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Selective mass scaling for multi-layer solid-shell discretization of thin-walled structures

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Abstract. *The computational burden of an explicit dynamic analysis of thin-walled structures discretized with solid-shell elements can be very high, since the stability condition leads to extremely low time steps because of the small thickness. A selective mass scaling procedure can be introduced to overcome this limitation. The technique proposed by Cocchetti et al. for single-layer 8-node solid-shell elements is here generalized to the case of multi-layer shells.*

Keywords: Explicit dynamics; selective mass-scaling; solid-shell elements, multi-layer.

1 INTRODUCTION

The use of solid-shell elements is particularly suitable for simulating the mechanical response of multi-layered shell structure, in particular when fracture and delamination phenomena are involved. The presence of different layers can be modelled employing one or more elements per layer through the thickness. In addition, they allow for the implementation of fully three-dimensional constitutive behaviours, being formulated in displacement degrees of freedom only. The main drawback is the high computational burden associated to the use of solid-shell elements in an explicit dynamic simulation framework. The Courant-Friedrichs-Lewy (CFL) condition leads, indeed, to extremely low values of the stable time step, since the thickness dimension is sensibly smaller than the in-plane ones. To overcome this issue, it is possible to introduce a mass scaling, modifying the solid-shell element mass matrix in order to artificially scaling down the highest structural eigenfrequencies, without significantly altering the lowest ones. Several techniques for selective mass scaling have been developed in recent years, see, for instance, [1], [2], [3]. In this paper, the selective mass scaling procedure, proposed in [4] and [5] and specifically conceived for solid-shell elements, is reconsidered for application to layered shells, where several solid-shell elements are used through the shell thickness. The adopted mass scaling leads to a critical time step size which is determined by the element in-plane dimensions only, independent of the layers number, with negligible accuracy loss, both in small and large displacement problems.

2 SELECTIVE MASS SCALING PROCEDURE

The reference solid shell element is the solid 8-node brick element shown in figure 1 and characterized by a thickness dimension significantly smaller than the in-plane size. The element middle surface is coloured in grey. Let us define the corner fibers as the segments connecting corresponding pairs of nodes belonging to the lower and upper surfaces and the corner nodes as the four nodes belonging to the middle surface of the element. As in [4] and [5], the starting point of the proposed procedure is the definition of a transformed set of degrees of freedom, corresponding to the variables related to the corner nodes and to the corner fibers. Let us introduce the vectors of classical and transformed nodal accelerations, $\mathbf{a}_e$ and $\widehat{\mathbf{a}}_e$:

$$\mathbf{a} = \begin{bmatrix} \mathbf{a}_{1-4} \\ \mathbf{a}_{5-8} \end{bmatrix}, \quad \widehat{\mathbf{a}} = \begin{bmatrix} \mathbf{a}^m \\ \Delta\mathbf{a} \end{bmatrix}, \tag{1}$$

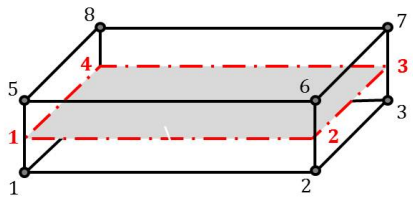

Figure 1: Eight-node solid shell element

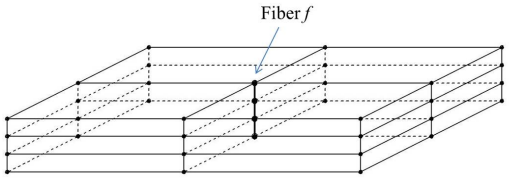

Figure 2: Fiber in a multi-layered shell structure

$\mathbf{a}_{1-4}$ and $\mathbf{a}_{5-8}$ represent the vectors gathering the accelerations of the nodes belonging to the lower and upper surface of the element, while $\mathbf{a}^m$ and $\Delta\mathbf{a}$ collect the corner nodes and the corner fibers dofs. A linear transformation can be introduced to map the original nodal accelerations into the transformed ones.

$$\mathbf{a}_e = \mathbf{Q}\widehat{\mathbf{a}}_e. \tag{2}$$

with:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{I}_{12} & -\mathbf{I}_{12} \\ \mathbf{I}_{12} & \mathbf{I}_{12} \end{bmatrix} \tag{3}$$

where $\mathbf{I}_{12}$ is the 12×12 identity matrix.
Let us consider the balance momentum equation written in variational form by the principle of virtual work:

$$\delta\mathbf{a}_e^T\mathbf{M}_e\mathbf{a}_e = \delta\mathbf{a}_e^T\mathbf{f}_e, \tag{4}$$

By introducing the transformation 3 into equation 4, it is possible to define a transformed element mass matrix $\widehat{\mathbf{M}}_e$.

$$\widehat{\mathbf{M}}_e = \mathbf{Q}^T\mathbf{M}_e\mathbf{Q} = \begin{bmatrix} \left(\mathbf{m}^{up} + \mathbf{m}^{low}\right) & \left(\mathbf{m}^{up} - \mathbf{m}^{low}\right) \\ \left(\mathbf{m}^{up} - \mathbf{m}^{low}\right) & \left(\mathbf{m}^{up} + \mathbf{m}^{low}\right) \end{bmatrix}_e \tag{5}$$

being $\mathbf{m}^{low}$ and $\mathbf{m}^{up}$ the diagonal matrices collecting the mass coefficients of the nodes belonging to the lower and upper surfaces, respectively. The off-diagonal terms are usually small if compared with the diagonals ones and can, thus, be neglected ([5]), obtaining a lumped form of the transformed mass matrix.
The dynamical response of an inertia dominated problem is mainly ruled by element translational rigid body modes, which correspond to the corner nodes dofs. The basic idea of the selective mass scaling technique relies on a local modification of the coefficients of the lumped transformed mass matrix. Only the coefficients corresponding to the corner fibers dofs are increaed, so that the inertia associated to the rigid body motion is left unaltered. Let us introduce the scaled transformed mass matrix $\widehat{\mathbf{M}}^{\alpha}_{e_{lumped}}$ as:

$$\widehat{\mathbf{M}}^{\alpha}_{e_{lumped}} = \begin{bmatrix} \left(\mathbf{m}^{up} + \mathbf{m}^{low}\right) & \mathbf{0} \\ \mathbf{0} & \alpha_e\left(\mathbf{m}^{up} + \mathbf{m}^{low}\right) \end{bmatrix}_e, \tag{6}$$

being α_e is the element mass scaling parameter. The weak form of the momentum balance equation of an undamped element becomes:

$$\delta\widehat{\mathbf{a}}_e^T\widehat{\mathbf{M}}^{\alpha}_{e_{lumped}}\widehat{\mathbf{a}}_e = \delta\widehat{\mathbf{a}}_e^T\widehat{\mathbf{f}}_e. \tag{7}$$

The choice of the optimal scaling parameter has been discussed in details in [5]: it is computed by showing that mass scaling is equivalent to a geometrical scaling and imposing that the geometrical scaling is such that the element size in the thickness direction becomes comparable to the in-plane dimension.

When a multi-lareyed shell problem is addressed, the fiber is a multilayer segment formed by the set of corner fibers of the solid-shell elements stacked up through the shell thickness at the same in-plane position (fig. 2). Consequently, it is necessary to introduce an assembly through the thickness to properly define the fiber degrees of freedom. Let us consider the inverse of the transformation, defined in 2.

$$\widehat{\mathbf{a}}_e = \mathbf{a}_e \mathbf{Q}^{-1} = \frac{1}{2}\mathbf{a}_e \mathbf{Q}^T \tag{8}$$

being,

$$\mathbf{Q}^{-1} = \frac{1}{2}\mathbf{Q}^T = \frac{1}{2}\begin{bmatrix} \mathbf{I}_{12} & \mathbf{I}_{12} \\ -\mathbf{I}_{12} & \mathbf{I}_{12} \end{bmatrix} \tag{9}$$

and introduce it into equation 7, in order to express the weak virtual balance equation in terms of nodal degrees of freedom.

$$\delta\mathbf{a}_e^T \underbrace{\mathbf{Q}\frac{1}{2}\widehat{\mathbf{M}}^{\alpha}_{e_{lumped}}\frac{1}{2}\mathbf{Q}^T}_{\mathbf{M}^{\alpha}_e}\mathbf{a}_e = \delta\mathbf{a}_e^T\mathbf{Q}^T\frac{1}{2}\widehat{\mathbf{f}}_e \tag{10}$$

The scaled element mass matrix of a generic solid shell element $\mathbf{M}^{\alpha}_e$ is, thus, defined as:

$$\mathbf{M}^{\alpha}_e = \frac{1}{4}\left(\mathbf{Q}\widehat{\mathbf{M}}^{\alpha}_{e_{lumped}}\mathbf{Q}^T\right) = \frac{1}{4}\begin{bmatrix} (1+\alpha_e)\left(\mathbf{m}^{low}+\mathbf{m}^{up}\right) & (1-\alpha_e)\left(\mathbf{m}^{low}+\mathbf{m}^{up}\right) \\ (1-\alpha_e)\left(\mathbf{m}^{low}+\mathbf{m}^{up}\right) & (1+\alpha_e)\left(\mathbf{m}^{low}+\mathbf{m}^{up}\right) \end{bmatrix} \tag{11}$$

Let us focus on a single fiber. The mass matrix of a single layer $\mathbf{M}^{\alpha}_l$ can be defined as the assembly of the mass matrices of the elements of the support of the fiber belonging to the same layer *l*:

$$\mathbf{M}^{\alpha}_l = \sum_e \mathbf{M}^{\alpha}_e = \begin{bmatrix} \mathbf{m}^{LL}_l & \mathbf{m}^{LU}_l \\ \mathbf{m}^{UL}_l & \mathbf{m}^{UU}_l \end{bmatrix} \tag{12}$$

being:

$$\mathbf{m}^{LL}_l = \mathbf{m}^{UU}_l = \sum_e (1+\alpha_e)\left(\mathbf{m}^{up}_e + \mathbf{m}^{low}_e\right) \tag{13a}$$

$$\mathbf{m}^{LU}_l = \mathbf{m}^{UL}_l = \sum_e (1-\alpha_e)\left(\mathbf{m}^{up}_e + \mathbf{m}^{low}_e\right) \tag{13b}$$

The overall solution can be, thus, computed simply solving a set of subsystems in the form:

$$\mathbf{M}^{\alpha}_f \mathbf{a}_f = \mathbf{F}_f \tag{14}$$

where the fiber mass matrix $\mathbf{M}^{\alpha}_f$ is built by assembling the mass matrices $\mathbf{M}^{\alpha}_l$ arising for each layer *l* along the fiber, namely:

$$\mathbf{M}^{\alpha}_f = \sum_{l=1}^{N_l} \mathbf{M}^{\alpha}_l \tag{15}$$

The resulting scaled fiber mass matrix $\mathbf{M}_f$ is a tridiagonal matrix, whose dimensions are directly related to the number of layers, which are in general in a limited number through the shell thickness. The global mass matrix $\mathbf{M}^{\alpha} = \sum_{f=1}^{N_f} \mathbf{M}^{\alpha}_f$ becomes a diagonal block matrix, each block corresponding to the degrees of freedom of a single fiber *f*. Even though accelerations cannot be computed explicitly, the solution of the small linear system providing accelerations of nodes belonging to the same fiber is inexpensive and the small additional burden is by far compensated by the largest stable time step which can be used in the computation.

The goal of the procedure is to scaling down the highest structural eigenfrequencies, so that the critical time step is determined only by the in-plane size of the elements, as with standard four-nodes shell meshes. Moreover, the resulting critical time step is independent of the number of layers used for the through-the-thickness discretization.

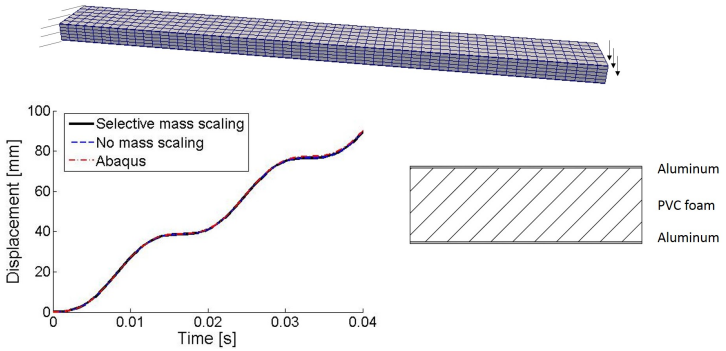

Figure 3: Sandwich beam: geometry, material layers, boundary conditions and tip displacement.

3 NUMERICAL EXAMPLE: SANDWICH CANTILEVER BEAM

The bending response of a cantilever sandwich beam is addressed in this example. As shown in figure 3, the beam is completely clamped at one side and loaded at the opposite one by a surface load linearly varying in time from 0 to 0.05 MPa. The beam length is equal to 600 mm, while its rectangular cross section is 60 mm wide and 20 mm deep. The sandwich beam is composed of two external aluminum thin face sheets (E=72400 MPa, ν=0.3, ρ = 2700 kg^3) and of a soft core of low-density PVC foam (E= 58 MPa, ν=0.33, ρ = 60 kg^3). Both aluminum layers are 0.5 mm high, while the thickness of the soft core is equal to 19 mm. Each aluminum layer has been discretized by only one solid-shell element through-the-thickness, while five solid-shell elements of equal thickness are stacked up to model the soft core. Two different values of the mass scaling parameter have to be defined since different materials and thicknesses are present: α = 400 and α = 6.93 has been considered for the aluminum and for the soft core respectively. The analysis is run for 0.04 s. The stable time step is equal to 1.126 μs or to 0.079 μs, depending on whether the selective mass scaling technique is applied or not. In figure 3, the vertical displacement of the loaded end of the beam computed with the proposed technique is compared both with the numerical result obtained without applying the selective mass scaling procedure and with Abaqus.

4 CONCLUSIONS

The proposed scaling reduces the highest eigenfrequencies, while the lower ones, associated to the rigid body translations, remain unaffected. As a result, when the dynamical behavior is governed by the lowest frequencies, the structural response is well reproduced. The adopted mass scaling leads to a critical time step size which is determined by the element in-plane dimensions only, independent of the layers number, with negligible accuracy loss, both in small and large displacement problems. The resulting scaled mass matrix is not perfectly diagonal. However, the introduced coupling is shown to be limited to the nodes belonging to the same fiber through the thickness, so that the additional computational burden is almost negligible and by far compensated by the larger size of the critical time step.

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