# Evaluation of the dynamic behaviour of steel staircases damped by the presence of people

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People acting on a slender structure can affect the dynamic behaviour of the structure they occupy, in addition to being a source of forcing. In such cases, the use of the dynamic properties of the empty struc-ture to estimate the structural response can lead to an erroneous estimation of the amplitudes of vibra-tion. This work proposes an approach to improve the prediction of the structural response due to the presence of people. The method is based on the identification of an equivalent set of frequency response functions to represent the dynamic behaviour of the joint structure-moving people system. The method starts from the modal model of the empty structure, i.e., natural frequencies, damping ratios and mode shapes. No restriction on the number of degrees of freedom of the structure is required. Each subject is modelled through an equivalent apparent mass and is introduced on the model of the empty structure to obtain a model of the joint structure-moving people system. An active force is then applied to the equivalent model to obtain a prediction of vibration levels. The effectiveness of the approach was verified through experimental tests performed in controlled conditions. Two lightly damped steel staircases were used as test cases. A comparison between the amplitudes of the measured vibration and those predicted using the proposed methodology is presented. The results show that the use of the empty structure model can lead to a high overestimation of the vibration amplitudes. Conversely, the results obtained with the proposed approach are in agreement with the experimental data.

*Keywords*
Human–structure interaction
Slender structure
Structure vibration
Damping
Staircase

## 1. Introduction

In recent years, problems related to in-service vibrations have gained growing attention [1]. Since brand new structures have become more and more slender, an increasing number of problems related to the unexpected high vibration amplitudes have been recorded. Reported problems have occurred in various types of structures, such as footbridges [2,3], football stadia [4,5] and long-cantilevered structures [6]. Indeed, the prediction of in-service vibration amplitudes of structures occupied by people is a complex task. In particular, at least two main critical challenges can be identified.

The first aspect regards the correct characterisation of the active forces induced by people on the structure. The majority of standards and codes suggests modelling human-induced forces as deterministic harmonic forces [7,8]. However, this assumption is too simplistic and does not reflect the real trend of human-induced forces. This problem was addressed in many works, such as Refs. [9,10], and approaches to correctly identify such forces were proposed [11–16].

The second aspect regards the influence of people on the dynamic properties of the structure. Indeed, it is well known that people acting on a pedestrian structure behave as dynamical systems capable of modifying the dynamics of the structure itself [16]. In the literature, few attempts have been made to include the effects of people. Particularly, a paper by Qin et al. [17] faced the problem of a pedestrian-bridge interaction using a bipedal walking model. The proposed method consisted of a feedback con-trol force applied by the pedestrian. The results of the numerical study showed that the effect of people increased with the ampli-tude of vibration. However, despite an increase of damping ratios due to the presence of people, the results showed an increase of predicted amplitudes of vibration using the model of Human–Structure Interaction (HSI), in contrast to experimental evidence. A work by Pavic and Reynolds [18] proposed the use of a three degrees-of-freedom (DOF) model to represent the dynamics of a structure occupied by passive and active subjects. In the proposed model, the three DOF represented the structure, the passive crowd and the active crowd. The model was used to predict the response

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of a stadium grandstand with good results. Another work by Shahabpoor et al. [19] proposed the use of a mass-spring-damper (MSD) model of the human body to predict the effect of walking pedestrians on the dynamic properties of a structure. In their work, the authors reported a theoretical analysis of the proposed approach. However, as evidenced by the authors, experimental data were required to validate the methodology. A common assumption and limitation of the last two above-mentioned approaches is the modelling of the structure and the people as single DOF (SDOF) systems.

Although experimental evidence suggests that appropriate dynamic models of human occupants should be used to obtain an accurate description of HSI, at the design stage, it is common practice to consider people interacting with a structure as only a source of force [20–22]. However, this approach may lead to an erroneous estimation of the vibration levels at the design stage for cases where the influence of people is not negligible. To over-come this issue, a recent guidance [23] (Joint Working Group, 2008), regarding the dynamic performance requirements for per-manent grandstands subjected to crowd action, recommends the consideration of HSI at the design stage. Indeed, the guidance underlines that if the effects due to HSI are ignored in calculations, the response of the structure will be incorrectly represented in the analysis. In Ref. [23], an analytical method for treating human structure interactions is proposed. This approach was developed using the most recent research and available experimental data, with the aim of reproducing the patterns of behaviour observed in actual structures subjected to dynamic crowd loading. However, as evidenced in the guidance, the method is not able to address all the variations in human behaviour and physical characteristics that affect the structural dynamics. When brand new structures fail the vibration serviceability check, an a-posteriori mitigation of the vibration amplitudes is often required [24,25]. Thus, many types of solutions have been proposed in the past years [26,27]. However, such solutions imply additional costs. A better knowl-edge of the effect of people on the dynamic behaviour of structures would allow a more accurate evaluation of the vibration ampli-tudes at the design stage [28]. As a consequence, the cost and effort to mitigate vibration amplitudes a-posteriori could be avoided or at least reduced. In this context, there are still grounds for general methods to account for the presence of people.

This work focuses on the analysis of vertical vibrations of a slender structure and proposes an approach to predict such vibrations. Regarding vertical vibrations, experimental evidence suggests that people interacting on a structure is a source of added damping [29–32]. If the damping ratios change significantly, considering the dynamic properties of the empty structure for estimating the structural response can lead to a high overestimation of the amplitudes of vibration [32]. Therefore, this work validates an approach to include the effect of people on the dynamic behaviour of a structure in terms of changes in the modal parameters. In the proposed method, no restriction on the number of DOFs of the structure and of people's model is required. This would allow for obtaining a reliable prediction of the structural response even in the case of a high modal density. The proposed method is a generalisation of a recent work [33] proposing an approach to evaluate the influence of passive people on the dynamic behaviour of a structure. The present work proposes an extension of this approach to the case of moving people. The idea behind such an extension is the identification of an equivalent model to represent the dynamic behaviour of the joint structure-moving people system. An appropriate active force is then applied to this equivalent model to obtain a prediction of vibration levels.

The key aspects of the methodology proposed in [33] are revised in Section 2, and its extension to the case of moving people is proposed. Sections 3 and 4 explain the calculations of passive ground reaction forces and active ground reaction forces, respectively. The ground reaction forces are treated in the literature as the total forces exchanged between the structure and the people, and in these two sections the split into passive and active forces is explained. Section 5 details the whole procedure used to predict the vibration of the structure with the theoretical approach described in Section 2. Finally, Section 6 discusses the experiments carried out to validate the method proposed herein.

All the performed tests were carried out in accordance with The Code of Ethics of the World Medical Association (Declaration of Helsinki) for experiments involving humans.

## 2. An approach to evaluate structural responses due to the presence of people

A recent work [33] proposed a model to evaluate the effect of passive people on the dynamic behaviour of a slender structure.

The method suggested in Ref. [33] requires the knowledge of the dynamic behaviour of the empty structure. This is represented by the matrix of Frequency Response Functions (FRFs), $\mathbf{G}(\omega)$ (Eq. (1)):

$$\mathbf{G}(\omega) = \sum_{k=1}^{n} \frac{\boldsymbol{\phi}_k \boldsymbol{\phi}_k^T}{\omega_k^2 - \omega^2 + 2j\zeta_k \omega \omega_k} \tag{1}$$

where $\boldsymbol{\phi}_k$ is the $j$th mode shape vector (scaled to the unit modal mass) measured/evaluated at discrete points, $\omega_k$ is the natural frequency of the $k$th mode, $\zeta_k$ is the $k$th non-dimensional damping ratio and $n$ is the (arbitrary) number of considered modes; $T$ indicates the transpose, $j$ is the imaginary unit and $\omega$ is the circular frequency. Because the eigenvectors are known at discrete ($m$) points, the matrix $\mathbf{G}(\omega)$ is the $m \times m$ matrix containing the Frequency Response Functions (FRFs) for these points.

If $\mathbf{x}(\omega) = [x_1(\omega), \ldots, x_m(\omega)]^T$ is a vector which contains the responses of the structure in the considered points, and $\mathbf{f}(\omega) = [f_1(\omega), \ldots, f_m(\omega)]^T$ is a vector containing all the external forces applied to the structure, the structural response can be expressed in Eq. (2) as:

$$\mathbf{x}(\omega) = \mathbf{G}(\omega)\mathbf{f}(\omega) \tag{2}$$

When a person is in contact with a point of a structure, he/she produces a Ground Reaction Force (GRF). The GRF is the total force exchanged between the person and the structure. If we consider a passive person, the GRF is just due to the dynamic characteristics of the person and the structure. This force is termed as the Passive Ground Reaction Force (PGRF) and will be indicated by the symbol $f^{GR}$. The PGRF is a force which is generated by structural movement; when an external force $f$ acts on the structure, this vibrates and excites the person. If we consider a person to be a dynamic system, it starts to vibrate as well. The consequence is that a force (i.e., PGRF) is exerted by the person on the structure. Fig. 1a shows the situation related to the case mentioned above; the structure and the person are described as SDOF systems to simplify the sketch, and both can be described by multi-DOF systems. Fig. 1b shows the GRF, which is a PGRF in this case. On the other hand, if we con-sider a moving person (active people), the GRF can be split into two components, i.e., GRF = PGRF + AGRF (where AGRF is the Active Ground Reaction Force). The PGRF is due to the dynamic character-istics of the person and the structure, while the AGRF is due to the active force generated by the person's active movement. The AGRFs do not depend on (and are not generated by) the vibration of the structure behind the person and are only due to the active move-ment of the person. We can see AGRF as the force exerted by a mov-ing person on a structure with an infinite stiffness (i.e., $\mathbf{x} = 0$, which corresponds to a null PGRF). Fig. 1c shows an example scheme of

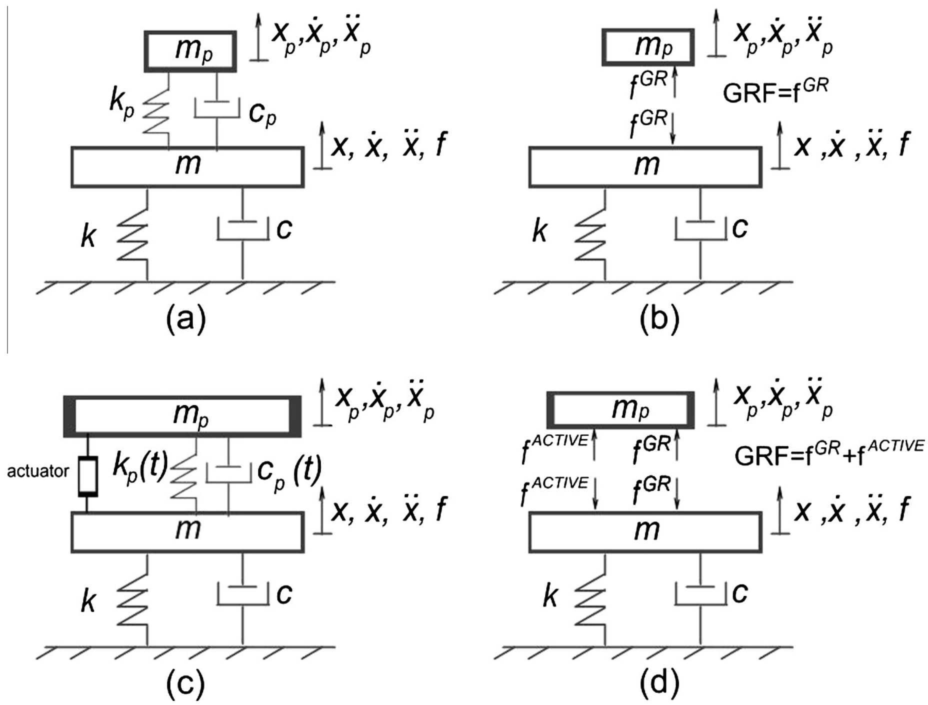


**Fig. 1.** Joint human–structure system: passive people + structure (a) and corresponding forces (b), moving people + structure (c) and corresponding forces (d). The person is seen as a SDOF system with parameters $m_p$, $c_p$ and $k_p$; the structure is seen as a SDOF system with parameters $m$, $c$ and $k$. Furthermore, an actuator is added to the scheme of the person to take into account of the AGRFs.

this situation, where the AGRF is described by an active actuator (which produces $f^{ACTIVE}$). It is noticed that in this case the dynamic properties of the person are time dependent because the person changes his/her position during the time period. Fig. 1d shows the corresponding force components.

The PGRF depends on the dynamic characteristics of the person and on the motion of the structure, as mentioned before. The dynamic features of the person are taken into account by describing his/her frequency behaviour by means of his/her apparent mass curve $M^*(\omega)$, which is the point frequency response function between the acceleration at the contact point and the force $f^{GR}(\omega)$.

Thus, the apparent mass, defined as $M^*(\omega) = \frac{f^{GR}(\omega)}{\ddot{x}(\omega)}$, describes the relationship between the acceleration at the contact point and the PGRF. It is noticed that $M^*(\omega)$ is a complex function defined in frequency.

Accordingly, the GRF ($f_i^{GR}(\omega)$) of each passive subject connected to the $i$th point of the structure can be expressed in terms of the apparent mass $M^*(\omega)$, as in Eq. (3):

$$f_i^{GR}(\omega) = M^*(\omega)\ddot{x}_i(\omega) = -M^*(\omega)\omega^2 x_i(\omega) = H(\omega)x_i(\omega)$$
$$\Rightarrow H(\omega) = \frac{f_i^{GR}(\omega)}{x_i(\omega)} \quad (3)$$

If $\mathbf{w_i}$ identifies the connection of the person to the structure, as shown in Fig. 2, then the PGRFs can be expressed in terms of the full displacement vector $\mathbf{x}(\omega)$ as in Eq. (4):

$$\mathbf{f}^{GR}(\omega) = H(\omega)\mathbf{w}_i\mathbf{w}_i^T\mathbf{x}(\omega) \quad (4)$$

In the case of $m$ people on the structure, the PGRFs can be expressed as:

$$\mathbf{f}^{GR}(\omega) = \mathbf{WHW}^T\mathbf{x}(\omega) \quad (5)$$

where $\mathbf{W} = [\mathbf{w}_1, \ldots, \mathbf{w}_m]$ represents the connection of $m$ subjects, and $\mathbf{H}$ is the (diagonal) frequency response function matrix, ontaining the $H_i(\omega)$ FRFs of each subject.

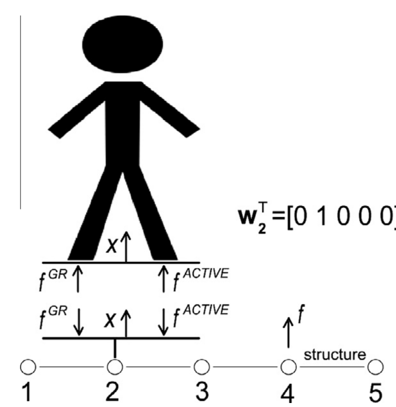


**Fig. 2.** Connection of a subject to the structure.

In addition to the PGRFs, people moving on a structure also exert an active force (AGRF) on the structure. Thus, Eq. (2) becomes (refer to Fig. 2 for the force directions):

$$\mathbf{x}(\omega) = \mathbf{G}(\omega)\left(-f^{ACTIVE}(\omega) - \mathbf{f}^{GR}(\omega) + \mathbf{f}(\omega)\right) \quad (6)$$

Combining Eqs. (6) and (5), (7) is obtained:

$$[\mathbf{G}^{-1}(\omega) + \mathbf{WHW}^T]\mathbf{x}(\omega) = \mathbf{G}_H^{-1}(\omega)\mathbf{x}(\omega) = -f^{ACTIVE}(\omega) + \mathbf{f}(\omega) \quad (7)$$

In Eq. (7) the modified matrix of FRFs $\mathbf{G}_H(\omega)$ includes the passive GRFs, while the vector $f^{ACTIVE}(\omega)$ includes all the active forces exerted by people on the structure.

Therefore, the problem of estimating the response of a structure occupied by moving people is decoupled to two main tasks, i.e., the identification of the PGRFs of moving people and their AGRFs.

In summary, the steps to obtain an estimation of the structural response of a structure due to the presence of people are:

I. find an equivalent transfer function $\mathbf{G}_H(\omega)$ to represent the dynamic behaviour of the joint structure-moving people system (named H-S system in the following);
II. identify the AGRFs induced by moving people on the structure;
III. apply the AGRFs (Eq. (7)) and the possible external forces $\mathbf{f}$, if they are significant, to estimate the structural response.

Contrary to passive people, the PGRF of a moving person significantly changes with time because the posture changes. The proposed approach assumes that average PGRFs can be used to identify a single matrix of FRFs $\mathbf{G}_H(\omega)$. This is a reasonable assumption as the number of people increases. Although this is an approximate approach, it will be shown to provide results much more reliable than those achieved using an empty structure model to predict structural vibrations. Nevertheless, a significant amount of effort is necessary to identify a correct equivalent model of PGRFs to represent human behaviour during motion.

The next section explains how to estimate this equivalent model of PGRFs.

## 3. Estimation of passive ground reaction forces

As explained in Section 2, an equivalent matrix of FRFs $\mathbf{G}_H(\omega)$ has to be identified to apply the proposed approach. $\mathbf{G}_H(\omega)$ represents the dynamic behaviour of the H-S system and accounts for the PGRFs at the contact points. Therefore, a general approach to estimate equivalent PGRFs and an equivalent matrix of FRFs $\mathbf{G}_H(\omega)$ is proposed in this section.

As outlined in Section 2, the PGRF can be expressed in terms of the apparent mass $M^*(\omega)$ (Eq. (3)). The apparent mass depends on many factors, such as (a) the particular subject (inter-subject variability), (b) his posture, and (c) the amplitude of vibration. Papers related to the topic, such as Refs. [34,35], analysed such influences. Particularly:

(a) the posture was found to have a high influence on the apparent mass;
(b) as for the inter-subject variability, the apparent mass depended on the characteristics of the particular person. However, for practical applications it was reasonable to assume that the average behaviour of a high number of people was properly modelled using average values of apparent mass;
(c) the vibration magnitude was found to have a smaller relevance, as also shown in Ref. [33] for the case of passive people.

Ultimately, for the purpose of this work, the parameter mostly affecting the apparent mass is the posture. Regarding passive people, it is likely to assume that the posture does not change with time. This assumption is definitely not true for the case of moving people. To address this case, a possible approach is proposed, and the steps are outlined as follows:

I. identify one cycle $T$ (i.e., the time elapsing between two touches on the ground of the same foot) of the particular motion;
II. divide the cycle to an appropriate number of postures, $P$. These postures must be representative of the overall behaviour during motion;
III. identify an average apparent mass $M^*_{a,i}(\omega)$ (with $i = 1, 2, \ldots, P$) for each posture. $M^*_{a,i}(\omega)$ is obtained by averaging the apparent mass curves of different people in the same posture. The apparent mass curve of each person and for each posture is measured using an electro-dynamic shaker as described in Ref. [33];
IV. define an equivalent apparent mass $M^*_{eq}(\omega)$ as the weighted average of the apparent masses $M^*_{a,i}(\omega)$: $M^*_{eq}(\omega) = \sum_{i=1}^{P}\alpha_i M^*_{a,i}(\omega)$ (where the $\alpha_i$ coefficients are the weights: $0 \leqslant \alpha_i \leqslant 1$ and $\sum_{i=1}^{P}\alpha_i = 1$).

In other words, we freeze $P$ postures within the movement cycle and treat each of them as a static posture. Then, we define $M^*_{eq}(\omega)$. The determined equivalent apparent mass $M^*_{eq}(\omega)$ is used to define the equivalent PGRF, as shown in Eq. (3), and subsequently the equivalent matrix of FRFs $\mathbf{G}_H(\omega)$ (Eq. (7)). The aim of this approach is to use a sort of mean apparent mass curve (i.e., $M^*_{eq}$), representative of the average behaviour of the human body during one cycle of his/her motion. Obviously, the number of postures $P$ considered must be as high as possible, to provide a dense description of the motion cycle. One aim of this paper is to check if the use of this equivalent apparent mass curve $M^*_{eq}$ provides reliable results because this is a strong assumption.

As for the $M^*_{a,i}(\omega)$ of each posture, these average frequency functions can be estimated by averaging the frequency functions measured for different people. As for the coefficients $\alpha_i$, they are chosen to properly describe the amount of time spent by people in the corresponding posture within the cycle time $T$. Indeed, we can assume that we are sampling the motion, and each posture is considered to be maintained by the person for a given amount of time $\Delta t$ (of course, this is an approximated way to see the method, and $\Delta t$ must not be considered as a fixed sampling frequency).

The values of the coefficients $\alpha_i$ can be different for each person because each human being moves differently from the others. We tested the influence of changes of the values of the coefficients $\alpha_i$ for the case of people walking on the spot. This case was chosen because this type of movement allows the use of data already available in the literature. Indeed, if we split the motion into three postures (i.e., $P = 3$): standing, one leg (left) and one leg (right) (the "one leg" position is described in Ref. [34] as standing on a straight leg with comfortable and upright upper-body), the corresponding $M^*_{a,i}(\omega)$ can be found in the literature [34]. Actually, the referenced work gives the expression of $M^*_{a,i}(\omega)$ normalised over the mass of the person. Hence, the $M^*_{a,i}(\omega)$ curves are achieved by multiplying the curves given in Ref. [34] for the mass of the person considered.

The equivalent apparent mass is obtained as in Eq. (8)

$$M^*_{eq}(\omega) = \alpha_1 M^*_{a,standing} + \alpha_2 M^*_{a,one\ leg(left)} + \alpha_3 M^*_{a,one\ leg(right)} \tag{8}$$

During one cycle, each subject was assumed to stand on two legs for an amount of time between $0.05{\cdot}T$ and $0.15{\cdot}T$ (estimated by means of an image analysis of people walking on the spot) and on one leg (left or right) for the remaining time. The coefficients $\alpha_i$ were varied accordingly to investigate their influence on the results. We calculated $M^*_{eq}(\omega)$ for different cases: (1) $\alpha_1 = 0$, $\alpha_2 = \alpha_3 = 0.5$; (2) $\alpha_1 = 0.1$, $\alpha_2 = \alpha_3 = 0.45$; (3) $\alpha_1 = 0.2$, $\alpha_2 = \alpha_3 = 0.4$.

Fig. 3 reports the yielded curves (considering a person of 80 kg) and compares them with the curve achieved for a standing person. The curves related to the three configurations mentioned are similar to each other but very different from the curve obtained for a standing person. This result means that a very accurate choice of the $\alpha_i$ coefficients is not needed, and a rough estimation is enough.

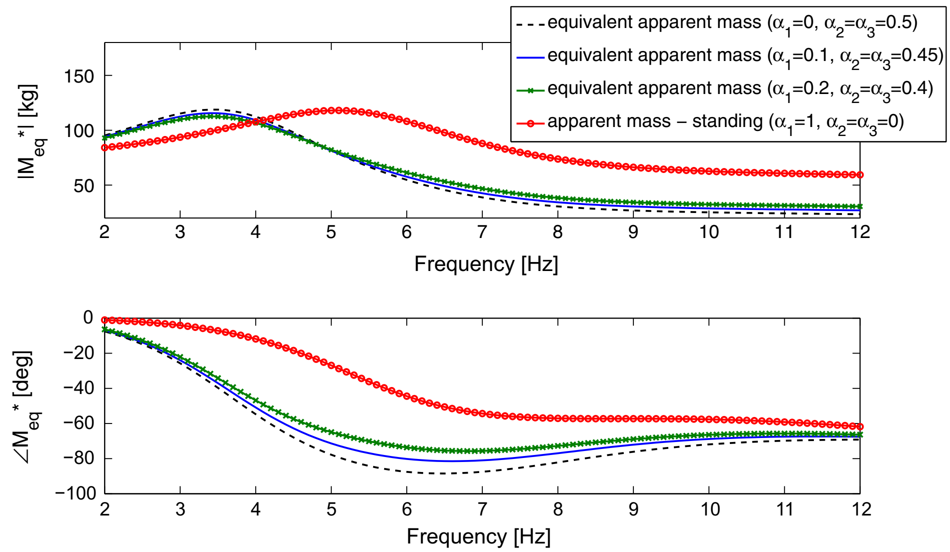


**Fig. 3.** Equivalent apparent masses for a person walking on the spot compared to the apparent mass of a person standing (the curve with $\alpha_1 = 1, \alpha_2 = \alpha_3 = 0$).

## 4. Estimation of active ground reaction forces

The estimation of the AGRFs is a key point to predict the vibration of a structure occupied by moving people. We consider the case of people walking on staircases, as mentioned previously. Therefore, we need to measure or model the AGRFs exerted by people when ascending or descending the steps. Although some data are already available in the literature (e.g., Ref. [12]), we built a dedicated test set-up to collect time-histories of AGRFs and build a database of possible force signals.

A force plate and two accelerometers were used to perform the experimental tests. The force plate was located on a wooden auxiliary step at the end of a staircase (Fig. 4). A second wooden auxiliary step was placed after the force plate. The two accelerometers were placed on each of the two auxiliary steps.

The acquired data were used to obtain a characterisation of the AGRFs exerted by people when ascending and descending stairs. The wooden steps and the dynamometric plate were designed so that the vibration of the plate was almost negligible for many tens of Hertz. This means that the force measured by the plate is just due to AGRFs, and PGRFs are avoided (see Section 2).

A total of 26 people were involved in the tests. Each person was asked to ascend and descend the step three times with the right foot and three times with the left foot. An overall amount of 312 force time-histories was measured. Fig. 5 reports the 12 force time-histories exerted by one test subject when ascending and descending the step with the left and right foot, as an example.

In addition, the collected data were also used to determine the step frequencies. For this purpose, the cross-correlation functions of the accelerations measured in the two subsequent steps were used. Fig. 6 shows the estimated step frequencies.

The recorded time histories and the estimated step frequencies were used to simulate the structural response. The simulation procedure and the results are discussed in the next sections.

## 5. Application of the model to the case of operating conditions

Section 2 explained that the model presented in this paper aims at foreseeing the dynamic response of a structure occupied by moving people by estimating the PGRFs and AGRFs. Subsequently, Section 3 showed how to estimate the PGRFs of moving people, and Section 4 explained how data on the AGRFs of people moving (ascending or descending on a staircase in the case considered; nonetheless, the same can be applied to different structures and different movements) were stored. Now, the time-response of a structure occupied by moving people is simulated to predict its level of vibration. We start by taking into account of the PGRFs (see Section 5.1) and then consider the AGRFs (see Section 5.2).

### *5.1. Passive ground reaction forces*

The PGRF due to each person can be calculated using $M^*_{eq}(\omega)$ in Eq. (3). There are no ways to know in advance the people occupy-

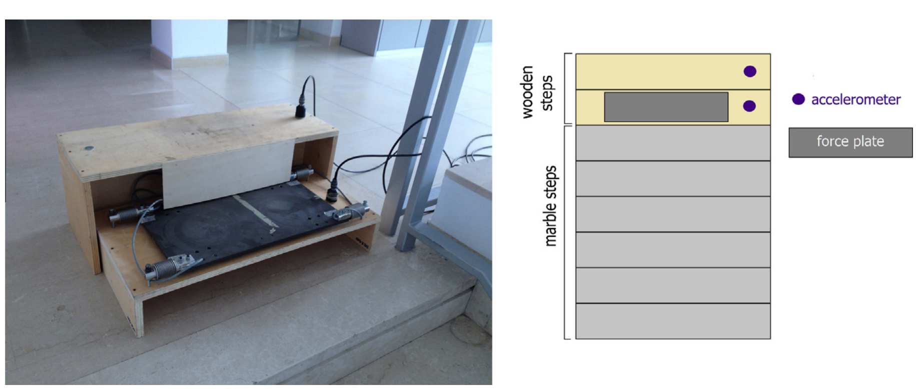


**Fig. 4.** Experimental set-up to measure the AGRFs.

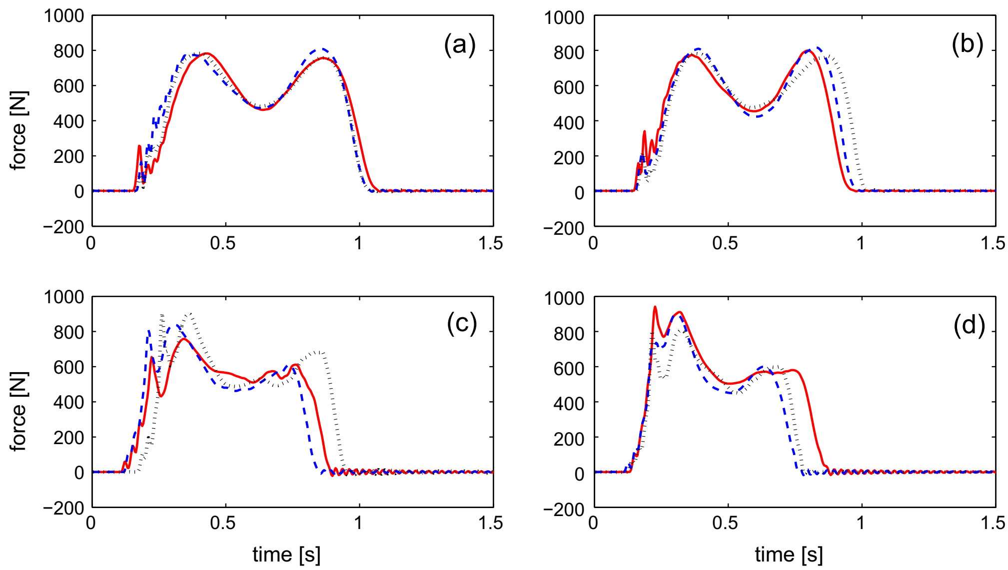


**Fig. 5.** Example of recorded forces: ascending – right leg (a), ascending – left leg (b), descending – right leg (c), descending – left leg (d).

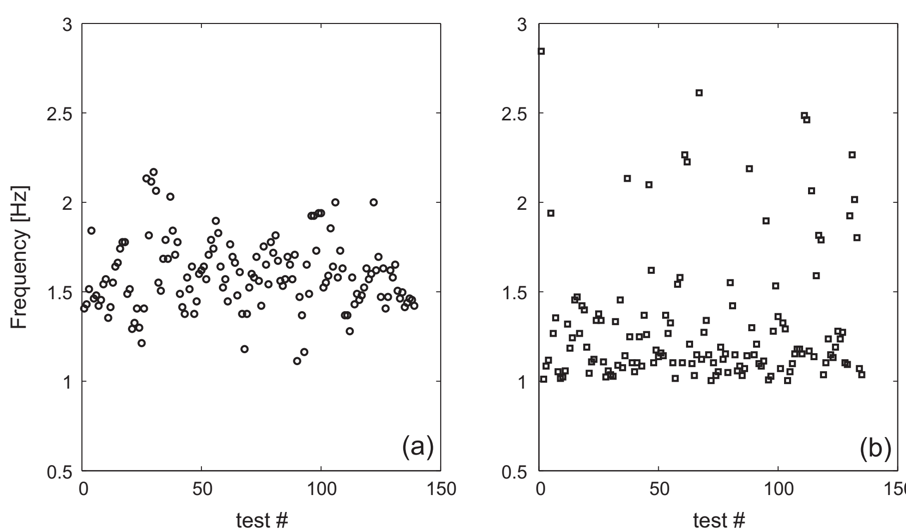


**Fig. 6.** Step frequency for some of the tests: ascending (a) and descending (b).

ing the structure, and thus average data must be used. Then, the PGRFs can be applied to the structure in two different ways:

I. each $f_i^{GR}(\omega)$ (see Eq. (3), one for each person on the structure) is considered as a moving excitement. Hence, the equivalent matrix $\mathbf{G}_H(\omega)$ changes with time because the matrix **W** in Eq. (7) changes with time;

II. we compute a fraction of the apparent mass $m^*_{fr}(\omega)$:

$$m^*_{fr}(\omega) = \frac{m}{n_d} M^*_{eq}(\omega) \tag{9}$$

where $m$ is the number of people on the structure, and $n_d$ is the number of points in which the structure is discretised. Then, $m^*_{fr}(\omega)$ is applied to each of the $n_d$ points. Thus, the PGRF in each point can be expressed as:

$$f_i^{GR}(\omega) = m^*_{fr}(\omega)\ddot{x}_i(\omega) = -m^*_{fr}(\omega)\omega^2 x_i(\omega) \tag{10}$$

In terms of the full displacement vector $\mathbf{x}(\boldsymbol{\omega})$, Eq. (10) becomes:

$$\mathbf{f}^{GR}(\omega) = \mathbf{W_n H W_n^{\mathit{T}} x}(\omega) = -\omega^2 m^*_{fr}(\omega)\mathbf{W_n x}(\omega) \tag{11}$$

where $\mathbf{W_n}$ is a $n_d \times n_d$ identity matrix, and $\mathbf{H}(\omega)$ is a $n_d \times n_d$ diagonal matrix containing the fractions of the equivalent apparent mass (i.e., $\mathbf{H}(\omega) = -\mathbf{W_n}\omega^2 m^*_{fr}(\omega)$). If Eq. (11) is substituted into Eqs. (6), (12) is obtained (neglecting **f**):

$$[\mathbf{G}^{-1}(\omega) + \omega^2 m^*_{fr}(\omega)\mathbf{W_n}]\mathbf{x}(\omega) = \mathbf{G}_H^{-1}(\omega)\mathbf{x}(\omega) = -\mathrm{f}^{ACTIVE}(\omega) \tag{12}$$

where $\mathbf{G}(\omega)$ is the $n_d \times n_d$ matrix containing the FRFs of the empty structure, and $\mathbf{G}_H(\omega)$ is the $n_d \times n_d$ matrix representing the equivalent set of FRFs representing the dynamic behaviour of the H-S system. Obviously, the behaviour of this joint system is an average behaviour due to the use of the variable $m^*_{fr}$.

The second approach of the previous numbered list assumes a fixed form of $\mathbf{G}_H(\omega)$ with time. Therefore, this assumption makes the simulation of the structure response easier and faster under a computational point of view. Furthermore, when the number of people on the structure increases, the accuracy of this simplified approach is expected to increase as well. We will refer to this easy-to-apply approach when we show the experimental validation of the method proposed in this paper (see Section 6).

### 5.2. Active ground reaction forces

After the matrix $\mathbf{G}_H(\omega)$ is calculated (see Section 5.1), the AGRFs must be applied to the modified system (see Eq. (12)). In predicting the response of the structure, we do not know how many people will occupy the structure, the speed of their walk, etc. Therefore, a statistical approach is needed to estimate the structural response. The database of AGRFs collected and shown in Section 4 can be used for this purpose. The statistical procedure requires at first which scenario is to be simulated. This means that the number of people $m$ on the structure must be fixed, as well as the time-length of the simulation. Then, the following procedure is carried out for each of the $m$ people:

I. one subject from the available database (26 subjects, see Section 4) is chosen randomly from a uniform distribution;
II. the starting point (1 to $n_d$, Fig. 7 shows an example of a staircase where $n_d = 78$; this staircase is one of the two considered in Section 6) is chosen randomly from a uniform distribution;
III. the initial foot (i.e., left or right) is chosen randomly;
IV. the direction (i.e., ascending or descending) is chosen randomly. Actually, in this case we decided to link this choice to the result of the extraction of the starting point (see point II of this list): descending if $1 \leqslant$ starting point $\leqslant n_d/2$ and ascending if $(n_d/2) + 1 \leqslant$ starting point $\leqslant n_d$;
V. we have 3 measured footsteps to choose at this point. For each person, we recorded 12 footsteps: 3 ascending-right foot, 3 ascending-left foot, 3 descending-right foot and 3 descending-left foot. We can now choose among 3 measured footsteps because we have already extracted the person, the foot and the direction. The footstep is chosen randomly among the three available. Then, the force signal corresponding to this footstep is applied to the starting step;
VI. the subsequent iteration is applied to the subsequent step ($\pm 1$ depending on the direction), and the foot is changed. Again, 3 footsteps are available and one of them is extracted randomly and applied to the step. The time elapsed between the application of two subsequent steps is chosen according to a random extraction of the step frequencies reported in Fig. 6;
VII. when the person reaches the step number $n_d/2$ or $n_d$, the direction is changed.

This procedure is applied to each of the $m$ people, taking care that each person does not superimpose another person on the same step. The procedure to generate the AGRFs for one person is exemplified in Fig. 8. In this example, the starting step is 45 (refer to Fig. 7).

When the time-histories of the AGRFs are generated for each person and for the whole time-length of the simulation, these AGRFs are applied to the FRFs of the H-S system (i.e., $\mathbf{G}_H(\omega)$). The simulation is carried out in the time domain. The Impulse Response Functions (IRFs) for all the $n_d$ points of the structure are calculated starting from the FRFs in the matrix $\mathbf{G}_H(\omega)$. Then, the structural response is computed by convolving the IRFs with the AGRFs.

The procedure explained so far is based on some random extractions. Hence, the same procedure must be repeated many times (100 in this paper) to achieve a good statistical reliability for the results. To simulate the structural response statistically, each of the 100 simulations required approximately 60 s on a normal laptop for the structures considered.

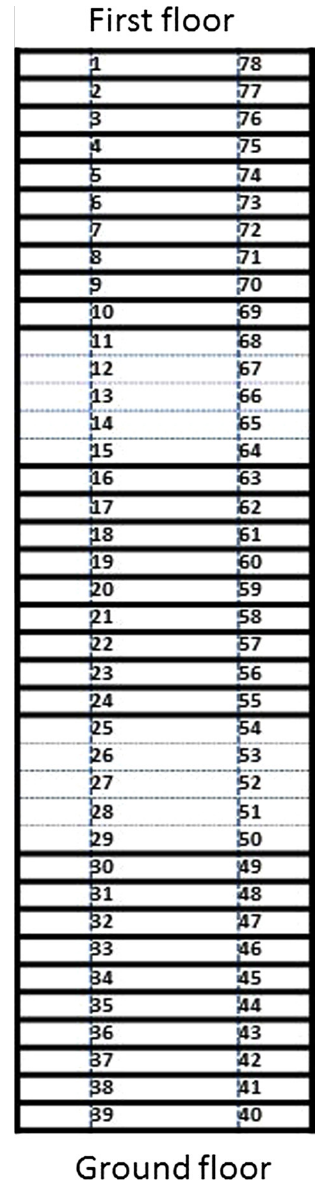


**Fig. 7.** Discretisation of a structure.

This section explained how to predict the vibration of a structure occupied by moving people. The next section shows the experimental tests carried out to validate both the model and the procedure described so far.

## 6. Experimental tests and model validation

Two test-case structures were used to validate the proposed approach (STRUCT1 and STRUCT2). STRUCT1 and STRUCT2 are staircases (Fig. 9) connecting the ground and the first floors in the main building of the campus Bovisa of Politecnico di Milano and the ground and the basement floors of building U2 at Univer-sità degli Studi Milano-Bicocca, respectively. Details on the two structures are provided in Table 1.

A modal characterisation of the structures without people was first carried out. Accelerations were measured in 23 points in the vertical direction for STRUCT1 and in 24 points in the vertical direction for STRUCT2. The structures were forced by accelerating a known mass with an electro-mechanical shaker. The force exerted on the structure by the moving mass could be estimated

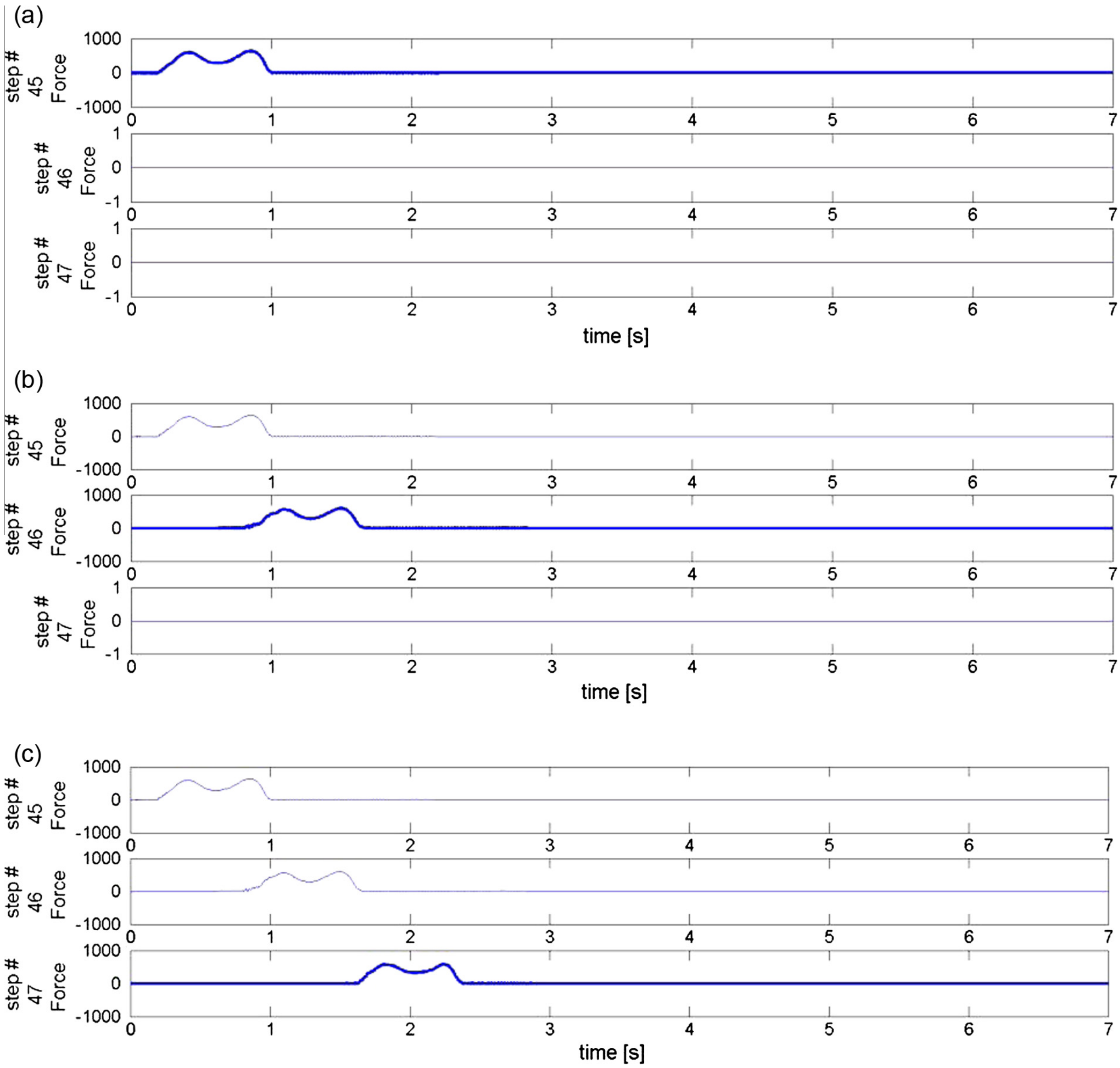


**Fig. 8.** Generation of AGRFs: iteration 1 (a), iteration 2 (b) and iteration 3 (c). The force on the vertical axis is in Newton.

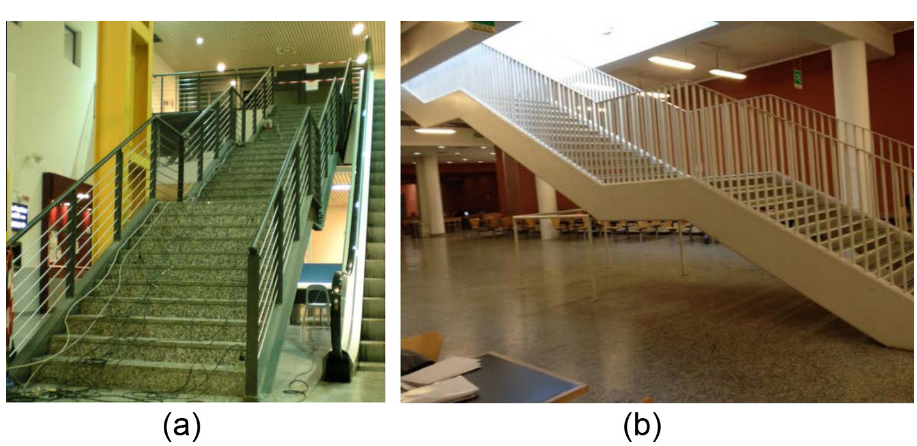


**Fig. 9.** Tested staircases: STRUCT1 (a) and STRUCT2 (b).

**Table 1**
Nominal data of STRUCT1 and STRUCT2.

| | Material | Length [m] | Width [m] | Height [m] |
|---|---|---|---|---|
| STRUCT1 | Steel and marble | 12.03 | 1.80 | 5.22 |
| STRUCT2 | Steel with marble just over the steps | 13.30 | 1.80 | 5.60 |

by multiplying the value of the mass and its acceleration. The acceleration of this mass was measured with an additional accelerometer placed on the mass itself. Fig. 10 shows the experimental set-up for STRUCT1.

The modal parameters were identified by means of Experimental Modal Analysis (EMA) techniques [36,37], and they are reported in Table 2. Only the modes up to approximately 15 Hz are taken into consideration because the apparent masses tend to have a null influence for higher frequency values (see Section 6.1). Fig. 11 shows an example of FRF of the empty structure (STRUCT1).

Furthermore, interpolation was applied to the identified eigenvector components to extend the information to other points of the structure. Several interpolation methods were used and robustness checks were performed to verify the appropriateness of the obtained mode shapes. The low-influence of the interpolation method was verified via simulations. Figs. 12 and 13 show the mode shapes associated with the two modes of STRUCT1 reported in Table 2.

Next, another series of tests was carried out. In this case, people were asked to walk on the structure, and the shaker was removed (previous tests showed that the presence of the shaker has negligible effects on the modal parameters of the structure in the frequency range of interest) so that the main excitation to the structure was induced by the moving people. The experimental setup described previously was changed to allow people to walk freely on the structure. Particularly, only the accelerometers placed at the two sides of the staircases were left. Two types of test were performed for STRUCT1: 3 subjects walking on the structure and 9 subjects walking on the structure. As for STRUCT2, the tests were performed with 5 and 10 people walking on the structure.

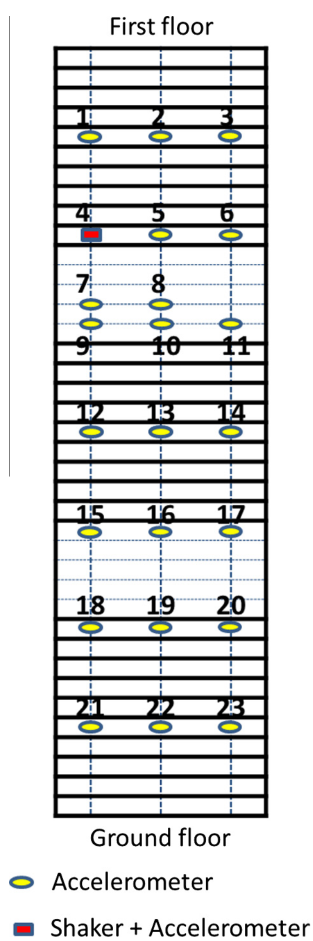


**Fig. 10.** Experimental set-up for STRUCT1.

Before presenting the experimental results and the comparison with the numerical predictions, a further task must be carried out. That is, the $M^*_{a,i}$ curves (i.e., the average apparent mass curves for the postures considered, see Section 3) for people walking on a staircase must be estimated. Hence, Section 6.1 discusses the tests to find these curves and their results.

### *6.1. Experimental identification of the average apparent mass curves*

To apply the proposed approach, the PGRFs exerted by people during motion must be identified. As outlined in Section 3, the $M^*_{a,i}$ curves must be modelled or measured to estimate $M^*_{eq}$ and subsequently the PGRFs (see Section 5.1). Therefore, an image analysis was performed to identify a given number of postures during one cycle of the ascending and descending motions. Fig. 14 shows an example of frames extracted from one test performed to investigate such motions.

The motion can be divided into an arbitrary number of postures (as shown in Fig. 14) which can be used to determine an equivalent apparent mass $M^*_{eq}$. The positions must be representative of the

**Table 2**
Modal parameters of the empty structures. $f_i$ are the eigenfrequencies, and $\zeta_i$ are the non-dimensional damping ratios.

| Modal parameter | STRUCT1 | STRUCT2 |
|---|---|---|
| Mode 1 – $f_1$ [Hz] | 7.84 | 6.70 |
| Mode 1 – $\zeta_1$ [%] | 0.33 | 0.33 |
| Mode 2 – $f_2$ [Hz] | 8.89 | 9.55 |
| Mode 2 – $\zeta_2$ [%] | 0.43 | 0.28 |
| Mode 3 – $f_3$ [Hz] | – | 10.75 |
| Mode 3 – $\zeta_3$ [%] | – | 0.29 |
| Mode 4 – $f_4$ [Hz] | – | 11.21 |
| Mode 4 – $\zeta_4$ [%] | – | 0.17 |

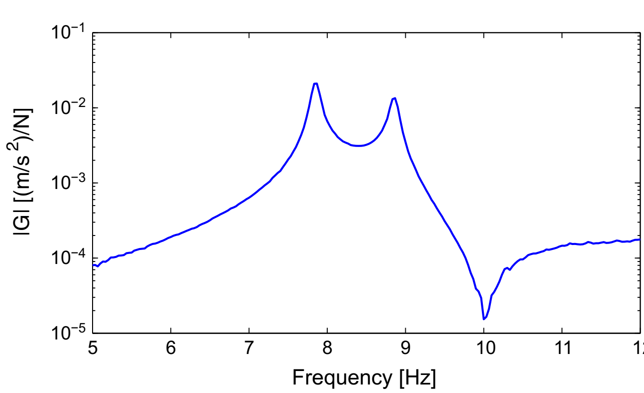


**Fig. 11.** FRF of the empty structure (STRUCT1).

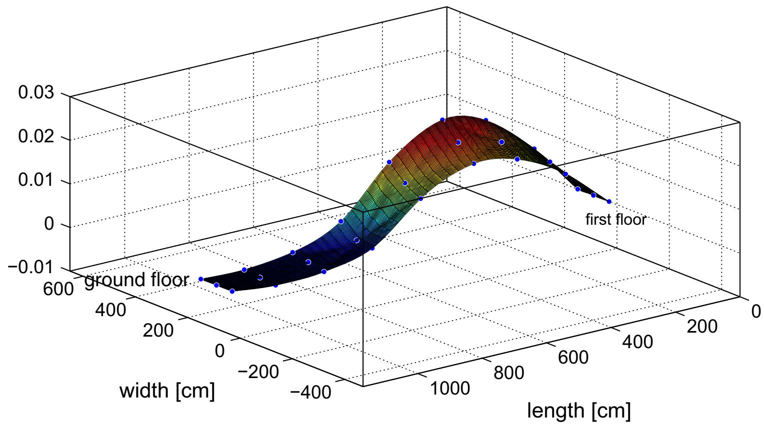


**Fig. 12.** Interpolated mode shape (Mode 1 of STRUCT1).

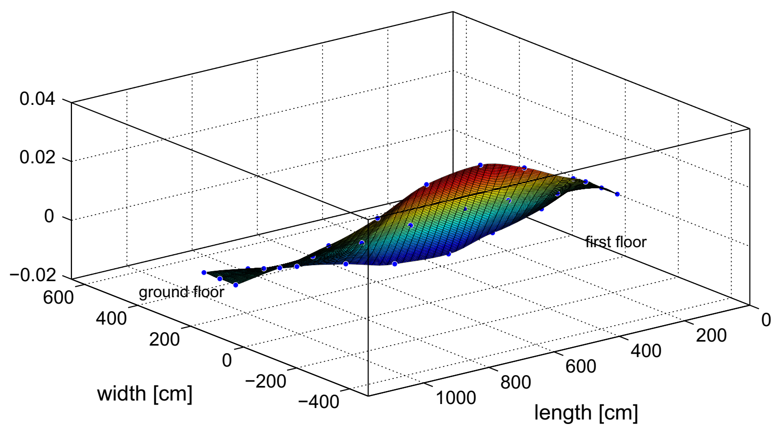


**Fig. 13.** Interpolated mode shape (Mode 2 of STRUCT1).

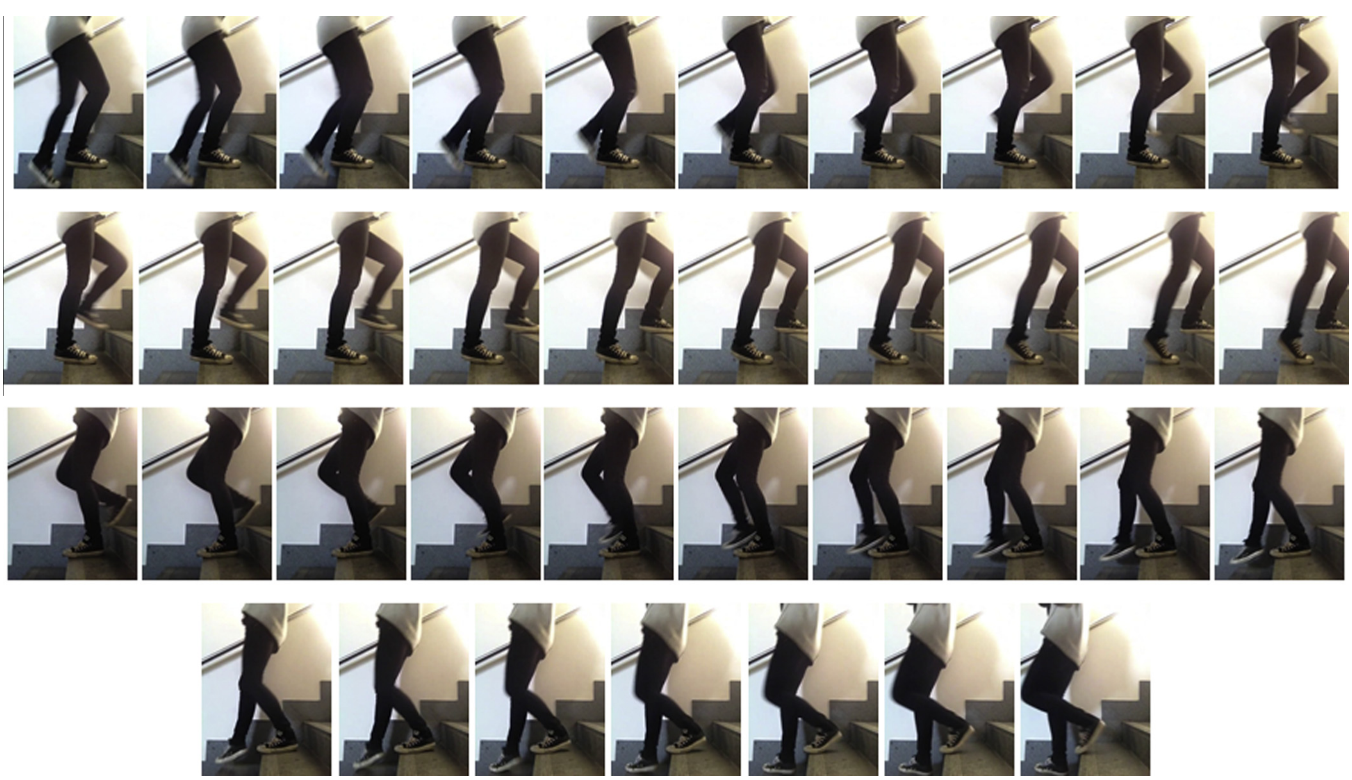

**Fig. 14.** Image analysis of the motion of a person ascending and descending stairs.


**Fig. 15.** Selected positions.

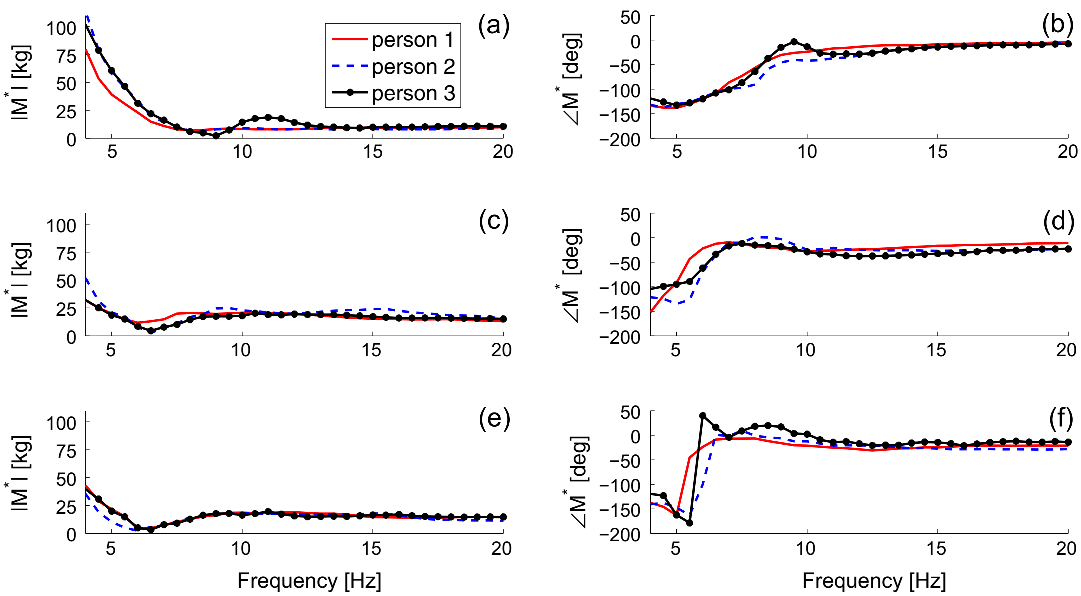


**Fig. 16.** Apparent masses of 3 subjects for the right foot: Position 1 – amplitude (a), Position 1 – phase (b), Position 2 – amplitude (c), Position 2 – phase (d), Position 3 – amplitude (e), Position 3 – phase (f).

overall behaviour during the motion. In this case, 8 positions were chosen to describe the motion of people when ascending and descending stairs ($\alpha_i = \frac{1}{8}$ with $i = 1 \div 8$). The first 6 positions are shown in Fig. 15 and are named with numbers from 1 to 3, left and right leg. The other chosen posture is the one defined as "one leg" in Ref. [34], i.e., standing on a straight leg with comfortable and upright upper-body, as mentioned previously. The apparent mass was measured while the subject was standing on the left and right leg for each posture. Three people were involved in the tests, and a total of 24 apparent mass curves were measured (3 subjects, 4 postures, left and right feet). Then, the $M^*_{a,i}$ curves were computed for each posture, and finally $M^*_{eq}$ was yielded (see Section 3).

The collected data were also used to analyse the influence of inter-subject variability (i.e., different people do not have the same dynamic behaviour) and posture on the apparent mass values. For this purpose, the apparent masses of different people in the same posture were first compared.

Fig. 16 shows the measured apparent masses, for the right foot, related to the postures reported in Fig. 15.

Fig. 17 shows a comparison among the apparent masses measured in the four postures considered in this analysis for a single person.

It can be reasonably assumed that the apparent mass values are highly influenced by the person's posture. Conversely, the inter-subject variability seems to have a lower influence on the results (see Figs. 16 and 17). The obtained results confirm the experimental evidence reported in the literature [35], i.e., the high influence of posture on the apparent mass values, and support the use of average apparent mass values to express the PGRFs.

The next section shows the results of the experimental tests with people walking on STRUCT1 and STRUCT2 and compares them to the numerical expectations.

### 6.2. Experimental results and comparison with numerical simulations

The tests with walking people were simulated numerically using Eq. (12) and the procedure described in Sections 5.1 and 5.2. The variable $n_d$ (number of points by which the structure is discretised) was fixed to 78 for STRUCT1 (see Fig. 7) and 86 for STRUCT2 (i.e. 2 points for each step). The eigenvector components related to the $n_d$ points were found from the interpolation of the identified mode shapes (see Section 6).

A comparison between the experimental results and those obtained by means of simulations is provided here. In addition to the modal model of the H-S system (described by the matrix $\mathbf{G}_H$, see Eqs. (7) and (12)), other models were also used for the numerical simulations. Particularly, three different modal models were used to simulate the structural response and to compare the obtained predictions with the experimental results.

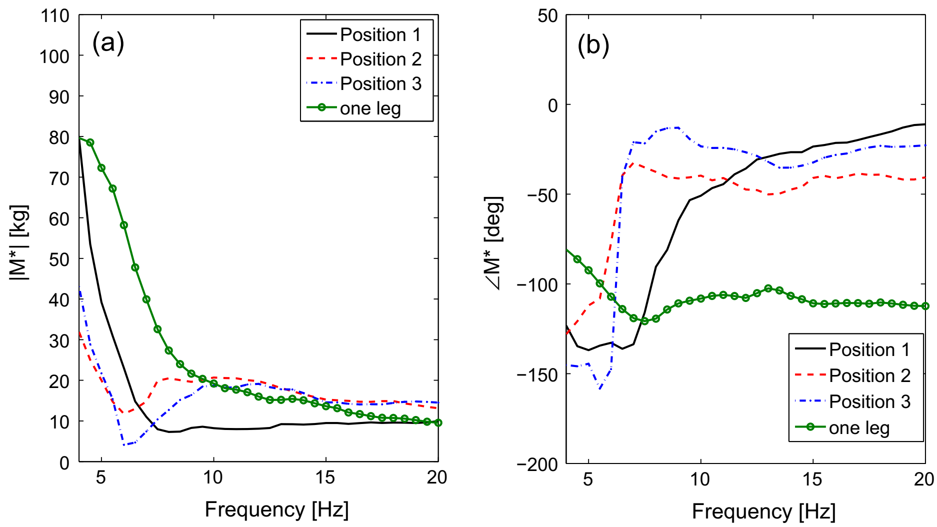


**Fig. 17.** Apparent masses of person 1 (see Fig. 16) for the right foot: amplitude (a) and phase (b).

**Table 3**
Modal parameters – STRUCT1.

| | Test with 3 people | | | Test with 9 people | | |
|---|---|---|---|---|---|---|
| | **G** | $\mathbf{G}_H$ | $\mathbf{G}_{H,exp}$ | **G** | $\mathbf{G}_H$ | $\mathbf{G}_{H,exp}$ |
| $f_1$ [Hz] | 7.84 | 7.82 | 7.84 | 7.84 | 7.79 | 7.82 |
| $f_2$ [Hz] | 8.89 | 8.88 | 8.86 | 8.89 | 8.87 | 8.85 |
| $\zeta_1$ [%] | 0.33 | 0.54 | 0.79 | 0.33 | 0.94 | 1.18 |
| $\zeta_2$ [%] | 0.43 | 0.47 | 0.44 | 0.43 | 0.56 | 0.83 |

**Table 4**
Modal parameters – STRUCT2.

| | Test with 5 people | | | Test with 10 people | | |
|---|---|---|---|---|---|---|
| | **G** | $\mathbf{G}_H$ | $\mathbf{G}_{H,exp}$ | **G** | $\mathbf{G}_H$ | $\mathbf{G}_{H,exp}$ |
| $f_1$ [Hz] | 6.70 | 6.68 | 6.60 | 6.70 | 6.67 | 6.59 |
| $f_2$ [Hz] | 9.56 | 9.55 | 9.47 | 9.56 | 9.54 | 9.44 |
| $f_3$ [Hz] | 10.75 | 10.75 | 10.75 | 10.75 | 10.75 | 10.74 |
| $f_4$ [Hz] | 11.21 | 11.21 | 11.18 | 11.21 | 11.20 | 11.18 |
| $\zeta_1$ [%] | 0.33 | 0.64 | 0.75 | 0.33 | 0.95 | 1.25 |
| $\zeta_2$ [%] | 0.28 | 0.33 | 0.42 | 0.28 | 0.37 | 0.60 |
| $\zeta_3$ [%] | 0.29 | 0.30 | 0.22 | 0.29 | 0.30 | 0.32 |
| $\zeta_4$ [%] | 0.17 | 0.20 | 0.20 | 0.17 | 0.23 | 0.24 |

The considered models are:

1. modal model of the empty structure $\mathbf{G}(\omega)$ (i.e., $\mathbf{G}^{-1}(\omega)\mathbf{x}(\omega) = -\mathrm{f}^{\mathrm{ACTIVE}}(\omega)$);
2. modal model $\mathbf{G}_H(\omega)$ (i.e., $\mathbf{G}_H^{-1}(\omega)\mathbf{x}(\omega) = -\mathrm{f}^{\mathrm{ACTIVE}}(\omega)$);
3. experimental modal model of the joint human–structure system. Here, the modal parameters of this joint system were estimated from the experimental structural responses collected during the tests with the walking people via Operational Modal Analysis (OMA) techniques [37] (i.e., $\mathbf{G}_{H,exp}^{-1}(\omega)\mathbf{x}(\omega) = -\mathrm{f}^{\mathrm{ACTIVE}}(\omega)$). Particularly, $\mathbf{G}_{H,exp} = \sum_{k=1}^{n} \frac{\phi_k \phi_k^T}{\omega_{k,exp}^2 - \omega^2 + 2j\zeta_{k,exp}\omega\omega_{k,exp}}$ where $\omega_{k,exp}$ and $\zeta_{k,exp}$ are the $k$th eigenfrequency and the non-dimensional damping ratio estimated by means of the OMA, respectively (the mode shapes do not show any significant change when compared to those of the empty structure [33]).

Table 3 reports the modal parameters associated with the three mentioned modal models above for STRUCT1. As for STRUCT 2, the results are shown in Table 4.

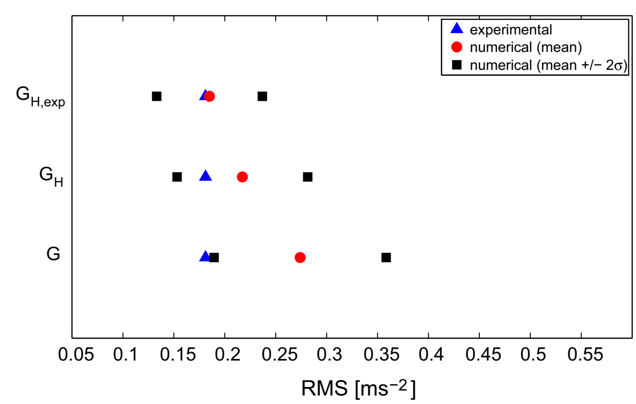


**Fig. 18.** Test with 3 people walking on the staircase (STRUCT1).

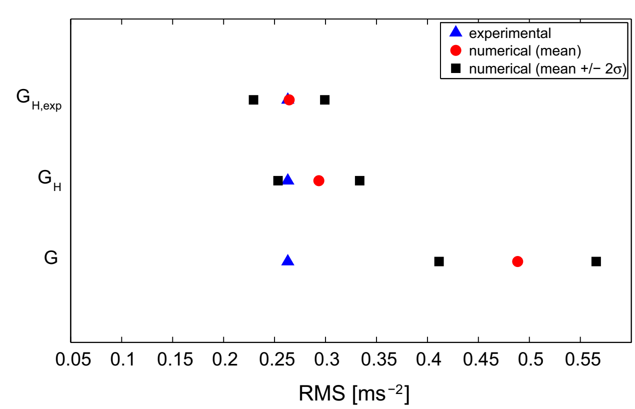


**Fig. 19.** Test with 9 people walking on the staircase (STRUCT1).

As for the test with 3 people on STRUCT1, the experimental modal parameters (i.e., $\mathbf{G}_{H,exp}$) show a moderate increase of damping ratios and a slight decrease of natural frequencies. The predicted modal parameters (i.e., $\mathbf{G}_H$ model) are close to the experimental values (i.e., $\mathbf{G}_{H,exp}$). However, because the forces exerted by people are not white noise, it is important to notice that the identification via OMA techniques might be subject to biases, especially for the test with 3 people.

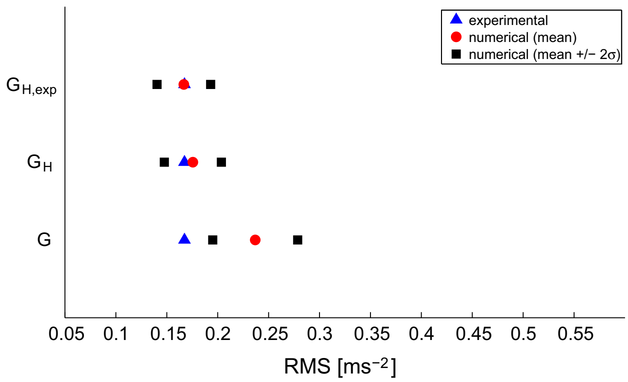


**Fig. 20.** Test with 5 people walking on the staircase (STRUCT2).

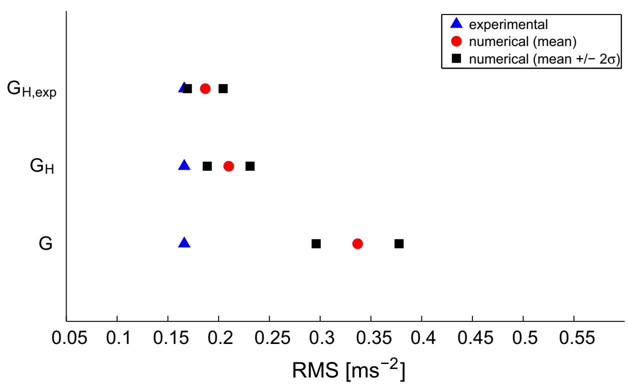


**Fig. 21.** Test with 10 people walking on the staircase (STRUCT2).

As for the test with 9 people on STRUCT1, the experimental modal parameters also show a consistent increase of damping ratios and a slight decrease of natural frequencies. The model of the joint H-S system (i.e., $\mathbf{G}_H$) provides non-dimensional damping ratios which slightly underestimate the experimental values (i.e., $\mathbf{G}_{H,exp}$). However, the results obtained with the H-S model are much closer to the experimental values than the modal parameters of the empty structure. Similar observations can be noted for STRUCT2 (see Table 4). The results shown so far clearly indicate the consistent effect of people (even a few people) on the value of the damping ratios.

Next, the simulated results were compared with the experimental measurements in terms of the Root Mean Square (RMS) of the accelerations of the structure. The RMSs were compared in the frequency range of 0–15 Hz. Figs. 18 and 19 show a comparison among the experimental and predicted RMSs of the point of maximum acceleration for STRUCT1 (accelerometer 6 in Fig. 10). The points reported in Fig. 18 (test with 3 people) and Fig. 19 (test with 9 people) refer to:

- the experimental RMS (triangle);
- the mean of the 100 simulated RMSs (circle);
- the mean RMS ± 2$\sigma$ (squares), where $\sigma$ is the standard deviation of the 100 simulated RMSs.

Figs. 18 and 19 report the results related to the modal model of the empty structure (i.e., $\mathbf{G}$), the model of the H-S system (i.e., $\mathbf{G}_H$) and the experimental model (i.e., $\mathbf{G}_{H,exp}$), respectively.

Figs. 20 and 21 show the comparison of RMSs for STRUCT2 at the point of maximum acceleration. The experimental time-history in this point can be compared to one of the simulated histories in Fig. 22 for STRUCT2 and the test with 10 people. The simulated time-response was computed using $\mathbf{G}_H$.

The results obtained confirm that the use of the empty structure model causes an overestimation of the predicted structural vibrations. Conversely, using the experimental modal parameters, the accelerations are well predicted. This result supports the validity of the simulation method and shows that if the modal parameters of the joint system are correctly predicted, the structural vibrations can be obtained via superposition of the effects (i.e., PGRFs+ AGRFs). Indeed, when the actual modal parameters are used (i.e., $\mathbf{G}_{H,exp}$ is used), the mean value of the numerical RMSs is always very close to the experimental RMS. The use of the model of the H-S system (i.e., $\mathbf{G}_H$) to predict the structural response significantly improves the results when compared using of the empty structure model because it allows a good estimation of the modal parameters of the system composed by the structure and the people. Indeed in this case, the results of the simulations are compatible with the experimental values. Therefore, a key point for the success of the proposed approach is the identification of a correct set of equivalent FRFs accounting for the PGRFs of moving people. If the modal parameters of the H-S system are correctly predicted, the proposed approach can provide reliable predictions of vibra-tion amplitudes.

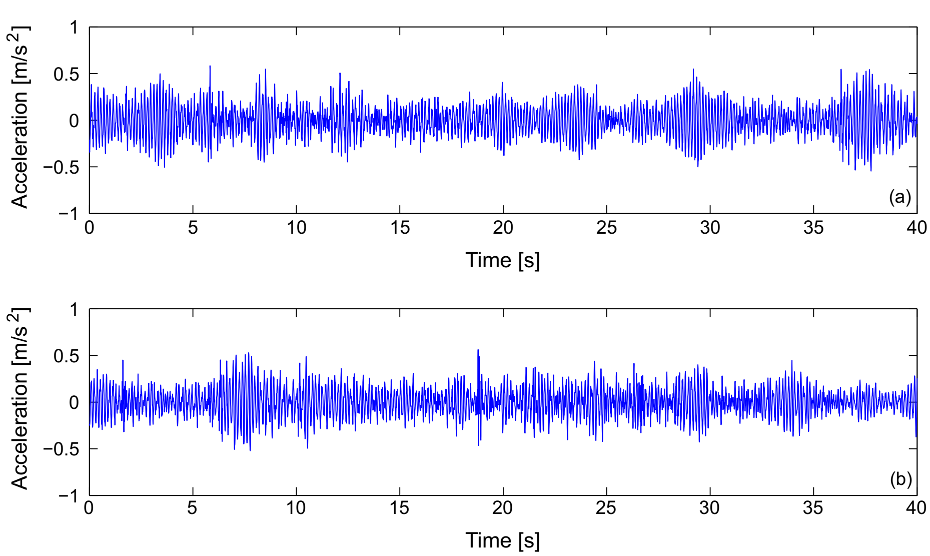


**Fig. 22.** Pieces of response time-histories (at the point of maximum response) for STRUCT2 for the test with 10 people: Numerical (a) and experimental (b). The time-histories are low-filtered with a second order Butterworth filter (cut-off frequency at 20 Hz).

The identification of a correct model $\mathbf{G}_H$ is thus a key point of this approach. Nonetheless, the AGRFs have a significant influence on the result. The resulting RMS of each simulation depends on the set of AGRFs used and is proven by the non-negligible value of $\sigma$ in Figs. 18–21. This means that a statistical analysis is always needed to take into account of the many different possible situations and to have a description of the vibrational behaviour of the structure in many different cases.

## 7. Conclusions

This work aimed at proposing and validating an approach to predict the structural response of staircases due to the presence of moving people. Furthermore, this approach can be applied to every type of structure, not only a staircase, because the analytical and numerical approaches are of general validity. The methodology is based on the superposition of two contributions produced by people acting on a structure. Particularly, the effect of people is decoupled into passive and active ground reaction forces (GRF). The passive GRFs (PGRFs) are used to find an appropriate equivalent model to represent the dynamics of a structure occupied by moving people. The active GRFs (AGRFs) are then applied to this modified model to obtain a prediction of the structural vibrations.

Two main problems were investigated in this work. The first issue was related to the identification of PGRFs that were representative of the average influence of moving people in terms of changes of the modal parameters. For this purpose, a set of apparent masses, representative of various postures taken by people during motion, was measured. Thus, an equivalent apparent mass was obtained and used to assess the influence of people. The second issue was related to the identification of appropriate AGRFs. Under normal operating conditions, because the actual force exerted by people could not be measured, an appropriate set of possible forces was measured. Thus, a statistical approach was used to simulate the structural response. An appropriate simulation procedure was used to obtain predictions of the structural vibrations. The results showed that the modal model of the empty structure gave vibration amplitudes which overestimated the actual structural response. Conversely, by using the approach proposed in this work, results were in agreement with the experimental measurements.


## Acknowledgment

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