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SPACECRAFT MAGNETIC ATTITUDE CONTROL USING APPROXIMATING SEQUENCE OF RICCATI EQUATIONS

Antonio L. Rodriguez-Vazquez, *Graduate Student Member, IEEE*

Maria A. Martin-Prats, *Senior Member, IEEE* and Franco Bernelli-Zazzera, *Senior Member, AIAA*

Abstract

This paper presents the results of a spacecraft attitude control system based on magnetic actuators designed for Low Earth Orbits. The control system is designed using a non-linear control technique based on the Approximating Sequence of Riccati Equations (ASRE). The behavior of the satellite is discussed under perturbations and model uncertainties. Simulation results are presented where the control system is able to guide the spacecraft to the desired attitude in a variety of different conditions.

Index Terms

satellite, attitude control, magnetic actuator, ASRE, Riccati Equation, Low Earth Orbit

I. INTRODUCTION

Magnetic Attitude Control systems are a research topic since 1960s. In [1], the Earth's Magnetic field is studied in order to design a control law with 3 axis and 2 axis coils as actuators. It is shown that coils could be used on board satellites for momentum dumping and coarse attitude control. Several advantages of magnetic coils over other types of actuators quickly showed up [2]:

- There is no need for special propellant. Thus, there is a saving in launch mass and a longer operational lifetime.
- They have no moving parts, so an improved reliability is achieved with absence of catastrophic failure modes.
- The electric power needed to command the actuator can be easily generated by the solar panels onboard.
- Smooth continuous operation in contrast to the impulsive character of mass expulsion systems.

PhD Student, Electronic Engineering Department, Universidad de Sevilla, Av. de los Descubrimientos SN - 41092 Sevilla, Spain. Email: antoniorv@us.es

Associate Professor, Electronic Engineering Department, Universidad de Sevilla, Av. de los Descubrimientos SN - 41092 Sevilla. Email: mmprats@us.es

Full Professor, Department of Aerospace Science and Technology, Politecnico di Milano, Via La Masa, 34 - 20156 Milano, Italy. Email: franco.bernelli@polimi.it

Due to these advantages, several control techniques have been proposed and implemented. These Magnetic Control Techniques, can be classified as passive and active [3]. A literature review of passive and active magnetic techniques is presented below. Passive magnetic attitude control methods are based on a fixed magnet rigidly mounted on the structure of the satellites. Similarly to the gravity gradient technique, a magnet generates a dipole moment which, like a simple compass, tends to align itself along the local direction of the Earth's magnetic field [4].

Although first proposed in 1963, this method of stabilization is still in use nowadays. In [5] and [6], a full description and design example of the permeable rods effect on the attitude is reviewed. In [7] or [8], another use of a passive magnetic attitude control system can be found. Finally, in [9], a full mathematical analysis of the magnetic hysteresis damper is presented.

A. Active Magnetic Attitude Control

Active magnetic control methods are an evolution of the passive methods. The key idea is to have a *time varying magnet* instead of a fixed one. This is easily achievable using a set of three mutually orthogonal magnetic coils fixed to the satellite structure [10].

The main problem of a magnetic control scheme is that a three axis control of a satellite cannot be performed using typical control algorithm. This is due to the nature of the magnetic actuation. The magnetic actuators generate a magnetic dipole in an arbitrary direction of the space which tends to align with an external magnetic field, typically the Earth's magnetic field. Therefore, the torque that this dipole can produce is contained in a plane perpendicular to the external magnetic field vector. This produces a loss of one degree of control, meaning that independent actions on the three degrees of freedom cannot be generated.

This problem has been usually worked out by using magnetic control techniques in conjunction with another control technique or anther actuators. One of the first and very well explained works regarding 3 axis control on a spinning satellite is [2]. In this work, the authors present an active magnetic control system that, together with a flywheel that gives momentum bias, can perform the initial acquisition [1], nutation damping, precession control and momentum Bias Regulation. Several modifications of the technique in [2] have been proposed for different uses. In [11] and [12] active attitude control systems for Sun pointing momentum biased satellites are designed. In [13] the same control system is applied to a small university satellite. In [14], the problem of optimal reaction wheel desaturation maneuver of a satellite using internal magnetorquers is discussed. In [15] another set of algorithms for spin stabilized spacecraft are proposed for three mission stages. In a more recent work [16][17], a spin stabilization control law by the use of magnetic only actuation is presented. It is shown that, with adequate orbit inclination, the control law globally asymptotically stabilizes a three inertial spacecraft, leading it to a desired spin condition in the inertial frame.

Gravity gradient has also been used together with active magnetic control. In [18], a detailed design report of a satellite mission using gravity gradient and magnetorquers is presented. In [19], a comparison of the attitude control system described in [18] and other control algorithms is presented. Those algorithms are based on fuzzy logic and Linear Quadratic Regulation. All of the controllers were able to keep the pointing accuracy of the satellite

to within 1 degree, although the best accuracy is achieved with fuzzy controller and the controller in [18]. In [20], a state feedback PD control law for the magnetic attitude stabilization of a nadir pointing spacecraft is proposed. In presence of gravity gradient, the control law is proven to guarantee stability for obit inclinations greater than 0 degrees.

B. Purely Magnetic Attitude Control

All the techniques presented so far are based on an inherent stability of the satellite's attitude. The stability can be because of gyroscopic stiffness (momentum biased satellites) or due to the design of the moment of inertia of the satellite (gravity boom). However, a bigger problem arises when the satellite has no mechanical nor physical stability.

This case is the main study of this work and has also been the subject of several works over the last few years. In [21], a good review of these techniques is available up to 2003. In this work, the authors classify the control techniques under three categories:

- Linear design methods
- Non-Linear design methods
- Predictive control

The main difference between these methods is the linearization of the attitude kinematics and dynamics of the attitude equations of a rigid solid in orbit in space. Despite the presentation of the equations of motion, all the techniques presented so far can be also classified depending on the design assumption made for problem statement. Therefore, the following classification is suggested:

- Periodic controllers
- Projection based controllers
- Full non-linear model controllers

1) Periodic Controllers: Periodic controllers are based on the periodic approximation of the magnetic field of the Earth as seen from the orbit of the spacecraft. The characteristics of the magnetic field will be reviewed later in section III. This is a first order approximation and is quite good for high altitude orbits.

Assuming the periodic nature of the magnetic field, the problem can be transformed into a periodic linear problem which can be stabilized by state periodic feedback or the more classical output feedback. A very good discussion and design of periodic controller and periodic control theory is presented in [22]. In this work, the author presents a wide range of controllers, form constant gain linear periodic controller to energy approaches for attitude control. Other works by the same author describe a finite horizon controller [23] and the design of H_2-optimal periodic controller[24]. Others designs of optimal linear periodic controllers are presented in [20], [25], [26], [27], [28].

The main advantage of periodic linear controllers is the use of Floquet's theory to prove stability[22], [29]. Floquet stability analysis computes the closed-loop state transition matrix for one period of the system and verifies that all of its eigenvalues have a complex magnitude less than unity.

2) Projection Based Controllers: Projection based controllers are based on the idea presented in [1], which is fairly simple. The main drawback of active magnetic control is the lack of one degree of freedom for 3 axis torque generation. In fact, torque can only be produced in the plane orthogonal to the local direction of the magnetic field ($\vec{B}$). Therefore a possible design rule is, given the ideal torque (T_{ideal}) for a fully actuated spacecraft, project that torque over the orthogonal plane of $\vec{B}$ obtaining the effective magnetic torque (T_{mag}). The moment generated by this technique and the torque applied are given by equations 1 and 2, where $S(\vec{B})$ is the skew-symmetric matrix of vector $\vec{B}$.

$$\vec{m} = \frac{1}{|\vec{B}|^2}S(\vec{B})'\vec{T}_{ideal} \tag{1}$$

$$\vec{T}_{mag} = S(\vec{B})\vec{m} = \frac{1}{|\vec{B}|^2}S(\vec{B})S(\vec{B})'\vec{T}_{ideal} \tag{2}$$

Examples of projection based controller can be found in the aforementioned [1] or [2]. In [21] a predictive controller which uses this technique is also suggested for attitude control. In a more recent work[30], the MATLAB toolbox for periodic system is used to design discrete controllers based on both optimal periodic controller and averaging techniques controller.

3) Non-linear controllers: Perhaps the most popular non linear controller proposed for attitude control is the B-dot detumbling algorithm. In [31], a related line of work has been devoted to the nonlinear analysis of a magnetic scheme based on the sole measurement of the magnetic field vector $\vec{B}$

Other nonlinear control systems are presented in [32]. In this work, the authors implement and compare six different algorithms for attitude control for nutation damping, coarse reorientation, spinning and fine reorientation of the satellite.

The present work introduces a purely magnetic control scheme for satellite attitude control. It is particularized for a nadir pointing spacecraft. Within this scheme, a new model which includes the satellite attitude dynamics and kinematics joined with the magnetic field of the Earth is introduced. The model is written in the State-Dependent-Coefficient (SDC) form. The control problem is formulated via the Approximating Sequence of Riccati Equation (ASRE) algorithm [33], and solved as a two-point boundary value problem. This algorithm was originally developed to solve guidance problems, whereas in this application it is used to solve sequences of control problems with fixed horizons. The main advantage of this approach is to retain the non-linearities inside the model without the need to assume periodic dynamics of the system. At the same time, the adoption of a fixed horizon scheme allows to promptly react to any unmodeled disturbance. The behavior of the control system under perturbations and model uncertainties is presented using simulations. Furthermore, several Low Earth obits have been tested in order to explore the stability of the control technique.

The paper is organized as follows. In section II the satellite dynamics and kinematics equations of motion are presented and particularized for the use of magnetic torquers. In section III, the Earth magnetic field model which

has been used through the paper is discussed. Sections IV and V are dedicated to the factorization of the problem in order to be solved by the ASRE control technique. Finally, some simulation results are presented in section VI.

II. Spacecraft Attitude Dynamics Model

In this section, the satellite equations of motions are used to derive a dynamic model. The attitude control algorithm will be designed to operate using only the magnetic torques generated by the actuators [34]. Consequently the attitude equations will consider three magnetic torquers, each one aligned with one principal axis.

A. Reference Systems

There are several coordinate systems that will be used through the paper. These coordinate systems are presented in figure 1 and described below.
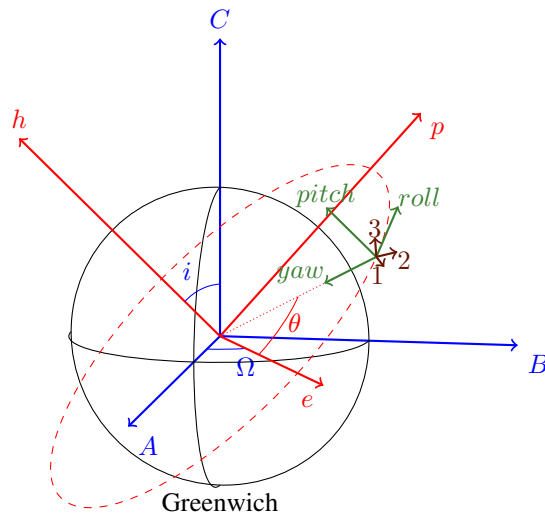

Figure 1. Reference Systems. Ω is the Right Ascension of the Ascending Node (RAAN), i is the orbit inclination and θ is the satellite anomaly.

- **Body Coordinate System** (pi), axes 1,2,3: This coordinate reference system is associated to the satellite body. It is centered in the center of mass of the satellite and its axis are oriented along the principal axis of the satellite.
- **Local Vertical Local Horizontal** (LVLH), axes roll, pitch, yaw: This coordinate system is centered in the center of mass of the satellite. The yaw axis points toward the center of the earth (nadir), the pitch axis is perpendicular to the orbital plane in the direction of the angular velocity and the roll axis is perpendicular to both pitch and yaw resulting in the tangential direction to the the orbit and in the sense of the instantaneous velocity.
- **Orbit Perifocal** (orb), axes e,p,h: The orbit perifocal coordinate system is a system for which the plane of the spacecraft orbit is the equatorial plane of the coordinate system. It is centered in the center of the Earth. The

e axis is parallel to the line from the center of the Earth to the Ascending Node (AN) of the spacecraft orbit, the h axis is parallel to the orbit normal and the p axis can be found using the right hand rule.

- **Earth Centered Earth Fixed** (ECEF), axes A,B,C: This coordinate system is fixed to the Earth. Its Z axis is aligned with the rotation axis of the Earth, the A axis point towards the intersection of the Greenwich meridian with the Equator and the B axis is perpendicular to both C and A and completes the right hand rule.

B. Attitude Dynamics Equation of Motion

The satellite attitude dynamics equation of motion [35] is

$$I\dot{\omega}_I = -\omega_I \wedge I\omega_I + N \tag{3}$$

where ω_I is the angular velocity of the satellite expressed in the pi reference with respect to (w.r.t.) an inertial frame, I is the inertia matrix and N are the external and control torques expressed in the pi frame.

The control system is particularized for a nadir pointing spacecraft. Therefore, the $LVLH$ reference frame is introduced as reference, because the three axes of this reference system are exactly the target attitude. A_{lvlh}^{pi} represents the transformation matrix from $LVLH$ to the pi and ω_{pi} is the angular velocity of the spacecraft w.r.t the $LVLH$ frame.

ω_I can be expressed as:

$$\omega_I = \omega_{pi} + A_{lvlh}^{pi}\omega_{lvlh} \tag{4}$$

and its derivative:

$$\dot{\omega}_I = \dot{\omega}_{pi} + \dot{A}_{lvlh}^{pi}\omega_{lvlh} + A_{lvlh}^{pi}\dot{\omega}_{lvlh} \tag{5}$$

where ω_{lvlh} is the angular velocity of the $LVLH$ frame w.r.t. an inertial frame. In order to obtain a simpler analytical solution of the problem, the orbit is considered to be circular. Due to this consideration, $\dot{\omega}_{lvlh} = 0$. Therefore:

$$\dot{\omega}_I = \dot{\omega}_{pi} - \omega_{pi} \wedge \left(A_{lvlh}^{pi}\omega_{lvlh} \right) \tag{6}$$

Substituting equation 6 and equation 4 into equation 3, the dynamic equation particularized for a nadir pointing spacecraft is obtained:

$$I\left[\dot{\omega}_{pi} - \left(\omega_{pi} \wedge A_{lvlh}^{pi}\omega_{lvlh} \right) \right] = -\left(\omega_{pi} + A_{lvlh}^{pi}\omega_{lvlh} \right) \wedge I\left(\omega_{pi} + A_{lvlh}^{pi}\omega_{lvlh} \right) + N \tag{7}$$

Furthermore, as a full magnetic control is designed, ignoring perturbation effects, $N = m \wedge B$, where m is the magnetic moment generated by the magnetic torquers and B is the magnetic field of the Earth in the principal inertia frame. These considerations permit to obtain the following dynamic equation of motion:

$$I\left[\dot{\omega}_{pi} - \left(\omega_{pi} \wedge A_{lvlh}^{pi}\omega_{lvlh}\right)\right] = -\left(\omega_{pi} + A_{lvlh}^{pi}\omega_{lvlh}\right) \wedge I\left(\omega_{pi} + A_{lvlh}^{pi}\omega_{lvlh}\right) + m \wedge B \tag{8}$$

C. Kinematic Equation of Motion

The kinematic equation can be found with different representations in literature [36]. In this work, the authors will use the quaternions [37]. A quaternion q is defined as:

$$q = [q_0, q_1, q_2, q_3]^T = [q_0, q_v^T]^T = \left[\cos\left(\frac{\alpha}{2}\right), \hat{e}^T \sin\left(\frac{\alpha}{2}\right)\right]$$

where $\hat{e}^T$ is the vector representing the unit rotation axis and α is the angle of rotation. Therefore, the kinematic equation using quaternions [35] is:

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -q_1 & -q_3 & -q_2 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix} \cdot \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{9}$$

Equations 8 and 9 will be discussed later in detail to derive the State Dependent Coefficient form, in order to clarify the motivations behind the proposed model.

III. MAGNETIC FIELD MODEL

The Earth's Magnetic Field can be represented as the gradient of the scalar potential function V

$$V(R, \theta_R, \lambda_R) = R_e \sum_{n=1}^{k} \left(\frac{R_e}{R}\right)^{n+1} \sum_{m=0}^{n} (g_n^m \cos m\lambda_R + h_n^m \sin m\lambda_R) P_n^m(\cos\theta_R) \tag{10}$$

Thus,

$$B = -\nabla V \tag{11}$$

where R_e is the equatorial radius of the Earth (6371.2 adopted for the international Geomagnetic Field, IGRF [38]); g_n^m and h_n^m are Gaussian coefficients; and R, θ_R and λ_R are the geocentric distance, co-elevation, and East longitude from Greenwich which define any point in space. P_n^m is the associated Legendre function of the first kind of degree n and order m:

$$P_n^m(x) = \frac{(1-x^2)^{m/2}}{2^n \cdot n!} \frac{\delta^{n+m}}{\delta x^{n+m}} (x^2 - 1)^n \tag{12}$$

These equations provide a complete framework for simulation purposes. However, for analytic purposes, it is convenient to obtain a *dipole model* by expanding the field model to first degree (n=1) and all orders (m=0,1). In this case, eq. 10 becomes:

$$
\begin{aligned}
V\left(R, \theta_R, \lambda_R\right) & = \frac{R_e^3}{R^2}\left[g_1^0 P_1^0\left(\cos \theta_R\right) + \left(g_1^1 \cos \lambda_R + h_1^1 \sin \lambda_R\right) P_1^1\left(\cos \theta_R\right)\right] \\
& = \frac{R_e^3}{R^2}\left(g_1^0 \cos \theta_R + g_1^1 \cos \lambda_R \sin \theta_R + h_1^1 \sin \lambda_R \sin \theta_R\right)
\end{aligned}
\tag{13}
$$

The $\cos \theta$ term is just the potential due to a dipole of strength $g_1^0 R_e^3$ aligned with the polar axis. Similarly the $\sin \theta$ terms are dipoles aligned with the x and y axes. Relying on the principle of linear superposition, these terms are just the Cartesian components of the dipole component of the Earth's magnetic field. For year 2010:

$$
\begin{aligned}
g_1^0 & = -29496.5 nT \\
g_1^1 & = -1585.9 nT \\
h_1^1 & = 4945.1 nT
\end{aligned}
$$

Therefore, the total dipole strength is:

$$
R_e^3 H_0 = R_e^3 \sqrt{\left[\left(g_1^0\right)^2 + \left(g_1^1\right)^2 + \left(h_1^1\right)^2\right]} = 7.746 \times 10^{15} Wb \cdot m
\tag{14}
$$

The coelevation of the dipole can be expressed as:

$$
\theta_m' = \arccos\left(\frac{g_1^0}{H_0}\right) = 170.0^\circ
\tag{15}
$$

The East longitude of the dipole is the following:

$$
\phi_m' = \arctan\left(\frac{h_1^1}{g_1^1}\right) = 107.8^\circ
\tag{16}
$$

Now the magnetic field of the Earth can be approximated as due to a dipole vector, m, whose magnitude and direction are given by eqs. 14 to 16. Thus,

$$
B\left(R\right) = R_e^3 H_0 \left[\frac{3\left(m \cdot R\right) R}{\|R\|^5} - \frac{m}{\|R\|^3}\right]
\tag{17}
$$

Where R is the position vector of the point at which the field is desired.

IV. State-Dependent Coefficients (SDC) factorization

Merging in one single model the attitude dynamics equations of motion, the kinematics and the dipole model of the magnetic field, the system dynamics can be factorized in State Dependent Coefficients (SDC) form:

$$
\dot{x} = A\left(x\right) x + B\left(x\right) u
\tag{18}
$$

where A models the spacecraft system dynamics and B models the actuators effect.

Let $x = \left[\omega_{pi}^T, q_v^T\right]^T$ be the state vector. A reduced quaternion q_v is considered to describe the kinematics [39] :

$$
\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \sqrt{1 - q_1^2 - q_2^2 - q_3^2} & -q_3 & q_2 \\ q_3 & \sqrt{1 - q_1^2 - q_2^2 - q_3^2} & -q_1 \\ -q_2 & q_1 & \sqrt{1 - q_1^2 - q_2^2 - q_3^2} \end{bmatrix} \cdot \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{19}
$$

where q_0 has been substituted by $\sqrt{1 - q_1^2 - q_2^2 - q_3^2}$.

Equation 8 can be rewritten using the property $a \wedge b = S(a) \cdot b$ where $S(\cdot)$ is defined as the operator which obtains the skew-symmetric matrix using the coefficients of a given vector. Let $a = [a_1, a_2, a_3]^T$. Then:

$$
S(a) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{20}
$$

There are several options to factorize the non-linear problem into the SDC form. Among the possible, the following form has been selected:

$$
A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix} \tag{21}
$$

$$
A_{11} = -S(A_{lvlh}^{pi} \omega_{lvlh}) - I^{-1} \cdot S(w_{pi}) \cdot I + I^{-1} \cdot S(I \cdot A_{lvlh}^{pi} \omega_{lvlh}) - I^{-1} \cdot S(A_{lvlh}^{pi} \omega_{lvlh}) \cdot I
$$

$$
A_{12} = \begin{bmatrix} 2\xi \omega_{orb}^2 q_0 \frac{Iyy - Izz}{Ixx} & 0 & 2\xi \omega_{orb}^2 q_2 \frac{Iyy - Izz}{Ixx} \\ 0 & 2\xi \omega_{orb}^2 q_0 \frac{Ixx - Izz}{Iyy} & 2\xi \omega_{orb}^2 q_1 \frac{Izz - Ixx}{Iyy} \\ -4\omega_{orb}^2 \frac{Ixx - Iyy}{Izz} (q_0 q_2 - q_1 q_3) q_0 & 0 & -4\omega_{orb}^2 \frac{Ixx - Iyy}{Izz} (q_0 q_2 - q_1 q_3) q_2 \end{bmatrix}
$$

$$
A_{21} = \frac{1}{2} [S(q_v) + q_0 I_3]
$$

where $\xi = q_0^2 - q_1^2 - q_2^2 + q_3^2$ and I_3 is the identity matrix of order 3. Matrix B becomes:

$$
B = -I^{-1} S(\beta_{pi}) \tag{22}
$$

where β_{pi} is the Magnetic field in the principal axis reference frame. To calculate β_{pi} equation 17 is used. This is a vectorial equation that is valid in whatever the reference frame that is used provided that both R and m are centered in the reference system. As a circular orbit has been considered, R is very easy to calculate in the Orbit reference system:

$$
R_{lnb} = R_{orbit} \begin{bmatrix} \cos(\theta_p) \\ \sin(\theta_p) \\ 0 \end{bmatrix} \tag{23}
$$

where θ_p is the satellite anomaly. The orientation of the magnetic dipole can be derived using standard rotations. Therefore:

$$m_{lnb} = \begin{bmatrix} \sin(\theta'_m) \cdot \cos(\Omega - \alpha_m) \\ -\sin(\theta'_m) \cdot \cos(i) \cdot \sin(\Omega - \alpha_m) + \cos(\theta'_m) \cdot \sin(i) \\ \sin(\theta'_m) \cdot \sin(i) \cdot \sin(\Omega - \alpha_m) + \cos(\theta'_m) \cdot \cos(i) \end{bmatrix} \tag{24}$$

where Ω is the Right Ascension of the Ascending Node (RAAN), i is the orbit inclination and $\alpha_m = \phi'_m + \alpha_G$. α_G is the angle of rotation of the Earth and θ'_m and ϕ'_m are given by equations (15) and (16). Substituting equations 23 and 24 into equation 17, the magnetic field in the Orbit Perifocal frame (B_{lnb}) is given. Thus, β_{pi} is:

$$\beta_{pi} = A_{lnb}^{pi} \cdot B_{lnb} \tag{25}$$

where A_{lnb}^{pi} is the transformation matrix between the Orbit Perifocal reference frame and the Principal Inertia frame and is given by:

$$A_{lnb}^{pi} = \left[(q_0^2 - q_v^T q_v) I_3 + 2 q_v q_v^T - 2 q_0 S(q_v) \right] \begin{bmatrix} \cos(\theta_P) & \sin(\theta_P) & 0 \\ -\sin(\theta_P) & \cos(\theta_P) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{26}$$

V. Description of the Applied Control Technique (ASRE)

The model written in the SDC form is given by equation 18. However, the matrix B depends on the state $x = \left[\omega_{pi}^T, q_v^T \right]^T$, on the satellite anomaly (θ_p) and on the angle of rotation of the Earth (α_G). The latter two parameters, can be easily made time dependent by the equations 27 and 28.

$$\theta_p = \omega_{orb} \cdot t + \theta_{p0} \tag{27}$$

$$\alpha_G = \omega_{Earth} \cdot t + \alpha_{G0} \tag{28}$$

Thus, the nonlinear system dynamics is given by

$$\dot{x} = A(x, t) x + B(x, t) u \tag{29}$$

This model can be used to solve a nonlinear control problem, formulated as to minimize the nonlinear objective function of the form

$$J = \frac{1}{2} x^T(t_f) S(x(t_f), t_f) x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} x^T Q(x, t) x + u^T R(x, t) u \, dt \tag{30}$$

where S, Q are two positive-semidefinite matrices and R is a 3x3 positive definite matrix. This problem is solved with a method named approximating sequence of Riccati equations (ASRE) [33]. This method has been used in the literature to solve the nonlinear nonaffine control problem [40]. The method is based on transforming the problem into an equivalent time variant problem with the introduction of the iterative sequence corresponding to the system dynamics:

$$\dot{x}^0(t) = A(x_0, t)x^0(t) + B(x_0, t)u^0(t)$$

$$\dot{x}^i(t) = A(x^{i-1}(t), t)x^i(t) + B(x^{i-1}(t), t)u^i(t) \tag{31}$$

$$x^i(t_0) = x_0 \quad i = 1, \ldots, N$$

and the iterative sequence corresponding to the cost function to be minimized:

$$J^i = \frac{1}{2}x^{iT}(t_f)Sx^i(t_f) + \frac{1}{2}\int_{t_0}^{t_f} x^{iT}Q\left(x^{i-1}, t\right)x^i + u^{iT}R\left(x^{i-1}, t\right)u^i \, dt \tag{32}$$

The problem described by Equations 31 and 32 can be solved as a sequence of *Two Points Boundary Value Problems*. This iterative method was firs designed to solve guidance problems as described in detail in Reference [41]. In this work, the control algorithm is used to solve sequences of control problems with fixed horizons. This application scheme seems suitable to problems where model uncertainties are relevant so that the controller can correct the non perfect system modeling.

VI. SIMULATION RESULTS

In order to check the validity of the method, simulations have been performed to analyze the performance of the control scheme for a nadir pointing satellite. Numerical parameters for the orbit and inertia properties of the satellite are representative of a small satellite in a LEO orbit. Due to the model properties, the satellite must be spinning and its orbit can not be equatorial in order to get a controllable system. Specifically, the inertia moments of the satellite are:

$$I_{11} = 1.0 \, [kg \cdot m^2]$$
$$I_{22} = 1.2 \, [kg \cdot m^2] \tag{33}$$
$$I_{33} = 1.5 \, [kg \cdot m^2]$$

The orbit parameters are:

$$i = 15 \cdot pi/180 \, [rad]$$
$$\Omega = 0 \, [rad] \tag{34}$$
$$R_{orbit} = R_{Earth} + 600 \, [km]$$

Given the inertia and orbit parameters of the spacecraft, the performance of the control system will be highly dependent on the way the performance index is defined (Equation 32) and on the control time span $t_f - t_0$. For the case under consideration a control time span of 5 seconds has been adopted, which means that one orbit is covered in 1160 time spans.

The matrices Q, S and R have been chosen as:

$$S = Q = diag\left(\left[\begin{array}{cc} \frac{1}{\bar{\omega}^2} & \frac{1}{\bar{q}^2} \end{array}\right]\right) \quad R = I_3 \tag{35}$$

where $\bar{w} = \omega_{orb}$ is the value of the nominal angular velocity in pitch and $\bar{q} = \sin(3\pi/180)$ is the quaternion value for an angle of 3 degrees. This choice, common in the definition of optimal control problems, assumes that:

- equal importance is given to each axis;

- three degrees angular error and angular velocity error equal to the nominal pitch angular velocity are the maximum admissible;

- the maximum dipole moment generated by the actuators should be $1 \, [A \cdot m^2]$, that is within the performances of commercial magnetic actuators for this class of satellites.

A. Perturbation Model

In order to test the performance of the algorithm in a more realistic environment, two main perturbations have been considered for the simulation. First, a model uncertainty in the magnetic field has been introduced. This in principle could affect the performance of the actuator since the control is evaluated on the basis of the model of magnetic field included in matrix B. The effective action instead depends on the real magnetic field that is different from the one included in the model. As described before in section III, the Earth's magnetic field can be reproduced very accurately with the model introduced by the equations (10) to (12). However, only the first grade of the polynomial, which is equal to a dipole, is taken into account in the control design. Therefore as an example of model uncertainty a thirteenth order polynomial has been introduced in the simulation as the real magnetic field surrounding the spacecraft. The magnitude of the Earth's magnetic field modeled by the dipole and by the 13th order model are shown in figure 2
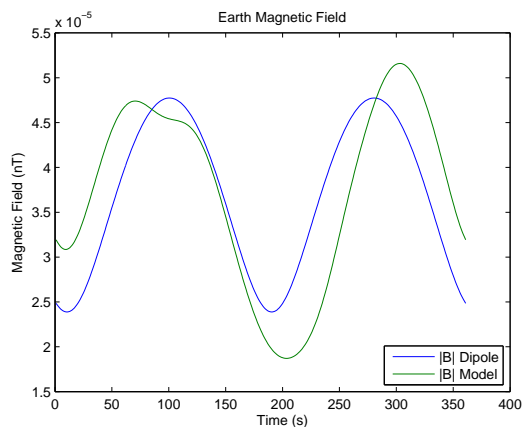

Figure 2.   Magnitude of the magnetic field during one orbit

The other perturbation considered is the gravity gradient. In this model, it is representative of a net external torque applied to the spacecraft which is not included in the control model. As such, its effect on the dynamics can be considered equivalent to internal disturbances like those due to internal magnetic fields that interact with the actuators. Every rigid object which is not symmetrical is subject to a torque produced by the effect of the

gravitational force over each point of the object. Assuming the Earth as a point-mass, the gravitational force $d\vec{F_i}$ over an infinitesimal point i of mass dm_i on the satellite is:

$$dF_i = -\frac{\mu \vec{R_i} dm_i}{R_i^3} \tag{36}$$

where μ is the Earth gravity constant, $\vec{R_i}$ is the vector Earth-to-satellite point and R_i is the module of $\vec{R_i}$. Therefore the torque produced by the Earth gravity is:

$$\vec{M_g} = \int_B \vec{r_i} \wedge d\vec{F_i} \tag{37}$$

where $\vec{r_i}$ is the vector between the center of gravity of the satellite and the point i within the satellite body. This torque could also be included inside the dynamic model of the system, but it has been used as an external perturbation to show the effectiveness of the control algorithm under a more demanding environment. It is worth to point out that with the inertia moment defined in equation (33), the gravity gradient effect is destabilizing.

Simulations have been performed first with small error on the initial conditions and second with large initial conditions which might be the result of a detumbling control technique such as Bdot.

B. Small Initial conditions

The initial conditions for these simulations are initial angular velocity error one order of magnitude less than nominal angular velocity (ω_{orb}) and an error in the target nadir pointing attitude of 2 degrees in roll, 1 degree in yaw and 0 degrees in pitch, the axis of the nominal angular velocity.

A simulation of the behavior of the system has been performed for 10 orbits. For the whole control period the control algorithm has reached convergence in at most two iterations. The results of the simulations are shown in Figures from 3 to 6.

Figures 3 and 4 represent the evolution over time of the states of the system in the ideal case and considering perturbations. The largest angular error considered in this simulations is greatly reduced in one orbit. After it, a lower damping slowly drives the error to zero. It can be noticed that in this simulation the effects of external perturbation is beneficial in terms of damping.

In Figures 5 and 6 the control signal is shown. Figure 5 reports only the value of the control at the beginning of each time span which turns out to be also the maximum value within each time span as reported in detail in Figure 6. Analysis of the envelope of the control demonstrates that the control actions are well within the target limits assumed in the definitions of the weight matrix R. This analysis considers only the magnetic moment generated by the actuators, assuming that the spacecraft is able to supply the corresponding electric power.

Figure 6 shows in detail how the control signal is generated and applied within each time span. Every 5 seconds the control problem defined by Equations 31 and 32 is solved with initial conditions equal to the final conditions at the end of the previous time span. It can be noticed that the oscillations in the control signal are not important
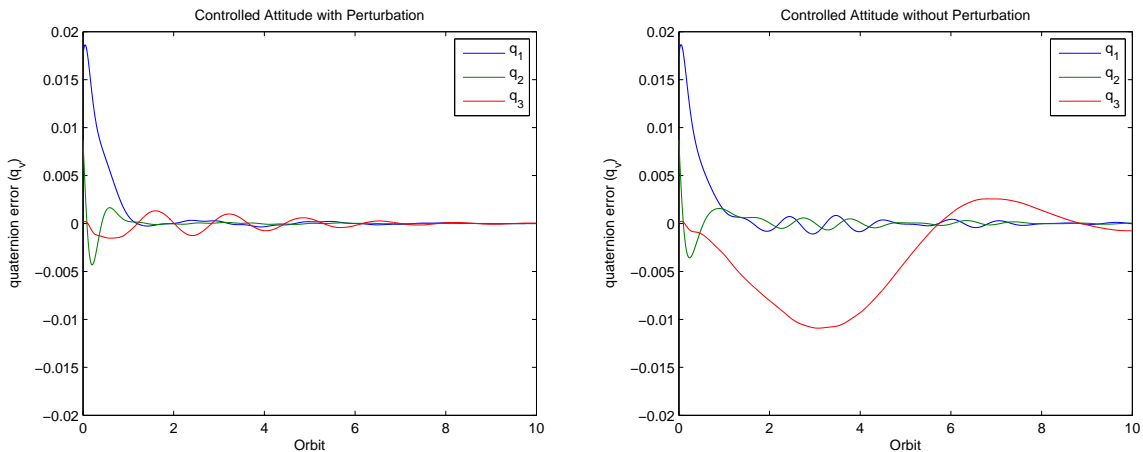
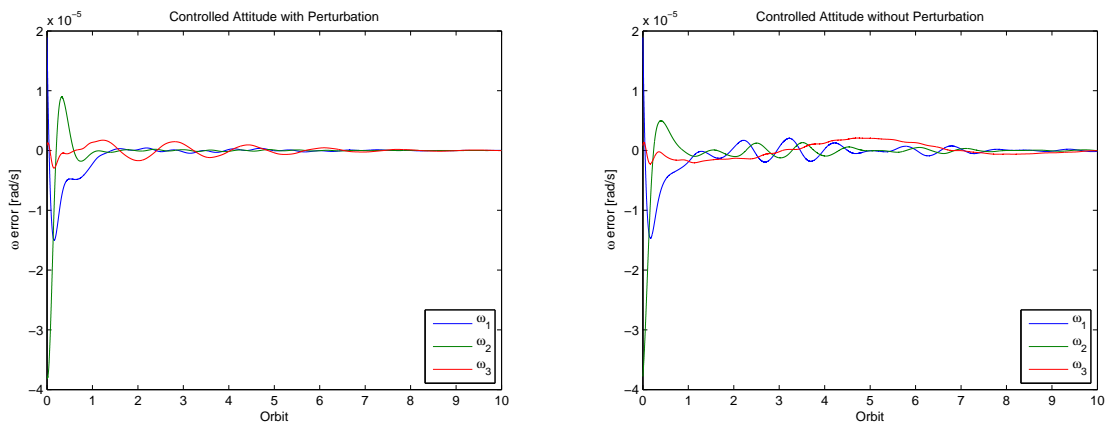Figure 3. Attitude quaternion error

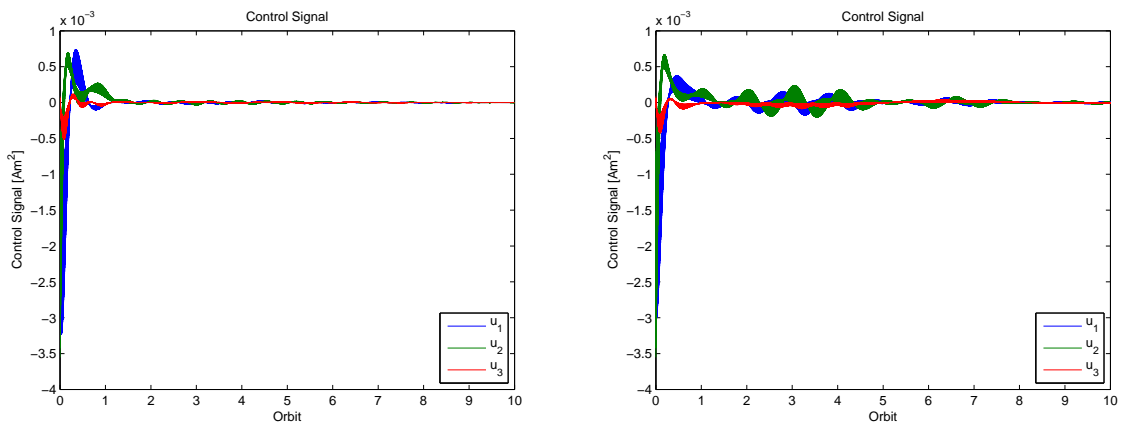Figure 4. Angular velocity error

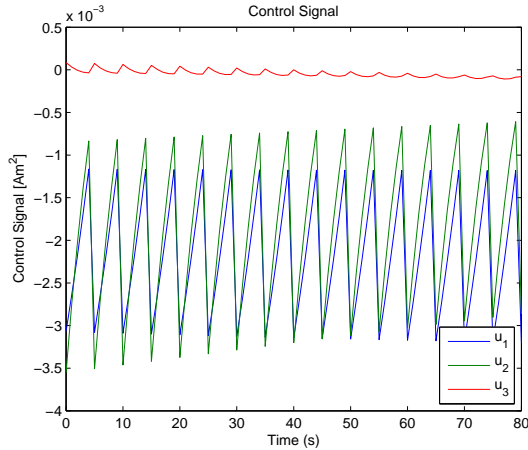Figure 5. Envelope of the Control Signal

Figure 6.   Detail of the Control Signal

in this context, because they only represent a change in the current flow through the coils and do not imply any mechanical motion inside the actuators.

C. Large Initial conditions

Another set of simulations have been carried out in order to test the behavior of the attitude control system in more realistic conditions. Such condition are chosen as a set of angular velocities that could result at the end of detumbling with a B-dot like algorithm [42]. The B-dot algorithm can usually carry the spacecraft to a total angular rate in the order of the nominal angular rate of the nadir pointing spacecraft. To be conservative, the following initial angular rates, which are an order of magnitude higher than required, have been chosen:

$$\omega_{ini} = \begin{bmatrix} 0.06 \\ 0.08 \\ 0.06 \end{bmatrix} [rad/s] \tag{38}$$

In these conditions, the results of the simulations are presented in figures 7 to 9 with and without perturbations. Although the initial conditions in angular velocity are large, the control system is able to reduce the angular velocity in two of the three axes very quickly, although it takes some orbits to reduce the angular velocity in the pitch axis. Nevertheless the results show that it is feasible to implement this 3 axis control system together with a Bdot control law.

D. *Sensitivity to orbit*

The results presented so far show a good performance for a certain orbit and several initial conditions. A set of simulations have been carried out in order to test the control system for several orbital conditions.

Two performance parameters have been defined to present the results and evaluate the behavior of the satellite.
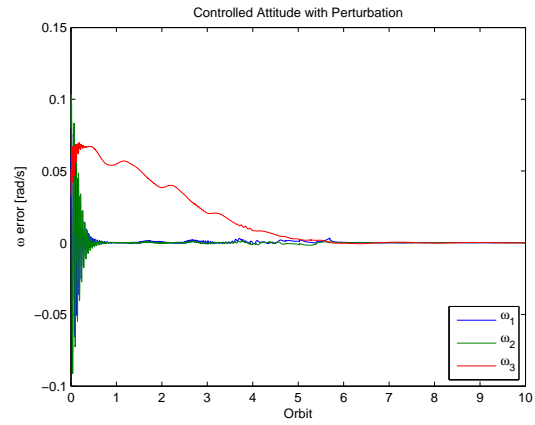
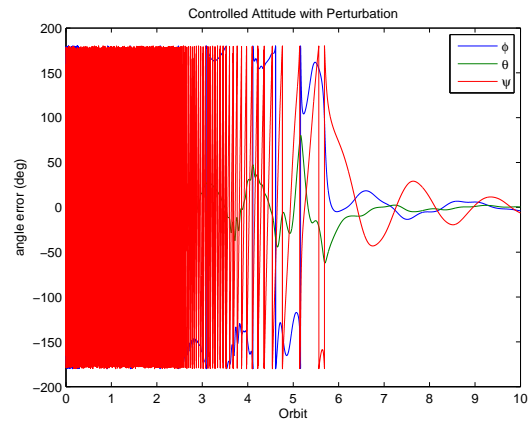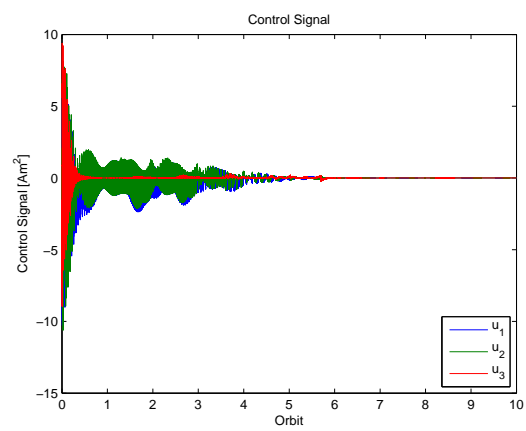Figure 7.   Angular velocity error

Figure 8.   Angle error

Figure 9.   Envelope of the Control Signal

The first parameter takes into account the *control time*, defined as the time for which all components of the quaternion vector become permanently less than 0.005; it means $|q_i| \leq 0.005$ (see Figure 10).
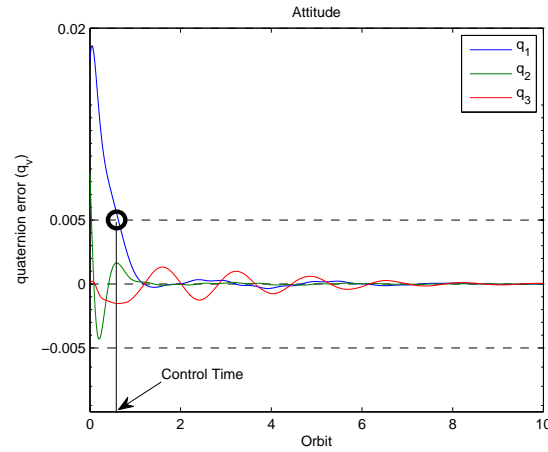

Figure 10.   Control Time Definition

The second parameter is intended to measure the *effectiveness of the control*. Therefore, the mean value of the quaternion error vector during the 10th orbit is chosen.

RAAN	0	45	90	135	180	225	270	315
inclination	Time [orbits]							
30	4,43	5,55	6,11	5,62	4,15	2,03	2,01	2,17
45	2,02	2,16	3,46	1,91	1,77	1,7	1,71	1,83
60	1,7	1,77	1,72	1,62	1,53	1,49	1,51	1,58
75	1,49	1,53	1,5	1,41	1,3	1,27	1,34	1,42
90	1,32	1,35	1,27	1,05	1,03	**1,02**	1,05	1,2

Table I

CONTROL TIME, ORBIT HEIGHT = 500 KM

The numerical results are resumed in tables II and I, where the best values are in bold. These results show that the control system works for a wide variety of orbits and that a better performance is achieved with high inclination orbits. Orbits with higher inclinations allow the satellite to move in a magnetic field varying much more than in orbits with inclinations closer to Earth Magnetic equator.

VII. CONCLUDING REMARKS

In this work, a new algorithm for magnetic satellite attitude control system design is presented. In this method, only magnetic actuators are needed and three axis pointing accuracy is achieved using a non linear technique called *Approximating Sequence of Riccati Equations (ASRE)*. This technique is based on transforming the nonlinear control

RAAN	0	45	90	135	180	225	270	315
inclination	Quaternion vector mean value [*1e-7]							
30 [*1e-3]	0,08	0,08	0,30	0,46	0,24	0,31	0,12	0,07
45 [*1e-5]	0,04	0,22	0,91	1,48	0,66	0,10	0,02	0,01
60	0,03	0,07	0,07	0,15	0,21	0,09	0,03	**0,01**
75	0,09	0,04	0,02	0,02	0,03	0,06	0,10	0,12
90	0,15	0,15	0,15	0,13	0,11	0,11	0,12	0,14

Table II

QUATERNION VECTOR MEAN VALUE, ORBIT HEIGHT = 500 KM

problem into an equivalent time variant problem with the introduction of the iterative sequence corresponding to the system dynamics and another iterative sequence corresponding to the cost function to be minimized. The new problem can be solved as a sequence of two point boundary value problems using the costate transformation as a soft Constrained Problem.

ASRE technique can solve the control problem for a given horizon time. However, it is not efficient at all to solve the problem for a whole orbit, because the simulated state will not match the real state due to perturbation or model uncertainties. Therefore, a time span is introduced. Then, the control will be calculated in every time span period. Reducing the time span also reduces the convergence time for a given hardware setup. However, reducing the time span also reduces the time period of the calculated control, thus the algorithm should be rerun at a higher frequency. Indeed, another possibility might be to introduce a receding horizon technique. If the algorithm is to be implemented on board a given hardware, testing of the time the hardware takes to solve the ASRE problem should be undertaken.

The control system has been intensively proven for a wide variety of orbits and initial conditions. Model uncertainties and perturbations have been also taken into account. The results show that the control system works for a wide variety of orbits and that a better performance is achieved with high inclination orbits. Furthermore the control strategy is able to control the satellite in case of large angular rates such as the ones that will remain after a detumbling phase using the well known Bdot algorithm.

The performance of the algorithm is presented for several orbits and different initial conditions. A direct comparison of the performances with other control systems has not been done. However, on a qualitative basis, it can be stated that the performances are similar to those found in the literature, where the time scale of the damping is measured in terms of orbits.

It is also shown that the control actions are well within the limits of the actuators for a satellite with similar inertia moments as the one under test. However the performances can be tailored to the available control power.

Finally, as a control algorithm, there are some restrictions on the computational time that any control algorithm can take in order to be implemented in real time. The time to get to a solution for the control signal is in the order of hundreds of milliseconds for the simulations presented in this paper. These simulations have been run using

Matlab in a first generation i7 processor at a clock speed of 2.66 Ghz. In any case, the algorithm can be tailored to the available hardware by posing a limit in the number of iterations for every step and by adjusting the time span.

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