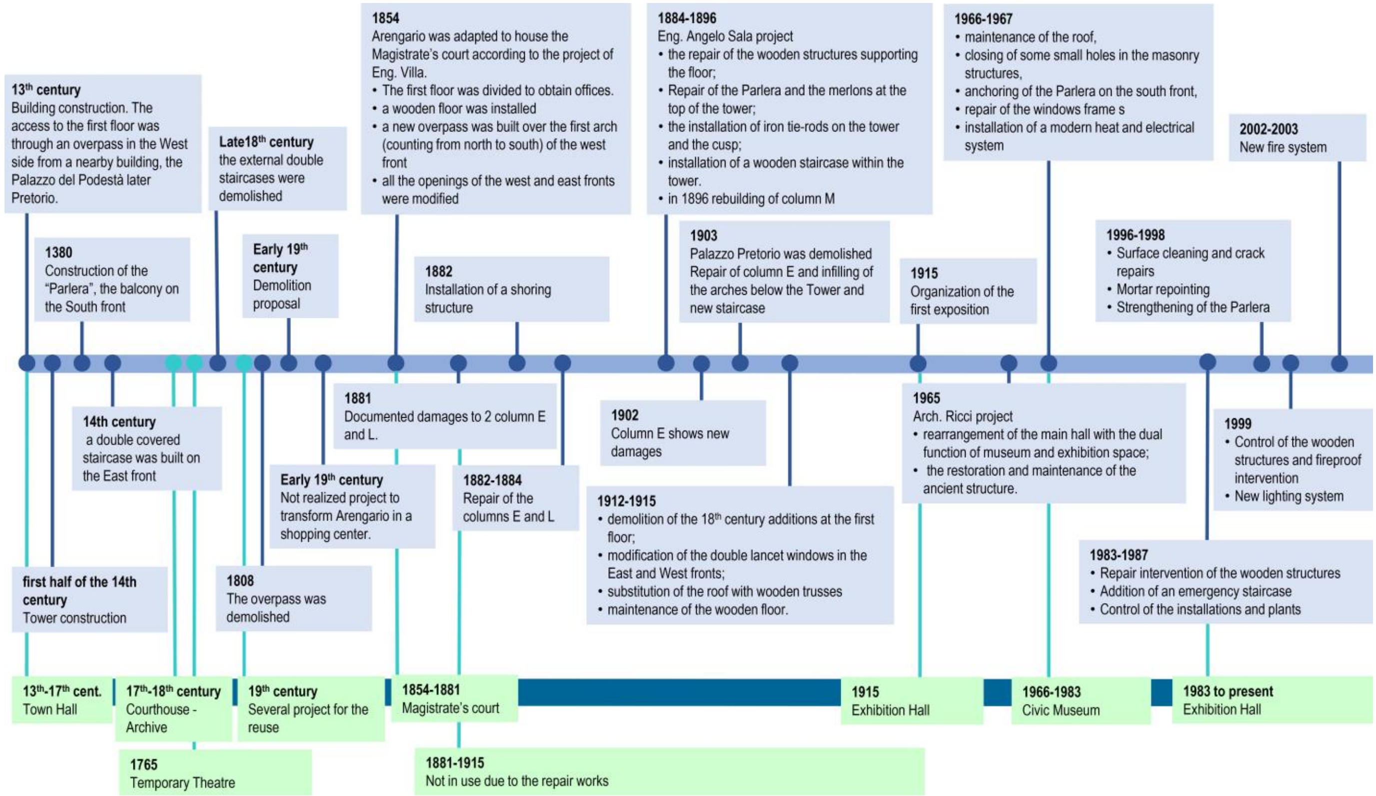
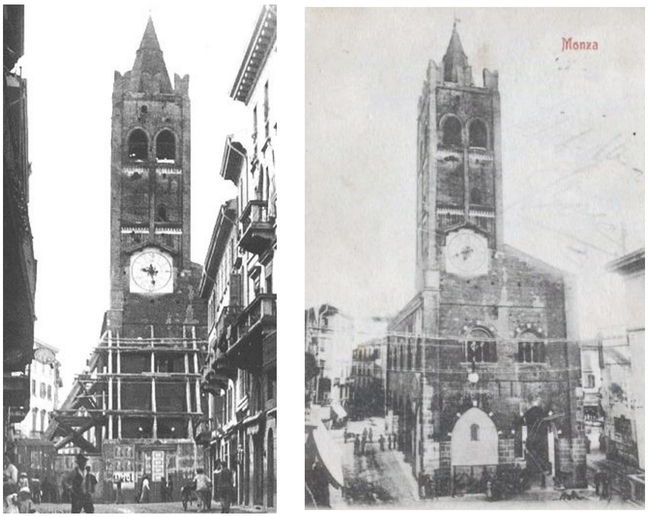


Figure 4. Timeline of the main interventions and uses of the building, according to the historical and documentary research.

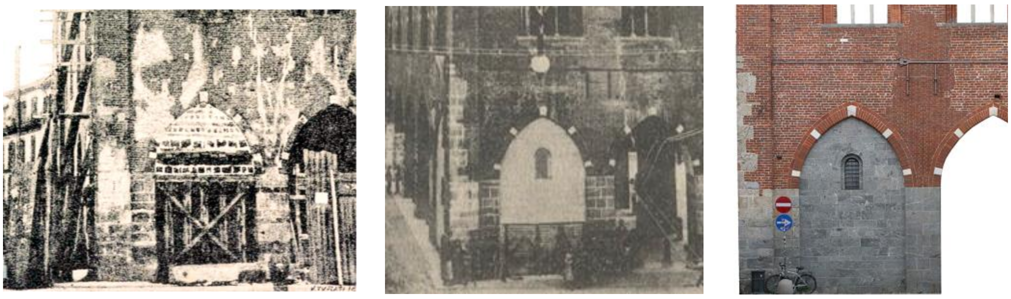
In December 1881 [48], serious damages to the columns supporting the tower—particularly on the north and east sides—raised concerns (Figures 5 and 6). A technical committee was appointed and composed of Archimede Sacchi—one of the first professors of the Politecnico di Milano—and Giovanni Ceruti. Due to the extensive historical documentation analyzed, the monograph by Sacchi and Ceruti [48] remains today the foremost source for studying the Arengario.

The committee promptly installed a shoring structure (Figure 5a) to support the four arches supporting the tower, the arch between column E (the most damaged) and column D, and the wall on the east side, preventing the out-of-plumb of the façade. The conventional names of the columns (A, B, C, . . .) are summarized in Figure 7. During the operations, it was observed that: (i) the tower had a regular tapering of walls, without any out-of-plumb, and (ii) the wall built over the arch between column E and column L (Figure 7) was separated from the rest of the structure by some cracks on the corners. Consequently, the cause of the column damage was identified being caused by excessive stress (estimated at about 2.1–2.4 MPa) rather than the soil settlement.



(**a**) (**b**)

Figure 5. In 1881, serious damages to the north and east sides raised concerns and a temporary structure was built (**a**); a wide repair intervention was carried out to repair the damaged walls (**b**).



(**a**) (**b**) (**c**)

Figure 6. Details of the north front during the repair works (**a**,**b**) and of the current state (**c**).

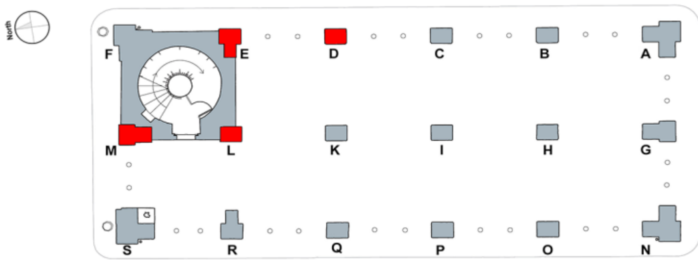


Figure 7. Plan of the ground floor. In order to strengthen the tower, the supporting arches at the ground floor were infilled and columns D, E, L and M were rebuilt.

The works started in 1881 showed that the columns were previously partially reconstructed: the core built was from a local soft and porous sandstone called "Ceppo", coated with a 20 cm thick layer of "Saltrio's Stone" (i.e., a compact limestone). The E and L columns (Figure 7) were then rebuilt with squared blocks of a local high-strength Gneiss, called "Serizzo", respectively, in 1882 and 1884. The cracked masonry was locally dismantled and re-built, as it is easily readable through the changes of the masonry texture (Figure 6c).

The works continued almost without interruption until the beginning of the 20th century, involving: (i) the renovation of the wooden structures supporting the floors; (ii) the restoration of the Parlera and the merlons at the top of the tower; (iii) the installation of iron tie-rods on the tower and the conical roof; (iv) in 1896, the rebuilding of column M (Figure 7); and (v) the installation of a wooden staircase in the tower.

Notwithstanding the several strengthening works, in 1902, new cracks were identified on column E (Figure 7), and new interventions were planned. At the same time, it was decided to move the courthouse from the area of the Arengario and demolish the Palazzo del Pretorio. Consequently, it was necessary to find new access to the Arengario's first floor; until that time, it was indeed provided by the overpass from the Palazzo del Pretorio.

The new works proposed by Arch. Brusconi [48] started in 1903, together with the demolitions of the Palazzo del Pretorio. The strengthening of the tower involved the: (i) consolidation of the arches supporting the tower; (ii) construction of new walls at the selected arches; and (iii) installation of new stairs within the new space at the ground floor of the tower.

From 1903 [48], some internal demolition and the removal of internal plasters were performed, revealing the position of the original openings of the east façade. Since the building was empty and (temporarily) without a function, it was possible to start an overall restoration project (1912–1915) involving (Figure 8a,b): (i) the demolition of the 18th century additions on the first floor; (ii) the substitution of the double-lancet windows with triple-lancet windows; (iii) the transformation of the openings on the east façade following the shapes identified after the plaster removal showing an unregular distribution; and (iv) the substitution of the roof with wooden trusses and the maintenance of the wooden floor.

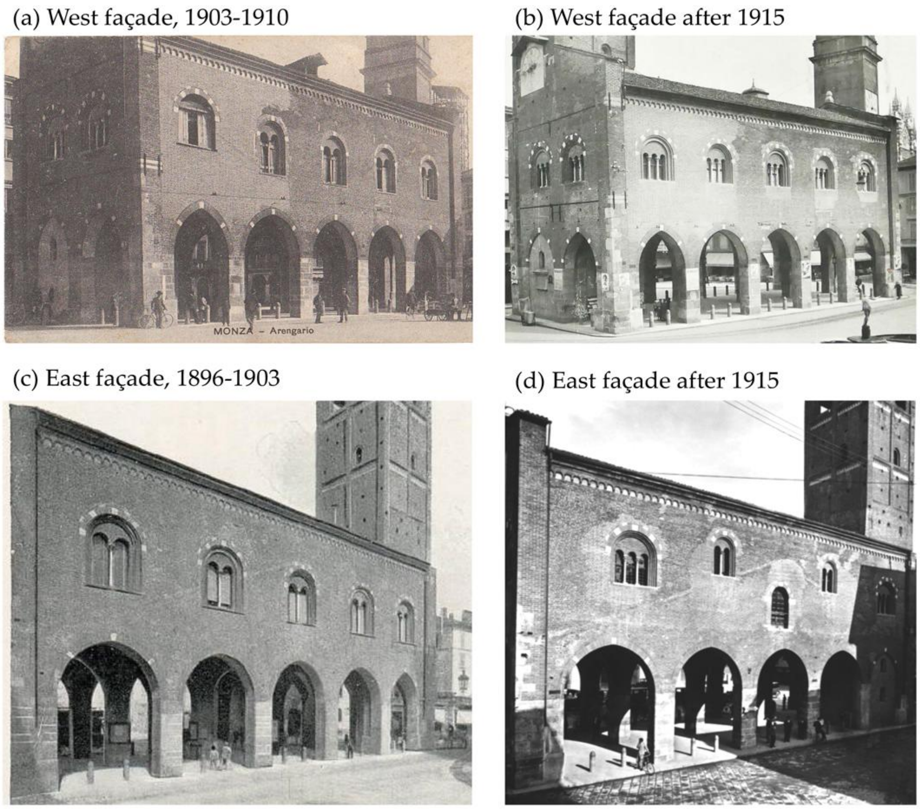


Figure 8. Historical pictures and postcards of the east (**a**,**b**) and west (**c**,**d**) fronts, before (**a**,**c**) and after (**b**,**d**) the restoration works of 1912–1915. The size, shape, and position of the several openings were modified.

More recently, new restoration works were carried out and a new emergency staircase was added on the east front [49].

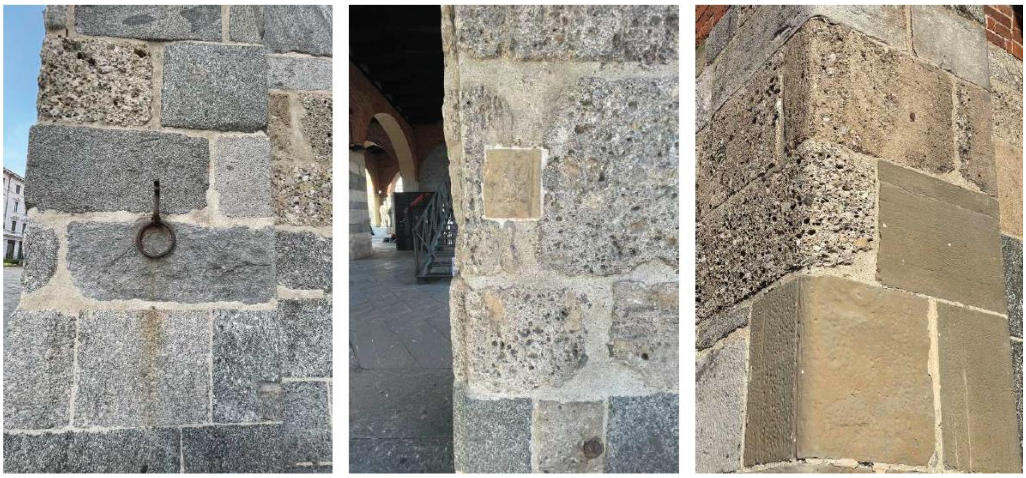
2.3. The Building Materials and the Damage

During the present investigation, the materials and the masonry discontinuities were accurately surveyed and reported in thematic drawings. The investigation procedures applied to the architectural heritage must be non-destructive, and the material sampling, when possible, must be limited. Thematic maps related to surface damage or crack patterns can assist in the evaluation of the building's state of preservation, supplemented with a stratigraphic survey [2,6,8,47].

The structure is composed of brick masonry supported by stone masonry columns on the ground floor (Figure 9). The mortars are presumably composed of lime, despite several instances of joint repointing and local repairs occurring over time.

Figure 9. View of the columns NOPQ (on right) and GHIK (on left) (See Figure 7). The GHIK columns are built with alternate white and grey stone blocks, similar to the façade of the Monza Cathedral.

The masonry of the perimetrical columns appears to be composed of different stone blocks, such as soft sandstone, Ceppo, and high-strength Serizzo (Figure 10). The Serizzo masonry was used to infill the arch and the ground floor and for the local repair of some columns.



(**a**) (**b**) (**c**)

Figure 10. The masonry of the perimetrical columns appears to be composed of different stone blocks, such as Gneiss, Ceppo (i.e., a local soft and porous sandstone), and soft sandstone. (**a**) Detail of stone masonry column mainly composed of Serizzo; (**b**) mainly in Ceppo and (**c**) column mainly in Ceppo and sandstone.

The brick masonry shows several irregularities due to the past repairs and window redistribution (Figures 8 and 11). The stratigraphic survey of all the fronts enhanced the complex sequence of interventions that were partially confirmed by the historical research (Figure 11).

(**a**) (**b**)

Figure 11. Detail of the north side (**a**) and part of the stratigraphic survey with the main interventions on the north façade (**b**). During the intervention of the late 19th century, the masonry was locally demolished and rebuilt. This is clearly readable by the changes in the masonry texture, the brick color, and the surface compactness.

Furthermore, visual inspections of the tower highlighted the presence of several cracks (Figure 12). Some of them were repaired but the cracks are still open. This suggests the necessity of the installation of a static monitoring of the main cracks. In fact, thermal actions can re-open cracks even without the progress, further damage progress. The distinction between thermal effects and damage is a safety priority.

(**a**) (**b**) (**c**) (**d**)

Figure 12. Detail of the cracks surveyed on the tower. Despite the past interventions, several cracks are still present (**a**). Some of them were repaired but the cracks are still open (**b**–**d**).

The floor and the roof trusses, rebuilt during the interventions between the end of the 19th and the beginning of the 20th century, seem in good condition, also thanks to the continued maintenance. The beam layout and the corbels give a relevant stiffness to the floor.

As illustrated by Sacchi [48], the foundations of the Arengario are constituted by four perimeter walls (thickness 1.50 m) and an inner longitudinal wall (thickness 1.45 m) based 2.50 m below the street level. The masonry of the foundations is constituted by a rubble masonry composed of a mix of mortar, brick and stones. In addition, the foundations on the east side are larger in correspondence with the columns, demonstrating the existence of an arched system that supported the aforementioned double staircases. On the other hand, these latter foundations are less regular, based at 1.50 m below the street level, and are constituted by re-used bricks, demonstrating that the staircases were built later with respect to the rest of the building.

The research highlights the key role of carefully checking the results of the historic analysis through direct inspections of the building and stratigraphic reconstruction. The identification of the building transformations can highlight possible local vulnerability, changes in the structural characteristics, or constrains that should be considered in the assessment and could suggest the necessity of further controls.

Material characterization should be performed to define the properties of the masonry for a more reliable structural model and assessment. Furthermore, material characterization provides information useful for the design of compatible interventions or maintenance processes. Flat-jack tests can evaluate the local state of compression in selected areas and measure the elastic properties of the masonry.

More accurate investigation concerning the stone columns is necessary [50–53]. The procedure does not consider the possible complexity of the section layout, which can produce unexpected structural behavior and specific vulnerability. Pulse sonic tests or radar surveys can explore the section morphology. The external texture appears to be composed of large stone blocks. However, the internal organization is unknown, as is the presence of a rubble masonry core.

3. Ambient Vibration Tests (AVTs)

As is known, modal identification is generally carried out by acquiring the vibration induced by micro-tremors and wind.

AVTs are a reliable non-destructive technique used for the identification of the dynamic properties of a structure. They involve the processing of output-only records through operational modal analysis (OMA) techniques [54]. In the technical literature, the application of AVTs to historic structures and specifically slender structures like towers or minarets [16–18,21,26] has constantly increased as they are non-destructive and are useful for evaluating the overall building behavior response to vibration. Furthermore, the use of AVTs is often extended to continuous monitoring systems, with preventive conservation and/or structural health monitoring purposes [27–32,55].

In the research carried out on the Arengario, an AVT was applied for the diagnosis of the building and to collect information for model calibration [16,17,20,21,24–26].

The vibration tests were carried out on 21 October 2021 with eight seismometers (electro-dynamic velocity transducers, SARA SS45), placed on four different levels (Figure 13), in order to better estimate the mode shapes of the tower [17].

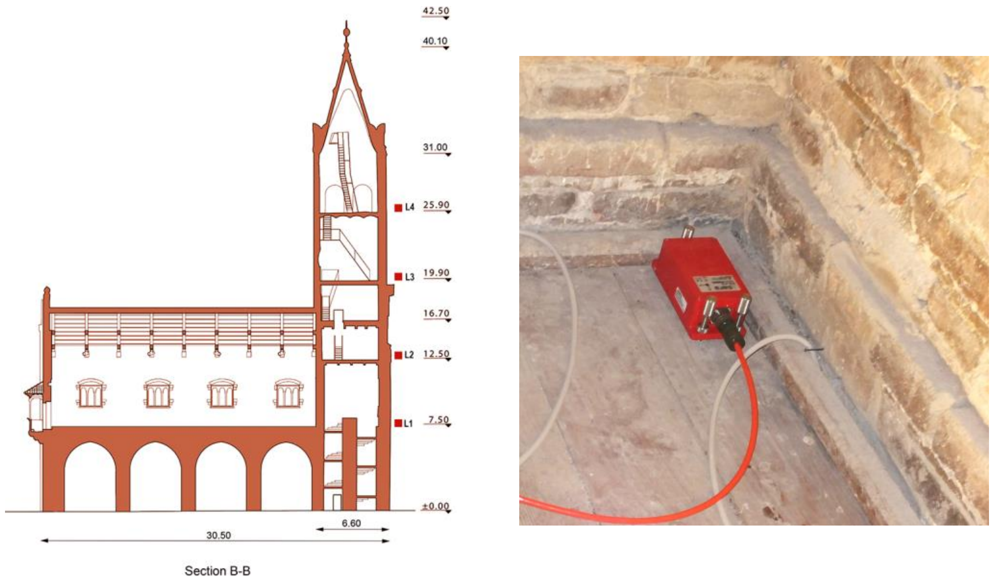
During the tests, the temperatures ranged from +10.3 °C to +12.6 °C.

The data processing of the series of 3600 s was carried out with different output-only identification algorithms, including Frequency Domain Decomposition (FDD) [56] and data-driven Stochastic Subspace Identification (SSI-data) [57].

The data processing allowed us to identify five modes in the frequency range of 0–5 Hz. The mode shapes of the tower, shown in Figure 14 (under the assumption of rigid floors), include:

- Two modes involving dominant bending in the N-E/S-W plane (1.63 and 1.69 Hz), along the diagonal of the building. The presence of the hall and of the side walls in the north and east fronts can affect the behavior;
- Two modes involving dominant bending (3.18 Hz and 4.14 Hz) in the E/W plane (without a diagonal contribution);
- One torsion mode (4.43 Hz).

The sequence of the first five modes only partially agrees with former research on historic towers [17,19,26], with closely bending modes in opposite directions, a torsion mode, and another two bending modes.



(**a**) (**b**)

Figure 13. Instrumented cross-sections (**a**) and sensors (**b**) used during the tests.

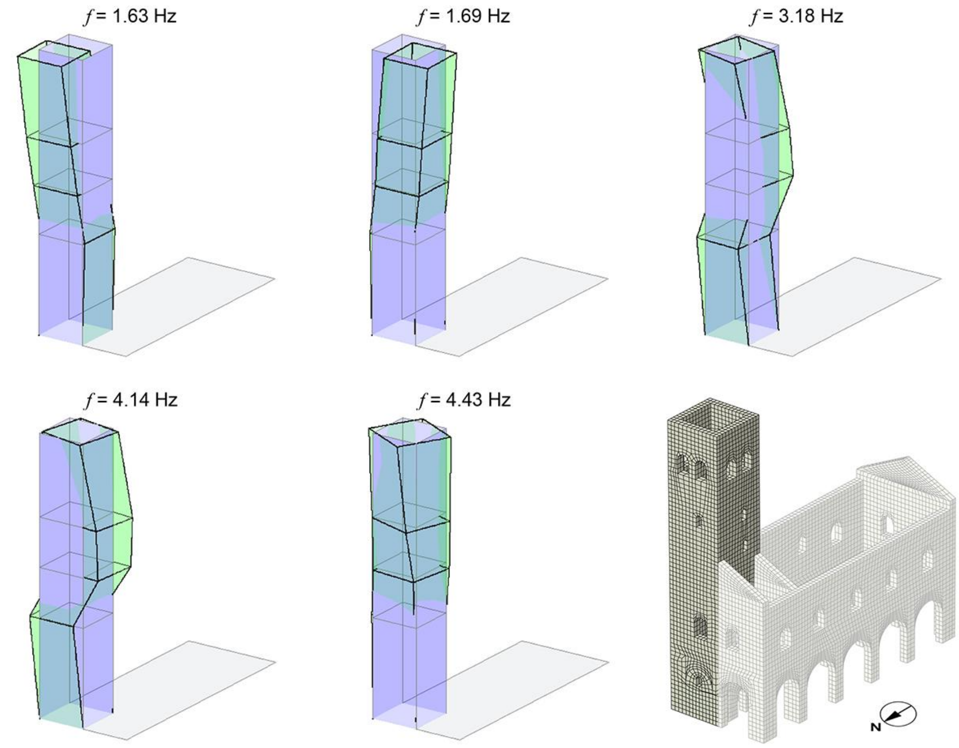


Figure 14. Identified vibration modes. (Modes at 1.63 and 1.69 Hz involve bending in the N-E/S-W plane along the diagonal of the Tower; modes at 3.18 Hz and 4.14 Hz are dominant bending in the E/W plane; torsion mode at 4.43 Hz).

4. The Structural Model

A preliminary 3D FE model of the Arengario (Figure 15) was developed in ABAQUS software using eight-node brick elements (C3D8). The regular distribution of masses, the accurate description of geometrical details, and the negligible mesh sensitivity on natural frequencies were obtained using a relatively large number of elements. Overall, the FE model consisted of 16,338 brick elements with 78,066 degrees of freedom and an average mesh size of 0.5 m.

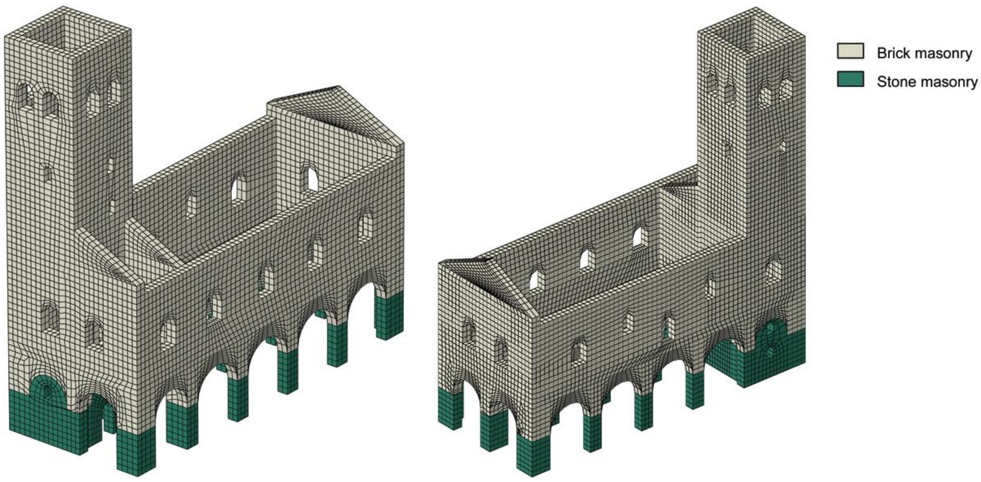


Figure 15. First FE model of the Arengario.

The geometry of masonry structures was obtained from the geomatic survey: 2D drawings were extracted from the point cloud, and—after necessary simplifications and regularizations—the 3D model of the masonry structures was developed in Autocad. The timber floors were not considered in this preliminary model. Subsequently, the 3D Autocad model was imported into ABAQUS/CAE 2017, and the mesh was generated.

Two main materials were identified in the FE model: (1) masonry bricks for the arches and walls and (2) stone masonry pillars and walls on the ground floor. The metal ties, not visible in Figures 15 and 16, were modelled by two-noded rods. In addition, the following assumptions were selected: (a) the effect of soil-structure interaction was neglected, and the building was assumed to be fixed at the base; (b) a linear elastic isotropic material was adopted for the brick and stone masonry; and (c) the mass density and Poisson's ratio of the masonry were set equal to 18 kN/m^3 (brick masonry) and 20 kN/m^3 (stone masonry), and 0.15, respectively.

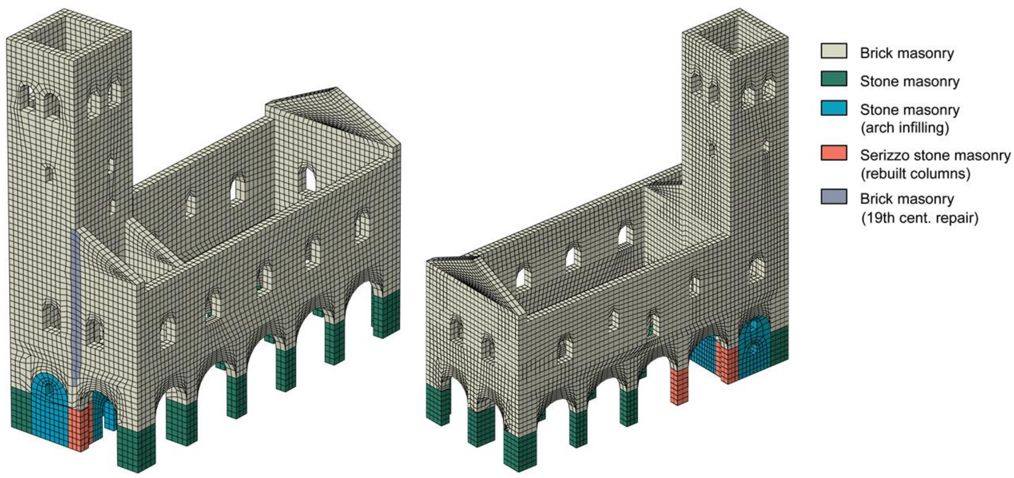


Figure 16. Refined FE model of the Arengario considering the transformations of the last 19th century.

The next phase was the calibration of uncertain structural parameters based on the AVT results. In detail, the Young's modulus of brick and stone masonry was calibrated to minimize the discrepancy between the modal parameters computed with the FE model and identified in the AVT.

In the following steps of the model updating, the results of the historic research and of the decay survey were more accurately considered (Figure 16).

More specifically, the properties of the arch infillings and of the rebuilt columns are changed and initially considered as 18 kN/m^3 and 22 kN/m^3.

The model tuning of the models was carried out step by step considering the correlation between the numerical and experimental response according to other case studies in the scientific literature [16,17,19,20,24–26], starting from: (i) the initial FE modeling based on the available geometry and selected assumptions; (ii) the choice of the uncertain structural parameters of the FE model; (iii) the identification of the optimal parameters by minimizing the difference between the model responses and the experimental responses using surrogate models [25].

The developed model is linear elastic, and the characteristics of the materials were roughly calibrated using the dynamic characteristics identified at an (almost) constant temperature. If a continuous monitoring system is installed in the building, data related also to the effects of thermal variations [19,26–29] will become available and will be accounted for in numerical simulations [58]. One of the future developments of the present research is, in fact, repeating the dynamic tests and installing some accelerometers and temperature sensors on the building to investigate the changes of dynamic characteristics associated with environmental variability [19,26–29].

5. Conclusions

This research documents how multidisciplinary research could collect information for the development of a reliable structural assessment, focusing on the importance of historic research.

Information merging is the core of the activity, as crosschecking the evidence with the visual inspection of the building drives us toward the estimation of qualitative and quantitative data for the development of reliable numerical models of the structure.

The marks of the historical transformations should be always recognized directly on the building, highlighting the potential local vulnerability or the structural problems to monitor or to mitigate. Furthermore, dating the transformations helps in the identification of the building technology and the potential discontinuities.

Surveys and documentation concerning construction technology and materials, visible changes in the surface texture, and discontinuities are important pieces of structural information that are often not directly considered in safety analysis. Nevertheless, a detailed and rigorous organization of the collected information can directly affect the reliability of the assessment of the structure. Stratigraphic surveying, a method developed for architecture archeology, can supplement direct surveying with the local observation of the superficial transformation in order to reconstruct the building evolution.

A good knowledge of the structural geometry and the results of AVTs should allow us to establish numerical models for a quantitative assessment of structural and seismic vulnerability.

This paper explores the main items of the historical information within the structural assessment of the Arengario in Monza. The interventions carried out between the end of the 19th century and the beginning of the 20th produced meaningful changes in the material properties of the building. Future steps of the research will focus on the recognition of the interventions in the model tuning.

The procedure herein proposed was developed to define a robust framework for a fast structural assessment to be extensively applied at the urban level, for prioritization purposes. General investigations with local tests and laboratory tests on sample materials are often time-consuming and/or not compatible with the available resources. The novelty of the research, instead, is to propose and calibrate a prompt robust investigation strategy to develop reliable assessments and initial structural models. This activity might be supplemented by subsequent research, to be performed once the availability of a reliable set of information has allowed for the definition of priorities at an urban scale.

Nevertheless, the procedure has some limitations. In fact, historic and documentary research sometimes cannot be properly performed in unorganized archives, particularly in the case of diffuse architectural heritage.

Being a preliminary analysis, tests on materials and local non-destructive tests were not carried out. Information related to materials are estimated according to the Guidelines for Heritage Structures of the Italian Code.

Furthermore, a more accurate investigation of the stone column is necessary, and recent information on the foundation system is not available. On the other hand, the foundation inspection in a historic building is often a challenging operation, and it is not generally included among the first diagnostic activities performed on site, such as those described in the paper.

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