A mathematical model of "Pride and Prejudice"

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INTRODUCTION

Love stories are dynamical processes that develop in time starting, in general, from a state of indifference. They reach a stationary, cyclic, or chaotic regime where they often remain for a long time before fading or being interrupted by the disappearance of one of the partners (Rey, 2010). The evolution of romantic relationships is marked by all typical features known in nonlinear system theory.

Numerous love stories tend, sometimes after turbulent transients, towards a stationary regime where the involvements of the two partners remain practically constant. One of the most known examples of this kind is the story between Scarlett and Rhett described in "*Gone with the Wind*" (Mitchell, 1936) that has inspired the most popular film of the entire cinematographic history. A cyclic love story between a poet (Francesco Petrarca) and his mistress (Laura De Sade) is described in the "*Canzoniere*" (Petrarca, 1374), the most celebrated collection of love poems in western culture (Jones, 1995). Among the numerous chaotic love stories reported in literature, perhaps the most interesting one is described in "*Jules et Jim*" (Roché, 1953), a novel that became famous after the release in 1962 of the homonymous film by François Truffaut, one of the leaders of the Nouvelle Vague.

The existence of alternative stable states (each one with its basin of attraction) – a typical property of nonlinear dynamical systems – is detectable in many romantic relationships. Indeed, it is known that the quality of pairwise relationships can drop from a high to a low regime and remain there for a long time, if not forever, after the temporary infatuation of one of the two partners for another person. An important example of a couple with alternative stable states is described in "*Cyrano de Bergerac*" (Rostand, 1897), the masterpiece of romantic French literature, where it is shown how difficult it can be to jump from an unfavorable sentimental regime into the basin of attraction of a more favorable (and, hence, more desirable) regime.

Another important feature of nonlinear dynamical systems is the possibility that very small (in the limit not perceivable) variations of some strategic parameter can trigger relevant discontinuities in the feelings of the partners. In other words, small discoveries can have great consequences in love affairs. From a formal point of view these discontinuities are nothing but *catastrophic bifurcations* (Strogatz, 1994). They are important because they are, in general, associated with great emotions, emerging when there are dramatic breakdowns or enthusiastic explosions of interest. All relationships prevalently based on sex are potential examples of the first kind. Indeed, sexual appetite inexorably decreases in time (Rinaldi, Della Rossa, & Fasani, 2012) so that a point of unavoidable and unexpected separation can easily be reached if the individuals are mainly interested in sex. A well known example of the second kind is that of a playboy who systematically insists in reinforcing his appeal until his prey suddenly falls in love with him.

All above properties, as well as others, can be well understood by putting them in a rational frame through the use of mathematical models. This is what has been done up to now, starting with a naive model presented in a seminal paper (Strogatz, 1988) and continuing through the analysis of a long series of general and abstract models of romantic relationships (Ahmad & El-Khazali, 2007; Barley & Cherif, 2011; Bielczyk, Bodnar, & Foryś, 2012; Bielczyk, 2013; Buder, 1991; Feichtinger, Jorgensen, & Novak, 1999; Felmlee, 2006; Gragnani, Rinaldi, & Feichtinger, 1997; Liao & Ran, 2007; Ozalp & Koca, 2012; Rey, 2010, 2013; Rinaldi, Della Rossa, & Dercole, 2010; Rinaldi & Gragnani, 1998a, 1998b; Rinaldi, 1998a; Son & Park, 2011; Sprott, 2004, 2005; Wauer, Schwarzer, Cai, & Lin, 2007). However, we must admit that in order to reinforce the analysis and make it more credible, it is desirable, if not mandatory, to refer to specific and well documented love stories, because the possibility of successfully describing a complex romantic relationship with a mathematical model can not be given as granted. In this respect, the existing literature is still quite poor, because only three studies, where love stories are satisfactorily described with mathematical models, are available today. They deal with “Gone with the Wind” (Rinaldi, Della Rossa, & Landi, 2013), the “Canzoniere” (Rinaldi, 1998b) and “Jules et Jim” (Dercole, 1999). Moreover, modeling studies dealing with love stories characterized by catastrophic bifurcations have never been carried out so far. To cover these deficiencies, we present in this paper the detailed study of the well known love story between Elizabeth and Darcy described in “*Pride and Prejudice*” (Austen, 1813).

THE LOVE STORY BETWEEN ELIZABETH AND DARCY

"Pride and Prejudice" (Austen, 1813) is a very popular English novel in which the love story between Elizabeth and Darcy is described. The novel has inspired a number of films, the most successful of which was released in 2005, starring Keira Knightley as Elizabeth and Matthew Macfadyen as Darcy. Sixteen short segments of the film (PP1,...,PP16) available at home.deib.polimi.it/rinaldi/PrideAndPrejudice/film.html are described below to support the mathematical interpretation of the love story. Still frames of the sixteen film segments are shown in Fig. 1 where the initial and final times of each segment are reported on the corresponding panel.

Insert Figure 1 About Here

Elizabeth, the twenty-years-old second born of an impoverished family, lives modestly in a lovely country house (PP1). At a ball she is introduced to Darcy, an elegant young man with quite arrogant tones belonging to the rich local nobility (PP2). Without being seen, Elizabeth hears Darcy making a rather unfair statement on her appeal (PP3). Her reaction is strong and pungent (PP4). This is only the first of a series of disputes in which she brightly sustains ideas which are strongly in conflict with those of Darcy (PP5) or members of his family (PP7). Sometimes these disputes reach the intensity of real fights, as when they dance together for the first time (PP6). However, Darcy is fascinated by her grace and skill (PP8) to the point of showing the first symptoms of involvement when they remain alone for a short time (PP9). Soon after that, Darcy declares his love, mentioning, however, the difficulty that the difference in their social status implies for him (PP10). Once more, Elizabeth does not appreciate these arguments and refuses to marry him (PP10). Stressed by this refusal, Darcy writes a letter to Elizabeth in which he apologizes and justifies, with detailed and convincing argumentations, those points of his past behavior that she definitely dislikes (PP11). This letter is the turning-point of the entire story because it removes the obstacles that prevented her from fully appreciating his rectitude. Her sentimental involvement inexorably grows from a state of initial confusion (PP12) to the acknowledgment of his appeal (PP13) and from jealousy (PP14), when she suspects he is in love with another young lady, to happiness (PP15), when she discovers that her jealousy is unjustified. Thus, the awareness of a fully requited love is gradually reached and culminates in the last scene of the film (PP16).

A GENERAL MODEL OF STANDARD COUPLES

The majority of the models proposed in the past for the description of love stories are composed of two Ordinary Differential Equations (ODEs), one for each partner, i.e.,

$$\begin{aligned} \dot{x}_1(t) &= f_1(x_1(t), x_2(t), A_2) \\ \dot{x}_2(t) &= f_2(x_1(t), x_2(t), A_1). \end{aligned} \tag{1}$$

In these models the state variables $x_i(t),\ i = 1{,}2$, are the *feelings* at time t of the individuals for their partner (see Gottman, Murray, Swanson, Tyson, & Swanson (2002) for a relevant exception), while A_1 and A_2 are their *appeals*. The appeals, as well as all parameters specifying the psychical characteristics of the individuals, are assumed to be time-invariant at least on specified time intervals. In accordance with Levinger (1980), positive values of the feelings range from sympathy to passion, while negative values are associated with antagonism and disdain. In a number of contributions it is shown how model (1) can be conceptually justified and potentially derived from the general theory of stochastic processes (Bellomo & Carbonaro, 2006, 2008; Carbonaro & Giordano, 2005; Carbonaro & Serra, 2002) or from very general principles, like those of Quantum Mechanics (Bagarello & Oliveri, 2010; Bagarello, 2011). However, from an operational point of view, these approaches do not seem to be very promising if the aim of the study is to model specific love stories. This is why in the majority of the contributions, as well as in this paper, the functions $f_i,\ i = 1{,}2$, appearing in model (1), which are the unbalances between regeneration and consumption flows, are empirically defined as explained below.

The appeal of individual i has various components A_i^h, like physical attractiveness, age, social position, richness and others, which are independent of the feeling x_i. If λ_j^h is the weight that individual j ($j \neq i$) gives to the h-th component of the appeal of his/her partner, we can define the appeal of i (perceived by j) as

$$A_i = \sum_h \lambda_j^h A_i^h.$$

Thus, the appeal is not an absolute character of the individual, but rather a value perceived by his/her present or future partner. As such, the appeal A_i of individual i varies discontinuously each time the partner j discovers some relevant hidden aspect of the character of i.

Two persons meeting for the first time at $t = 0$, are, in general, indifferent one to each other, i.e. $x_1(0) = x_2(0) = 0$. Then, the feelings evolve in accordance with equations (1) where the rates of change f_i are dictated by the unbalance between regeneration and consumption processes.

The consumption process is *oblivion*, which explains why individuals gradually lose memory of their partners after separating. As done in almost all fields of science, losses are assumed to occur at exponential rate, i.e., $x_i(t) = x_i(0)\exp(-\alpha_i t)$ or, equivalently, in terms of ODEs, $\dot{x}_i(t) = -\alpha_i x_i(t)$, where α_i is the so-called *forgetting coefficient*. In contrast, the regeneration processes are of two distinct kinds, namely *reaction to appeal* and *reaction to love*. The flow of interest generated in individual j by the appeal of the partner is obtained by multiplying A_i by a factor ρ_j identifying the *sensitivity* of individual j to appeal, while the second regeneration process, the reaction to the love of the partner, is described by a function $R_i(x_j)$. The most standard individuals, often called *secure*, are those who like to be loved. An individual i belonging to this class is formally characterized by an increasing function $R_i(x_j)$ that identifies the flow of interest generated in individual i by the love x_j of the partner. In the first studies on love dynamics, reaction functions of secure individuals were linear (Rinaldi, 1998a; Strogatz, 1988), while, later, they have been assumed to be bounded (Rinaldi & Gragnani, 1998b; Rinaldi et al., 2010), as shown in Fig. 2, where the two graphs correspond to

$$R_1(x_2) = \frac{e^{x_2} - e^{-x_2}}{e^{x_2}/R_1^+ - e^{-x_2}/R_1^-} \ , \ R_2(x_1) = \frac{e^{x_1} - e^{-x_1}}{e^{x_1}/R_2^+ - e^{-x_1}/R_2^-}$$

with $R_1^+ = 1$, $R_2^+ = 2$, and $R_1^- = R_2^- = -1$. The reason for this change is that unbounded functions do not capture the psychophysical limitations present in all individuals.

Insert Figure 2 About Here

Thus, in conclusion, a reasonable model for couples of secure individuals, in any period of constant perception of the appeals, is

$$\begin{aligned} \dot{x}_1(t) &= -\alpha_1 x_1(t) + \rho_1 A_2 + R_1(x_2) \\ \dot{x}_2(t) &= -\alpha_2 x_2(t) + \rho_2 A_1 + R_2(x_1). \end{aligned} \tag{2}$$

The reason why we limit our attention to model (2) with increasing and saturating reaction functions (see Fig. 2) is that such a model is perfectly suited for mimicking the characters of Elizabeth and Darcy. All properties of model (2), including bifurcations, are summarized below. They have been derived through numerical bifurcation analysis and thoroughly discussed in Rinaldi et al., (2010), where the interested reader can find all details.

Insert Figure 3 About Here

Model (2), as well as any other realistic model, says that if the two individuals are indifferent one to each other when they meet for the first time at $t = 0$, the story is initially dominated by the appeals, since

$$\dot{x}_1(0) = \rho_1 A_2, \qquad \dot{x}_2(0) = \rho_2 A_1.$$

Thus, in particular, the story has a positive start, in the sense that $x_i(t) > 0$, $i = 1,2$, for all sufficiently low values of t, if and only if both individuals are appealing, as shown in Fig. 3a. At first glance, Fig. 3a seems to suggest that non appealing persons cannot be loved by other persons, a rather crude conclusion which is at odd with real life observations. Indeed, there are plenty of individuals perceived as non appealing but living steadily in a high quality relationship. This apparent conflict between model and life is due to two different reasons, one trivial and one subtle.

The trivial reason is that, as already remarked, the appeal is not an absolute character of an individual but rather a subjective value perceived by the partner. Thus, it is not important to be judged as appealing by the majority of the people of a given community, but rather to be appreciated by the partner. This explains why individuals perceived as non appealing in their community can be involved in positive love stories.

The second reason for the apparent conflict between model and life is more subtle as explained below. It is related with the fact that dynamical systems can have long term behaviors which have only little to share with short time behaviors. In particular, the signs of the feelings in the long term can be different from those shown in Fig. 3a characterizing the initial phase of the romantic relationship. In the case of model (2) the divergence (equal to $-(\alpha_1 + \alpha_2)$) does not change sign, so that the attractors (capturing the long term behaviors of the system) can only be equilibria – limit cycles being excluded by Bendixon's theorem (Strogatz, 1994). The detailed analysis of model (2) shows that there are two alternative stable states if the appeals belong to a region from now on called ASS (Alternative Stable States). Such a region can be determined through numerical bifurcation analysis, as shown in Rinaldi et al., (2010). For example, if the reaction functions are as in Fig. 2 and $\alpha_1 = 0.1$, $\alpha_2 = 0.3$, $\rho_1 = \rho_2 = 1$, the regions ASS turns out to be the white compact region shown in Fig. 3b. In contrast, when the point (A_1, A_2) does not fall in ASS only one stable equilibrium exists and the signs of its components are as specified in Fig. 3b. Obviously, the signs of the feelings can be determined also for pairs (A_1, A_2) falling in region ASS if reference is made to only one of the two alternative stable states. For example, if reference is made to the equilibrium reached when the two individuals are initially indifferent one to each other, the signs of the feelings in the long term are as in Fig. 3c. This last figure points out a very important property: couples with positive appeals are guaranteed to evolve toward a high quality relationship, but this is possible also for many other couples in which one of the two individuals is not appealing. In conclusion, the comparison of Fig. 3a with Fig. 3c supports the following important statement: “love stories involving individuals who are not appealing can develop positively after an initial phase of antagonism” or, more crudely, “individuals can be happy in the long term even if they are not appealing” .

Insert Figure 4 About Here

As shown in Rinaldi et al., (2010), the points of the boundary of ASS are *saddle-node* bifurcation points (Strogatz, 1994) and there are no other bifurcations in the system. More precisely, for (A_1, A_2) in region ASS there are three equilibria x', x'', and x''' that are ordered from low to high, in the sense that $x' < x'' < x'''$. Figure 4 refers to one example of this kind and shows the null-clines $\dot{x}_1 = 0$ (red), $\dot{x}_2 = 0$ (blue) and the trajectories in the space of the feelings for model (2) with reaction functions as in Fig. 2 and $\alpha_1 = 0.1$, $\alpha_2 = 0.3$, $\rho_1 = \rho_2 = 1$, $A_1 = 0.3$ and $A_2 = 0.5$. The intermediate equilibrium x'' is always unstable (a saddle), while the two others are stable. On the lower boundary of ASS $x'' = x'''$, while on the upper boundary of the same region $x' = x''$. Thus, consider a couple represented by a point (A_1, A_2) in region ASS close to its upper boundary and assume that the couple is in its unfavorable stable state x'. Then, if one or both appeals increase by a small amount, the new point (A_1, A_2) will be just above the upper boundary of ASS where the favorable state x''' is the unique equilibrium point, as shown in Fig. 3b. This means that a very small variation of the appeal can trigger a remarkable variation in the feelings – the transition from the unfavorable to the favorable state. For this reason, the saddle-node bifurcation is often called catastrophic.

Many real love stories reported in the technical literature, as well as many fictitious stories portrayed in novels and films, show that catastrophic transitions are frequent. This can be due to two different reasons. The first occurs when the appeals vary slowly over time due to ageing or adaptation, thus following very slowly a line in the space (A_1, A_2), that crosses the boundary of ASS. The second occurs when a series of discoveries force the two individuals to change from time to time their perception of the appeal of the partner up to the point of crossing the boundary of ASS. The last is the case described in “Pride and

Prejudice" where Darcy discovers gradually the grace and brightness of Elizabeth and finally demonstrates to her his moral uprightness.

THE MODEL OF ELIZABETH AND DARCY

The love story between Elizabeth and Darcy can be perfectly interpreted with model (2), provided suitable values are assigned to the parameters of the model. The appeal A_1 of Elizabeth (perceived by Darcy) is initially low (PP3) but then increases at each encounter, where she shows, with no exception, her grace and talent (PP5,...,8). In contrast, the appeal A_2 of Darcy (perceived by Elizabeth) remains negative, in view of the prejudices she has against the rich and the nobel. It is therefore licit to imagine that the evolution of the love story is the result of a recursive increase of A_1 followed by a final sudden increase of A_2 (due to the revealing letter of Darcy), as shown in Fig. 5a. At the beginning of the story (PP2,3,4) the representative point in the space of the appeals is point 1 in the red region of Fig. 5a where Elizabeth and Darcy are in an antagonistic relationship. When the perception A_1 of Elizabeth's appeal increases (see points 2,3,4,5 in Fig. 5a) nothing relevant occurs though Darcy's involvement is positive at point 5 (see PP9). At that point Elizabeth is still antagonistic and, indeed, she refuses to marry him (PP10). It is only Darcy's letter that suddenly reveals to Elizabeth the rectitude of her lover. This is represented as a vertical jump in the space of the appeals from point 5 to point 6 in the green region of Fig. 5a where the two lovers can only be in a positive relationship. In other words, as a consequence of the letter the upper saddle-node bifurcation curve is crossed from below and this crossing implies a discontinuous jump from x' to x''' in the feelings of Elizabeth and Darcy (PP12,...,15).

Insert Figure 5 About Here

The analysis of the null-clines in the love space reported in Fig. 5b is also interesting. It shows that the blue null-cline shifts upward when A_1 increases (from point 1 to point 5 in Fig. 5a). This implies that the involvement of Darcy increases gradually and finally becomes positive at point 5, where, however, Elizabeth's involvement is still negative. But the letter of Darcy suddenly shifts the red null-cline so much to the right that the two null-clines intersect only at a single point, namely point 6 in the positive quadrant.

CONCLUSION AND DISCUSSION

We have shown that the evolution of a love story can be characterized by a sudden surprise due to the inevitable explosion of the feelings of the partners. This has been done by interpreting with a mathematical model the love story described in “Pride and Prejudice” (Austen, 1813). The paper is interesting not only because it deals for the first time with bifurcations in a specific love story, but also because it enriches the list of examples showing that love stories can be well described with simple ODEs models (Dercole, 1999; Rinaldi & Gragnani, 1998b; Rinaldi et al., 2013). Another study concerning bifurcations in romantic relationships is discussed in a companion paper (Rinaldi, Landi, & Della Rossa, 2013) dedicated to the mathematical analysis of the love story portrayed by Walt Disney in the film “Beauty and The Beast”.

It is interesting to note that the analysis points out a general property – often known from personal experiences – namely that a series of small discoveries can give rise to a sudden turning-point in the development of a love story. In mathematical terms these turning-points are nothing but so called catastrophes, which, in the case of “Pride and Prejudice”, are technically revealed by the existence of a saddle-node bifurcation. As far as we know, Jones (1995) was the first to invoke, on a purely intuitive ground, the use of catastrophe theory in the study of romantic relationships, while Gragnani et al. (1997) were the first to discover catastrophic bifurcations in an abstract model of love dynamics.

It is also important to note that the same model used here for the description of a love story could, in principle, be used to deal with the dynamics of other kinds of interpersonal relationship. For example, the evolution of friendship can certainly be described with the same model, provided the appeals do not have anything to do with physical attractiveness. In this case the saddle-node bifurcation would explain the sudden emergence or rupture of a friendship.

Linear models, i.e. models like the one discussed in this paper but with linear reaction functions, have first been studied by Rinaldi (1998a) in the context of love dynamics and then proposed to study other social interactions in couples by giving different interpretations to the appeals (Felmlee & Greenberg, 1999). These linear models have been used in Psycophysiology to interpret time-series of heart rate and respiration (Ferrer & Helm, 2013) and in Marketing to show that successful and lasting relational exchanges are those in which partners go beyond short-term transactional benefits, and incorporate behavioral factors such as trust and commitment (Fruchter & Sigué, 2009; Fruchter, 2014).

REFERENCES

Ahmad, W. M., & El-Khazali, R. (2007). Fractional-order dynamical models of love. *Chaos, Solitons & Fractals*, *33*, 1367–1375.

Austen, J. (1813). *Pride and Prejudice*. London, England: T. Egerton, Whitehall.

Bagarello, F. (2011). Damping in quantum love affairs. *Physica A*, *390*, 2803–2811.

Bagarello, F., & Oliveri, F. (2010). An operator-like description of love affairs. *SIAM Journal on Applied Mathematics*, *70*, 3235–3251.

Barley, K., & Cherif, A. (2011). Stochastic nonlinear dynamics of interpersonal and romantic relationships. *Applied Mathematics and Computation*, *217*, 6273–6281.

Bellomo, N., & Carbonaro, B. (2006). On the modelling of complex sociopsychological systems with some reasoning about Kate, Jules, and Jim. *Differential Equations and Nonlinear Mechanics*, *2006*, 1–26.

Bellomo, N., & Carbonaro, B. (2008). On the complexity of multiple interactions with additional reasoning about Kate, Jules, and Jim. *Mathematical and Computer Modelling*, *47*, 168–177.

Bielczyk, N. (2013). Dynamical Models of Dyadic Interactions with Delay. *The Journal of Mathematical Sociology*, *37*(4), 223–249.

Bielczyk, N., Bodnar, M., & Foryś, U. (2012). Delay can stabilize: Love affairs dynamics. *Applied Mathematics and Computation*, *219*, 3923–3937. doi:10.1016/j.amc.2012.10.028

Buder, E. H. (1991). A Nonlinear Dynamic Model of Social Interaction. *Communication Research*, *18*, 174–198. doi:10.1177/009365091018002003

Carbonaro, B., & Giordano, C. (2005). A second step towards a stochastic mathematical description of human feelings. *Mathematical and Computer Modelling*, *41*, 587–614.

Carbonaro, B., & Serra, N. (2002). Towards mathematical models in psychology: a stochastic description of human feelings. *Mathematical Models and Methods in Applied Sciences*, *10*, 1453–1490.

Dercole, F. (1999). *A Modeling Interpretation of the Novel "Jules et Jim."* Department of Electronics and Information, Politecnico di Milano, Milano, Italy.

Feichtinger, G., Jorgensen, S., & Novak, A. J. (1999). Petrarch's Canzoniere: Rational addiction and amorous cycles. *Journal of Mathematical Sociology*, *23*(3), 225–240.

Felmlee, D. (2006). Application of dynamic systems analysis to dyadic interactions. In A. D. Ong & M. van Dulmen (Eds.), *Handbook of Methods in Positive Psychology* (pp. 409–422). New York, USA: Oxford University Press.

Felmlee, D., & Greenberg, D. (1999). A dynamic systems model of dyadic interaction. *Journal of Mathematical Sociology*, *23*(3), 155–180.

Ferrer, E., & Helm, J. L. (2013). Dynamical systems modeling of physiological coregulation in dyadic interactions. *International Journal of Psychophysiology*, *88*(3), 296–308.

Fruchter, G. E. (2014). Relationships in marketing and optimal control. *International Series in Operations Research & Management Science*, *198*, 95–106.

Fruchter, G. E., & Sigué, S. P. (2009). Social relationship and transactional marketing policies-maximizing customer lifetime value. *Journal of Optimization Theory and Applications*, *142*(3), 469–492.

Gottman, J. M., Murray, J. D., Swanson, C. C., Tyson, R., & Swanson, K. R. (2002). *The Mathematics of Marriage: Dynamic Nonlinear Models*. Massachussetts Institute of Technology: Bradford Book.

Gragnani, A., Rinaldi, S., & Feichtinger, G. (1997). Cyclic dynamics in romantic relationships. *International Journal of Bifurcation and Chaos*, *7*, 2611–2619.

Jones, F. J. (1995). *The Structure of Petrarch's Canzoniere: A Chronological, Psychological and Stylistic Analysis*. Cambridge, UK: Brewer.

Liao, X., & Ran, J. (2007). Hopf bifurcation in love dynamical models with nonlinear couples and time delays. *Chaos, Solitons & Fractals*, *31*(4), 853–865.

Mitchell, M. (1936). *Gone with the Wind*. New York, USA: Macmillan Publishers.

Ozalp, N., & Koca, I. (2012). A fractional order nonlinear dynamical model of interpersonal relationships. *Advances in Difference Equations*, *189*, 510–544.

Petrarca, F. (1374). *Rerum {V}ulgarium {F}ragmenta*. Padova, Italy.

Rey, J.-M. (2010). A mathematical model of sentimental dynamics accounting for marital dissolution. *PLoS ONE*, *5*(3), e9881.

Rey, J.-M. (2013). Sentimental equilibria with optimal control. *Mathematical and Computer Modelling*, *57*(7), 1965–1969.

Rinaldi, S. (1998a). Love dynamics: The case of linear couples. *Applied Mathematics and Computation*, *95*, 181–192.

Rinaldi, S. (1998b). Laura and Petrarch: An intriguing case of cyclical love dynamics. *SIAM Journal on Applied Mathematics*, *58*, 1205–1221.

Rinaldi, S., Della Rossa, F., & Dercole, F. (2010). Love and appeal in standard couples. *International Journal of Bifurcation and Chaos*, *20*(8), 2443–2451.

Rinaldi, S., Della Rossa, F., & Fasani, S. (2012). A conceptual model for the prediction of sexual intercourse in permanent couples. *Archives of Sexual Behavior*, *41*(6), 1337–1343.

Rinaldi, S., Della Rossa, F., & Landi, P. (2013). A mathematical model of "Gone with the Wind." *Physica A*, *392*(15), 3231–3239.

Rinaldi, S., & Gragnani, A. (1998a). Minimal models for dyadic processes: A review. In F. Orsucci (Ed.), *The Complex matters of Mind* (pp. 87–104). Singapore: World Scientific.

Rinaldi, S., & Gragnani, A. (1998b). Love dynamics between secure individuals: A modeling approach. *Nonlinear Dynamics, Psychology, and Life Sciences*, *2*, 283–301.

Rinaldi, S., Landi, P., & Della Rossa, F. (2013). Small discoveries can have great consequences in love affairs: The case of Beauty and The Beast. *International Journal of Bifurcation and Chaos*, *23*(11).

Roché, H.-P. (1953). *Jules et Jim*. Paris, France: Gallimard.

Rostand, E. (1897). *Cyrano de Bergerac*. Paris, France: Fasquelle.

Son, W. S., & Park, Y. J. (2011). Time delay effect on the love dynamical model. *Journal of the Korean Physical Society*, *59*(3), 2197–2204.

Sprott, J. C. (2004). Dynamical models of love. *Nonlinear Dynamics, Psychology, and Life Sciences*, *8,* 303–314.

Sprott, J. C. (2005). Dynamical models of happiness. *Nonlinear Dynamics, Psychology, and Life Sciences*, *9*, 23–36.

Strogatz, S. H. (1988). Love affairs and differential equations. *Mathematics Magazine*, *61*, 35.

Strogatz, S. H. (1994). *Nonlinear Dynamics and Chaos*. Reading, Massachussets: Addison-Wesley.

Wauer, J., Schwarzer, D., Cai, G. Q., & Lin, Y. K. (2007). Dynamical models of love with time-varying fluctuations. *Applied Mathematics and Computation*, *188*, 1535–1548.

FIGURE CAPTIONS

Figure 1. Still frames of the twelve film segments (PP1,...,PP16) described in the text (courtesy of Universal Studios).

Figure 2. Typical reaction functions $R_1(x_2)$ (red) and $R_2(x_1)$ (blue) of two secure individuals (see Rinaldi et al., (2010)).

Figure 3. The signs of the feelings in model (2) for all possible values of the appeals A_1 and A_2. In the green [red] regions the appeals are both positive [negative], while in the yellow regions they are of opposite sign. (a) The signs of the feelings during the initial phase of the love story. (b) The signs of the feelings in the long term. (c) The signs of the feelings in the long term when the two individuals are initially indifferent one to each other.

Figure 4. Null-clines and trajectories of model (2) in the space (x_1, x_2) of the feelings. Trajectories are vertical [horizontal] on the null-cline $\dot{x}_1 = 0$ [$\dot{x}_2 = 0$].

Figure 5. Interpretation of the love story between Elizabeth and Darcy with model (2). (a) The evolution of the perceived appeals (point 2: PP4; point 3: PP6; point 4: PP7; point 5: PP8; point 6: PP11). (b) The null-clines $\dot{x}_1 = 0$ $\dot{x}_2 = 0$ for the points 1,…,6 in (a).