Design of stabilizing strategies for discrete-time dual switching linear systems

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Abstract

Discrete-time dual switching linear systems are piecewise linear systems subject to both stochastic and deterministic commutations. Stochastic jumps, well-suited to account for unpredictable events like faults or abrupt changes in the parameters, are modeled by means of Markov chains. The deterministic switches are dictated by a scheduling signal, used as a control variable in order to achieve stochastic stability and guaranteed input/output performance. We derive sufficient conditions for the existence of a state-feedback switching law attaining these goals. Further, the more challenging co-design problem is addressed, namely the joint synthesis of a linear state-feedback controller and a stabilizing switching strategy ensuring a prescribed performance. The results are illustrated by means of a numerical example concerning a networked control system under communication failures.

Keywords: Markov Jump Linear Systems, Dual switching, Stability, $\mathscr{H}_2$ performance, $\mathscr{H}_\infty$ performance, Networked control.

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1. Introduction

Dual switching systems are characterized by the simultaneous presence of a deterministic switching mechanism and a second stochastic switching signal giving rise to "jumps" occurring at random times. The stochastic jumps, often modeled by resorting to Markov chains, are well suited to model faults/repairs and several other kinds of system changes due to exogenous uncontrollable events. In this context, the family of piecewise linear systems whose changes are governed by a Markov chain has been extensively investigated under the name of Markov Jump Linear Systems (MJLS), see e.g. [7], [4], [8]. It is worth remarking that the analysis and control of such systems must cope with their stochastic nature: for instance, there exist different definitions of stochastic stability, a feature that renders stability analysis and system stabilization more varied and challenging than in the deterministic case. Systems subject to a deterministic switching signal have also been the object of several research papers dealing with issues that range from stability analysis and stabilization to guaranteed performance, see the books [12], [15], the survey papers [6], [13] and the references therein.

The study of deterministically switching systems subject to stochastic changes, typically associated with faults or other unpredictable events, leads directly to the class of dual switching systems, characterized by the interplay between switching signals of very different nature. For a real world example consider a wind turbine connected to an energy storage device. The transition between the operating modes of the turbine (standby, power-optimization, power-limitation) can be reasonably regarded as governed by a deterministic switching signal whose schedule is decided by the controller. Conversely, the transitions between the modes of the storage device (charging, discharging, disconnected) depend on causes exogenous to the wind generation system and are better described by a stochastic model, e.g. a Markov chain. Another application example is given by a multi-loop networked control system (NCS) exploiting a shared communication channel with limited capacity and affected by failures. Again, the random failures of the communication network can be given a stochastic Markov chain description, while, at each time instant, the scheduling signal selects which control loop is currently attended.

The class of dual switching linear systems has already been the object of some previous studies. Under the assumption of dwell-time constraints on the deterministic switching signal, both the mean-square and the almost-sure stability properties of the overall system were investigated, [1, 2, 3]. More recently, in [3] the design of stabilizing switching signals ensuring a guaranteed performance has also been studied. While most contributions so far are concerned with continuous-time systems, the goal of this paper is to extend and generalize the results of [3] to the discrete-time case. The discrete-time framework is more suitable to address NCS applications on digital communication networks.

First, we study the problem of mean-square stabilization, via switching, of the origin of an unforced dual switching system. A state-feedback solution is found assuming feasibility of suitable coupled matrix inequalities, parameterized by a free design matrix parameter. Further, the design of state-feedback switching laws guaranteeing the fulfillment of $\mathscr{H}_2$ and $\mathscr{H}_\infty$ performance requirements is carried out. Again, the sufficient conditions for the existence of such switching strategies are expressed in terms of parameterized Linear Matrix Inequalities (LMI). Optimization of the design parameter can be performed to tighten the performance bounds.

In the second part of the paper, we introduce an additional control input and tackle the co-design problem of determining both the controller gains and the switching strategy to attain mean-square stability and prescribed performance measures. Interestingly, the resulting stabilizing strategy can be implemented either in closed-loop, assuming perfect knowledge of the state, or in open-loop through a randomly generated switching signal. The two strategies ensure the same performance bounds, but in closed-loop the actual performance is in general better.

The paper is organized as follows. Section 2 provides the problem formulation for the unforced system and the definition of the performance indices. In section 3 we design switching strategies ensuring stability and guaranteed $\mathscr{H}_2$ and $\mathscr{H}_\infty$ performance. The problem of co-design of the state-feedback controller gains and the switching strategy is addressed in Section 4. Section 5 illustrates the results of the paper in a significant example regarding a failure-prone NCS. The paper ends with some conclusions and perspectives in Section 6.

The notation adopted in the paper is fairly standard. In particular, the set of all discrete-time signals with finite 2-norm is denoted by ℓ_2. Moreover, $\mathscr{T}_N$ is the set of right-stochastic matrices of size N, i.e. unit row-sum nonnegative square matrices of size N. For a stochastic variable x, the notation $E[x]$ represents its expected value. For symmetric matrices, the symbol $\star$ stands for each of its symmetric blocks. The symbol $\otimes$ denotes the Kronecker product.

2. Problem formulation

Consider the class of discrete-time dual switching linear systems described by

$$x(t+1) = A_{\sigma(t)}^{\gamma(t)} x(t) + B_{\sigma(t)}^{\gamma(t)} w(t), \quad x(0) = x_0 \tag{1}$$

$$z(t) = C_{\sigma(t)}^{\gamma(t)} x(t) + D_{\sigma(t)}^{\gamma(t)} w(t) \tag{2}$$

where t is the discrete time index, $x(t) \in \mathscr{R}^n$ is the state, $w(t) \in \mathscr{R}^m$ is a deterministic disturbance, with $w(\cdot) \in \ell_2$, $z(t) \in \mathscr{R}^p$ is the performance output, $\gamma(t)$ is a switching signal taking values in the finite set $\mathscr{M} = \{1, 2, \ldots, M\}$, and $\sigma(t)$ is a time homogeneous Markov process taking values in the set $\mathscr{N} = \{1, 2, \ldots, N\}$, with transition probability matrix Λ. More precisely, the entry $\lambda_{ij} \geq 0$ of Λ represents the probability of a transition from mode i to mode j, namely

$$\lambda_{ij} = \Pr\{\sigma(t+1) = j | \sigma(t) = i\}$$

Of course Λ is a right-stochastic matrix (unit row-sum nonnegative matrix), i.e. $\Lambda \in \mathscr{T}_N$. Letting $\pi(t)$ denote the probability distribution at time t, it is well-known that its evolution is governed by the difference equation

$$\pi(t+1)' = \pi(t)'\Lambda \quad , \quad \pi(0) = \pi_0$$

In the sequel, we assume that Λ is irreducible and aperiodic, so that the Markov process admits a unique stationary (strictly positive) probability distribution $\bar{\pi}$ satisfying $\bar{\pi}' = \bar{\pi}'\Lambda$, see e.g. [5].

In summary, the system is subject to both stochastic jumps governed by the form process $\sigma(t)$ and deterministic switches dictated by the control signal $\gamma(t)$. Therefore, the state dynamics of the overall system is characterized by NM quadruples $(A_i^r, B_i^r, C_i^r, D_i^r), i \in \mathscr{N}, r \in \mathscr{M}$.

In accordance with standard notions of stochastic stability, for a given deterministic switching signal $\gamma(t)$, system (1) is mean-square stable (MS-stable) if, for $w(t) = 0$, it follows that

$$\lim_{t\to\infty} E\left[\|x(t)\|^2\right] = 0$$

for any initial condition x_0 and any initial probability distribution π_0. Here and afterwards, the symbol $E[\cdot]$ will denote the expectation with respect to the stationary distribution $\bar{\pi}$.

We will consider two performance indices inspired by the standard $\mathscr{H}_2$ and $\mathscr{H}_\infty$ norms of deterministic LTI systems. Precisely, let $x_0 = 0$, $\gamma(t)$ be given, and define $z^{(k)}(t)$ as the impulse response generated by $w(t) = \delta(t)e_k$, where $\delta(t)$ is the unit discrete-time impulse function and e_k is the k-th column of the $m \times m$ identity matrix. The $\mathscr{H}_2$ performance associated with $\gamma(t)$ is defined by the following expected quadratic cost

$$J_2(\gamma) = E\left[\sum_{k=1}^{m}\sum_{t=0}^{\infty} z^{(k)}(t)'z^{(k)}(t)\right] \tag{3}$$

As for $\mathscr{H}_\infty$-type performance, we consider $x_0 = 0$ and the worst-case measure of disturbance attenuation

$$J_\infty(\gamma) = \sup_{w\in\ell_2, w\neq 0} \frac{E\left[\sum_{t=0}^{\infty} z(t)'z(t)\right]}{\sum_{t=0}^{\infty} w(t)'w(t)} \tag{4}$$

Letting $\rho > 0$ be the prescribed level of disturbance attenuation, the $\mathscr{H}_\infty$ requirement is guaranteed if $J_\infty(\gamma) < \rho^2$.

The design of an optimal switching signal $\gamma(t)$ yielding the minimum of (3) is a formidable task which would require the use of the Maximum Principle for optimal stochastic control. In the following, we will consider the simpler problem of designing a suboptimal feedback control $\gamma(t)$ so that the overall system is mean-square stable and an upper bound $\bar{J}_2$ of the optimal cost is guaranteed. As for the $\mathscr{H}_\infty$-type performance, we will work out a switching design procedure ensuring $J_\infty(\gamma) < \rho^2$. In both cases, sufficient conditions will be provided.

Remark 2.1. *In a full information context, the switching signal $\gamma(t)$ can exploit the knowledge of both $x(t)$ and $\sigma(t)$, namely we can design $\gamma(t) = f(x(t), \sigma(t))$. This is the case that will be considered in the paper. For what concerns the case of partial information, two situations are possible, depending whether just $\sigma(t)$ or $x(t)$ is accessible.*

In the former case, the system is described by

$$x(t+1) = A_{\sigma(t)}^{f(\sigma(t))} x(t) + B_{\sigma(t)}^{f(\sigma(t))} w(t) \tag{5}$$

$$z(t) = C_{\sigma(t)}^{f(\sigma(t))} x(t) + D_{\sigma(t)}^{f(\sigma(t))} w(t) \tag{6}$$

and a possible control design strategy consists in constructing the static decision map $f : \mathscr{N} \to \mathscr{M}$. Note that, for a given map $f(\cdot)$, system (5), (6) is a standard MJLS, for which stability analysis and performance assessment can be carried out by means of well-established tools, [7]. In order to find the optimal map, an exhaustive combinatorial search can be rather easily implemented.

When just $x(t)$ is available for feedback, a possible strategy would consist in using present and past values of $x(t)$ to reconstruct the current value of $\sigma(t)$, see e.g. [10, 16] in continuous-time. Then, relying on a kind of certainty equivalence principle, one might select $\gamma(t)$ as a function of $x(t)$ and the estimate $\hat{\sigma}(t)$, along with the techniques developed later in this paper. Proving stability and performance properties of this heuristic approach is an interesting open issue.

3. Switching strategies design

The first result of this section deals with the design of a state-feedback switching strategy ensuring MS-stability of system (1) when the disturbance $w(t)$ is absent.

Theorem 3.1. *Consider system (1) with $w(t) = 0$. Assume that there exist positive definite matrices P_i^r, $i \in \mathcal{N}$, $r \in \mathcal{M}$ and a right-stochastic matrix $\Phi = [\varphi_{rs}] \in \mathcal{T}_M$ satisfying, $\forall i, r$, the matrix inequalities*

$$P_i^r > (A_i^r)' \left(\sum_{j=1}^{N} \sum_{k=1}^{M} \lambda_{ij} \varphi_{rk} P_j^k \right) A_i^r \tag{7}$$

Then, the feedback switching law

$$\gamma^* = g(x, \sigma) = \mathrm{argmin}_r x' P_\sigma^r x$$

makes the closed-loop system MS-stable.

Proof. Consider the stochastic Lyapunov function $V(x,i) = \min_r x' P_i^r x$ and compute its expected one-step difference at time t with the positions $x(t) = x$, $\sigma(t) = i$ and $g = \mathrm{argmin}_r x' P_i^r x$. For brevity, the event $\sigma(t) = i$ and the joint event $(x(t), \sigma(t)) = (x, i)$ will be indicated by $\mathcal{E}_i$ and $\mathcal{E}_{xi}$, respectively. It results that:

$$\begin{aligned} E[\Delta V(x,i)] &= E[V(x(t+1), \sigma(t+1) | \mathcal{E}_{xi}] - V(x,i) \\ &= E[\min_r x(t+1)' P_{\sigma(t+1)}^r x(t+1) | \mathcal{E}_{xi}] - \min_r x' P_i^r x \\ &= E[\min_r x' (A_i^g)' P_{\sigma(t+1)}^r A_i^g) x | \mathcal{E}_i] - x' P_i^g x \end{aligned}$$

Recall now that the expected value of the minimum of a function is not greater than the minimum of the expectation. Moreover,

$$E[P_{\sigma(t+1)}^r | \mathcal{E}_i] = \sum_j \lambda_{ij} P_j^r$$

Therefore, it follows

$$E[\Delta V(x,i)] \leq \min_r x' (A_i^g)' \sum_j \lambda_{ij} P_j^r A_i^g x - x' P_i^g x$$

Now, being $\varphi_{gk} \geq 0$ and $\sum_k \varphi_{gk} = 1, \forall g$, it holds that

$$\min_r x' (A_i^g)' \sum_j \lambda_{ij} P_j^r A_i^g x \leq x' (A_i^g)' \sum_j \lambda_{ij} \sum_k \varphi_{gk} P_j^k A_i^g x$$

Thanks to (7) we obtain

$$E[\Delta V(x,i)] < x' P_i^g x - x' P_i^g x = 0$$

and MS-stability follows from standard results on stochastic discrete-time Lyapunov functions, see e.g. [11]. $\square$

By slightly strengthening the conditions of Theorem 3.1, it is possible to design a stabilizing switching strategy which yields a guaranteed $\mathscr{H}_2$ performance.

Theorem 3.2. *Consider system (1), (2) with $x_0 = 0$, $B^r_\sigma = B_\sigma$ and $D^r_\sigma = D_\sigma$, $\forall r \in \mathscr{M}$, and the performance index (3). Assume that there exist positive definite matrices P^r_i, $i \in \mathscr{N}$, $r \in \mathscr{M}$ and a right-stochastic matrix $\Phi = [\varphi_{rs}] \in \mathscr{T}_M$ satisfying $\forall i, r$ the matrix inequalities*

$$P^r_i > (A^r_i)' \left(\sum_{j=1}^{N} \sum_{k=1}^{M} \lambda_{ij} \varphi_{rk} P^k_j \right) A^r_i + (C^r_i)' C^r_i \tag{8}$$

Then, the feedback switching law

$$\gamma^* = g(x, \sigma) = \mathrm{argmin}_r x' P^r_\sigma x \tag{9}$$

makes the closed-loop system MS-stable and guarantees that

$$\begin{aligned} J_2(\gamma^*) < \bar{J_2} &= \min_r E[\mathrm{trace}(B'_\sigma P^r_\sigma B_\sigma + D'_\sigma D_\sigma)] \\ &= \min_r \sum_{i=1}^{N} \mathrm{trace}(B'_i P^r_i B_i + D'_i D_i) \bar{\pi}_i \end{aligned}$$

Proof. First observe that feasibility of inequalities (8) implies feasibility of inequalities (7), so that the system is MS-stable.

Using again the stochastic Lyapunov function

$$V(x, i) = \min_r x' P^r_i x$$

and applying the same arguments (for $t \geq 1$) as in the proof of Theorem 3.1, it can be shown that

$$E[\Delta V(x, i)] < -x' (C^g_i)' C^g_i x$$

Consider now the trajectories of system (1), (2) when $w(t) = \delta(t) e_k$ and $x_0 = 0$ and let $x^{(k)}(t)$ be the associated state variable. In view of the discrete-time version of the Dynkin's Formula [14], one obtains (recall that the expectation is taken with respect to the stationary distribution of $\sigma(t)$)

$$E[V(x(k)(\infty), \sigma)] - E[V(x(k)(1), \sigma)] < -E\left[\sum_{t=1}^{\infty} z^{(k)}(t)' z^{(k)}(t)\right]$$

Thanks to stability and noticing that

$$E[V(x(k)(1),\sigma)] = E[\min_r e_k' B_\sigma' P_\sigma^r B_\sigma e_k]$$

one can conclude that

$$\begin{aligned}
J_2(\gamma^*) &= E\left[\sum_{k=1}^{m}\sum_{t=0}^{\infty} z^{(k)}(t)' z^{(k)}(t)\right] \\
&< E\left[\sum_{k=1}^{m} \min_r e_k' B_\sigma' P_\sigma^r B_\sigma e_k\right] + E\left[\sum_{k=1}^{m} e_k' D_\sigma' D_\sigma e_k\right] \\
&< E\left[\sum_{k=1}^{m} e_k' B_\sigma' P_\sigma^{\bar{r}} B_\sigma e_k\right] + E\left[\sum_{k=1}^{m} e_k' D_\sigma' D_\sigma e_k\right]
\end{aligned}$$

for any $\bar{r} \in \mathscr{M}$. Hence

$$J_2(\gamma^*) < \min_r E\left[\mathrm{trace}(B_\sigma' P_\sigma^r B_\sigma + D_\sigma' D_\sigma)\right] \tag{10}$$

so that the result follows. □

In the theorem above we have assumed that the matrices B_σ^γ and D_σ^γ do not depend on the controlled switching signal $\gamma(t)$. This was done for simplicity. Indeed, when dealing with impulse responses, the values of these matrices are relevant only at time 0. If such matrices did depend on γ, the value $\gamma(0)$ would be an additional degree of freedom in minimizing the cost. To be precise, the feedback switching law (9) would be valid for $t > 0$ and (10) would become

$$J_2(\gamma^*) < \min_r E\left[\mathrm{trace}((B_\sigma^{\gamma(0)})' P_\sigma^r B_\sigma^{\gamma(0)} + D_\sigma^{\gamma(0)})' D_\sigma^{\gamma(0)})\right]$$

so that a minimization with respect to $\gamma(0)$ could be further performed.

Observe that the performance bound (10) depends both on $\Phi \in \mathscr{T}_M$ and the matrices P_i^r satisfying the bilinear matrix inequalities (8). In order to strengthen this bound, an optimization procedure can be worked out, e.g. by gridding the free parameters of Φ in the finite box $[0,1]^M$ and solving, for each selected Φ, a convex optimization problem.

Remark 3.1. *An alternative bound to $J_2(\gamma^*)$ can be obtained by duality, making use of positive definite matrices S_i^r satisfying the inequalities*

$$S_i^r > \sum_{j=1}^{N}\sum_{k=1}^{M} \lambda_{ji}\varphi_{kr} A_j^k S_j^k (\hat{A}_j^k)' + B_i(B_i)'\bar{\pi}_i \tag{11}$$

It is a matter of tedious but easy computation to show that

$$J_2(\gamma^*) < \tilde{J}_2 = \min_r \sum_{i=1}^{N} \mathrm{trace}(C_i^r S_i^r (C_i^r)' + D_i D_i')$$

Finally, consider the $\mathscr{H}_\infty$ performance associated with the index (4). We can prove the following result.

Theorem 3.3. *Consider system (1), (2) with* $x_0 = 0$ *and the performance index (4) with a given value of* $\rho > 0$. *Assume that there exist positive definite matrices* P_i^r, $i \in \mathscr{N}$, $r \in \mathscr{M}$ *and a right-stochastic matrix* $\Phi = [\varphi_{rs}] \in \mathscr{T}_M$ *satisfying* $\forall i, r$ *the matrix inequalities*

$$\begin{bmatrix} (A_i^r)'\mathscr{P}_i^r A_i^r + (C_i^r)'C_i^r - P_i^r & (A_i^r)'\mathscr{P}_i^r B_i^r + (C_i^r)'D_i^r \\ \star & -\rho^2 I + (B_i^r)'\mathscr{P}_i^r B_i^r + (D_i^r)'D_i^r \end{bmatrix} < 0 \qquad (12)$$

where $\mathscr{P}_i^r = \sum_{j=1}^N \sum_{k=1}^M \lambda_{ij}\varphi_{rk} P_j^k$. *Then, the feedback switching law*

$$\gamma^* = g(x,\sigma) = \mathrm{argmin}_r x' P_\sigma^r x$$

makes the closed-loop system MS-stable and guarantees that $J_\infty(\gamma^*) < \rho^2$.

Proof. First of all, feasibility of (12) implies feasibility of (7), so that the switching law γ^* guarantees MS-stability.

Now, consider again the stochastic Lyapunov function $V(x,i) = \min_r x' P_i^r x$, and compute its expected one-step difference $E[\Delta V(x,i)]$ along the systems trajectories. Straightforward computation leads to

$$\begin{aligned} E[\Delta V(x,i)] &< -\begin{bmatrix} x' & w' \end{bmatrix} \begin{bmatrix} P_i^g - (A_i^g)'\mathscr{P}_i^g A_i^g + (C_i^g)'C_i^g & (A_i^g)'\mathscr{P}_i^g B_i^g + (C_i^g)'D_i^g \\ \star & \rho^2 I - (B_i^g)'\mathscr{P}_i^g B_i^g - (D_i^g)'D_i^g \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \\ & \quad -z'z + \rho^2 w'w \le -z'z + \rho^2 w'w \end{aligned}$$

Using again the Dynkin's formula, and recalling that $x_0 = 0$, it results that, for all $w \in \ell_2$,

$$0 < -E\left[\sum_{t=0}^{\infty} z(t)'z(t)\right] + \rho^2 \sum_{t=0}^{\infty} w(t)'w(t)$$

so that the thesis follows. □

An alternative, yet equivalent, formulation of Theorem 3.3 which is amenable for controller synthesis is obtained by reformulating inequalities (12) in terms of the unknowns $X_i^r = (P_i^r)^{-1}$. Indeed, it is just a matter of standard manipulation to obtain the following result.

Theorem 3.4. *Consider system (1), (2) with $x_0 = 0$ and the performance index (4) with a given value of $\rho > 0$. Assume that there exist positive definite matrices P_i^r, $i \in \mathscr{N}$, $r \in \mathscr{M}$ and a right-stochastic matrix $\Phi = [\varphi_{rs}] \in \mathscr{T}_M$ satisfying $\forall i, r$ the matrix inequalities*

$$\begin{bmatrix} X_i^r & 0 & (A_i^r X_i^r)' \Upsilon_i^{r,1} & \cdots & (A_i^r X_i^r)' \Upsilon_i^{r,M} & (C_i^r X_i^r)' \\ \star & \rho^2 I & (B_i^r)' \Upsilon_i^{r,1} & \cdots & (B_i^r)' \Upsilon_i^{r,M} & (D_i^r)' \\ \star & \star & \Xi^1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & 0 \\ \star & \star & 0 & \cdots & \Xi^M & 0 \\ \star & \star & \star & \cdots & \star & I \end{bmatrix} > 0 \tag{13}$$

where

$$\Upsilon_i^{r,k} = \begin{bmatrix} \sqrt{\lambda_{i1}\varphi_{rk}} I & \sqrt{\lambda_{i2}\varphi_{rk}} I & \cdots & \sqrt{\lambda_{iN}\varphi_{rk}} I \end{bmatrix}$$

and $\Xi^k = \mathrm{diag}\{X_i^k, i = 1, 2, \ldots, N\}$. Then, the feedback switching law

$$\gamma^* = g(x, \sigma) = \mathrm{argmin}_r x' (X_\sigma^r)^{-1} x$$

makes the closed-loop system MS-stable and guarantees that $J_\infty(\gamma^*) < \rho^2$.

4. Switching and control co-design

In this section, we discuss a more challenging problem. Given a discrete-time dual switching linear system with an additional control input variable, we aim at developing a procedure to jointly design a set of feedback controllers and a switching strategy so as to guarantee either an upper bound on the $\mathscr{H}_2$ performance or a prescribed bound on the $\mathscr{H}_\infty$ performance. More precisely, consider the following system:

$$x(t+1) = A_{\sigma(t)}^{\gamma(t)} x(t) + B_{\sigma(t)}^{\gamma(t)} w(t) + G_{\sigma(t)}^{\gamma(t)} u(t), \quad x(0) = x_0 \tag{14}$$

$$z(t) = C_{\sigma(t)}^{\gamma(t)} x(t) + D_{\sigma(t)}^{\gamma(t)} w(t) + H_{\sigma(t)}^{\gamma(t)} u(t) \tag{15}$$

where $u(t) \in \mathscr{R}^{m_u}$ is a control input and all remaining variables are defined as in Section 2. For simplicity, assume again that both the state $x(t)$ and the Markov process $\sigma(t)$ are available for feedback, and the input $u(t)$ is generated by the closed-loop control law

$$u(t) = K_{\sigma(t)}^{\gamma(t)} x(t) \tag{16}$$

The closed-loop system resulting from the application of the control law (16) can be rewritten as

$$x(t+1) = \hat{A}_{\sigma(t)}^{\gamma(t)} x(t) + B_{\sigma(t)}^{\gamma(t)} w(t), \quad x(0) = x_0 \tag{17}$$

$$z(t) = \hat{C}_{\sigma(t)}^{\gamma(t)} x(t) + D_{\sigma(t)}^{\gamma(t)} w(t) \tag{18}$$

with $\hat{A}_i^r = A_i^r + G_i^r K_i^r$ and $\hat{C}_i^r = C_i^r + H_i^r K_i^r$, $i \in \mathscr{N}$, $r \in \mathscr{M}$.

4.1. $\mathscr{H}_2$ performance

In this subsection, we aim at finding a set of matrices K_i^r, $i \in \mathscr{N}$, $r \in \mathscr{M}$, and a feedback switching strategy $\gamma = g(x, \sigma)$ such that the dual switching system (17), (18) is MS-stable and its $\mathscr{H}_2$ performance is ensured to be less than an upper bound $\bar{J}_2$. To address this problem, it is useful to recall some results on the $\mathscr{H}_2$ performance of a standard MJLS, i.e. when $\gamma(t) = r$ is fixed. In this respect, we have the following result, adapted from [7], Proposition 4.8, that links the $\mathscr{H}_2$ performance with the reachability Gramian.

Theorem 4.1. *Consider system (17), (18) with $\gamma(t) = r, \forall t$, K_i^r, $i \in \mathscr{N}$ given, and the performance index (3). The system is MS-stable if and only if there exist positive definite matrices S_i^r, $i \in \mathscr{N}$, satisfying, $\forall i$, the matrix equations*

$$S_i^r = \sum_{j=1}^{N} \lambda_{ji} \hat{A}_j^r S_j^r (\hat{A}_j^r)' + B_i^r (B_i^r)' \bar{\pi}_i \tag{19}$$

Moreover, its $\mathscr{H}_2$ performance can be computed as

$$J_2^r = \text{trace}\left(\sum_{j=1}^{N} \left(\hat{C}_j^r S_j^r (\hat{C}_j^r)' + D_j^r (D_j^r)' \right) \bar{\pi}_j \right)$$

Note that the matrices S_i^r appearing in eq. (19) can be interpreted as the reachability Gramians, i.e.

$$S_i^r = \sum_{k=1}^{m} \sum_{t=1}^{\infty} E[x^{(k)}(t) x^{(k)}(t)' | \sigma(t) = i] \bar{\pi}_i$$

An equivalent dual formulation is provided next.

Theorem 4.2. *Consider system (17), (18) with $\gamma(t) = r, \forall t$, K_i^r, $i \in \mathscr{N}$ given, and the performance index (3). The system is MS-stable if and only if there exist positive definite matrices P_i^r, $i \in \mathscr{N}$, satisfying, $\forall i$, the matrix equations*

$$P_i^r = (\hat{A}_i^r)' \left(\sum_{j=1}^{N} \lambda_{ij} P_j^r \right) \hat{A}_i^r + (\hat{C}_i^r)' \hat{C}_i^r \tag{20}$$

Moreover, its $\mathscr{H}_2$ performance can be computed as

$$J_2^r = \text{trace} \left(\sum_{j=1}^{N} \left((B_j^r)' P_j^r B_j^r + (D_j^r)' D_j^r \right) \bar{\pi}_j \right)$$

Note that P_i^r appearing in eq. (20) can be interpreted as the generator of the cost-to-go function, i.e.

$$E[x^{(k)}(t)' P_i^r x^{(k)}(t)] = E[\sum_{\tau=t}^{\infty} z^{(k)}(\tau)' z^{(k)}(\tau) | \sigma(t) = i], \quad t \geq 1$$

As for the design of the gain matrices K_i^r when r is fixed, the gains optimizing the $\mathscr{H}_2$ performance are provided by the following convex optimization procedure. The proof can be found in [9].

Theorem 4.3. *Consider system (17), (18) with $\gamma(t) = r, \forall t$, and the performance index (3). Assume that there exist positive definite matrices S_i^r, W_i^r, $i \in \mathscr{N}$ and matrices Y_i^r, $i \in \mathscr{N}$, satisfying, $\forall i$, the matrix inequalities*

$$\begin{bmatrix} S_i^r - B_i^r (B_i^r)' \bar{\pi}_i & \Omega_i^r \\ \star & \Sigma^r \end{bmatrix} > 0 \tag{21}$$

$$\begin{bmatrix} S_i^r & (Y_i^r)' \\ \star & W_i^r \end{bmatrix} > 0 \tag{22}$$

where

$$\Omega_i^r = \left[\sqrt{\lambda_{1i}} (A_1^r S_1^r + G_1^r Y_1^r) \quad \cdots \quad \sqrt{\lambda_{Ni}} (A_N^r S_N^r + G_N^r Y_N^r) \right]$$

and $\Sigma^r = \text{diag}\{S_i^r, i = 1, 2, \ldots, N\}$. Then, letting $K_i^r = Y_i^r (S_i^r)^{-1}$, the system is MS-stable and its $\mathscr{H}_2$ performance is

$$J_2^r = \inf_{S_i^r, W_i^r, Y_i^r} \sum_{i=1}^{N} \text{trace} \left(\begin{bmatrix} C_i^r & H_i^r \end{bmatrix} \begin{bmatrix} S_i^r & (Y_i^r)' \\ \star & W_i^r \end{bmatrix} \begin{bmatrix} (C_i^r)' \\ (H_i^r)' \end{bmatrix} + D_i^r (D_i^r)' \bar{\pi}_i \right)$$

Now, we are in a position to formulate the co-design result for the $\mathscr{H}_2$ performance.

Theorem 4.4. *Consider system (17), (18) with $x_0 = 0$, $B^r_\sigma = B_\sigma$ and $D^r_\sigma = D_\sigma$, $\forall r \in \mathscr{M}$, and the performance index (3), and select a right-stochastic matrix $\Phi = [\varphi_{rs}] \in \mathscr{T}_M$. Assume that there exist positive definite matrices S^r_i, W^r_i, $i \in \mathscr{N}$, $r \in \mathscr{M}$, and matrices Y^r_i, $i \in \mathscr{N}$, $r \in \mathscr{M}$, solving the following convex optimization problem:*

$$\bar{J_2}(\Phi) = \min_r \inf_{S^r_i, W^r_i, Y^r_i} \sum_{i=1}^{N} \text{trace}\left(\begin{bmatrix} C^r_i & H^r_i \end{bmatrix} \begin{bmatrix} S^r_i & (Y^r_i)' \\ \star & W^r_i \end{bmatrix} \begin{bmatrix} (C^r_i)' \\ (H^r_i)' \end{bmatrix} + D^r_i (D^r_i)' \bar{\pi}_i \right)$$

with

$$\begin{bmatrix} S^r_i - B^r_i (B^r_i)' \bar{\pi}_i & \Psi^1_i(r) & \Psi^2_i(r) & \cdots & \Psi^M_i(r) \\ \star & \Sigma^1_i & 0 & \cdots & 0 \\ \star & 0 & \Sigma^2_i & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \star & 0 & 0 & \cdots & \Sigma^M_i \end{bmatrix} > 0, \quad \forall i, r \tag{23}$$

$$\begin{bmatrix} S^r_i & (Y^r_i)' \\ \star & W^r_i \end{bmatrix} > 0, \quad \forall i, r \tag{24}$$

where

$$\Psi^k_i(r) = \begin{bmatrix} \sqrt{\lambda_{1i}\varphi_{kr}} (A^k_1 S^k_1 + G^k_1 Y^k_1) & \cdots & \sqrt{\lambda_{Ni}\varphi_{kr}} (A^k_N S^k_N + G^k_N Y^k_N) \end{bmatrix}$$

and $\Sigma^k_i = \text{diag}\{S^k_i, i = 1, 2, \ldots, N\}$. Then, letting $K^r_i = Y^r_i (S^r_i)^{-1}$, the system is MS-stable under the switching law

$$\gamma^* = g(x, \sigma) = \text{argmin}_r x' P^r_\sigma x$$

where matrices P^r_i solve the equations

$$P^r_i = (\hat{A}^r_i)' \left(\sum_{j=1}^{N} \sum_{k=1}^{M} \lambda_{ij} \varphi_{rk} P^k_j \right) \hat{A}^r_i + (\hat{C}^r_i)' \hat{C}^r_i \tag{25}$$

Moreover the $\mathscr{H}_2$ performance of the closed-loop system is $J_2 < \bar{J_2}(\Phi)$.

Proof. Since its rationale is rather standard, below only a sketch of the proof is given. For given MS-stabilizing gains K_i^r, one can resort to Theorem 8 and Remark 3.1, with $A_i^r \to \hat{A}_i^r$, $C_i^r \to \hat{C}_i^r$, to obtain upper bounds $\bar{J}_2$ and $\tilde{J}_2$ to the $\mathscr{H}_2$ performance of the closed loop system. When the gains appearing in the statement are applied, it can be shown that inequality (11) coincides with inequality (23) and the upper bound $\tilde{J}_2$ coincides, after optimization, with $\bar{J}_2(\Phi)$. □

Remark 4.1. *In the previous theorem, the solution of the co-design problem was derived for a given matrix $\Phi \in \mathscr{T}_M$ by solving a convex optimization problem. The performance upper bound might therefore be optimized by a proper choice of Φ. Note however that this would lead to a bilinear problem. While in lower dimensions gridding techniques could be viable, most sophisticated techniques, like Cone Complementarity Methods, may be needed. At the cost of some additional conservatism, one might also reduce the number of free parameters in matrix Φ.*

Remark 4.2. *It is worth noting that the Eqs. (25) can be compactly rewritten as*

$$\bar{P}_k = (\bar{A}_k)' \left(\sum_{s=1}^{NM} \bar{\lambda}_{ks} \bar{P}_s \right) \bar{A}_k + (\bar{C}_k)' \bar{C}_k$$

where, for any $i = 1, \ldots, N$, $k = (i-1)M + 1, \ldots, iM$,

$$\bar{P}_k = P_i^{k-(i-1)M}, \quad \bar{A}_k = \hat{A}_i^{k-(i-1)M}, \quad \bar{C}_k = \hat{C}_i^{k-(i-1)M} \tag{26}$$

and λ_{ks} are the entries of matrix $\bar{\Lambda} = \Lambda \otimes \Phi, \in \mathscr{T}_{NM}$. These equations correspond to the coupled Lyapunov equations of the extended MJLS

$$\begin{aligned} \bar{x}(t+1) &= \bar{A}_{\bar{\sigma}(t)} \bar{x}(t) + \bar{B}_{\bar{\sigma}(t)} w(t) \qquad (27) \\ \bar{z}(t) &= \bar{C}_{\bar{\sigma}(t)} \bar{x}(t) + \bar{D}_{\bar{\sigma}(t)} w(t) \qquad (28) \end{aligned}$$

where $\bar{B}_k = B_i^{k-(i-1)M}$, $\bar{D}_k = D_i^{k-(i-1)M}$, $i = 1, \ldots, N$, $k = (i-1)M + 1, \ldots, iM$ and $\bar{\sigma}(t)$ is a Markov process with transition probability matrix $\bar{\Lambda}$. It is easily seen that system (27), (28) coincides with the closed-loop system (13), (14), (15) when $\gamma(t)$ randomly switches according to a Markov chain (independent of $\sigma(t)$) with probability transition matrix Φ. The overall system (27), (28) evolves as a MJLS with transition probability matrix $\bar{\Lambda}$, whose set of modes is the Cartesian product $\mathscr{N} \times \mathscr{M}$. In view of (26), the performance provided by this random switching strategy is equal to $\bar{J}_2(\Phi)$. It is also worth noting that the actual performance of the state-feedback switching strategy of Theorem 4.4, being bounded by $\bar{J}_2(\Phi)$, may well be better.

4.2. $\mathscr{H}_\infty$ performance

The co-design problem treated in this subsection consists of finding a set of matrices K_i^r, $i \in \mathscr{N}$, $r \in \mathscr{M}$, and a feedback switching strategy $\gamma = g(x, \sigma)$ such that the dual switching system (17), (18) is MS-stable and its $\mathscr{H}_\infty$ performance is ensured to be less than a prescribed upper bound ρ^2.

The co-design $\mathscr{H}_\infty$ problem is addressed in the following theorem, whose proof relies on Theorem 2.4 by replacing $A_i^r X_i^r$ with $\hat{A}_i^r X_i^r = A_i^r X_i^r + G_i^r Y_i^r$, and $C_i^r X_i^r$ with $\hat{C}_i^r X_i^r = C_i^r X_i^r + H_i^r Y_i^r$.

Theorem 4.5. *Consider system (17), (18) with $x_0 = 0$ and the performance index (4), and select a right-stochastic matrix $\Phi = [\varphi_{rs}] \in \mathscr{T}_M$. Assume that there exist positive definite matrices X_i^r and matrices Y_i^r, $i \in \mathscr{N}$, $r \in \mathscr{M}$, satisfying the following inequalities:*

$$\begin{bmatrix} X_i^r & 0 & (A_i^r X_i^r + G_i^r Y_i^r)' \Upsilon_i^{r,1} & \cdots & (A_i^r X_i^r + G_i^r Y_i^r)' \Upsilon_i^{r,M} & (C_i^r X_i^r + H_i^r Y_i^r)' \\ \star & \rho^2 I & (B_i^r)' \Upsilon_i^{r,1} & \cdots & (B_i^r)' \Upsilon_i^{r,M} & (D_i^r)' \\ \star & \star & \Xi^1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & 0 \\ \star & \star & 0 & \cdots & \Xi^M & 0 \\ \star & \star & \star & \cdots & \star & I \end{bmatrix} > 0 \tag{29}$$

where

$$\Upsilon_i^{r,k} = \begin{bmatrix} \sqrt{\lambda_{i1}\varphi_{rk}} I & \sqrt{\lambda_{i2}\varphi_{rk}} I & \cdots & \sqrt{\lambda_{iN}\varphi_{rk}} I \end{bmatrix}$$

and $\Xi^k = \mathrm{diag}\{X_i^k, i = 1, 2, \ldots, N\}$. Then, letting $K_i^r = Y_i^r (X_i^r)^{-1}$, the system is MS-stable under the switching law

$$\gamma^* = g(x, \sigma) = \mathrm{argmin}_r x' (X_\sigma^r)^{-1} x$$

Moreover the $\mathscr{H}_\infty$ performance of the closed-loop system is $J_\infty < \rho^2$.

5. Scheduling design in networked control systems

In order to illustrate an application of the dual switching framework, consider a scheduling problem for a multi-loop networked control system subject to packet dropout. More precisely, assume that M linear (possibly unstable) plants have to be controlled by a single regulator exchanging input-output data through a shared network, as depicted in Figure 1. The regulator is allowed to attend only one

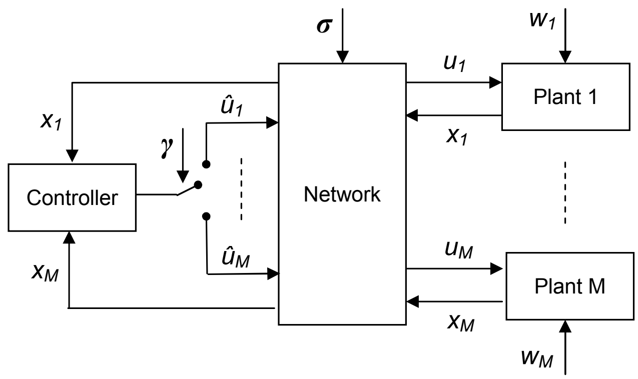

Figure 1: The NCS considered in the application.

plant at a time, according to the scheduling signal $\gamma(t)$ taking values in the set $\mathscr{M}$. Transmission of actuator data over the network is subject to random failures, modeled by the Markov process $\sigma(t)$ taking values in the set $\mathscr{N}$. For simplicity, we assume that full state information is transmitted by each sensor without failures, so that the regulator has complete access to the state information of all plants. As for the regulator-actuator channel, let $\sigma(t) = 1$ stand for the no-fault mode, when all packets are correctly transmitted, and $\sigma(t) = 2$ stand for packet dropout mode, when no packet is delivered. A Markov chain model with $N = 2$ and transition probability matrix Λ is assumed to describe the jumps between these two modes. Of course, more complex models could fit within the given framework. For instance, one could increase the number of logical states to cope with packet loss also in the sensor-regulator channel, augment the state to account for time delay effects, and introduce suitable state observers in the regulator when full-state information is not available.

We first assume that the regulator is equipped with M control laws tailored to the individual plants and only the scheduling signal has to be designed so as to satisfy stability and performance requirements. If the regulator has access to the value of $\sigma(t)$, the scheduling design problem can be cast in the formulation of Section 3.

In a second step, we will exploit the co-design methods of Section 4 assuming that both the controller gains and the scheduling signal γ are to be designed.

Consider, for simplicity, a Networked Control System (NCS) with two plants

($M = 2$) described by the sampled-data models

$$\begin{aligned} x_i(t+1) &= F_i x_i(t) + G_i u_i(t) + L_i w_i(t), \quad i = 1,2 && (30) \\ y_i(t) &= C_i x_i(t) && (31) \end{aligned}$$

where the output $y_i(t)$ enter the definition of the performance variable $z(t)$. More precisely, the performance output $z(t)$ is such that

$$z(t)'z(t) = y_1(t)'y_1(t) + y_2(t)'y_2(t) + \mu^2(u_1(t)'u_1(t) + u_2(t)'u_2(t))$$

so as to weigh the output energy of both plants and the actual control effort in the cost function.

The control law issued by the regulator is modeled as

$$\hat{u}_i(t) = \begin{cases} K_i x_i(t), & \text{if } \gamma(t) = i \\ 0, & \text{if } \gamma(t) \neq i \end{cases}$$

and the true actuator signals, affected by random packet loss, are given by

$$u_i(t) = \begin{cases} \hat{u}_i(t), & \text{if } \sigma(t) = 1 \\ 0, & \text{if } \sigma(t) = 2 \end{cases}$$

In other words, the control signal applied to the system in the faulty mode is set to zero. Let $x(t) = [x_1(t)' \quad x_2(t)']'$ be the state, $u(t) = [u_1(t)' \quad u_2(t)']'$ the control vector and $w(t) = [w_1(t)' \quad w_2(t)']'$ the disturbance vector.

The objective of the scheduling design (or co-design) is to guarantee simultaneous MS-stabilization of both plants, along with the fulfillment of the following $\mathscr{H}_\infty$-like performance specification when $x(0) = 0$:

$$J_\infty(\gamma) = \sup_{w \in \ell_2, w \neq 0} \frac{E\left[\sum_{t=0}^{\infty} z(t)'z(t)\right]}{\sum_{t=0}^{\infty} w(t)'w(t)} < \rho^2$$

Hereafter, we assume that plant P1 is a double integrator, described by

$$F_1 = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}, \quad G_1 = L_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

and plant P2 is a marginally stable system described by

$$F_2 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad G_2 = L_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The parameter μ in the cost function is set to $\mu = \sqrt{10}$.

Finally, let the transition rate matrix Λ of the Markov process $\sigma(t)$ be given by

$$\Lambda = \begin{bmatrix} 0.9 & 0.1 \\ 0.8 & 0.2 \end{bmatrix}$$

Note that the stationary probability distribution of this Markov chain is $\bar{\pi} = [8/9 \quad 1/9]'$.

5.1. Scheduling design

We first consider the case when the controller gains are assigned and given by

$$K_1 = \begin{bmatrix} 1 & -2 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0 & -1 \end{bmatrix}$$

which would provide deadbeat control on each individual loop in absence of failures and scheduling constraints. The overall system can be written as in (1), (2) with

$$\begin{aligned} A_1^1 &= \begin{bmatrix} F_1 + G_1 K_1 & 0 \\ 0 & F_2 \end{bmatrix}, \quad A_1^2 = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 + G_2 K_2 \end{bmatrix} \\ A_2^1 &= A_2^2 = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}, \quad B_i^r = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix}, \quad D_i^r = 0, \quad i, r = 1, 2 \end{aligned}$$

In order to represent the performance, take $C_{\sigma(t)}^{\gamma(t)}$ such that

$$(C_{\sigma(t)}^{\gamma(t)})' C_{\sigma(t)}^{\gamma(t)} = \tilde{C}' \tilde{C} + \mu^2 (K_{\sigma(t)}^{\gamma(t)})' K_{\sigma(t)}^{\gamma(t)}$$

where

$$\tilde{C} = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$$

$$K_{\sigma(t)}^{\gamma(t)} = \begin{cases} \begin{bmatrix} K_1 & 0 \\ 0 & 0 \end{bmatrix}, & \text{if } \sigma(t) = 1, \gamma(t) = 1 \\ \\ \begin{bmatrix} 0 & 0 \\ 0 & K_2 \end{bmatrix}, & \text{if } \sigma(t) = 1, \gamma(t) = 2 \\ \\ 0, & \text{if } \sigma(t) = 2 \end{cases}$$

Our aim is to design a scheduling strategy so as to minimize the bound ρ^2 on the $\mathscr{H}_\infty$ performance, taking into account also the effects of packet dropout. To this purpose, we apply the results of Theorem 3.4. After some tuning of the design parameters, we obtained that, in correspondence of

$$\Phi = \begin{bmatrix} 0.92 & 0.08 \\ 0.91 & 0.09 \end{bmatrix}$$

the resulting switching strategy γ^* is stabilizing and the guaranteed attenuation level is $\rho = 38$.

5.2. Scheduling and controller co-design

We consider now the co-design problem. In this case the system is modeled by (14), (15) with matrices B_i^r and D_i^r, $i, r = 1, 2$, defined as above and

$$\begin{aligned} A_i^r &= \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}, \quad C_i^r = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \\ 0 & 0 \end{bmatrix}, \quad i, r = 1, 2 \\ G_1^1 &= \begin{bmatrix} G_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad G_1^2 = \begin{bmatrix} 0 & 0 \\ 0 & G_2 \end{bmatrix}, \quad G_2^1 = G_2^2 = 0 \\ H_1^1 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \mu & 0 \end{bmatrix}, \quad H_1^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \mu \end{bmatrix}, \quad H_2^1 = H_2^2 = 0 \end{aligned}$$

Making reference to Theorem 4.5, after some tuning we obtained that, in correspondence of

$$\Phi = \begin{bmatrix} 0.65 & 0.35 \\ 0.9 & 0.1 \end{bmatrix}$$

the designed gains are

$$K_1 = \begin{bmatrix} 0.8508 & -1.0686 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.0000 & -0.6261 \end{bmatrix}$$

that, together with the switching strategy γ^*, ensure the attenuation level $\rho = 12$, which greatly improves on the result achievable by just scheduling γ with fixed gains, as done in the previous subsection.

For illustrative purposes a comparative simulation was carried out using either scheduling design or scheduling/controller co-design. In all simulations, the same realization of the Markov process $\sigma(t)$ was considered. On each plant the disturbance $w_1(t) = w_2(t) = \sin(0.2t)$, truncated at time $t = 40$, was applied. The results are shown in Figures 2 and 3, reporting the time pattern of $\sigma(t)$ and $\gamma^*(t)$, the plant outputs $y_1(t)$ and $y_2(t)$, and the control signals $u_1(t)$ and $u_2(t)$.

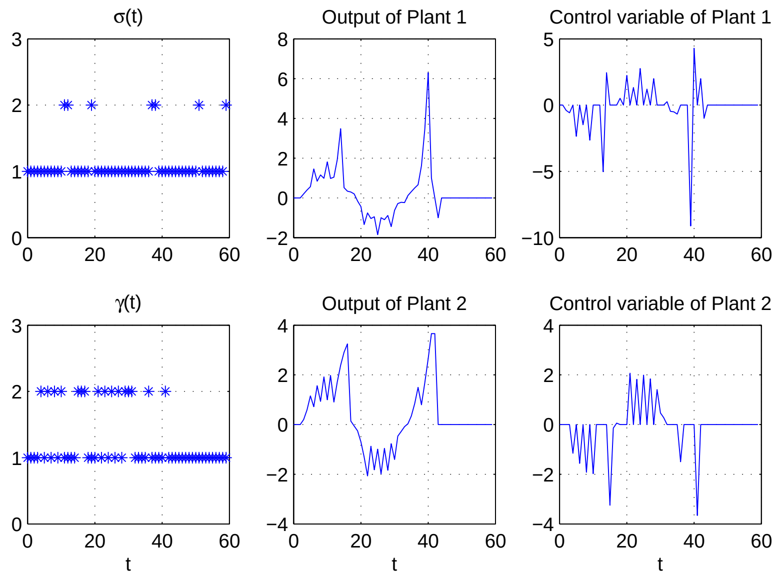

Figure 2: Results of the example with the scheduling technique of Section 5.1.

It is apparent that in the co-design strategy the control effort is much reduced thanks to the coordinated action of $\gamma(t)$ and the properly designed gains K_i. The actual attenuation level was computed in both simulations, yielding $J_\infty(\gamma) = 62.71$ for the scheduling design strategy and $J_\infty(\gamma) = 57.79$ for the scheduling/controller co-design strategy. For comparison, the open-loop switching strategy of Remark 4.2 was also tested. The results are displayed in Figure 4 and the achieved performance is $J_\infty(\gamma) = 78.76$, worse than using the state-feedback switching but still below the guaranteed bound on the average cost. Note that a distinct advantage of the open-loop random scheduling strategy is that it does not need any information on the current mode of the Markov process $\sigma(t)$.

6. Concluding remarks

The problem of designing a state-feedback switching law for discrete-time dual switching linear systems subject to Markov jumps has been solved. Design specifications include mean-square stability and the achievement of guaranteed $\mathscr{H}_2$ and $\mathscr{H}_\infty$ costs. These results may prove useful in several contexts, such as scheduling problems for NCS's with capacity limitations and random faults. Further research will address the same problems in case only partial information on

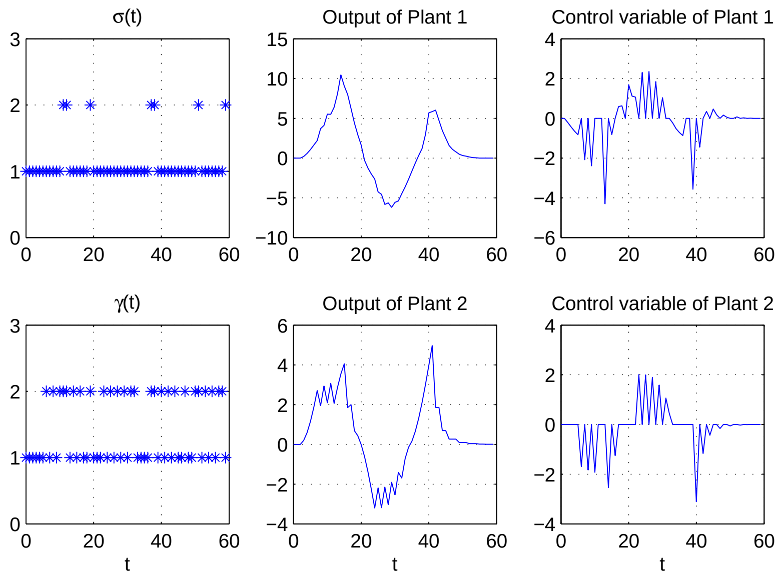

Figure 3: Results of the example with the scheduling and controller co-design technique of Section 5.2.

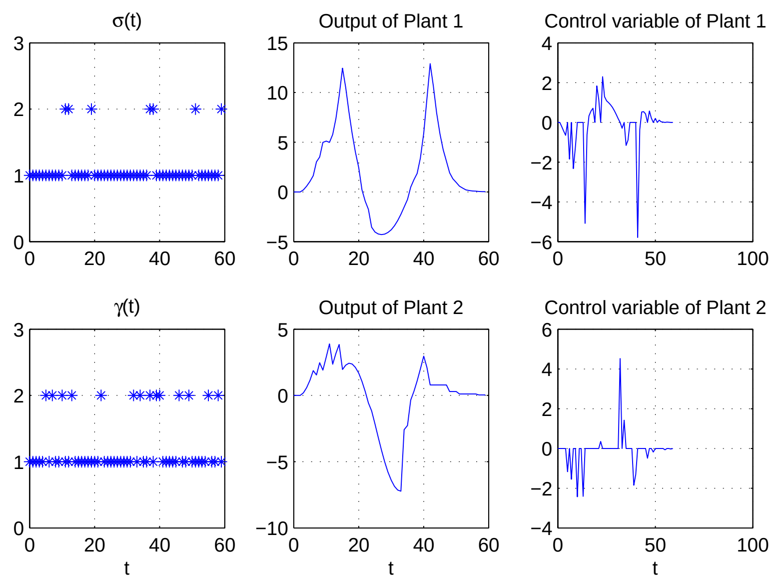

Figure 4: Results of the example with random scheduling and controller co-design technique of Section 5.2.

the system state is available.

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