Data-driven modelling of vertical dynamic excitation of bridges induced by people running

Vitomir Racica,*, Jean Benoit Morinb

aDepartment of Civil & Structural Engineering, University of Sheffield, Sir Frederick Mappin Building, Sheffield S1 3JD, UK
bFaculty of Science and Technology, Université Jean Monnet, Saint-Etienne, France

Article history:
Received 29 January 2013
Received in revised form
25 June 2013
Accepted 6 October 2013
Available online 2 November 2013

1. Introduction

In the last 15 years substantial developments in workmanship and structural materials have enabled bridge engineers to promote much more slender designs than ever. These reduce the mass and stiffness of the structure, hence the bridge is more likely to have a natural frequency well below 5 Hz for spans greater than 50 m [1]. As a result, modern bridges are at a considerable risk of developing resonance when occupied and dynamically excited by people moving during ordinary activities, such as walking, jumping and running [2]. Excessive dynamic responses recorded worldwide on at least a
demonstrated by the significance of footfall vibration within the civil engineering context. For example, signals has yet been found that can be used for development of their statistically reliable characterisation for application studies in very small laboratories [18]. However, no database of running forces in the form of continuously recorded forces has been in the domain of biomechanics as the contact forces between the feet and supporting ground (thus generally called “ground reaction forces” – GRFs) provide useful diagnostics for medical and sports applications [19]. Medics introduced instrumented force measuring treadmills which can be used to carry out even high-speed running studies in very small laboratories [18]. However, no database of running forces in the form of continuously recorded signals has yet been found that can be used for development of their statistically reliable characterisation for application in the civil engineering context.

The lack of fundamental forcing data and their reliable mathematical models has resulted in a total lack of formal design guidance regarding running excitation of bridges. As a result, bridge designers are faced nowadays with a great deal of uncertainty about dynamic performance of the structure under a number of relevant loading case scenarios related to people running, such as increasingly popular marathon events in urban environments [9,20]. The present study aims to change this situation by bringing together: (1) a comprehensive database of continuously measured vertical running loads generated by individuals on flat stationary surfaces (i.e. without the vertical bridge motion and hence discounting effects of structure dynamic interaction), and (2) two mathematical characterisations of the measurements. The first modelling strategy is based on a deterministic perfectly periodic Fourier-based approach adapted from a popular design guideline for walking loading [20], while the second model features more realistic stochastic-based modelling of narrow-band signals. The models could be applied to cases of incipient instability, i.e. when vertical bridge motion is still not significantly perceptible to the runner, and is ideally suited for considering cases of multiple runners in a group or crowd.

Realistic modelling of running force signals, based on direct observation and measurement, is an essential prerequisite to predicting bridge vibrations whether or not they are perceptible. McMahon and Greene [21] showed that a decrease in surface stiffness resulted in a decrease in a runner’s metabolic rate and a decrease in the vertical GRF, but without affecting running mechanics. Also, there is strong evidence that peripheral stimuli are an equally important factor influencing timing of footfalls [9] and that resulting GRFs are inevitably a narrow-band random process [22]. Hence, the present study focuses on appropriate modelling of this process in order to build a framework that can generate the correct interface forces between the runners and the structure.

2. Definition of running

Of all factors that influence running GRFs, such as the body morphology [23], gait style [24], mechanical properties of the supporting surface [25], texture and stiffness of footwear [26], speed is the most prominent [27]. Typical running force-time history is a series of pulses separated by zero-force periods, which correspond to the “flying” phase of running when both feet are off the ground (Fig. 1a). In contrast, during walking only one foot at a time leaves contact with the ground and there is a period of double-support (Fig. 1b). Running footfalls also have one dominant peak and therefore differ from the characteristic double-peak shape (also called “M shape”) of walking footfalls. Hence, people walking and running clearly represent two different dynamic loading case scenarios of footbridges.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_r$</td>
<td>footfall rate or running frequency</td>
</tr>
<tr>
<td>$F(t)$</td>
<td>force–time history</td>
</tr>
<tr>
<td>$G$</td>
<td>body weight</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>dynamic load factors (DLF)</td>
</tr>
<tr>
<td>$\varphi_i$</td>
<td>Fourier phases</td>
</tr>
<tr>
<td>$f_s$</td>
<td>sampling rate</td>
</tr>
<tr>
<td>$T_i, T_k$</td>
<td>cycle intervals</td>
</tr>
<tr>
<td>$\tau_i, \tau_k$</td>
<td>normalised cycle intervals</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>mean of $T_i$</td>
</tr>
<tr>
<td>$S(t), S(f)$</td>
<td>ASD of $\tau_i$ and $\tau_k$</td>
</tr>
<tr>
<td>$A_i, A_j, W_j$</td>
<td>Gaussian heights (weights)</td>
</tr>
<tr>
<td>$t_i, c_j$</td>
<td>Gaussian centres</td>
</tr>
<tr>
<td>$\delta_i, b$</td>
<td>Gaussian widths</td>
</tr>
<tr>
<td>$Z(t)$</td>
<td>template cycle</td>
</tr>
<tr>
<td>$E_i, E_k$</td>
<td>energy of weight normalised cycles</td>
</tr>
<tr>
<td>$E_{ic}$</td>
<td>energy of template cycle</td>
</tr>
<tr>
<td>$\Delta E_i$</td>
<td>disturbance term</td>
</tr>
<tr>
<td>$\xi_k$</td>
<td>scaling factor</td>
</tr>
<tr>
<td>$\rho_0, \rho_1$</td>
<td>coefficients of linear regression</td>
</tr>
<tr>
<td>$\rho$</td>
<td>coefficients of linear correlation</td>
</tr>
<tr>
<td>$N$</td>
<td>number of cycles</td>
</tr>
<tr>
<td>$T$</td>
<td>duration of force signal</td>
</tr>
</tbody>
</table>
There is some evidence from both measurements of oxygen consumption [28] and mechanical simulations of bodies in motion [29] that people select gait style at a given speed to optimise energy cost of locomotion. In fact, walking feels easiest at low speeds and running feels easiest at fast speeds [30]. Hence people switch from a walk to run at speeds close to the boundary at which walking ceases to be optimal [31,32]. However, in the proximity of this boundary both fast walking and slow running (i.e. jogging) are possible. From the perspective of bridge dynamics, jogging is potentially more severe loading scenario than walking since it is characterised by more bouncy (i.e. trotting) gait style yielding higher force amplitudes. The difference is illustrated in Fig. 2.

In the present study, all test subjects were encouraged to start jogging at the slowest possible but still comfortable speed, i.e. self-selected speed at which jogging feels natural. The test protocol is elaborated in the next section.

3. Experimental data collection

A database comprising many force records at a wide range of running speeds (i.e. from jogging to sprinting) is an essential component for their mathematical characterisation presented in Sections 4 and 6. In this paper, such a database is established using two force measuring treadmills, as elaborated in Sections 3.1 and 3.2.

Although there is no theoretical difference in the physics of treadmill and overground locomotion, some early experimental studies reported major discrepancies [33]. The reason for this was thought to be imposed “constant” speed of movement, controlled by rotation of the treadmill belt, as opposed to the overground locomotion when the gait speed has no similar control [34]. However, van de Putte et al. [35] and Riley et al. [36] showed more recently that difference between treadmill and overground locomotion is negligible for those who use treadmill regularly (e.g. to maintain fitness). In addition, based on a statistical comparison between three-dimensional knee kinematics and step lengths recorded on ten healthy test subjects, van de Putte et al. [35] showed that habituation of inexperienced test subjects to treadmill locomotion takes up to 10 min. These studies demonstrated the essential equivalence between treadmill and overground locomotion in biomechanical domain, such as measuring performance of healthy athletes [37] and design of “blade runners” for disabled athletes [38]. Hence, they justify treadmill-based force measurements in design of less delicate bridge structures.

3.1. Experimental setup

The tests were carried out in the Vibration Engineering Section Laboratory in the University of Sheffield (UK) and Laboratoire de Physiologie de l’Exercice in the University Jean Monnet Saint-Etienne (France). Continuously measured individual force records were collected on two state-of-the-art instrumented treadmills: ADAL3D-F in Sheffield (Fig. 3a) and ADAL3D-R in France (Fig. 3b).

All components of each treadmill, including brushless servo motors equipped with internal velocity controllers, belts and secondary elements, are mounted on a rigid metal frame and mechanically connected to the supporting ground only through four Kistler 9077B three-axial piezoelectric force sensors [39]. These transducers have high stiffness to avoid any possibility of system dynamic characteristics affecting measurements. The whole system is mechanically isolated, i.e. the sensors measure only external running forces, while the internal forces due to belt friction and belt rotation are not detected by the sensors [18]. More technical details and a full description of the force measuring process are elaborated in ADAL3D user manuals [40].

Speed of the belt rotation (here also called “treadmill speed”) can be controlled and monitored remotely either with a control panel or with bespoke software, run from the data acquisition PC. Similar to fitness treadmills, the remote control panel and the treadmill itself are equipped with a safety stop switch. Rotational speed of ADAL3D-F belts is in the
range 0.1–10 km/h, whereas the speed range of ADAL3D-R is extended up to 25 km/h. The difference in speed range gave rise to different test protocols used in Sheffield and Saint-Etienne, as elaborated in the next section.

3.2. Test sequence

Prior to the force measurements, the Research Ethics Committee of the University of Sheffield and the University Jean Monnet Saint-Etienne required each prospective test subject complete a Physical Activity Readiness Questionnaire and pass a preliminary fitness test (by satisfying predefined criteria for blood pressure and resting heart rate) to check whether they were suited for physical activity required during the experiment. Measurements of the body mass, age and height were taken for test subject who passed the preliminary test.

All participants wore comfortable running footwear. Those who had no experience with treadmill running were given a brief training prior to the force collection supervised by a qualified instructor. Each participant had at least 10 min long warming up, which included walking and jogging on the treadmill while the speed was varied randomly and controlled by the speed of rotation of the treadmill belts.

During each individual test participants were asked to run on the treadmill at a fixed speed, which was the average individual’s running speed. The latter could vary on the step-by-step basis around the given treadmill speed as 1 m long belts allowed test subjects to move forwards and backwards on the treadmill, thus to slow down and speed up while

---

**Fig. 2.** Force signals generated by the same individual (a) walking and (b) jogging at 8 km/h.

**Fig. 3.** Experimental setup in (a) Sheffield and (b) Saint-Etienne.
running. This made running feel natural and allowed variability of footfalls normally present in overground running [35]. Pacing rate was not prompted by any stimuli such as a metronome, and it was determined only from subsequent analysis of the generated force signals (see Section 4). Each test was completed when at least 64 successive footfalls were recorded. Rests were allowed between successive tests.

In Sheffield, acquisition of the force records started at the slowest self-selected speed at which jogging, rather than walking, is more comfortable. In the 95% of cases processed, this speed was 7 km/h, after being rounded to nearest integer. In each following test the running speed was increased by 0.5 km/h, until it added up to 10 km/h. In Saint-Etienne, the first test was run at 10 km/h and the speed was increased by 0.5 km/h in each following test until a participant reached either their maximum sprinting speed or 25 km/h. All recorded force signals were sampled at 200 Hz.

In total, 45 volunteers (30 males and 15 females, body mass 72.3 ± 14.2 kg, height 175.2 ± 7.1 cm, age 27.2 ± 8.1 years) were drawn from students, academics and technical staff of the University of Sheffield and the University Jean Monnet Saint-Etienne. All together they generated 458 vertical running force–time histories of kind illustrated in Fig. 2b. In the remaining part of the paper, this unique database is used to provide their first mathematical representations derived from the actual continuously measured force records. Section 4 takes a traditional yet conservative approach to modelling human-induced dynamic loading based on the deterministic representation of their dominant Fourier harmonics [41,42]. On the other hand, Section 5 presents a radically different model motivated by the existing numerical generators of earthquakes [43], human electrocardiogram signals [44] and speech recognition [45] which can accurately simulate the measurements.

4. Modelling approach based on Fourier series

Measured force–time histories of human-induced loading are invariably near-periodic [22], indicating their narrow band nature (Fig. 4b). However, to simplify their mathematical representations for design purposes, they are usually assumed perfectly periodic, deterministic and presentable via Fourier series:

\[
F(t) = G \sum_{j=1}^{m} \alpha_j \sin(2\pi f_j t - \phi_j)
\]  

Here \(F(t)\) represents the total force at time \(t\), with \(G\) representing the body weight in the same unit. A total of \(m\) harmonics are considered, capturing peaks \(\alpha_j\) from the Fourier amplitude spectrum (Fig. 4b) at integer multiples of the footfall rate \(f_r\) (in Hz, thus here also called “running frequency”). Commonly known as “dynamic load factors” (DLFs), \(\alpha_j\) coefficients have been intensively studied for walking, jumping and bouncing [22] and the results have been incorporated in the most advanced vibration serviceability guidelines. On the other hand, values of the phase angles \(\phi_j\) have never been publicised in detail and are usually ignored in Eq. (1).

A number of studies has shown that the Fourier modelling approach leads to significant loss of information and introduction of inaccuracies during the data reduction process [6,7,16,46,47]. For instance, Brownjohn et al. [46] reported differences as high as 50% between vertical vibrations of pedestrian structures due the imperfect real walking forces and periodic Fourier-based simulations, which were related to neglecting the energy around dominant harmonics in actual narrow band forces (Fig. 4b). Recent studies based on comparisons between the measured and simulated structural response due to the actual imperfect and artificial periodic pedestrian loading of footbridges [48] and floors [49], as well as due to jumping and bouncing loading of grandstands [50], all recognised the importance of modelling near-periodic nature of human-induced loads. Although a quality fit of the complete Fourier amplitude spectrum could be obtained, e.g. as suggested for the vertical walking forces elsewhere [2,46], widely varying harmonic phase lags \(\phi_i\) (Figs. 4c and 5) are very difficult to characterise analytically. If they are, however, assumed to be uniformly distributed in the range \([-\pi, \pi]\), the sum of Fourier harmonics normally does not match the actual force time history. This is illustrated by comparison of Fig. 4a–d. Hence, randomising phases is not the way forward. They seem to be a structured data set rather than an assembly of uniformly distributed random numbers. This clearly demonstrates that the Fourier series can only provide rudimentary conceptualisation of individual running loads. However, fitting Eq. (1) to the numerous database of running force records established in Section 3 can provide a valuable insight into their inter-subject variability, which this paper specifically aims to address in Section 7. The results of the fitting are summarised in the next section.

5. Inter-subject randomness of running forces

Graphs shown in Fig. 6 indicate that running force amplitudes generally increase as people increase their footfall rate. Due to the apparently large scatter of the DLF values at a fixed running frequency their relationship cannot be described reliably as a deterministic function, such as the linear and polynomial curve fits suggested for vertical walking loads elsewhere [20,41,42]. Moreover, histograms of DLFs corresponding to a narrow range 0.1 Hz around the most frequent footfall rate in the sample \(f_r = 2.7\) Hz do not resemble any known probability density function (Fig. 7), which is a key condition for probabilistic curve fitting [52]. The Kolmogorov–Smirnov (K–S) goodness of fit test [51] rejects the hypothesis (at 5% confidence level) that the DLF data come from a normal, lognormal, gamma, Rayleigh or exponential distribution.
Fig. 4. (a) An example of continuously measured running force signal and the corresponding Fourier (b) amplitude and (c) phase spectra derived from 30 s long force–time history. (d) The force–time history reproduced using the amplitude spectrum shown in (b) and uniformly distributed phases in the range $[-\pi, \pi]$.

Fig. 5. Histogram of the phases illustrated in Fig. 4c.
Fig. 6. Footfall rate $f_r$ vs. DLFs ($\rho_a = 0.37$, $\rho_b = 0.52$, $\rho_c = 0.47$, $\rho_d = 0.62$).

Fig. 7. Histograms of the first four DLFs in a narrow range 0.1 Hz around the most frequent footfall rate $f_r = 2.70$ Hz.
Table 1

Results of Kolmogorov–Smirnov goodness of fit test (at 5% confidence level) corresponding to the data shown in Fig. 7.

<table>
<thead>
<tr>
<th>p-value</th>
<th>Probability distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
</tr>
<tr>
<td>DLF α₁</td>
<td>0.293</td>
</tr>
<tr>
<td>DLF α₂</td>
<td>0.419</td>
</tr>
<tr>
<td>DLF α₃</td>
<td>0.087</td>
</tr>
<tr>
<td>DLF α₄</td>
<td>0.196</td>
</tr>
</tbody>
</table>

(Tables 1). Similar scatter in the histograms and outcome of the K–S test can also be observed for DLFs in the close proximity of fixed running speeds.

Fig. 8 proves that the body weight G and the fundamental DLF α₁ are uncorrelated variables (ρ = −0.07), indicating that the running style may be the single most important factor affecting amplitudes of running loads. As a result, arbitrary G values can be assigned to a synthetic G-normalised force signal as an independent random parameter. For instance, one can use probability density functions of the body weight, such as reported by Hermanussen et al. [53] for German, Austrian and Norwegian citizens, to generate random G values. This is an important aspect of the modelling strategy presented in Section 7.

Shapes of the histograms shown in Fig. 9 suggest that running footfall rate is not a normally distributed variable as in case of walking [2,13,47,54]. The normal probability plots in Fig. 10 and the results of the K–S test summarised in Table 2 further prove the point. Moreover, fitting a wide range of other probability density functions, such as the family of skewed distributions [52], showed a lack of statistical significance (Table 2).

Apart from the strong linear trend between running speed and footfall rate (ρ = 0.75), Fig. 11 illustrates a large scatter of the running frequencies at fixed running speeds, and vice versa. This observation can play a key role in simulating reliably running excitation of bridges due to dense crowd situations (e.g. marathon events) where running speed of individuals in a crowd can be imposed by spatial constrains and the crowd flow. As in the case of DLFs, visual inspection of the histograms in
Fig. 12 shows that distribution of the footfall rates at fixed running speeds cannot be approximated satisfactorily by any known probability density function.

The results presented so far have demonstrated large intra- and inter-subject randomness of running forces which cannot be modelled reliably using the popular deterministic Fourier series approach. To simulate more accurately the intra-subject variability of running, the next section offers a sophisticated modelling strategy featuring individual forces as a narrow band process. The inter-subject variability is addressed in Section 7, where the model is fitted to each of the 458 force records measured in Section 3 yielding a numerical generator of random narrow band running time series.

6. Modelling near-periodic force signals

Due to the naturally occurring variations in all repetitive human movements, the force measures vary on cycle-by-cycle basis. A running cycle can be defined between any two nominally identical events in the force–time history. In this study, the instant at which the feet hit the ground (also known as “initial contact”) leading to a new force pulse was selected as starting (and completing) event (Fig. 13).

From the 35 s long force signal (a fraction is shown in Fig. 13) and yielding about 94 cycles, a window comprising 76 successive cycles was extracted from the middle. Nine cycles (i.e. approximately 10% of all cycles measured) at the beginning and end of the force record were cast aside since their natural variability might be affected by the measuring process. In the next section, the selected 76 cycles are used to model morphology of the force signal (so called “template cycle”), while

<table>
<thead>
<tr>
<th>p-value</th>
<th>Normal</th>
<th>Lognormal</th>
<th>Gamma</th>
<th>Rayleigh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheffield data</td>
<td>0.634</td>
<td>0.461</td>
<td>0.385</td>
<td>0.076</td>
</tr>
<tr>
<td>Saint-Etienne data</td>
<td>0.661</td>
<td>0.283</td>
<td>0.801</td>
<td>0.081</td>
</tr>
<tr>
<td>All</td>
<td>0.393</td>
<td>0.069</td>
<td>0.180</td>
<td>0.069</td>
</tr>
</tbody>
</table>

Fig. 11. Running speed vs. footfall rate ($\rho=0.75$).
modelling variability of the cycle intervals and force amplitudes is outlined in Sections 6.2 and 6.3. The models are motivated by an existing numerical generator of electrocardiogram (ECG) signals [44], which have similar near-periodic features as running time series. Section 6.4 illustrates how the template cycle can be scaled (i.e. stretched and compressed) on the cycle-by-cycle basis along the synthetic time and amplitude axes to reflect the intra-subject variability present in the actual force–time history.

6.1. Template cycle

The selected 76 cycles have been normalised by the body weight $G$, aligned to their starting points and resampled to the length of the longest cycle. Visual inspection of Fig. 14 suggests that there is a common waveform which distorts along time and amplitude axes on a cycle-by-cycle basis. Their average would be an inadequate representative of the force morphology due to misalignments of the common events, such as positions of the local extreme values. Hence, before finding the average (i.e. the template cycle) all selected cycles had been lined up using dynamic time warping (DTW) method. Originally developed for speech recognition, the DTW nonlinearly warps two discrete signals to align similar events along time dimension and thereby minimise the sum of squared differences [45]. This process is illustrated in Fig. 15a. In a similar way

![Histograms of footfall rates at fixed running speeds](image1)

**Fig. 12.** Histograms of footfall rates at fixed running speeds (a) 7 km/h, (b) 11 km/h, (c) 15 km/h, and (d) 19 km/h.

![A portion of a 35 s long force–time history](image2)

**Fig. 13.** A portion of a 35 s long force–time history ($f_r=2.70$ Hz).
wavelet analysis linearly shifts and stretches so called “mother” wavelet [55]. However, due to the apparent nonlinear misalignments of the common events in the force data, linear shifting and stretching is considered unsuitable in this study.

The DTW is designed to warp only two trajectories at a time, so the cycles are warped individually onto a “reference cycle” (Fig. 15a). It is one of the measured cycles in Fig. 14 that minimises the sum of point-by-point Euclidean distances to their simple numerical point-by-point average [56]. The resulting warped cycles and template cycle are shown together in Fig. 15b.

The underlying shape of the template cycle can be modelled mathematically as a sum of four Gaussian functions (Fig. 16 and Table 3):

$$Z(t) = \sum_{j=1}^{4} A_j e^{-\left(t-t_j\right)^2/2\delta_j^2}$$

where $Z(t)$ is the curve fit, the parameter $A_j$ is the height (also called weight) of the $j$th Gaussian peak, $t_j$ is the position of the centre of the peak, and $\delta_j$ controls the width of the Gaussian bell functions. The number of Gaussians in the sum and values of the parameters are optimised using nonlinear least-square curve fit [57].

6.2. Variability of cycle intervals

Variations of cycle intervals $T_i$ ($i = 1, \ldots, 76$) can be represented by a dimensionless set of numbers $\tau_i$:

$$\tau_i = \frac{T_i - \mu_T}{\mu_T}$$

$$\mu_T = mean(T_i)$$

As $\tau_i$ series has zero mean value, its variance is the integral of its auto spectral density (ASD) [55]. Assuming that the variation of $\tau_i$ does not change for the given test subject, footfall rate and duration of running, the ASD of the actual $\tau_i$ data can be used to generate synthetic series $\tau'_k (k = 1, \ldots, N)$ of arbitrary length (e.g. $N \gg 76$) with the same statistical properties, such as standard deviation and “pattern” of variation between successive $\tau_i$ values. Empirical evidence is provided in Section 6.4.
An ordinary random number generator, such as a probability density function of $\tau_i$, cannot model the pattern as it cannot reflect the frequency content of $\tau_i$ series. The ASD of $\tau_i$ can be calculated as \[ S_{\tau}(f_m) = \frac{A_{\tau}(f_m)^2}{2\Delta f}, \quad f_m = \frac{m}{76}, \quad m = 0, \ldots, 37 \] where $A_{\tau}(f_m)$ is a single-sided discrete Fourier amplitude spectra and $\Delta f = 1/76$ is the spectral line spacing (Fig. 17). The ASD ordinates do not depend on the number of discrete data points $\tau_i$, but it is coarse due to their limited number. More points would result in a smoother ASD and might reveal a richer ASD structure but this requires more measured cycles. However, having longer force records would be ethically inappropriate. Guided by the Research Ethics Committees in the University of Sheffield and the University Jean Monnet Saint-Etienne the test duration was limited to avoid fatigue and discomfort, which can also influence natural variability of the force records.
Similarly to the template cycle (Section 6.1), the ASD $S_\tau(f_m)$ can be mathematically described by a sum of 38 equidistant Gaussian functions (Fig. 17):

$$S_\tau(f) = \sum_{j=1}^{38} W_j e^{-(f-f_c^j)^2/2b^2}$$

Here, parameter $W_j$ is the height of the $j$th Gaussian peak, $c_j$ is the position of the centre of the peak, and $b$ is the common width of the Gaussian function. The Gaussian centres $c_j$ are placed in each sample on the quasi-frequency axis to fit exactly the actual ASD (Fig. 17). For such fixed positions of $c_j$ and predefined widths $b = \Delta f$, Gaussian heights $W_j$ can be optimised using the nonlinear least-square method [57].

Generation of synthetic series $\tau'_k (k = 0, \ldots, N)$ starts by calculating $S_	au(f_n)$ values using Eq. (5) at discretely spaced frequency points $f_n = n\Delta f$, where $n = 0, \ldots, N/2 - 1$ and $\Delta f = 1/N$. The synthetic ASD amplitudes are then used in conjunction with Eq. (4) to generate a set of Fourier amplitudes $A_\tau(f_n) = \sqrt{2\Delta f S_\tau(f_n)}$. Finally, assuming random distribution of phase angles in the range $[-\pi, \pi]$, $A_\tau(f_n)$ are used in inverse FFT algorithm to generate a set of $N$ synthetic $\tau'_k$ values.

According to Eq. (2), scaling $\tau'_k$ by $\mu_T$ and adding the offset value $\mu_T$, results in a series of synthetic intervals $T'_k$ (Fig. 18). Assuming that the test subject does not vary significantly their running style in a narrow range of footfall rates, $\mu_T$ value can be slightly changed in the scaling process to generate cycle intervals at rates close to $1/\mu_T$. Empirical evidence is presented in Section 7.

6.3. Variability of force amplitudes

Energy of running cycles $E_i$ can be defined as the integral of the $G$-normalised force amplitudes over the corresponding cycle intervals $T_i$ (Fig. 13). Due to the fairly linear trend between these two parameters, it is possible to simulate the energy transfer between successive cycles using the following linear regression model (Fig. 19):

$$E_i = \rho_1 T_i + \rho_0 + \Delta E_i$$

Here, $\rho_1 = 0.498$ and $\rho_0 = 0.183$ are regression coefficients and $\Delta E_i$ is a disturbance term or error, which is usually modelled as a random Gaussian noise [52].

Given a set of synthetic cycle intervals $T'_k (k = 1, \ldots, N)$ generated as presented in Section 6.2, the corresponding synthetic series $E'_k$ can be calculated using Eq. (6). These energies can be assigned to $k$ template cycles by scaling their amplitudes by factors $\xi_k$:

$$\xi_k = \frac{E'_k}{E_{tc}}$$

where $E_{tc}$ is the energy of the template cycle. When sequenced along the time axis, the scaled template cycles also reflect the variation of the force amplitudes on a cycle-by-cycle basis, as illustrated in Fig. 20.

6.4. Synthetic force signals

Visual comparison of Fig. 20a to b and Fig. 21a to 20b shows the apparent similarity between the measured signal and an example of synthetic force signal in both time and frequency domain.

Two identical synthetic force signals can be generated only by chance since the modelling parameters $T'_k$ and $\Delta E'_k$ are random variables. However, a family of synthetic forces share three key properties inherited from the measured signal:
Fig. 20. (a) Measured and (b) an example of synthetic force–time series.

Fig. 21. Discrete Fourier amplitude spectra of the time series shown in Fig. 20.
(1) Shapes of the running footfalls are drawn from the same template cycle.
(2) Cycle intervals $T'_k$ are statistically equivalent since they share the same ASD.
(3) According to Eq. (6), the statistical equivalence of $T'_k$ series reflects directly equivalence of $E'_k$ energies.

The next section shifts focus of the study from intra- to inter-subject variability. It integrates the numerous database of individual force records established in Section 3, the knowledge on their randomness presented in Section 5 and their mathematical characterisation outlined in Section 6, to create a numerical generator of random narrow band running time series.

7. Numerical generator of random artificial force signals

Each of the 458 force records in the database established in Section 3 was processed using the concept described in Section 6. Multiple sets of information, such as parameters of the template cycle $Z_k(t)$, the ASD $S_\tau(f)$, disturbance term $\Delta E_i(t)$ and the regression coefficients $\rho_1$ and $\rho_0$, were extracted and stored in 458 MATLAB structure files ([58]). These are called “mat files” in the remaining part of the paper.

The mat files were classified into 15 clusters with respect to the running footfall rate, as shown in the histogram in Fig. 9c. Each bar in the histogram is only 0.1 Hz wide, thus it is reasonable to assume that any mat file in a cluster can be used to generate synthetic signals at any footfall rate within the cluster’s narrow frequency range. This implies that the modelling parameters extracted from the actual force measurements with a certain footfall rate can be used to generate artificial force signals at close rates. This is a key feature of the random modelling strategy adopted in this section.

The flow chart in Fig. 22 outlines the algorithm of creating synthetic force signals. For specified footfall rate $f_r$ and duration of running excitation $G$ the algorithm first calculates the total number of running cycles $N$ in the force signal. Then, from a cluster corresponding to the $f_r$, it selects randomly and equally likely a mat file. Using the information about the ASD function $S_\tau(f)$ stored in the file, the algorithm generates synthetic cycle intervals $T'_k$, $k = 1, \ldots, N$, as elaborated in Section 6.2. These are further used to calculate the corresponding scaling factors $\xi_k$ using Eqs. (6) and (7).

Having generated $T'_k$ and $\xi_k$ values, the template cycle $Z(t)$ is stretched and compressed along time and amplitude axes through $N$ iterations, yielding weight normalised running forces of a kind shown in Fig. 20b. Finally, these become equivalent running force time histories when their amplitudes are additionally multiplied by a random body weight.
Fig. 23 illustrates examples of force signals generated when the model was run twice in succession using the same input parameters: \( f_r = 3 \) Hz and \( T = 40 \) s. A visual comparison of the time series provides empirical evidence that the model accounts for the inter-subject variability. On the other hand, the ability to generate different degrees of the intra-subject variability is more obvious from comparison between the corresponding frequency spectra (Fig. 24). Greater “leaking” of energy around dominant harmonics in Fig. 24b relative to Fig. 24a indicates that the force signal in Fig. 23b is characterised by a greater degree of variability in amplitude and timing on the cycle-by-cycle basis.

8. Discussions and conclusions

The lack of fundamental forcing data and their reliable mathematical characterisation are the key reasons for a total lack of urgently needed formal design guidance regarding running excitation of bridges. To change this situation, this interdisciplinary study first established a unique database comprising 458 individual force signals collected from 45 individuals running on two biomedical instrumented treadmills at a wide range of speeds. Then, two attempts were made to mathematically describe the measurements. The first model featuring Fourier series was transferred and adopted from a popular contemporary design guideline on dynamic loads due to pedestrians walking. Based on the deterministic and perfectly periodic representation of human locomotion, this modelling approach was shown to oversimplify the actual running force signals since they are characterised by significant inter- and intra-personal randomness. The key intellectual
delivery of this study is a more comprehensive and robust data-driven mathematical model designed to generate random signals which can replicate both temporal and spectral features of the measured forces. The modelling strategy is motivated by the existing models of human electrocardiogram (ECG) signals and speech recognition, as well as stochastic models of wind and earthquake loading. Being derived from the force data recorded on stiff treadmills, the current version of the model represents reliably only individual running excitation of structures which do not vibrate perceptibly, i.e. cases when the ground reaction forces are not significantly affected by the structural vibrations. However, the model is an essential prerequisite for future quality models of dynamic loading induced by individuals and multiple runners under a wider range of conditions. From the structural engineer’s point of view, there is still a gap in the knowledge on the proportion of individuals in a group or crowd who coordinate their body motion and the scale and character of the resulting net dynamic loads on the structure, with or without influence of the motion of the structure itself. In fact, perceptible motion of the structure is just one type of cue or stimulus among several others affecting human-induced loads, such as different auditory, visual and tactile stimuli. To include all these parameters in the model, the variations in the force amplitudes, energy, timing and shape for successive running cycles should be modelled as a function of bridge dynamic response, human perception to vertical vibrations and different external stimuli. To the best knowledge of the authors, experimental data necessary to accomplish this challenge is currently not available anywhere in the world.

When compared to the state-of-the-art approach to vibration serviceability design of bridges based on the Fourier series models, the key novelties and advantages of the proposed model are:

1. There is a very little to choose between measured and artificially generated force signals. The model presented accounts for the near-periodic nature of running loads by simulating natural variability of timing, amplitudes and energy of the successive running footfalls, thus offers a more realistic and less conservative vibration serviceability assessment of bridges.
2. The forces are described as a random rather than deterministic process, similar to other random excitation of civil engineering structures, such as due to wind, waves or earthquakes.
3. The modelling strategy is numerically too complex to generate artificial forces manually, but it can be coded and distributed to structural designers as computer software. This offers a great potential to modernise the vibration serviceability assessment by shifting from traditional hand calculations towards fully computerised design. Moreover, as the model generates synthetic force signals in a fraction of a second even on a standard office PC configuration, the software would enable more efficient and cost-effective structural design.

Acknowledgements

The authors would like to acknowledge the financial support provided by the UK Engineering and Physical Sciences Research Council (EPSRC) for Grant reference EP/E018734/1 (“Human Walking and Running Forces: Novel Experimental Characterisation and Application in Civil Engineering Dynamics”) and to thank all test subjects for participating in the data collection.

References


