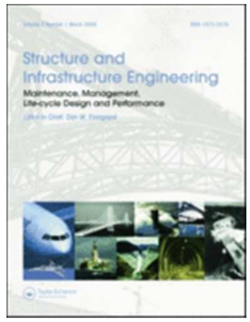


The use of a credibility index in life-cycle assessment of structures

Journal:	*Structure and Infrastructure Engineering*
Manuscript ID:	NSIE-2013-0117.R3
Manuscript Type:	Original Paper
Date Submitted by the Author:	n/a
Complete List of Authors:	Garavaglia, Elsa; Politecnico di Milano, Department of Civil and Environmental Engineering Sgambi, Luca; Politecnico di Milano, Department of Civil and Environmental Engineering
Keywords:	Life cycles, Predictions, Uncertainty principles, Bridges, Concrete structures, Infrastructure

The use of a credibility index in life-cycle assessment of structures

Elsa Garavaglia, Luca Sgambi

Department of Civil and Environmental Engineering, Politecnico di Milano, Milan, Italy

Elsa Garavaglia

Corresponding author

Department of Civil and Environmental Engineering, Politecnico di Milano

Piazza Leonardo da Vinci 32, 20133, Milan, Italy

e-mail: elsa.garavaglia@polimi.it

Luca Sgambi

Department of Civil and Environmental Engineering, Politecnico di Milano

Piazza Leonardo da Vinci 32, 20133, Milan, Italy

e-mail: luca.sgambi@polimi.it

The use of a credibility index to aid the probabilistic life-cycle assessment of bridges

The behavior of structural systems can change during their service lives due to unexpected loadings, environmental effects, and deterioration processes. In order to optimize maintenance interventions, the life-cycle of a structure has to be properly assessed.

Structural Health Monitoring (SHM) using collected experimental data provides a method for assessing the structural behavior over time. Since the cost related to SHM is substantial, sometime monitoring is limited in space and time. The modeling of the structural behavior based on few experimental data is characterized by uncertainty both in the choice of the appropriate model (epistemic uncertainty) and in parameters estimation (aleatory uncertainty).

This paper provides an original procedure to support decisions in the presence of epistemic uncertainty. The procedure provides the development of a credibility index able to catch, between two possible models, which is the most reliable to describe the evaluation of parameters of interest.

Considering as a case-study the occurrence of a foundation settlement in an arch bridge, the efficacy of the model proposed has been assessed. The approach can be applied to investigate the behavior of other aspects of the life-cycle assessment: the evolution of structural resistance, the failure time of an element or of the whole system.

Keywords: Life-Cycles Performance; Prediction Models; Uncertainty; Bridges; Infrastructure Management; Concrete Structures

Introduction

Several unforeseen causes can lead to a unexpected behavior of the structural systems over time: deterioration processes due to chemical attacks (Bertolini et al. 2004; Biondini et al. 2004; Saetta 2005; Biondini and Frangopol 2008), uncertainty in time-dependent behavior of concrete (Camossi et al. 2010; Malerba et al. 2011), foundation settlement (Terzaghi and Peck 1948; Das and Sivakugan 2007; Zhao et al. 2011), etc.. Therefore, the service life, aesthetics, and structural proprieties of the system under consideration may be compromised (Casciati and Faravelli 2010). Considering the above, it is clear that the structural analysis of existing buildings and constructions must be able to take into account some of these time-dependent phenomena. As a consequence, in the last decade, as a response to this demand, the Structural Life-Cycle Analysis (LCA) was developed and, nowadays, LCA, along with monitoring and planning methods, is playing an important role in civil engineering (Furuta et al. 2008; Furuta et al. 2011; Basso N. et al. 2012; Garavaglia et al. 2012, Basso P. et al. 2012, Okasha and Frangopol 2012, Decò and Frangopol 2013).

Any approach to LCA that does not consider time dependent effects could lead to a lack understanding of the assessment itself, and can potentially have as a consequence the improper planning of maintenance or repair actions, eventually leading even to the structural collapse of the system, with further expensive rehabilitation or rebuilding costs (Woodward 1988; Ciampoli 1998, Goins 2000; Proverbio et al. 2000). For this reason, LCA should not be implemented with a deterministic analysis, but it has to involve models and methods able to take into account the uncertainties present in it (Messervey et al. 2011, Elnashai and Tsompanakis 2012).

Over the last decade, different fuzzy or probabilistic approaches have been developed for studying the uncertainties involved in the problem (Biondini et al. 2004, Sgambi 2004, Akgul and Frangopol 2005, Pellissetti and Schüeller 2007, Pradlwarter et al. 2007, Bontempi et al. 2008, Melchers and Frangopol 2008, Möller and Beer 2008, Sgambi et al. 2012).

The uncertainty that can compromise a probabilistic modeling can be classified in: epistemic uncertainty and aleatory uncertainty (Vere – Jones and Ozaki 1982, Vere-Jones et al. 1995, Grandori et al. 1998, Ang 2004, Masri et al. 2009, Sankararaman and Mahadevan 2011, Ciampoli and Petrini 2012). The first one involves uncertainties that can be reduced through the attainment of more information and includes modeling choice and error. The second one involves statistical uncertainties regards the model's parameters estimation. Even if the model is correct it contains parameters that need to be estimated, often on the basis of the available experimental data.

In statistical analysis the choice between different models is usually supported by classic statistical tests, typically: maximum likelihood criterion or least squares method. Considering mainly the physics aspects of the phenomenon, these tests help to define the reliability of one model (or hypothesis), but a model isn't just a statistic consequence of a data set. Each distribution contains some physical interpretations and physical information that make the decision – making process easier and more aware (Guagenti et al. 2003, Blasone et al. 2008, Biondini et al. 2010, Goulet and Smith 2011, Garavaglia 2012).

Even if a model involves a subjective choice due to data interpretation, a careful analysis makes this choice reasonable and confident. Considering problems using small samples, a residual uncertainty range remains, in fact if few experimental data are available parameters estimation can be compromised.

In structural analysis, the evolution history of certain quantities can be modeled on the basis of monitoring data. However, only long-term and preferably continuous monitoring enables the best possible characterization of the parameter's variability over time. Because of the costs related to SHM are rather important, sometimes they are carried out within limited spatial and temporal bases and the data are recorded in a discrete way (only in some instants of the structural life span). In this case the estimation of the parameters involved in the modeling will be based on few available monitoring data and it could suffer from statistical uncertainty.

In this study, in order to investigate the error made in the modeling of a given structural quantity *C* in presence of few data, following the method proposed by Grandori (Grandori et al. 1998, 2003), a suitable credibility index has been developed. Objective of this index is to identify the most reliable model among some possible competitor models to foresee the behavior of a given parameter over the structure lifespan, even if the monitoring campaign is already terminated.

The developed methodology cannot solve the problem of the aleatory uncertainty affecting the estimation of parameters but it can be considered a valid supporting tool for dealing with decisions affected by epistemic uncertainty (e.g. the choice of the most reliable modeling for describing the evolution of a physical phenomenon).

This paper wants to point out the credibility index's effectiveness and potential with respect its possible engineering applications. The proposed methodology is applied to assess the service life of an arch bridge affected by unexpected foundation settlement *C*. Considering the structure of a Maillart's bridge, the beginning of foundation settlement is assumed to occur during its structural life. Since currently real monitoring data aren't available, it will be here assumed that the settlement varies following a known polynomial law. Applying a random procedure a possible dataset, simulating monitoring data, will be drawn and, on it, two evolution laws (*r* and *s*) will allow to foresee the structural behavior over the first decade of the bridge's service life, and in the following ones. Using the credibility index, it will be possible to point out which of the two models is closer to the actual behavior of the structure. Assuming two different trends of the settlement over time, the goal of this study is to verify the reliability of the index introduced.

The credibility index

Starting from a single data set concerning a given quantity *C*, the question is how to estimate the evolution of this quantity over time. The attention is focused on the error in estimating the quantity *C*. More generally, considering various and reasonable models, the goal is distinguishing the most reliable approach in order to estimate the quantity *C* studied. Following this objective and comparing two models a time, the procedure here presented is be able to give an opinion on which, between the two model investigated, is the most reliable in the evaluation of *C*.

Uncertainty in the evaluation of C in presence of few data and known evolution law F°.

If the evolution law F_C° of a given parameter *C* is completely defined over time (i.e. a probabilistic cumulative distribution), in each instant *t** the quantity *C* assumes the "*true value*" *C*° and it can be considered as the *statistical truth*. When the real process F_C° is well known but only samples of few data are available, an adequate modeling of them is necessary but, sometime, not simple. The measure of the error made using a wrong modeling can be investigated through the building of the credibility index Δ. Using a Monte Carlo simulation and following a random drawing procedure, several size ν samples can be drawn from the evolution law F_C°; they set up different conjectural realities, which the modeling must be based on. Assuming a plausible r – model, defined by its F_r law, each sample will be modeled with the r – model, and the value C_r will be evaluated for each instant *t**. Thus the random variable $\hat{C}_r$ is defined as the set of all C_r values obtained for each sample at every *t** time. The random variable's distribution is the sampling distribution of the parameter *C*.

The index:

$$\Delta_r^{\circ} = \Pr\left\{ C^{\circ} - h < \hat{C}_r \leq C^{\circ} + h \right\} \tag{1}$$

shows the probability the value $\hat{C}_r$, estimated with the $r-$model, is within a given range around the *true* value C° (h defines a conventional range around C°).

Considering a random size ν sample drawn from F_C°, Δ_r° indicates the probability that the model F_r leads to the C° value, with an error $\varepsilon_r^{\circ} \leq h$ (as absolute value)

$$\Delta_r^{\circ} = \Pr\left\{ \left| \varepsilon_r^{\circ} \right| \leq h \right\}. \tag{2}$$

Assuming a second plausible model, $s-$model (with law F_s), the same method can be applied and the index can be defined as:

$$\Delta_s^{\circ} = \Pr\left\{ C^{\circ} - h < \hat{C}_s \leq C^{\circ} + h \right\} \tag{3}$$

The difference

$$\Delta_{rs}^{\circ} = \Delta_r^{\circ} - \Delta_s^{\circ} \tag{4}$$

describes the *relative credibility* of the two models (Grandori et al. 1998, 2003). Evaluating C, if $r-$model is more reliable than $s-$model, the index Δ_{rs}° will be greater than zero, while if the $s-$model is the most reliable, Δ_{rs}° will be negative. Therefore, modeling a given quantity C, the sign of Δ_{rs}° is essential because it decides which model is the most reliable. The last statement underlines another property of the credibility index Δ. In Figure 1 the whole procedure is depicted.

[FIGURE 1 – HERE]

When C is the estimable quantity, starting from a small data set and considering *epistemic* uncertainty, the index Δ°_{rs} assesses the *credibility* of $r-$model in respect to F°_{C}; furthermore the index Δ°_{rs} enables to choose the most reliable model among two possible paths for modeling the given parameter *C*.

The index Δ°_{rs} cannot be considered as a complete validation of the model but instead an indicator of the goodness of the model chosen.

Uncertainty in the evaluation of C in presence of few data and unknown evolution law F°

When the real process F°_{C} is unknown and the modeling is only based on few monitoring data, investigating its credibility becomes very useful (Guagenti et al. 2003, and Garavaglia et al. 2010).). The procedure must be applied to the only available object: the data set, limited over space and time, and obtained by a monitoring action, operating on a given structure. Starting from the collected data, an experimental law F^{*}_{C} can be set up (e.g. a polyline); within this law, the monitoring instants are the points where the *C** values are estimated. Following the above-mentioned method, the empirical index Δ^{*}_{rs} can be assessed. This index is defined using a limited time of analysis; furthermore if no physical knowledge is available, the real behavior of the parameter in the future seems to be unpredictable. Therefore, it is necessary to investigate how much the index Δ^{*}_{rs} is *representative* of the real unknown Δ°_{rs}. Starting from the observed data, the conditional probability:

$$\Omega = \Pr\left\{ \Delta^{\circ}_{rs} > 0 \,\middle|\, \Delta^{*}_{rs} > 0 \right\} \qquad (5)$$

evaluates the posterior probability of hypothesis (Figure 2).

[FIGURE 2 – HERE]

Quantity Ω describes the probability that the unknown real Δ°_{rs} can be greater of zero if the experimental Δ^{*}_{rs} is greater of zero. In other words, Ω describes the probability that the more reliable model, actually, will be the *r*-model if $\Delta^{*}_{rs}>0$, and vice versa. Assessing Ω, all the possible realities must be pointed out, even if Ω cannot be exactly defined. However, it is possible to study several plausible realities (Guagenti et al. 2003). The above-mentioned simulation and estimation must be repeated for each of the studied realities. Therefore, the observed value Δ^{*} becomes much more than an indicator of goodness: it measures the winning probability of a model over another one.

Application to a case study

To evaluate the efficacy and the applicability of the procedure to a real structural problem, the authors proposed its application on an existing structure. The application is the first test of this procedure. Even if the case-study is a real structural system, with regard to the dangerous phenomenon studied, the foundation settlement, some assumptions in data and evolution laws must be made, since currently there are no real data which are completely available. The case-study chosen is the existing arch bridge over the Corace river (Calabria, Italy), built in 1955 (Galli and Franciosi 1955).

The bridge has suffered from foundation problems such as a recent landslide that occurred in 2010. For this reason, it is currently subject to a monitoring activity by the "Regione Calabria", but, currently, such data is not available.

Figure 3 shows bridge's dimensions (Franciosi 1971, Ronca and Cohn 1979, Biondini and Frangopol 2008b); the arch span is 80.7 m. Table 1 presents the distribution of steel reinforcement into different sections of the bridge.

[FIGURE 3 – HERE]

[TABLE 1 – HERE]

The bridge is expected to be affected by a variable foundation settlement over time. Assuming that during the first 10 years following the construction, few monitoring data concerning the foundation settlement can be made, it becomes relevant to investigate its evolution law over time starting from the few monitoring data collected. Referring to the methodology introduced above, the model to be judged consists in the evolution law F_C° of the foundation settlement $C(t)$ of the bridge analyzed, obtained throughout a small data set. Therefore, to extend the index Δ to the present case the quantity to estimate here is a "*value*": precisely the foundation settlement $C_s(t)$ at a given time t, assumed as a critical instant during the lifetime of the bridge.

Settlement is assumed to vary according to a known polynomial law. Two particular cases will be analyzed: the first one represents a stable settlement over time (consolidation of clay soil) while the second one represents a sinking increasing over time (unstable slope). Figure 4a shows the considered settlement trends over time.

These two different trends are very similar to each other within the first 20 years. However, these different models, each of them with different evolution laws, can provide results much similar in value over the first 20 years, but if a long term prediction must be conducted, different and potentially incorrect results might be attained by each single law used.

In the following, considering two evolution laws (*r* and *s*), the method validity and the bridge performance are investigated. In Figure 4b, two different parabolic laws are assumed to be representative of these models. Referring to a short-term, the *r*-model is closer to a soil stable behavior, while the *s*-model simulates an unstable behavior (Figure 4b).

[FIGURE 4 – HERE]

To assess the influence of the settlement on the bearing capacity of the bridge non-linear analyses were developed using ADINA software. For the assessment of the collapse multiplier λ_p, a numerical model with beam elements was developed on the basis of the geometrical characteristics showed in Figure 3. The bridge deck and the arch were modeled using beam elements, whereas the columns were modeled with truss elements. Material nonlinearity was introduced in beam elements through an appropriate moment-curvature relationship (Ballard and Sedarat 1999; Jones et al. 2004, Garavaglia et al. 2013). For each different section of beam elements (considering both the geometry and the reinforcement, like reported in Table 1) the authors assessed bending moment-curvature relationships for different values *N* of the axial force. This evaluation was performed using a home-made computer code able to perform sectional analysis of reinforced concrete sections. The relationships obtained were subsequently simplified into a tri-linear elastic perfectly plastic models and included into the numerical model of the bridge. Figure 5b shows the beam bending moment-curvature curves for different values *N* of axial force on the arch section.

[FIGURE 5 – HERE]

On the bridge, three loads were considered: *g* represents the dead load, *p* the service load (road traffic) and *C* the assumed settlement. Using non-linear analyses the collapse load multiplier of the service load λ_p was assessed. Briefly, defined the value of the settlement to the base of the arch, the bridge's model was loaded with its dead load (*g*) and with the settlement load (*C*). At the end of these steps the service load was introduced and increased up to the collapse of the bridge.

Figure 6 shows the static model of the bridge (Fig. 6a) and the outcomes of one of nonlinear analyses made (Fig. 6b). In the case shown in figure, the analysis stops with a λ_p= 1.9831, the collapse of the bridge due to a load equal to 1.9831 times the normal service load of the bridge. In Figure 6b are even marked the zones in which the sectional behavior went beyond the linear elastic field. Repeating the structural analysis for different settlement values, the influence of the settlement load on the bearing capacity of the bridge was obtained. This information is summarized in Figure 6c. If the foundation settlement is not present, λ_p is about 2.1 times the service load with a settlement of about 0.33 m the bridge is not able to sustain the service traffic load. Since the collapse multiplier is evaluated on the service loads, the authors assume a λ_p=1.5 as a safety threshold, that correspond to a settlement of 0.24 m.

[FIGURE 6 – HERE]

Assuming the value of the settlement *C* is known by monitoring, the abovementioned methodology is applied to the index Δ evaluation for predicting the credibility of the collapse load multiplier λ_p of the bridge over time as consequence of the time-dependent foundation settlement.

First results

To test the procedure for the settlement of the bridge foundations in Figure 3 it was supposed that five measurements were available, and that they were made between the fifth and tenth year of its life (see also Figure 7). Starting from this monitoring campaign, the evolution of the settlement *C* follows the discrete trend shown in Figure 4a. This information may be ambiguous: the law evolution of *C* can be stable or unstable. Basing on pure statistical fitting of experimental data, an error could affect *C* prediction at a given time. Extending the monitoring time and supporting the few available experimental data with a thorough knowledge of the physical phenomenon reduces the uncertainty and enables to quantify the error. The knowledge on foundation settlement suggests different evolution laws for *C*, and here two of them have been investigated as possible behavior of the bridge foundation settlement (Figure 4b). Since two possible behaviors are admitted, the credibility index Δ_{rs} can be very useful for the decision-making process.

Hypothesis: the evolution law of C is stable (Figure 4a)

The procedure introduced in the previous sections is here applied on a soil with stable behavior.

Firstly considering path *r* (Figure 4b) as the *true* evolution law of *C* (see table in Figure 4) and using a Monte Carlo simulation, combined with a random drawing procedure, 1000 samples of size ν = 5 were drawn (each sample consists in 5 values of $\hat{C}_r$ as the size of the monitoring dataset). In this first application, the distribution function used in the random drawing procedure was a uniform distribution function. The samples are drawn over 10 years and they can be considered a *conjectural reality*. Assuming

$h = [0.05 \cdot C^{\circ}]$, Eq. (1) evaluates index Δ_r° for each year between the tenth and the twentieth.

Afterwards considering path *s* as the *true* evolution law of *C*, the same procedure was applied and each value $\hat{C}_s$ between the tenth and the twentieth year was estimated for each sample drawn, so as the index Δ_s° (with $h = [0.05 \cdot C^{\circ}]$).

Eq. (4) provided the value and the sign of Δ_{rs}°. The results obtained are shown in Table 2.

[TABLE 2 – HERE]

The positive sign of Δ_{rs}° shown in Table 2 suggests that the *r* – model is the most reliable in the modeling of a *true* stable behavior. The last three columns of the table show the assessed values $\lambda°$, λ_r and λ_s of collapse multipliers for the "true" model r – model and s – model respectively. Note that for the next decade after the monitoring, both models (*r* and *s*) give a good answer: the prediction model based on law *r* is just a 10% better than the prediction model based on law *s*. Fifty years after the monitoring activity, comparing the behavior predicted by *s*–law with the actual behavior of the soil, the *s* – model becomes unreliable. While the *r* – model can predict the collapse multiplier with an error of about 5%, the forecast based on *s* – model is completely wrong (*s* – model predicts a collapse of the bridge, while the actual collapse load multiplier remains 1.83).

Hypothesis: the evolution law of C is unstable (Figure 4b)

For a soil with unstable behavior, the above described procedure is repeated and the results are shown in Table 3.

[TABLE 3 – HERE]

The negative sign of Δ°_{rs} points out the unreliability of the prediction law *r* which models the soil behavior. Over the next decade after the monitoring, the prediction laws *r* and *s* provide the same results. After fifty years, the unstable behavior of the soil causes the collapse of the bridge (the service loads multiplier at collapse results to be equal or less than zero). An analysis based on the prediction law *r* wrongly shows that the bridge is still safe, while the winning in credibility prediction *s* – model rightly predicts the non-safety of the structure. The index Δ°_{rs} shows a great effectiveness in catching the true behavior of the parameter investigated.

Real situation: the evolution law of C is unknown

In common practice, the evolution law F°_{C} of C° is unknown; therefore, F^{*}_{C}, $\hat{C}^{*}$ and Δ^{*}_{rs} must be experimentally *evaluated* through a given procedure, $\Delta^{*}_{rs} = \mathrm{E}_{rs}(F^{*}_{C})$ (E_{rs} is the procedure chosen). Figures 7a-b points out the difficulty in choosing the most reliable interpretation model of the foundation settlement in the portion analyzed.

Selecting a discrete polyline law F^{*}_{C}, the small data set can be modeled (Fig. 7a). Since the monitoring is time-limited, sometimes a simple linear regression may be sufficient to fit the experimental data. If no other information is available, this tendency could describe the evolution of parameter *C*, even if it's a strong simplification. However, here the linear regression is assumed and the tendency of the law has been investigated both over a 20-year and a 50-year period (Fig. 7b).

[FIGURE 7 – HERE]

If the simple regression law is assumed as evolution law, the regression branch at 50 year should lead to the value $\hat{C}^*(50)$ =0.249 m. Using both the $r-$model and $s-$model the small monitoring dataset has been modeled and indexes Δ_r^* and Δ_s^* have been evaluated. The results obtained are shown in Table 4.

These results show the difficulty in interpreting the phenomenon investigated when a short term monitoring has been carried out.

The index Δ_{rs}^* at 10th year is close to zero therefore the models reliability is quite the same. The index Δ_{rs}^* at 20th year results positive; therefore, over a 50 years period, the $r-$model is the most reliable and it results in $\hat{C}_r^*(50)$ =0.144517 m (Tab. 4). Nevertheless, considering 50th year the tendency changes: an index value $\Delta_{rs}^* < 0$ means that actually the $s-$model is the most reliable model over a 50 years period and it results in $\hat{C}_s^*(50)$ =0.339625 m (Tab 4).

Thus a wrong choice can lead up to underestimation or overestimation of $\hat{C}^*$ value (50) with significant consequences for maintenance and rehabilitation.

[TABLE 4 – HERE]

Figure 7a-b point out the mutual reliance between monitoring and reliability of the choice: longer the monitoring, higher the reliability of the choice. Therefore, a reasonably regular monitoring distributed over a time range between 10th and 30th year of the structural service life could suggest the behavior law of foundation settlement when it's uncertain.

Some remarks

By the first results here obtained it's noticeable that:

(1) When only a limited amount of monitoring data is available, there is a high risk of a wrong prediction of the evolution law of a given quantity that could influence the erroneous prediction of safety and or collapse of our structural systems.

(2) When the law describing a certain phenomenon is unknown, the credibility index provides valuable information on the correctness of the modeling process applied on data collected.

(3) The positive or negative value of Δ concurs in defining the process correctness. Furthermore it is necessary to consider the time Δ takes to reach 0: faster it reaches 0, smaller the reliability of both the processes is. Table 2 and 3 explain it.

(4) In both the cases studied, the index Δ enabled support the decision concerning the most reliable model to use in the prediction on the possible evolution law over time for the arch bridge investigated.

Conclusions

The structural behavior of a civil structure may differ from the expected one due to unforeseen environmental attacks; therefore an effective structural monitoring is necessary. The use of monitoring can help to enable the control of structural behavior and the residual life prediction of the structure. Thus, effective maintenance can be scheduled. Since monitoring data are usually affected by uncertainty and sometime, for different reasons, they are gathered over a short period of time, the knowledge of the physical phenomenon modeled must be deepened in a statistical or probabilistic way. Furthermore, the small size of available monitoring datasets could become a critical issue because of it can compromise the modeling of structural quantities over time and the prediction of its development over time.

In this paper an index has been introduced in order to identify between two plausible models, which is the most reliable to describe the evolution of phenomenon or quantity investigated. Basing on the credibility index, Δ, introduced by Grandori et al. (1998, 2003) for representing a probabilistic model in evaluating a static hazard quantity, the same index has been proposed as a possible credibility index to define the most suitable model in the modeling of the evolution law of a given structural quantity.

Following Grandori's approach the method is applied to a real case-study. This procedure has good effectiveness in identifying the most reliable evolution law for the settlement *C* of an arch bridge. Considering few monitoring dataset describing the physical phenomenon, the methodology developed appears to be suitable.

The procedure proposed, through the evaluation of the evolution of the foundation settlement *C* has shown how a wrong modeling of a given quantity can lead to a wrong estimation of the structural response with dangerous consequence. In fact, a wrong modeling of *C* can lead to a wrong estimation of the λ loads multiplier with a possible overestimation of the bridge bearing capability. It has shown like the procedure proposed is able to give information also on the error made in the estimation of a given quantity C in presence of few data and an unknown evolution law.

Acknowledgments

The second author wishes to acknowledge the support of the Regione Lombardia within the project “Dote Ricercatori”. Authors are very grateful to anonymous reviewers for their precious advices during the review process, Prof. Pier Giorgio Malerba, Dr. Noemi Basso and Dr. Konstantinos Gkoumas for their helpfulness and detailed discussions.

REFERENCES

Akgul, F., and Frangopol, D.M. (2005). Lifetime performance analysis of existing reinforced concrete bridges: I Theory, *Journal of Infrastructure Systems*, ASCE, **11**(2), 122-128.

Ang, A. H-S. (2004). Modeling and analysis of uncertainties for risk-informed decisions in engineering -- with application to bridges, *Keynote lecture, in Proc. of* IABMAS’04*, Second Int. Conf. on Bridge Maintenance, Safety and Management*, Ed. E. Watanabe, D.M. Frangopol, T. Utsunomiya, A.A. Balkema Publishers, Leiden, The Netherlands.

Ballard T.A., Sedarat H. (1999). SR5 Lake Washington Ship Canal Bridge pushover analysis. *Computers and Structures,* **72**, 63 – 80.

Basso, N., Garavaglia, E. and Sgambi, L. (2012). Selective maintenance planning based on a Markovian approach, *Pro. of* IALCCE 2012, *3rd International Symposium on Life-Cycle Civil Engineering*, Vienna, Austria, October 3-6, 2012 , 435-441.

Basso, P., Casciati, S., Faravelli, L. (2012). Estimating the remaining service life of a historical railway bridge, *Bridge Maintenance, Safety, Management, Resilience and Sustainability - Proceedings of the Sixth International Conference on Bridge Maintenance, Safety and Management*, 608-616.

Bertolini, L., Elsener, B., Pedeferri, P. and Polder, R.P. (2004). *Corrosion of Steel in Concrete: Prevention, Diagnosis, Repair*, WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim.

Biondini, F. and Frangopol, D.M. (2008). Long-term Performance of Structural Systems, *Structure and Infrastructure Engineering*, **4**(2), 72, Special Issue, Fabio Biondini (ed.), Taylor & Francis Publisher.

Biondini, F., and Frangopol, D.M. (2008b), Probabilistic Limit Analysis and Lifetime Prediction of Concrete Structures. *Structure and Infrastructure Engineering,* **4**(5), 399-412.

Biondini, F., Bontempi, F. and Malerba, P.G. (2004). Fuzzy reliability analysis of concrete structures, *Computers & Structures*, **82(**13-14), 1033-1052.

Biondini, F., Bontempi, F., Frangopol, D.M., and Malerba, P.G., (2004). Cellular Automata Approach to Durability Analysis of Concrete Structures in Aggressive Environments. ASCE *Journal of Structural Engineering,* **130**(11), 1724-1737.

Biondini, F., Frangopol, D.M. and Garavaglia, E. (2010). Credibility indicator for bridge service life prediction. *Proc. of* IABMAS’10 *5th Int. Conf. on Bridge Maintenance, Safety and Management*, Philadelphia, Pensylvania, USA, July 11-15, 2010, CD-ROM, 1363-1366.

Biondini, F., Frangopol, D.M. and Malerba, P.G. (2008). Uncertainty effects on lifetime structural performance of cable-stayed bridges, *Probabilistic Engineering Mechanics*, **23**(4) , 509-522.

Blasone, R.S., Vrugt J.A., Madsen, H. , Rosbjerg, D., Robinson, B.A. and Zyvoloski, G.A. (2008). Generalized likelihood uncertainty estimation (GLUE) using adaptive Markov Chain Monte Carlo sampling, *Advances in Water Resources* **31**, 630–648.

Bontempi, F., Gkoumas, K., and Arangio, S. (2008). Systemic approach for the maintenance of complex structural systems, *Structure and Infrastructure Engineering*, **4**(2), 77-94.

Camossi, G., Malerba, P.G., and Sgambi, L. (2010). Role of uncertainties on time dependent behavior of prestressed and cable stayed concrete bridges. *Proc. of* IABMAS'10 *5th Int. Conf. on Bridge Maintenance, Safety and Management*, Philadelphia, Pensylvania, USA, July 11-15, 2010, CD-ROM.

Casciati, S. and Faravelli, L. (2010). Vulnerability assessment for medieval civic towers. *Structure and Infrastructure Engineering*, **6**(1-2) , 193-203.

Ciampoli, M. (1998). Time dependent reliability of structural systems subject to deterioration. *Computers & Structures*, **67**(1-3), 29-35.

Ciampoli, M. and Petrini, F. (2012). Performance-Based Aeolian Risk assessment and reduction for tall buildings. *Probabilistic Engineering Mechanics*, **28**, 75–84.

Das, B.M. and Sivakugan, N. (2007). Settlements of shallow foundations on granular soil — an overview. *International Journal of Geotechnical Engineering*, **1**, 19–29.

Decò, A. and Frangopol, D.M. (2013). Life-cycle risk assessment of spatially distributed aging bridges under seismic and traffic hazards, *Earthquake Spectra*, **29**(1) , 127-153.

Elnashai, A.S. and Tsompanakis, Y. (2012). Uncertainties in life-cycle analysis and design of structures and infrastructures, *Structure and Infrastructure Engineering*, **8**(10), 891-892.

Franciosi V. (1971) (In Italian). Scienza delle costruzioni. Editore Liguori, Napoli.

Furuta, H., Frangopol, D.M. and Nakatsu, K. (2011). Life-cycle cost of civil infrastructure with emphasis on balancing structural performance and seismic risk of road network, *Structure and Infrastructure Engineering*, **7**(1), 65-74.

Furuta, H., Ishibashi, K., Nakatsu, K. and Hotta, S. (2008). Optimal Restoration Scheduling of Damaged Networks Under Uncertain Environment by Using Improved Genetic Algorithm, *Tsinghua Science and Technology*, **13**(SUPPL. 1), 400-405.

Galli, A., Franciosi, V. (1955). Il calcolo a rottura dei ponti a volta sottile ed impalcato irrigidente. *Giornale del Genio Civile*, 11, 686-700 (in Italian).

Garavaglia, E. (2012). The credibility measure of probabilistic approaches in life-cycle assessment of complex systems: a discussion. *Materials with Complex Behaviour II, Advanced Structured Materials* **16**(38), 619-636.

Garavaglia, E. and Sgambi, L. (2004). The credibility of lifetime assessment based on few experimental measurements, *Proc. of* IABMAS'04*, Second Int. Conf. on Bridge Maintenance, Safety and Management*, Ed. E. Watanabe, D.M. Frangopol, T. Utsunomiya, A.A. Balkema Publishers, Leiden, The Netherlands, CD-ROM.

Garavaglia, E., Guagenti, E., Pavani, R. and Petrini, L. (2010). Renewal Models for Earthquake Predictability, *Journal of Seismology*, **14**(1), 79-93.

Garavaglia, E., Sgambi, L. and Basso, N. (2012). Selective maintenance strategies applied to a bridge deteriorating steel truss, *Proc. of* IABMAS 2012, *Sixth Int. Conf. on Bridge Maintenance, Safety and Management*, Stresa, Lake Maggiore, Italy, July 8-12, 2012, 1764-1770.

Garavaglia E., Pizzigoni A., Sgambi L. and Basso N. (2013). "Collapse behaviour in reciprocal space frame structures", *Structural Engineering and Mechanics*, **46**(4): 533-547. Techno-Press, Korea.

Goulet, J.A. and Smith, I.F.C. (2011). Overcoming the limitations of traditional model-updating approaches. *Vulnerability Uncertainty and Risk Analysis Modeling and Management*, ASCE, **10(**21), 903-913.

Grandori, G., Guagenti, E. and Tagliani, A. (1998). A proposal for comparing the reliabilities of alternative seismic hazard models, *Journal of Seismology,* **2,** 27-35.

Grandori, G., Guagenti, E. and Tagliani, A. (2003). Magnitude distribution versus local seismic hazard, *Bull. of Seismological Society of America,* **93**(3), 1091-1098.

Guagenti E., Garavaglia, E. and Petrini, L. (2003). Probabilistic Hazard Models: is it Possible a Statistical Validation? in: "System – based Vision for Strategic and Creative Design" *Proc. of* ISEC02, *2nd Int. Structural Engineering and Construction Conf*erence, **2**, 1211 – 1216.

Jones M., Treyger S. and Pence P. (2004). Seismic analysis of the New Tacoma narrows suspension bridge. Proc. of *13th World Conference on Earthquake Engineering* , Vancouver, B.C., Canada.

Malerba, P.G., Garavaglia, E. and Sgambi, L. (2011). Fuzzy-Monte Carlo simulation for the safety assessment of bridge piers in presence of scouring. *Proc. of The 2001 World Congress on Advances in Structural Engineering and Mechanics*.

Masri, S.F. , Ghanem, R., Arrate, F., Caffrey, J.P. (2009). A data-based procedure for analyzing the response of uncertain nonlinear systems, *Structural Control and Health Monitoring* **16** (7-8), 724-750

Melchers, R.E. and Frangopol, D.M. (Editor) (2008). Probabilistic Modeling of Structural Degradation, *Special Issue of Reliability Engineering & System Safety* , Elsevier , **93**(3), 363-500.

Messervey, T.B., Frangopol, D.M. and Casciati, S. (2011). Application of the statistics of extremes to the reliability assessment and performance prediction of monitored highway bridges, *Structure and Infrastructure Engineering*, **7**(1), 87-99.

Möller, B. and Beer, M. (2008). Engineering computation under uncertainty – Capabilities of non-traditional models. *Computers & Structures*, **86**(10), 1024-1041.

Okasha, N.M. and Frangopol, D.M. (2012). Integration of structural health monitoring in a system performance based life-cycle bridge management framework, *Structure and Infrastructure Engineering*, **8**(11) , 999-1016.

Pellissetti, M.F. and Schuëller, G.I. (2007), The effects of uncertainties in structural analysis, *Journal of Structural Engineering and Mechanics,* **25**, 311-330.

Pradlwarter, H.J., Schuëller, G.I., Koutsourelakis, P.S. and Charmpis, D.C. (2007). Application of Line Sampling Simulation Method to Reliability Benchmark Problems, *Structural Safety,* **29**, 208-221.

Proverbio, E. and Ricciardi, G. (2000). Failure of a 40 years old post-tensioned bridge near seaside, *Proceedings of the International Conference Eurocorr 2000.*

Ronca, P. and Cohn, M.Z. (1979). Matrix-MP Method for the Analysis of Inelastic Arch Structures. *International Journal for Numerical Methods in Engineering*, 14, 703-725.

Saetta, A. (2005). Deterioration of Reinforced Concrete Structures due to Chemical-Physical Phenomena: Model-Based Simulation, *Journal of Materials for Civil Engineering*, ASCE, **17**(3), 313-319.

Sankararaman, S., and Mahadevan, S. (2011). Model validation under epistemic uncertainty, *Reliability Engineering and System Safety*. **96** (9), 1232 –1241.

Sgambi, L. (2004). Fuzzy theory based approach for three-dimensional nonlinear analysis of reinforced concrete two-blade bridge piers, *Computers & Structures*, **82(**13-14), 1067-1076.

Sgambi L., Gkoumas K. & Bontempi F. (2012). "Genetic algorithms for the dependability assurance in the design of a long span suspension bridge", *Computer-Aided Civil and Infrastructure Engineering*, **27**(9):655-675.

Terzaghi, K. and Peck, R.B. (1948). *Soil mechanics in engineering practice*, 1st Edition, John Wiley & Sons, New York.

Vere-Jones, D. and Ozaki, T. (1982). Some examples of statistical estimation applied to earthquake data – I. Cyclic Poisson and self-exciting models, *Ann. Inst. Statist. Math*., **34**, Part B, 189-207.

Woodward, R.J., and Williams, F.W. (1988). Collapse of Ynys-y-Gwas Bridge, West Glamorgan, *Proc. of the Institution of Civil Engineers*, **84**, 634 – 669.

Zhao, D., Zhao, Y. and Ye Y. (2011). Study on the frame structure under nonuniform settlement. *Applied Mechanics and Materials*, **94 – 96**, 830 – 833.

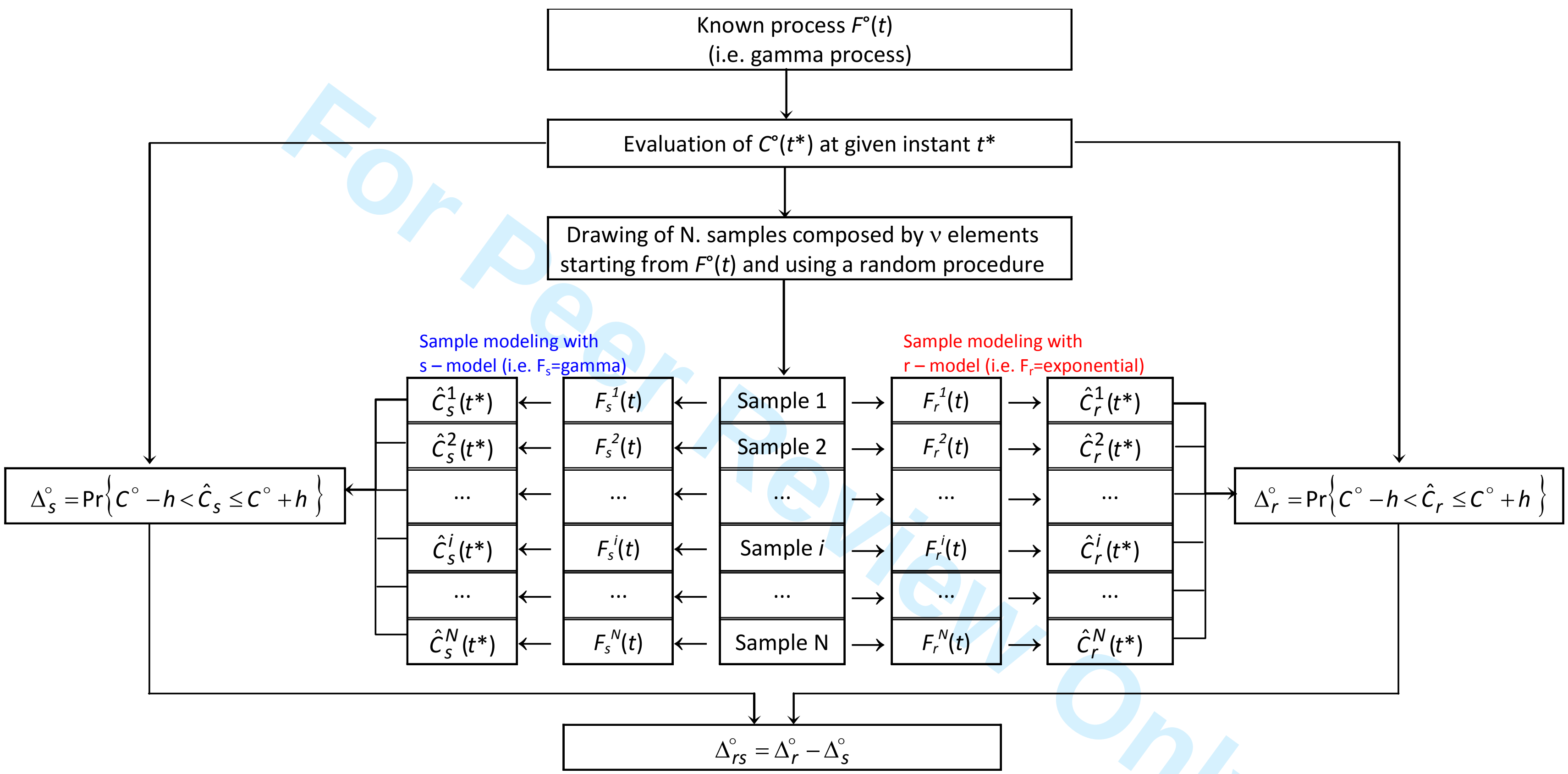


Figure 1. Credibility index: flowchart of the process.

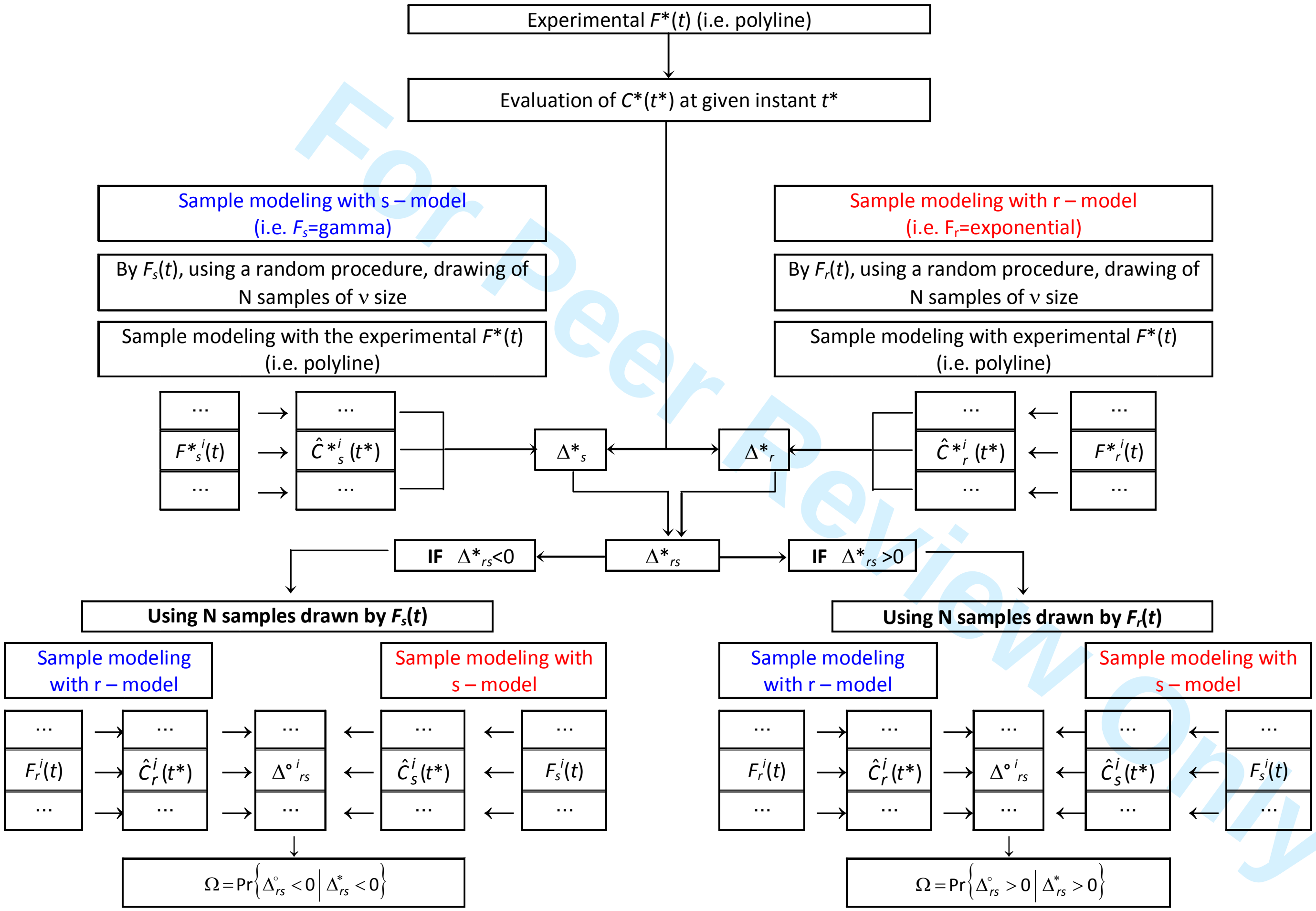


Figure 2. Flow chart showing the evaluation process of the posterior probability of hypothesis.

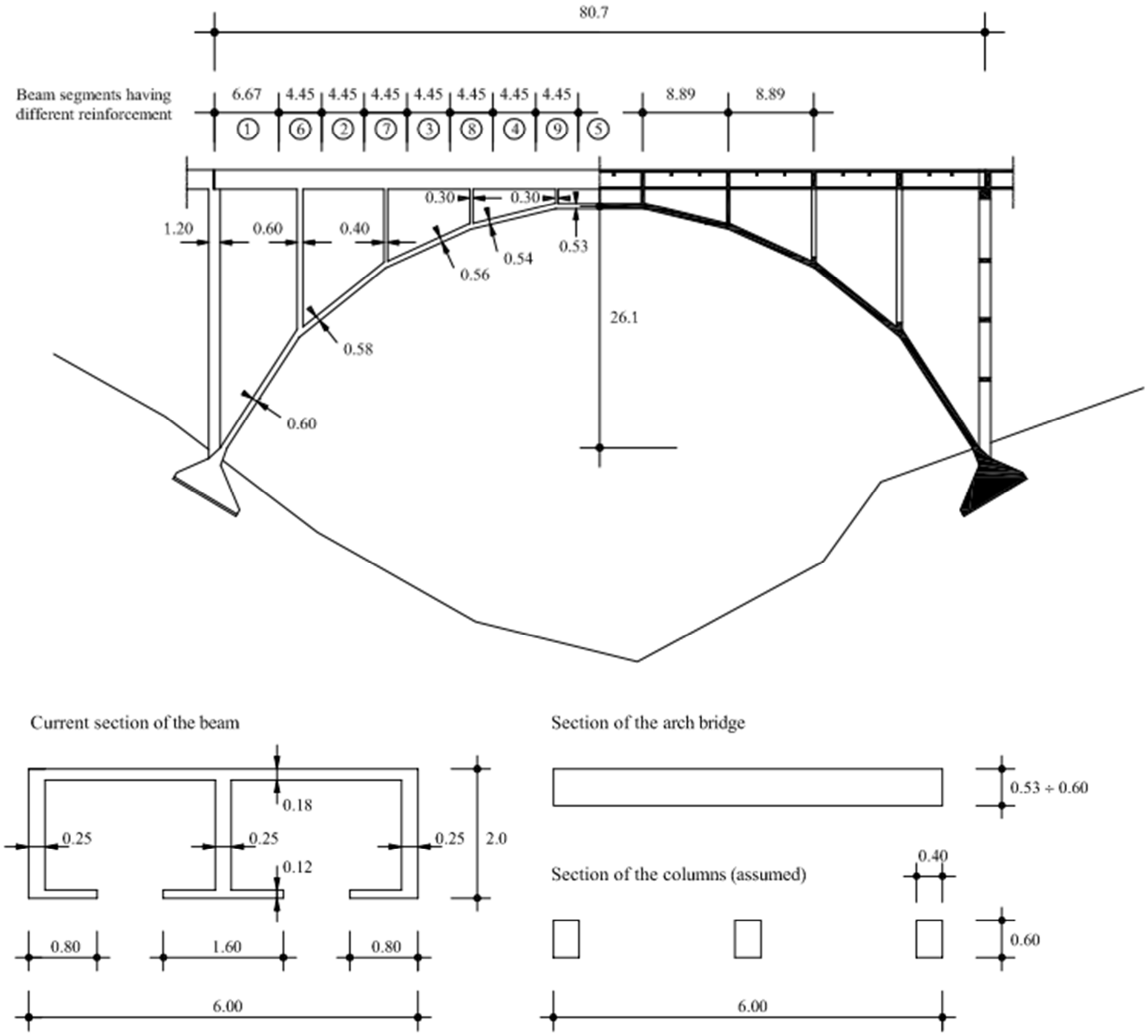
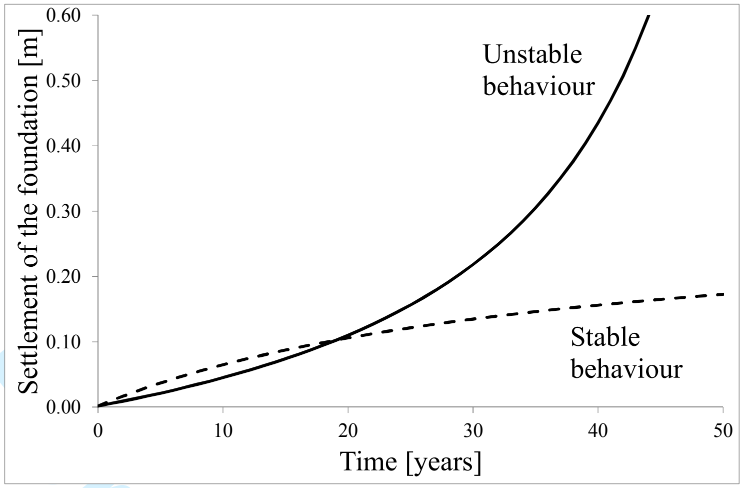
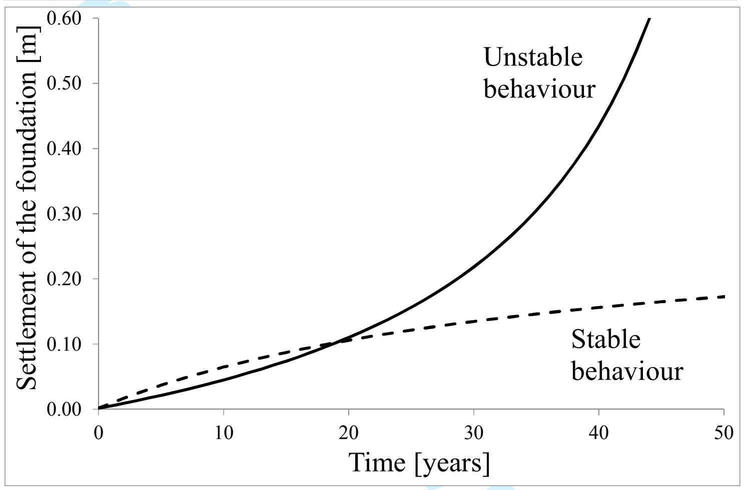


Figure 3. Overall dimensions of the arch bridge and detail of the current section of the beam, of the arch bridge and of the columns (measures expressed in meter).



a)



b)

	Settlement and prediction functions
Stable behaviour	$C(t) = \frac{1}{3} - \frac{1}{3+t}$
Unstable behaviour	$C(t) = \frac{13}{60-t} - \frac{13}{60}$
Prediction laws	$C(t) = a \cdot t^2 + b \cdot t$

Figure 4. Evolution laws (a) assumed to describe the foundation settlement over time and prediction laws (b). The model parameters (*a* and *b*) are estimated during the prediction phase with a least-squares procedure.

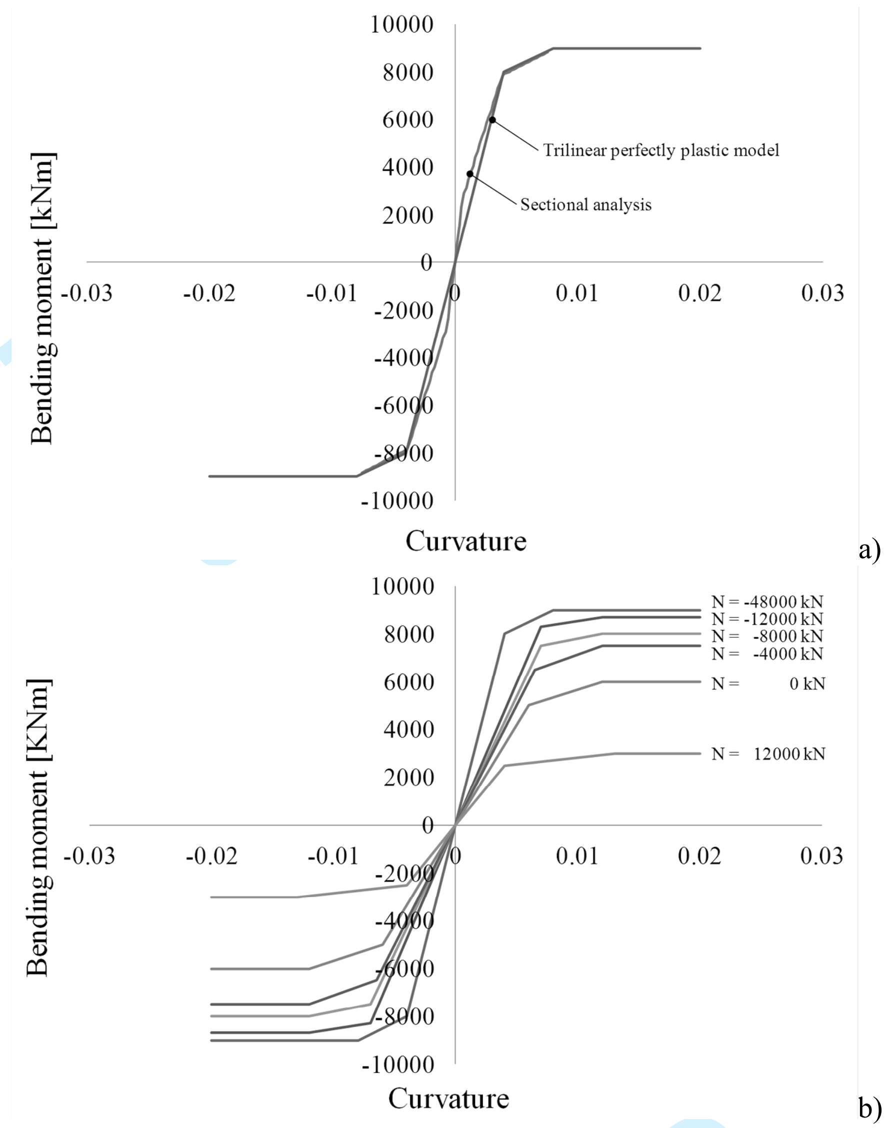
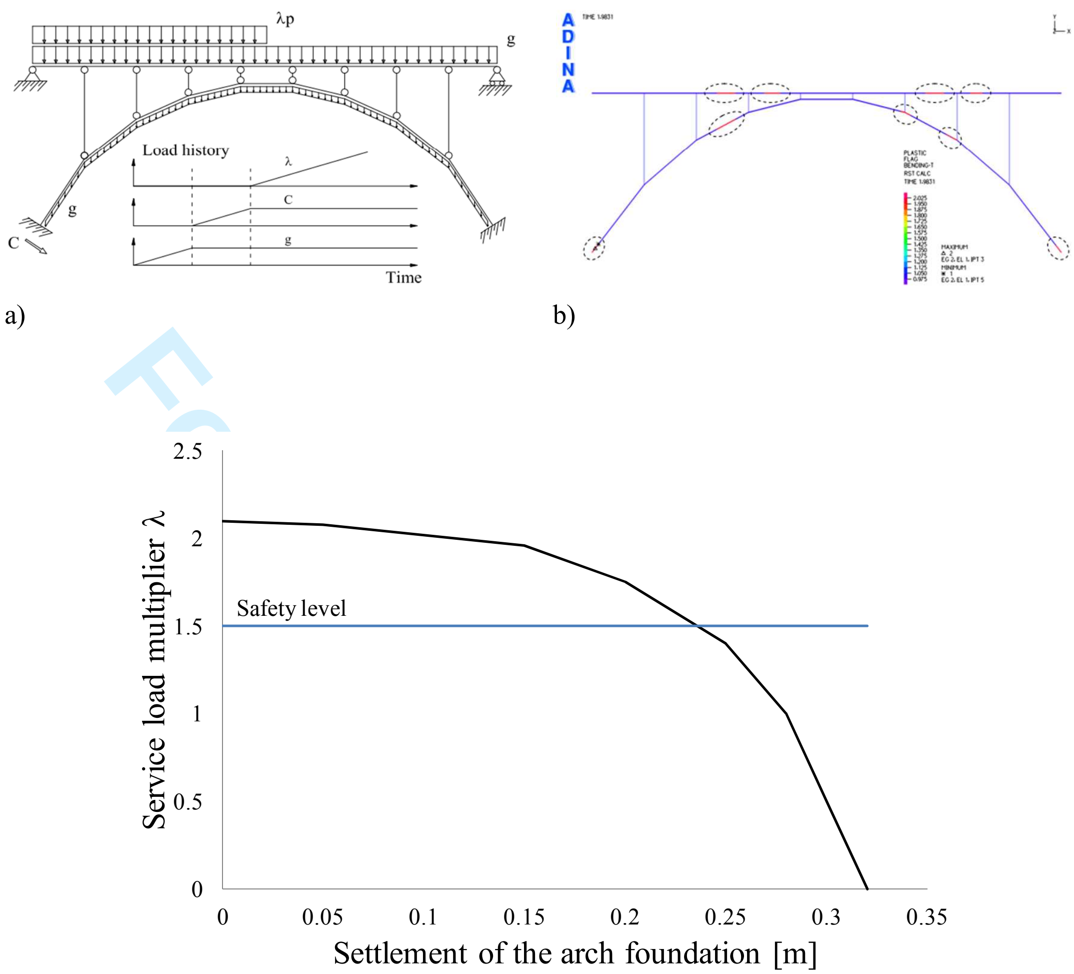


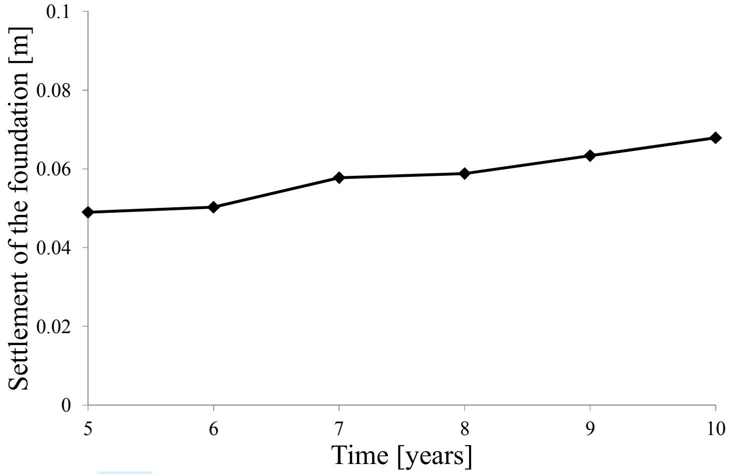
Figure 5. a) Bending moment-curvature for the arch section assessed with sectional analysis and tri-linear perfectly plastic simplification; b) series of bending moment-curvature curves for the arch section with different axial force (N).



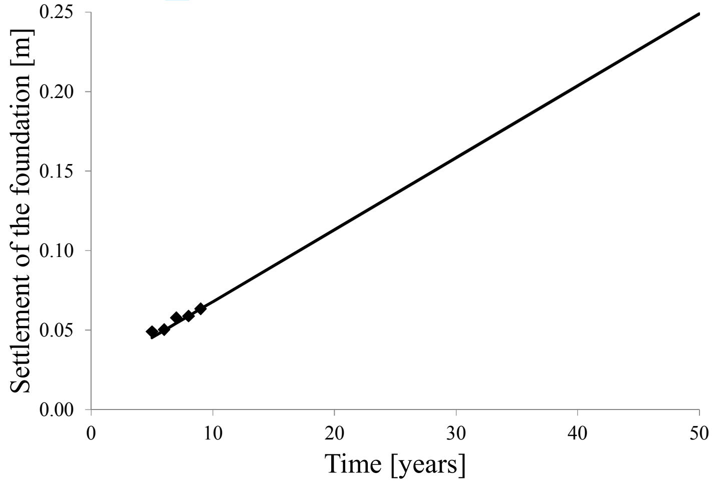
a) b)

c)

Figure 6. a) Load model of the bridge. The collapse load multiplier λ_p is 1.9831 (on the top-left in the figure). b) Outcomes of the nonlinear analysis performed. The figure shows the plasticized zones of the structure. c) Variation of the collapse load multiplier over settlement.



a)



b)

Figure 7. Case-study: small data set, a) polyline describing the tendency; b) simple linear regression of data describing the tendency over time.

Distribution at the top A_s' and at the bottom A_s reinforcement along the beam									
Section	1	2	3	4	5	6	7	8	9
A_s'	21Φ28 130Φ8	48Φ28 130Φ8	42Φ28 130Φ8	30Φ28 130Φ8	24Φ28 130Φ8	48Φ28 130Φ8	48Φ28 130Φ8	45Φ28 130Φ8	33Φ28 130Φ8
A_s	21Φ28	30Φ28	42Φ28	24Φ28	24Φ28	21Φ28	36Φ28	27Φ28	24Φ28
Reinforcement along the arch bridge: 45 (on the top) + 45Φ28 (on the bottom of the section)									
Reinforcement along the columns: 3 x 8Φ18 (assumed, each column in the 2D numerical model represents 3 columns of the bridge).									

Table 1. Distribution of the steel reinforcement in the beam, in the arch and in the columns (the upper, A_s', and downer, A_s, steel areas are expressed in number of steel bars per diameter).

Time of monitoring	Δ_r°	Δ_s°	Δ_{rs}°	λ°	λ_r	λ_s
11 years	1.00	1.00	0.00	2.05	2.05	2.05
12 years	1.00	0.98	0.02	2.05	2.04	2.02
13 years	0.99	0.89	0.11	2.04	2.04	1.99
14 years	0.97	0.59	0.39	2.04	2.03	1.96
15 years	0.92	0.23	0.69	2.03	2.03	1.93
16 years	0.74	0.03	0.71	2.03	2.02	1.90
17 years	0.66	0.00	0.65	2.02	2.02	1.87
18 years	0.60	0.00	0.60	2.02	2.01	1.84
19 years	0.53	0.00	0.53	2.01	2.01	1.81
20 years	0.49	0.00	0.49	2.01	2.01	1.78
60 years	---	---	---	1.83	1.97	< 0

Table 2. *Credibility evaluation: true* law F_C° representative of a stable settlement, *prediction model r* = stable behavior, *prediction model s* = unstable behavior.

Time of monitoring	Δ_r°	Δ_s°	Δ_{rs}°	λ°	λ_r	λ_s
11 years	0.99	1.00	-0.01	2.03	2.08	2.09
12 years	0.98	1.00	-0.02	2.03	2.07	2.09
13 years	0.96	1.00	-0.03	2.02	2.07	2.09
14 years	0.94	0.99	-0.05	2.02	2.06	2.08
15 years	0.90	0.99	-0.09	2.02	2.06	2.08
16 years	0.81	0.97	-0.16	2.02	2.05	2.08
17 years	0.50	0.84	-0.35	2.01	2.04	2.08
18 years	0.10	0.74	-0.64	2.01	2.04	2.08
19 years	0.00	0.68	-0.68	2.01	2.03	2.07
20 years	0.00	0.63	-0.63	2.00	2.03	2.07
60 years	---	---	---	< 0	1.12	< 0

Table 3. *Credibility evaluation: true* law F_C° representative of an unstable settlement, *prediction model r* = stable behavior, *prediction model s* = unstable behavior.

Time of monitoring	Δ_r^*	Δ_s^*	Δ_{rs}^*	$\hat{C}^*(t)$	$\hat{C}_r^*(t)$	$\hat{C}_s^*(t)$
10 years	0.998	0.999	-0.002	0.068	0.065	0.067
20 years	0.998	0.977	0.021	0.113	0.112	0.136
30 years	0.979	0.955	0.024	0.158	0.137	0.204
50 years	0.896	0.909	-0.014	0.249	0.144	0.339

Table 4. Real situation: the evolution law F_C^* is unknown and assumed to be a linear regression of the monitoring data *prediction model r* = stable behavior, *prediction model s* = unstable behavior.