TEMPORARY BLUFFING CAN BE REWARDING IN SOCIAL SYSTEMS: THE CASE OF ROMANTIC RELATIONSHIPS

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1. INTRODUCTION

Complex systems can often be in different stationary regimes, called *alternative stable states*. This is a well-established fact supported by empirical evidence in many fields of science, from physics to chemistry and from biology to economics. When a system has alternative stable states, it can happen that a microscopic perturbation

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triggers an unexpected macroscopic transition from one alternative stable state to another. These surprises often perceived as extremely positive/negative are called *catastrophes* ("revolutions" in ancient Greek). For example, the transitions from health to depression (and vice versa) can be abrupt and unexpected, and this is why their prediction is a major problem in psychiatry. When the alternative stable states are only two, one of them is often perceived as "good" and the other as "bad." Thus, if a system is trapped in its bad state, the problem is how to escape from it by moving into the basin of attraction of the good state. A brute force sol-ution of this problem is to modify permanently one or more characteristic traits of the system in such a way that the modified system has only one stable state, namely, the good one. A more subtle and often less costly solution is to temporarily apply a suitable control action that forces the system to enter into the basin of attraction of the most desired state. After this is accomplished, the control action can be interrupted because the dynamics of the uncontrolled system converge to the desired regime. The control action used to perform the switch is obviously very different from case to case, for example, a therapy for mental health hazards, a subsidy for promoting the production of a new crop, or the free access to a newly available service. In systems involving individuals or groups, bluffing or cheating is also frequently used to remain in or switch to favorable states: an athlete simulating to be in good shape during a competition, a union threatening a strike, a party freezing its support to the government are just a few examples. (For a rich and interesting discussion, see "Art of Lying, Cheating, Bluffing, Mind Stimulating Research," at http://answers.google.com/answers/threadview?id=287562.)

The existence of alternative stable states and catastrophes in a variety of social systems is described without the use of formal models in a great number of contributions, for example, collapse of ancient societies (Diamond, 2005; Tainter, 1990), mysterious sociological changes (Gladwell, 2000), resilience in human systems (Gunderson & Holling, 2002), and adaptive behavior in groups and societies (Scheffer & Westley, 2007). Unfortunately, the study of the same systems in rigorous mathematical terms is much more difficult and hence rare.

The existence of so many systems with alternative stable states and the relevant extent of temporary bluffing in our societies at any level of aggregation (from cou-ples to families, from parties to countries) suggest that temporary bluffing can often be used to promote the switch from a bad to a good state in social systems. Here a formal support is given to this conjecture by studying the simplest possible case, namely that of interpersonal relationships. The study is based on an already published mathematical model (Rinaldi & Gragnani, 1998a) in which the two individuals respond positively to any increase of the partner's love and can bluff by giving to the partner a biased impression of their involvement and appeal. Even if the study is focused on the simplest possible unit of interest in sociology—the couple—we believe that our results are of general value because we do not see conceptual obstacles preventing the extension of our analysis to interactions among larger social groups. In any case, the original contribution of our study is not to show that bluffing is a common practice in love affairs, a fact that is so well known, even from personal experiences, that it is not even discussed in the literature, but to explain why it is so. This distinction is very important in science, where knowing why something happens is often more important than knowing it happens.

2. A MATHEMATICAL MODEL OF LOVE AFFAIRS

Love stories are dynamical processes in which involvements (often called *feelings*) evolve in time, starting, in general, from a state of indifference. For this reason, love stories can be casted, at least in principle, in the formal frame of dynamical systems theory, where mathematical models are used to describe the time evolution of the variables of concern. The models most frequently used in science from the times of Newton are based on ordinary differential equations (ODEs), and in this respect *love dynamics* is not an exception. A naive model has been studied by Strogatz (1998) in a seminal paper, and the analysis has been extended to a series of more general ODE models of abstract romantic relationships (Femlee & Greenberg, 1999; Rinaldi, 1998; Rinaldi & Gragnani, 1998a, 1998b; Rinaldi, Della Rossa, & Dercole, 2010; Sprott, 2004, 2005). Complex issues involving optimal control theory (Feichtinger, Jorgensen, & Novak, 1999; Rey, 2010) have also been discussed. Continuous-time models based on ODEs or analogous discrete-time models have been used to describe emotions (Buder, 1991; Gottman, Murray, Swanson, Tyson, & Swanson, 2002; Gottman, Swanson, & Swanson, 2002) and physiological synchrony (Butler, 2011; Helm, 2013) in romantic partners. Love stories involving more than two individuals have also been studied (Dercole & Rinaldi, 2014; Sprott, 2004).

The majority of the models suggested in the past for the description of love stories are composed of two ODEs, one for each partner, that is,

$$\begin{aligned} \dot{x}_1(t) &= f_1(x_1(t), x_2(t), A_2) \\ \dot{x}_2(t) &= f_2(x_1(t), x_2(t), A_1). \end{aligned} \tag{1}$$

In these models the state variables $x_i(t)$, $i = 1, 2$, are the *feelings* at time t of the individuals for their partner (see Gottman, Murray, et al. [2002] for a relevant exception), while A_1 and A_2 are their *appeals*. In accordance with Levinger (1980), positive values of the feelings range from sympathy to passion while negative values are associated with antagonism and disdain. The appeals, as well as all other parameters specifying the psychical characteristics of the individuals, are assumed to be time-invariant, at least for long and specified intervals of time. In reality, these parameters, or some of them, can vary over time. Some of these variations are independent from the will of the individuals (e.g., the fading of physical attractiveness, or the loss of interest in sex), while others are consequences of conscious interventions of the individuals (e.g., the systematic attempt to hide romantic involvements). It is therefore sensible to ask which are the interventions that improve the quality of romantic relationships. The most simple interventions, here called *permanent*, are those due to a sudden and uninterrupted deviation of a parameter from its natural value. The consequences of permanent interventions can be determined through simulation or, more effectively, through bifurcation analysis. These analyses have already been performed and published by various authors. In particular, the consequences of permanent parametric variations in linear models have been derived analytically (Rinaldi, 1998) and then shown to hold true also in nonlinear models (Rinaldi & Gragnani, 1998a). A recurrent (and expected) result of these studies is that an increase of the appeal improves the quality of the romantic relationship. In this article we consider nonpermanent, that is, *temporary* (possibly short), interventions.

Since deviations from the natural individual behavior have, in general, a cost, or, equivalently, require a certain effort, it is natural to ask if a temporary deviation from the natural conditions, that is, a deviation for a limited but suitable amount of time, can be sufficient to generate positive consequences on the final quality of the romantic relationship. In other words, we like to know if a temporary deviation from the natural behavior, here called *temporary bluffing*, can improve the quality of the relationship in the long term, after the bluffing has ceased. For example, we like to know if "being on one's best behavior in early dating" pays in the long term.

The appeal of individual i has various components A_i^h like physical attractiveness, courage, education, wealth and others, which are independent of the feeling x_i. If λ_j^h is the weight that individual j ($j \neq i$) gives to the h -th component of the appeal of his/her partner, we can define the appeal of i (perceived by j) as

$$A_i = \sum_h \lambda_j^h A_i^h.$$

Thus, the appeal is not an absolute character of the individual but rather a value perceived by his/her present (or future) partner.

Two persons meeting for the first time at $t = 0$, are, in general, indifferent one to each other; that is, $x_1(0) = x_2(0) = 0$. Then, the feelings evolve in accordance with Eq. (1) where the rates of change f_i are dictated by the unbalance between regener-ation and consumption processes.

The consumption process, called *oblivion*, explains why individuals gradually lose memory of their partners after separating. As done in almost all fields of science, losses are assumed to occur at exponential rate, that is, $x_i(t) = x_i(0)\exp(-\alpha_i t)$ or, equivalently, in terms of ODEs, $\dot{x}_i(t) = -\alpha_i x_i(t)$, where α_i is the so-called *forgetting coefficient*. In contrast, the regeneration processes are of two distinct kinds, namely *reaction to appeal* and *reaction to love*. The flow of interest generated in individual j by the appeal of the partner is obtained by multiplying A_i by a factor ρ_j identifying the *sensitivity* of individual j to appeal, while the second regeneration process, the reaction to the love of the partner, is described by a function $R_j(x_i)$. The most standard individuals, usually called *secure*, are those who like to be loved. An individual i belonging to this class is formally characterized by an increasing function $R_i(x_j)$ that identifies the flow of interest generated in individual i by the love x_j of the partner. In order to capture the psychophysical limitations present in all individuals the reaction functions are assumed to be bounded as shown in Figure 1. The functional forms of these reaction functions are specified in the caption of Figure 1.

Thus, in conclusion, a reasonable model for couples of secure individuals is

$$\begin{aligned} \dot{x}_1(t) &= -\alpha_1 x_1(t) + \rho_1 A_2 + R_1(x_2) \\ \dot{x}_2(t) &= -\alpha_2 x_2(t) + \rho_2 A_1 + R_2(x_1). \end{aligned} \tag{2}$$

The reason why we limit our attention to model (2) with increasing and saturating reaction functions (see Figure 1) is that such a model is perfectly suited for mimicking the characters of the individuals involved in the love story described in the following. All properties of model (2), including bifurcations, are thoroughly discussed in Rinaldi and Gragnani (1998a) and Rinaldi et al. (2010), where the interested reader can find all

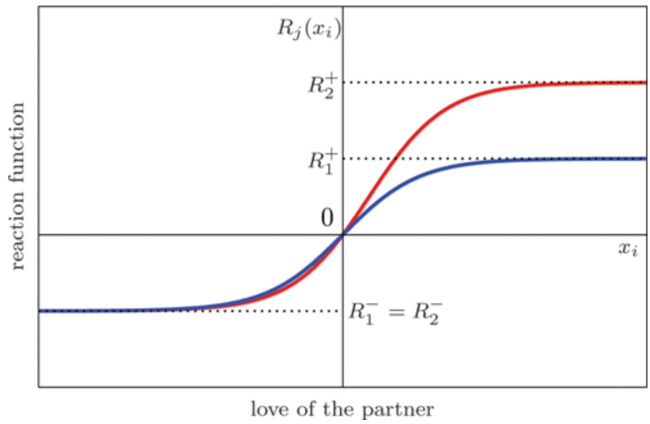

FIGURE 1 Typical reaction functions $R_1(x_2)$ (blue) and $R_2(x_1)$ (red) of two secure individuals (see Rinaldi et al., 2010). The two graphs correspond to $R_1(x_2) = \frac{e^{x_2} - e^{-x_2}}{e^{x_2}/R_1^+ - e^{-x_2}/R_1^-}$ and $R_2(x_1) = \frac{e^{x_1} - e^{-x_1}}{e^{x_1}/R_2^+ - e^{-x_1}/R_2^-}$; with $R_1^+ = 1$, $R_2^+ = 2$, and $R_1^- = R_2^- = -1$. These two functions are used in the following to describe Roxane and Cyrano.

mathematical details. Model (2) has also been used in Rinaldi, Landi, and Della Rossa (2013) and Rinaldi, Della Rossa, and Landi (2014) to show that small discoveries can have great and unexpected consequences in love affairs.

3. ALTERNATIVE STABLE STATES AND TEMPORARY BLUFFING

The divergence of model (2) is equal to $-(\alpha_1 + \alpha_2)$. Since it does not change sign, the attractors (capturing the long term behavior of the system) can only be equilibria—limit cycle being excluded by Bendixon's criterion (Strogatz, 1994). The two null-clines $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$ are given by

$$x_1 = \frac{\rho_1 A_2}{\alpha_1} + \frac{R_1(x_2)}{\alpha_1}, \qquad x_2 = \frac{\rho_2 A_1}{\alpha_2} + \frac{R_2(x_1)}{\alpha_2},$$

respectively, and their shapes (see Figure 2) imply that there are at most three equilibria x', x'', and x''', with $x' < x'' < x'''$.

The analysis of the Jacobian matrix proves that x' and x''' are stable nodes, while x'' is a saddle. Varying the appeal A_1 [A_2], that is, shifting the null-cline $\dot{x}_2 = 0$ [$\dot{x}_1 = 0$] vertically [horizontally], the equilibria x' and x'' can collide and disappear (see Figure 2b) and the same can occur to the equilibria x'' and x'''. These collisions are known as saddle-node bifurcations (Strogatz, 1994). Thus, for low values of the appeals there are two alternative stable states (see Figure 3a) while for large values of the appeals the equilibrium is unique and globally stable (see Figure 3b). The region in the space of the appeals giving rise to alternative stable states can be produced through numerical bifurcation analysis and continuation (Rinaldi et al., 2010). Such a region, indicated with "Alt. St. St.," is shown in Figure 4 and has $x'' = x'''$ on its lower boundary and $x' = x''$ on its upper boundary. The signs of the feelings at equilibrium are therefore uniquely determined outside region "Alt. St. St.," as indicated in Figure 4.

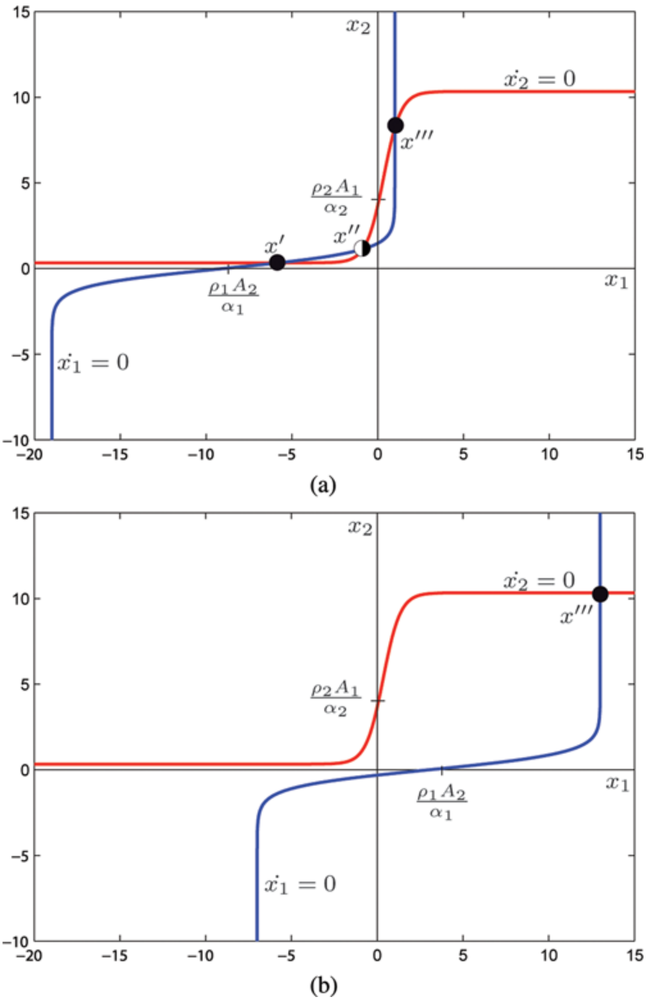

FIGURE 2 The null-clines $\dot{x}_1 = 0$ (blue) and $\dot{x}_2 = 0$ (red) for R_1 and R_2 as in Figure 1, $\rho_1 = 0.5$, $\rho_2 = 1$, $\alpha_1 = 0.1$, $\alpha_2 = 0.3$, $A_1 = 1.05$, and (a) $A_2 = -1.9$: three equilibria x', x'', and x''' ; (b) $A_2 = 0.5$: one equilibrium x'''.

Figure 4 shows that the most satisfactory situation for a couple is to be in region, ①, characterized by high values of the appeals. In fact, in all other couples at least one individual is not positively involved or has the risk of being so. This explains why common and sound advice given to the individuals of a couple is to improve one or more components of their appeals (in particular, those that are mostly appreciated by the partner) with the target of falling in region ①. Unfortunately, this advice requires a permanent change in one or more psychophysical traits and is difficult to follow without great and permanent efforts. For this reason, less costly solutions requiring only temporary efforts are often preferred. One of them is temporary bluffing.

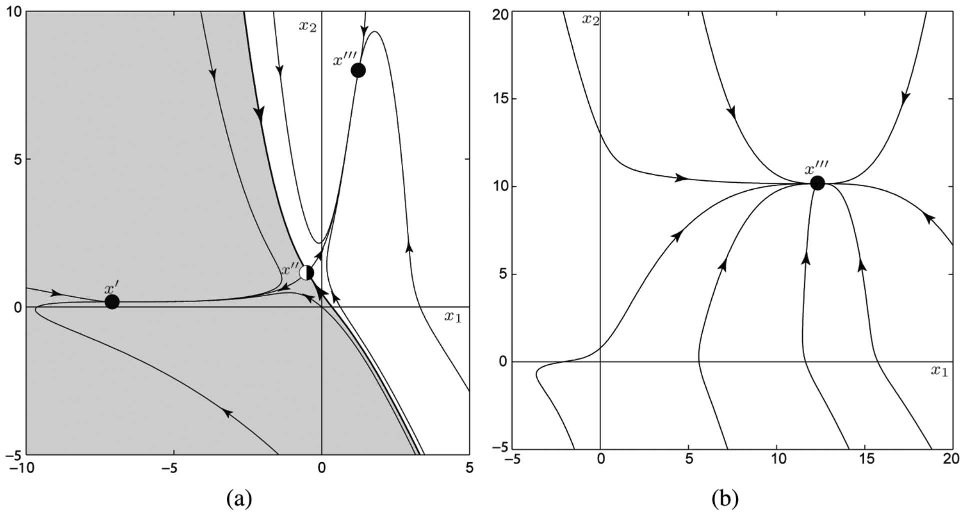

FIGURE 3 Trajectories in the space of the feelings: (a) parameter values as in Figure 2a; (b) parameter values as in Figure 2b. The grey region is the basin of attraction of the bad equilibrium x'.

The idea of temporary bluffing is very simple and can be easily captured by looking at Figures 2 and 3. If in natural conditions (i.e., without bluffing) the couple has reaction functions and parameters as in Figures 2a and 3a the feelings evolve unfavorably toward the state x' if the relationship starts from the state of indiffer-ence (because such a state is in the grey region of Figure 3a). However, the evolution would be favorable if the feelings were initially in the basin of attraction of x''' (white region of Figure 3a). Then the idea of modifying the initial values of the feelings with a suitable hidden artifact immediately comes to mind. The idea can be realized

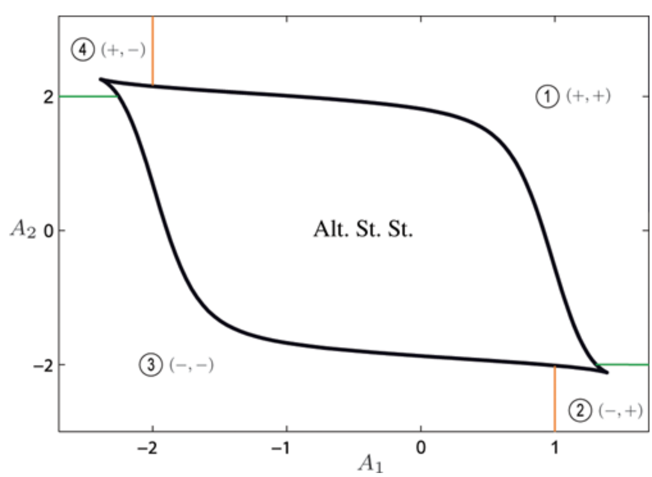

FIGURE 4 Region of alternative stable states (Alt. St. St.) in the space of the appeals and signs of the feelings in regions ①, ②, ③, and ④ outside Alt. St. St. Parameters and functions as in Figures 1 and 2.

through temporary bluffing, that is, by increasing for a certain time the appeals from A_i to

$$A_i^* = A_i + B_i,$$

where B_i is a measure of bluffing intensity of individual i. This form of bluffing has the effect of shifting the two null-clines. More precisely, if individual 1 [2] is bluffing (i.e., if A_1 [A_2] is increased) the null-cline $\dot{x}_2 = 0$ [$\dot{x}_1 = 0$] in Figure 2a moves upward [rightward]. Thus, if bluffing is sufficiently strong, the state portrait corresponding to the new temporary appeals (A_1^*, A_2^*) is like in Figure 3b, where the feelings can only evolve toward a positive state x'''. Once the state x''' has been approached, that is, once a state $\tilde{x}$ close to x''' has been reached, bluffing can be interrupted. Indeed, starting from state $\tilde{x}$ with the real appeals (A_1, A_2) the feelings will evolve as in Figure 3a along a trajectory tending toward the positive state x'''. In order to minimize the bluffing period, which is usually costly, the bluffing should possibly be initiated when the couple is close to the boundary of the basin of attraction of the good state x''' and then interrupted as soon as that boundary is crossed. For example, if the couple is as in Figure 3a, the bluffing could start when the couple is already in the bad state x', but a much more effective solution would be to initiate the bluffing at the very beginning of the love story when the couple is at the origin of the space of the feelings. Indeed, that point is almost on the boundary of the basin of attraction of the good state x''' ant this guarantees that the transition from the grey to the white region in Figure 3a will occur after a short time. This explains why the common practice of "being on one's best behavior in early dating" is absolutely justified.

A second form of bluffing, actually more common than the first, is realized when individual i systematically modifies his/her behavior in order to give to the partner j a biased impression of his/her involvement. In this way, the reaction of j to the love of i in model (2) is not $R_j(x_i)$, as it should be, but $R_j(x_i^*)$, where

$$x_i^* = x_i + B_i.$$

Thus, the two null-clines become

$$x_1 = \frac{\rho_1 A_2}{\alpha_1} + \frac{R_1(x_2 + B_2)}{\alpha_1}, \qquad x_2 = \frac{\rho_2 A_1}{\alpha_2} + \frac{R_2(x_1 + B_1)}{\alpha_2}.$$

If individual 1 [2] is bluffing (i.e., if x_1 [x_2] is increased of an amount B_1 [B_2]) the null-cline $\dot{x}_2 = 0$ [$\dot{x}_1 = 0$] shifts leftward [downward] of an amount B_1 [B_2]. Thus, the two null-clines do not move as in the previous case, but the final result is the same. Indeed, if bluffing is sufficiently strong, the two null-clines intersect only at a single point, as in Figure 2b, and the feelings evolve toward a positive regime. Interrupting the bluffing when that positive regime has been sufficiently approached, allows the couple to naturally evolve, as in Figure 3a, toward the desired satisfactory regime. Of course, the same result is obtained if the two forms of bluffing are realized at the same time.

In order to determine the couples for which temporary bluffing is rewarding, we partition the set alternative stable states in subsets ⓘ characterized by the signs of the feelings of the two alternative stable states x' and x'''. Thus, any subset ⓘ is identified by a pair of serial numbers, ranging from I to IV, indicating in which quadrant x' and x''' fall. For example, the case of Figure 3a corresponds to the subset (II,I) because x' is in the second quadrant while x''' is in the first quadrant. In order to discuss the role of temporary bluffing, we must also indicate for each subset ⓘ which one of the two equilibria has the basin of attraction containing the origin. This is simply done by writing the corresponding serial number in bold. For example, in the case of Figure 3a the subset is (**II**,I) because starting from the origin, i.e., from the state of indifference, the couple evolves toward the unsatisfactory state x'. Thus, (II,**I**) and (**II**,I) are different subsets of the partition of alternative stable states. The partition has already been determined through continuation algorithms (Rinaldi et al., 2010) and the result is shown in Figure 5. Couples with (A_1, A_2) in the subsets ⑫, ⑬, ⑭ evolve from the state of indifference toward a positive state x''' because their second cardinal number is **I**. This means that couples of this kind do not need any particular help or artifact to reach a positive regime. Conversely, couples with (A_1, A_2) in the subsets ⑥, ⑧, ⑩ in which the second cardinal number is I (grey region in Figure 5) converge to x' from the state of indifference and to x''' only if the initial feelings are sufficiently high, as explicitly shown in Figure 3a. Thus, couples in the grey region in Figure 5 need some form of temporary bluffing to reach the positive regime. For this reason, the grey region in Figure 5 is from now on called "bluffing region."

The following are a few remarks interpreting or complementing what we found on temporary bluffing.

- Temporary bluffing does not need to start at the beginning of the relationship, as it does in early datings or in computer-mediated romantic relationships (chatting).

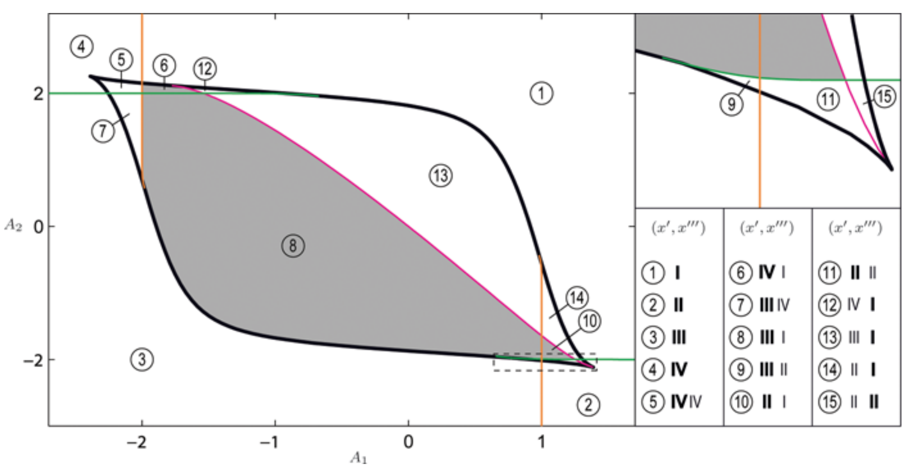

FIGURE 5 Partition of alternative stable states into subregions ⓘ, $i = 5, \ldots, 15$. In each subregion the two alternative stable states x' and x''' lie in the two quadrants reported in the legend, where the state reached from the state of indifference is in bold. The grey region ⑥∪⑧∪⑩ is the bluffing region.

However, it is difficult to imagine how bluffing could be activated in an already established romantic relationship.

- Temporary bluffing can be unilateral without lowering the chances of success, because the geometry of the bluffing region (see Figure 5) guarantees that it is always possible to reach points in region ① moving horizontally or vertically from any point of the bluffing region.
- Temporary bluffing is not needed in couples composed of very appealing individuals, because such couples always converge to the satisfactory regime (at least one of the two individuals has negative appeal in the bluffing region) (see Figure 5).
- Temporary bluffing is of no interest for couples with very low appeals, that is, for couples in regions ②, ③, ④ of Figure 5, because when bluffing is interrupted these couples tend inexorably toward a nonsatisfactory regime. This means that bluffing makes sense for these couples only if permanently adopted (Ben-Ze'ev, 2004).
- In real life the geometry of the bluffing region and the appeals of the individuals are perceived with great uncertainty. It is therefore not surprising (because conceptually consistent) if people with low self-esteem (who underestimate their appeal) have a higher tendency to bluff when wooing.

4. A FAMOUS EXAMPLE OF BLUFFING: ROXANE AND CYRANO DE BERGERAC

An example of successful temporary bluffing (actually an example of potentially successful temporary bluffing) is now briefly described and then analyzed in details with a mathematical model in the next section.

The story is described in *Cyrano de Bergerac*, an heroic comedy written in verse by Rostand (1897), a French neo-romantic poet and dramatist. The first performance of the play (Theatre de la Porte Saint-Martin, Paris, December 28, 1897) was a real triumph, and soon after the play was translated in many languages. Nowadays, *Cyrano de Bergerac* is considered the masterpiece of the French literature on love.

Cyrano de Bergerac has inspired a number of films, among which the most successful one was directed in 1990 by Jean-Paul Rappeneau, starring Anne Brochet as Roxane and Gérard Depardieu as Cyrano. The film is in verse and follows the play very closely.

In order to support the mathematical interpretation of the love story between Roxane and Cyrano, we have isolated nine significant segments of the film (CB1,...,CB9). These segments and the text of the play (in French) and its translation in English are available at home.dei.polimi.it/rinaldi/CyranoDeBergerac/film.html. Still frames of the segments are shown in the panels of Figure 6, where the initial and final times of each segment are reported in the corresponding panel.

The story develops in Paris, where Cyrano de Bergerac, a brilliant poet and swordsman, finds himself seriously attracted by his beautiful cousin Roxane. Besides his brilliance and charisma, Cyrano, who has a ginormous nose, considers himself too ugly to risk telling Roxane his feelings. One day Roxane meets Cyrano and confides in him that she is attracted by Christian, a young nice-looking Cadet of

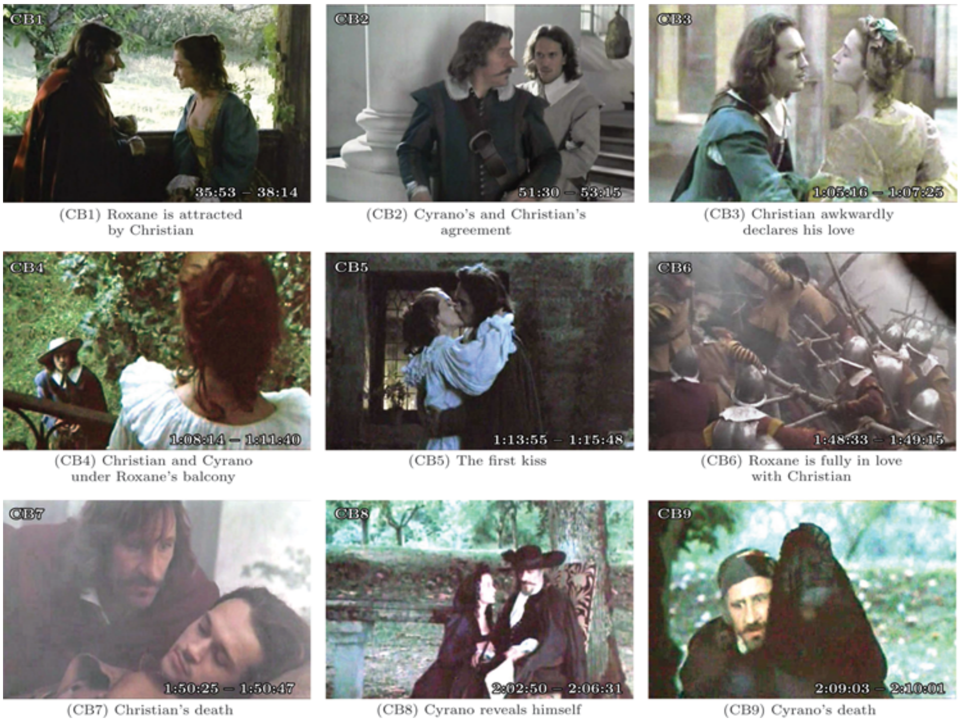

FIGURE 6 Still frames of the nine film segments (CB1, . . . ,CB9) described in the text. *Source*: home.dei.-polimi.it/rinaldi/CyranoDeBergerac/film.html, where the corresponding original text of the play (in French) and its translation in English can also be found.

Gascoyne (CB1). When Cyrano meets Christian (CB2), he embraces him and tells him about Roxane's feelings. Delighted at first, Christian then becomes distraught because he considers Roxane an intellectual while he is a simple, unpoetic man. Then, Cyrano has a brilliant idea: He can write to Roxane inspired letters signed by Christian who is happy to agree, welcoming the opportunity to reach Roxane's heart through this bluffing strategy. One day, Christian decides he no longer wants Cyrano's help, and then makes a fool of himself trying to speak seductively to Roxane (CB3). Unfortunately, all he can come up with is "I love you." Roxane is deceived and sends Christian away. At this point, Cyrano responds with a prompt move. Under cover of darkness, he makes Christian stand in front of Roxane's balcony and speak to her, while he stands hidden under the balcony, whispering to Christian what to say (CB4). At the end of this bluffing ballet, Cyrano forces Christian to jump on the balcony where Roxane kisses him for the first time (CB5). Soon after that, Roxane and Christian get married but are immediately separated because the Cadets of Gascoyne are sent to the front lines of the war. One day, at the front, Christian guesses Cyrano's secret feelings for Roxane and asks him to find out whom she chooses. When Cyrano meets Roxane she tells him that she is deeply involved with Christian because of his letters and would love him even if he were ugly (CB6). This is a clear indication of the success of Cyrano's bluffing. At the cusp of revealing his secret, Cyrano is interrupted by a gunshot that kills Christian. Cyrano whispers to him that Roxane loves him and he dies happy, while Roxane remains ignorant (CB7) and disappears. Thus, Cyrano misses the chance of transforming his successful bluffing into a successful love story.

Only 15 years later, Cyrano discovers that Roxane lives in a convent and visits her every week. One day he appears at the convent walking slowly and with a pained expression on his face because he was wounded in an ambush. As night falls, Cyrano asks to read Christian's last letter to her. He reads it, and when it is completely dark he continues to read, thus revealing to her that he knows the letter by heart. This is how she discovers that the man she has loved during her entire life is just in front of her (CB8). In this way, the bluffing is finally interrupted. Roxane exclaims that she loves him and that he can not die, but Cyrano collapses in her arms (CB9). In other words, Cyrano misses for the second time the chance of transforming his successful bluffing into a successful love story.

5. THE MODEL OF ROXANE AND CYRANO

In this section we show that the love story between Roxane and Cyrano can qualitatively but satisfactorily be interpreted with model (2), provided the parameters are fixed at suitable values. Since quantitative data on the love story do not exist, we can not use standard identification procedures for tuning the parameters. Instead, we are forced to fix them on the basis of purely subjective impressions. Assuming that individual 1 is Roxane and individual 2 is Cyrano, our choice is

$$\begin{array}{llllll} \text{Roxane} & \alpha_1 = 0.1 & \rho_1 = 0.5 & A_1 = 1.05 & R_1^+ = 1 & R_1^- = -1 \\ \text{Cyrano} & \alpha_2 = 0.3 & \rho_2 = 1 & A_2 = -1.9 & R_2^+ = 2 & R_2^- = -1. \end{array}$$

The parameter R_i^- has been given the same values for the two individuals because for that parameter even subjective impressions are missing. Quite low values have been assigned to the forgetting coefficients α_i because both Roxane and Cyrano keep memory of their love after a very long period (15 years) of separation (CB8). Roxane's forgetting coefficient is lower than that of Cyrano because one can imagine that she did not have many opportunities to forget her lover being segregated in a convent, while Cyrano did have higher chances to do so being still actively involved in the social life of the Cadets of Gascoyne. The maximum reaction R_2^+ and the sensitivity ρ_2 of Cyrano are higher than those of Roxane because this is the impression one gets when reading his wildly enthusiastic proposals. Finally, the two appeals are of opposite sign in view of the contrast between the grace of Roxane and the unpleasant physical aspect of Cyrano. The appeals A_1 and A_2 can be conveniently compared with Christian's appeal A_2^* which is assumed to be equal to 0.5.

The story can be subdivided into two real phases, (*b*) and (*d*). Phase (*b*) starts with the agreement between Cyrano and Christian (CB2) and ends with Christian's death (CB7). It is a bluffing phase because Cyrano's soul is hidden in the body of Christian. In other words, Cyrano succeeds in increasing with an artifact his appeal from A_2 to A_2^*. Phase (*d*) goes from the death of Christian to Roxane's discovery of the truth (CB8) and is simply a phase of separation where oblivion is the only active process.

For the sake of clarity, we also consider three virtual phases indicated with (*a*), (*c*), and (*e*), respectively. In phase (*a*) Christian does not exist and Cyrano reveals his interest to Roxane. At the beginning of phase (*c*), just after the death of Christian, Cyrano reveals the truth to Roxane and the feelings of the two lovers evolve toward an equilibrium. This is a virtual phase of no bluffing that ends in a positive equilibrium if the previous bluffing phase (*b*) was successful. In phase (*e*) Cyrano remains alive after Roxane's has discovered the truth in the convent.

The results obtained by applying the model in each one of the five phases are discussed below and summarized in Figure 7.

- *Phase* (*a*). For the parameter values we have proposed, the two null-clines $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$ are as in Figure 2a. The appeals (A_1, A_2) of the couple fall in subregion ⑩ of the bluffing region. If Cyrano would reveal his feelings to Roxane, their story would evolve along trajectory *a* in Figure 7, ending at point *A*, which is the unsatisfactory state x' of Figure 2a. Thus, Cyrano is right in hiding his feelings and in fusing with Christian to initiate a more promising phase of temporary bluffing.
- *Phase* (*b*). The null-clines are obtained from those in Figure 2a by simply shifting to the right the first one of an amount proportional to the bluffing $B_2 = A_2^* - A_2 = 2.4$ exerted by Cyrano. The resulting null-clines are as in Figure 2b and intersect only at a single point x''' in the first quadrant. This point is indicated with *B* in Figure 7. Thus, the feelings evolve along the trajectory *b* in Figure 7

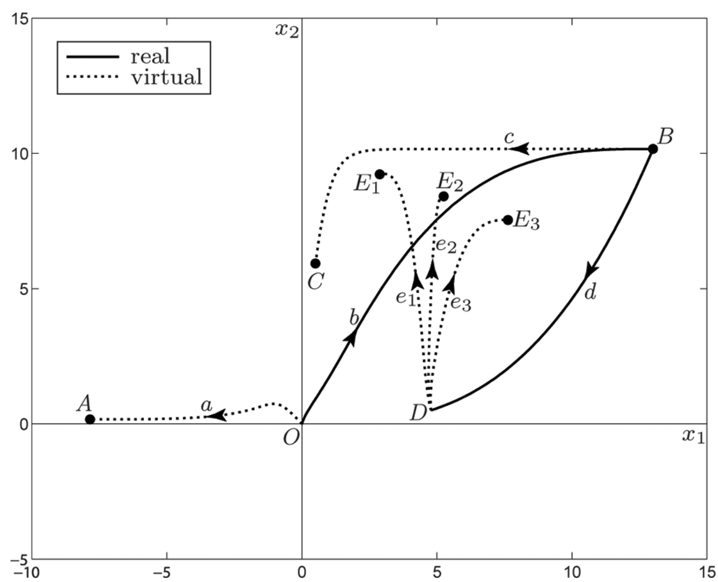

FIGURE 7 Trajectories of the feelings in each phase (*a*),...,(*e*) described in the text: (*a*) virtual no bluffing phase; (*b*) bluffing phase; (*c*) virtual phase of no bluffing after Christian's death; (*d*) separation phase; (*e*) virtual phase of no bluffing after Roxane's discovery of the truth in the convent: the three trajectories e_1, e_2, and e_3 are obtained with sensitivities ρ_1 and ρ_2 reduced of 25%, 50%, and 75%.

starting from the origin and ending close to point B when Christian dies (CB7). The positivity of the equilibrium B proves that this bluffing phase was successful.

- *Phase* (c). This no bluffing phase, corresponding to trajectory c in Figure 7, is virtual because it would be realized only if Roxane would have discovered the truth at Christian's death. Thus, the trajectory c starts from B, develops in agreement with model (2) where $A_2 = -1.9$ is the appeal of Cyrano, and ends at the equilibrium point C that coincides with x''' in Figure 2a.
- *Phase* (d). During this phase—starting at Christian's death—there are no reactions to love and appeal since the two lovers are separated. Thus, the couple is described by the reduced model

$$\begin{aligned} \dot{x}_1(t) &= -\alpha_1 x_1(t) \\ \dot{x}_2(t) &= -\alpha_2 x_2(t), \end{aligned}$$

  which implies that both feelings systematically decrease (see trajectory d in Figure 7 starting from B and tending toward the origin). This evolution is very slow, because the forgetting coefficients are low. Thus, when the truth is accidentally discovered by Roxane (CB8) the feelings of the two lovers are still relatively high, as shown by the terminal point D in Figure 7.
- *Phase* (e). Since in this virtual phase Cyrano remains alive and responds to Roxane's love and appeal, the model is again that used in phase (a), but with initial conditions represented by point D. However, one can reasonably suspect that the sensitivities ρ_1 and ρ_2 that the two lovers have for the appeal of their partners are now lower than these they had when they were younger. But no suggestions emerge from Rostand's play on this matter. For this reason, we have simulated the model for various reduced values of ρ_1 and ρ_2 and checked that in all cases the feelings of the two lovers tend toward positive values. Figure 7 shows three of these simulations (see trajectories e_1, e_2, and e_3) obtained for a 25%, 50%, and 75% reduction of the sensitivities ρ_1 and ρ_2. Thus, Roxane and Cyrano tend naturally, without the need of any bluff, toward a satisfactory romantic regime (see points E_1, E_2, and E_3 in Figure 7). This happy (although virtual) end of the overall story shows that temporary bluffing was successful in the love story between Roxane and Cyrano.

A sensible tradition in science is to validate proposed mathematical models using data that are independent from those used for estimating the parameters. Of course, this is impossible here because data do not exist. The best we can do is, therefore, to simply support the model through very qualitative arguments, which are, however, independent from those used to suggest the parameter values. Although we have only two such independent arguments, actually both weak, we believe that this can be considered a remarkably lucky case in a field like love dynamics.

The first argument is concerned with the fact that our model supports the idea that temporary bluffing was potentially successful. But Rostand's play, ending with Cyrano's death, obviously cannot say anything on this issue, even if Roxane reveals a definite interest in the future when she exclaims in the last scene that Cyrano can not die (CB9). We must therefore rely on the impression that the majority of those who have read the play or seen the film agree with the conclusion of our model.

Actually, a sort of confirmation of this impression can be found in another film inspired by Rostand's play, entitled *Roxanne* and directed in 1987 by Fred Schepisi, starring Daryl Hannah as Roxane and Steve Martin as Cyrano. The film is a free transposition of *Cyrano de Bergerac* in modern times (Cyrano is the chief of the fire unit in a small American town while Christian is a simple fireman in the same unit) and has the intent of reaching a large and not particularly sophisticated audience. In that film, in order to avoid missing the message on the effectiveness of temporary bluffing, Cyrano does not die and the story ends with Roxane kissing him despite his nose.

The second supporting argument is the agreement between the quantitative predictions of the model and the following qualitative but sharp statement of Roxane (Scene 4.8).

Roxane:	Je viens, ô mon Christian, mon maître!	O, Christian, my true lord, I come—
	Je viens te demander pardon.	—I come to crave your pardon.
	De t'avoir fait d'abord, dans ma frivolité,	For the insult done to you when, frivolous,
	L'insulte de t'aimer pour ta seule beauté!	At first I loved you only for your face!
Christian:	Ah! Roxane!	Roxane!
Roxane:	Et plus tard, mon ami, moins frivole,	And later, love— less frivolous—
	—Oiseau qui saute avant tout á fait qu'il s'envole,—	Like a bird that spreads its wings, but can not fly—
	Ta beauté m'arrêtant, ton âme m'entraînant,	Arrested by your beauty, by your soul
	Je t'aimais pour les deux ensemble!	Drawn close—I loved for both at once!
Christian:	Et maintenant?	And now?
Roxane:	Eh bien! toi-même enfin l'emporte sur toi-même,	Ah! you yourself have triumphed o'er yourself,
	Et ce n'est plus que pour ton âme que je t'aime!	And now, I love you only for your soul!

Roxane's statement contains two messages: The first is that her involvement has been initially a pure reaction to Christian's beauty. This is in agreement with the model because at the beginning of the story, when $t = 0$, we have $x_1(0) = x_2(0) = 0$ and $R_1(0) = R_2(0) = 0$, so that

$$\dot{x}_1(0) = \rho_1 A_2^*.$$

That is to say, Roxane starts to be involved because A_2^*—the appeal of Christian—is positive. However, it is fair to say that this agreement is not very significant because condition $R_1(0) = R_2(0) = 0$ guarantees the same conclusion for any choice of the reaction functions. More interesting is the second message contained in Roxane's statement, namely that at the end of phase (b) the soul of Christian is for her the dominant source of love. This can be compared with the prediction of the model. In fact, at the end of phase (b), just before Christian's death, we are close to the equilibrium point

$$B = (x_1''', x_2''') \simeq (13, 10)$$

of Figure 7, so that the two flows regenerating the love of Roxane are (see first equation of model (2))

$$\rho_1 A_2^* = 0.25, \qquad R_1(x_2''') \simeq 1.$$

Since the reaction to love is definitely greater than the reaction to appeal, the model is in agreement with Roxane's statement.

6. DISCUSSION

We have supported in this article the conjecture that temporary bluffing is often used in social systems to switch from a bad state to a good alternative state. This has been done by focusing on the simplest possible unit of interest in sociology—the couple—leaving the extension to larger social groups for future research.

More precisely, we have shown, through the use of an existing mathematical model, why temporary bluffing can be rewarding in love affairs. This discovery, consistent with observed behaviors in real life, allows one to consider temporary bluffing as a sort of therapy to be suggested to couples which, otherwise,would not be able to realize a satisfactory romantic relationship. This, in a sense, attenuates, in the context of love affairs, the negative moral value usually attached to bluffing in the context of social behavior.

A few technical but not less interesting consequences of the analysis are the following. First, unilateral temporary bluffing is virtually as promising as bilateral temporary bluffing, a fact that might be relevant for psychotherapists who might have serious difficulties in suggesting temporary bluffing to both partners. Second, couples with high appeals do not have any strategic advantage in bluffing because they are guaranteed to reach satisfactory romantic regimes in any case. Conversely, couples with too low appeals inexorably go back to an unsatisfactory romantic regime when the bluff is interrupted. For these couples only permanent actions (e.g, chatting and cosmetic surgery) can be considered valuable.

Many of the general properties discovered through the analysis of the model can be detected in the love story described by Edmond Rostand in his famous heroic comedy *Cyrano de Bergerac*. This comedy, a beautiful hymn to the power of temporary bluffing, shows how a couple can reach a high quality romantic regime despite of a very serious obstacle (the poor physical aspect of one of the partners). Since the *Cyrano de Bergerac* stresses the dichotomy between mind and body, its successful interpretation through a mathematical model gives hopes on the possibility of interpreting with formal mathematical tools other important poems where the same dichotomy is discussed. Along this line, we might even hope to be able to give answers to extremely complex dilemmas, like that described by Thomas Mann in his famous novel *The Transposed Heads*. In that novel, Sita is married with

Shridaman but is also attracted by Nanda, a good friend of her husband. One day, the heads of the two friends are magically restored, but put back to the wrong bodies. From that point on, Sita is confused. She does not know who is her husband and whom she prefers and she is forced to face an intriguing dilemma: Which creates and rules the person—the mind or the body? Although Sita's story is undoubtedly more complex than that of Roxane and Cyrano, it is licit to imagine that the use of a rational approach based on a mathematical model might enlighten the story and help solving the dilemma.

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