On the Complexity of Routing and Spectrum Assignment in Flexible Grid Ring Networks

Massimo Tornatore, Cristina Rottondi, Roza Goscien, Krzysztof Walkowiak, Giuseppe Rizzelli, and Annalisa Morea

I. INTRODUCTION

Today, fiber spectrum is typically divided using wavelength-division multiplexing (WDM) into separate channels with spacing of 50 GHz or 100 GHz. Optical signals are transmitted over each channel by transceivers supporting fixed line rates (e.g., 10, 40, or 100 Gb/s). As for the next generation of higher bit-rate transceivers (namely, at 400 Gb/s and 1 Tbps), they are expected not to fit the traditional grid at 50 or 100 GHz channel spacing due to insufficient inter-channel distance. Furthermore, innovative elastic transceiver architectures (capable of selecting their rates according to the network state by choosing the best combination among: modulation format, coding rate, and spectrum width) are being investigated. To exploit these elastic transceivers and to accommodate beyond-100 Gb/s rates, the conventional legacy grid has to evolve towards finer granularities, so that an arbitrary number of smaller frequency slices (e.g., 6.25 or 12.5 GHz [1]) can be used to create a channel serving the client demands with as much spectral resources as needed, provided that spectrum contiguity is ensured for all the slices belonging to the same channel (see Fig.1). Neighboring channels must be interleaved by guardbands to ensure proper filtering and switching, but such guardbands can be avoided/minimized in case of channels including multiple transceivers between the same source-destination node, forming a “superchannel”. In short, while in the fixed grid each wavelength is switched individually, in flexible networks all the slices in the same channel are managed as a single entity [2]. Consequently, the Routing and Wavelength Assignment (RWA) traditionally solved for fixed grids has to evolve in a Routing and Spectrum Assignment (RSA) accounting for the additional spectrum adjacency and guardbands placement constraints.

It is well understood that introducing flexibility in the optical grid implies additional complexity in the network design and control plane. In addition, complexity further increases if we consider additional optimization dimensions such as: traffic grooming (i.e., how to groom multiple low-rate flows by means of electronic processing to aggregate larger traffic units to be mapped onto higher-capacity optical channels), the assignment of different baud rates and modulation formats to different transceivers in the network, and the 3R regeneration in the optical layer. Intuitively, such additional dimensions increase the number of network configurations to be explored. However, we argue here that not in all network scenarios the advantages introduced by the above features are significant; e.g., a network scenario with several low-volume traffic requests would benefit more from grooming w.r.t. one with a few high-rate connections, or a network with short links would use less regeneration that a network with longer links.

This paper addresses the following questions: i) how much additional complexity is introduced in the RSA optimization by considering traffic grooming and/or regeneration and/or variable modulation formats and baud rates? ii) under which network and traffic conditions are the actual resource utilization savings worth the additional problem complexity? Complexity is evaluated in terms of i) number of variables and constraints of the ILP models, and ii) computational time required to obtain optimal network configurations. Resource utilization is expressed by the number of installed transceivers, \( N \), and by the overall spectrum occupation, \( S_o \).

To answer those question, we model and solve the RSA problem through Integer Linear Programming (ILP) formulations...
in a wide range of ring network scenarios. Backbone and metro network (possibly interconnecting large data centers) represent today the main arena for the adoption of flexible-grid optical technologies. These networks are typically ring-based or meshed with limited connectivity degree. Therefore, in order to limit the complexity of our approaches, in this study we focus only on the specific case of ring networks.

We consider modeling the problem by adopting either a slice-based or channel-based approach: the former handles each slice individually, whereas the latter uses precomputed subsets of contiguous slices of different bandwidths. Note that these are two most common modeling approaches for the RSA problem [3], [4], but an exhaustive comparison of the two approaches has never been performed. Moreover, in the performance comparison we also include a benchmark model designed for the traditional WDM fixed grid, which can as well support traffic grooming, multiple baud rates/modulation formats and 3R regeneration, but uses the traditional 50 GHz spectrum granularity and does not support channel/superchannel formation, thus not requiring spectrum contiguity and guardband insertion constraints. Similar conclusions as those obtained in this study for ring networks are expected to yield for mesh network, but a more thorough investigation of this aspect is left for future investigation.

The rest of the paper is organized as follows. Section II provides a brief overview of the related work. Section III describes the flexgrid network architecture, whereas the details of the ILP formulations to solve the RSA problem are reported in Section IV. The complexity and the performance assessment of the proposed models are evaluated in Section V and VI, respectively. Section VII concludes the paper.

II. RELATED WORK

The possible adoption of a flexible optical grid has stimulated a large number of studies in these last years (see [5] for a tutorial overview on physical layer technological and control plane aspects, as well as on network optimization approaches for flexible networks). In particular, the RSA problem has been widely studied and several exact models and heuristic approaches have been proposed (see Refs. [6]–[13]). The RSA problem has been proven to be NP-complete [3], even in case of chain and ring topologies [14], [15]. Ref. [14] also proposes approximated algorithms for binary trees and ring networks, and heuristic approaches for general mesh topologies. In Ref. [3], the complete Routing, Modulation Level and Spectrum Assignment RMLSA problem is investigated, considering that optical channels with different modulation formats can coexist; typically in RMLSA, the modulation is selected according to the traffic rates and channel length [16]. The ILP model proposed in [3] uses a slice-based approach that we will employ also in this paper. Another model that explicitly accounts for the spectrum adjacency constraint in RSA is proposed in [8], but it does not account for multiple modulation formats. The authors of [4] propose and evaluate several ILP formulations of the RSA problem, including a channel-based one, where channel is defined as a pair of adjacent frequency slices, such that the spectrum contiguity constraint is automatically satisfied. A detailed modeling approach is especially useful when the sets of traffic demands and routing paths are given in advance. Ref. [17] formulates four link-based and route-based linear programs for the RSA problem in elastic networks and compares their complexity and applicability in terms of solving approach (e.g. usage of column generation or branch and price techniques), but no considerations are drawn on the effect of other optimization dimensions beyond the basic RSA (e.g. regeneration, or grooming traffic, as in this paper). RSA with regeneration in flexible networks is also a quite recent topic [18], [19] but some preliminary works are appearing. E.g., Ref. [19] proposes an effective placement of regenerators that allows to obtain substantial gain of network capacity. Ref. [20] considers ILP models and heuristics for time-varying traffic scenarios. In this paper, we provide two main novel contributions with respect to existing studies: 1) we explore the trade-off between network cost and problem complexity under multiple assumptions in terms of traffic grooming, regeneration, modulation/baud rate assignment capabilities; 2) we compare the performance of two alternative models: the slice-based model vs. the channel-based model.

III. NETWORK ARCHITECTURE AND ASSUMPTIONS

Topology. We consider a ring physical topology with a radius of R km, N nodes, and E = 2N optical links, deployed in both clockwise and counterclockwise directions between each pair of contiguous nodes (see Fig. 2). Nodes are equipped with Flexible bandwidth optical switches.

Spectrum. We assume the optical spectrum to be partitioned in a grid of frequency slices of width FGHz ("granularity F GHz") as depicted in Fig. 1. The nominal central frequency M is placed in the middle of the slice, which imposes
that optical carriers used to transmit the optical signals are placed at predefined frequencies, which are regularly spaced along the spectrum every $M = F/2$ GHz, as mandated in [1]. The total spectrum width available on each optical link is $S = kF$ GHz, where $k$ is an integer number.

Transceivers. We consider coherent transceivers, that apply Digital Signal Processing (DSP) techniques for compensation of transmission distortion, such as chromatic and polarization mode dispersion, and support advanced modulation formats, i.e., different possible bit rates. Such transceivers have an electrical bandwidth of $e$ GHz and operate at fixed baud rate of $h = h_{net} + h_{fec}$ GBaud, where $h_{net}$ indicates the amount of net transmitted symbols per second and $h_{fec}$ accounts for the redundant error coding. Given the spectral efficiency $\eta$ (bit/s/Hz) of the modulation format in use (i.e., the net bit rate per Hertz), the transceiver capacity can be computed as $\eta h_{net}$. The channel bandwidth $B$ (GHz) indicates the width of the spectrum portion that has to be assigned to the optical signal generated by the transceiver, i.e., the bandwidth of the optical filter used to multiplex the signal\(^1\). To adapt to the granularity of the flexible grid, the channel bandwidth must be an integer multiple of the slice width $F$ (i.e., it must hold that $B = nF$ and $B > e$). Since $M = F/2$, $n$ can be either odd or even. Note that the performance of the transceiver highly depends on the choice of $n$: a high $h/B$ ratio allows for high spectrum utilization, but decreases signal reach, i.e. the distance that can be covered without electronically regeneration of the signal. The reduction in reach is mainly caused by additional crosstalk due to adjacent channel coherent interference, and is more pronounced with advanced modulation formats. Table I reports the maximum reaches for different modulation formats and various combinations of the baud rates and optical bandwidths used in this work. Reaches have been computed based on the results in [21].

Superchannel. In case of low traffic volumes, multiple low-rate flows can be electronically groomed in the capacity of a single transceiver. In case the volume of traffic requests exceeds the maximum capacity that a transceiver can support, traffic demands can be served by transmitting the signals using multiple transceivers. In this case, a channel obtained by placing continguously multiple transceivers is called superchannel and is handled by the optical switching equipment as a single entity, provided that each (super)channel is separated from the adjacent channels by a guard band $G = mF$ (GHz) (Fig. 1). Such frequency band between neighboring channels prevents overlapping and crosstalk among modulated signals. Note that the superchannel bandwidth, can be computed as $hB$, where $b$ is the number of transceivers used to serve the traffic request.

### IV. The RSA Problem

The RSA problem consists in assigning to each flow an optical path connecting the source and destination nodes and

\(^1\)Note that the channel bandwidth is usually wider than the electrical bandwidth.

We now start describing the ILP model for the RSA-RG setting supporting both multiple transceiver baud rates and modulation formats (Mi-Br). i.e., the most comprehensive case. Then we discuss how to adapt/downgrade the model to the other simpler settings.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>REACH VALUES FOR VARIOUS MODULATION FORMATS (KM) AND FOR 28 BAUD AND 14 BAUD TRANSCEIVERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$ (GHz)</td>
<td>$B$ (GHz)</td>
</tr>
<tr>
<td>28 (14)</td>
<td>31.25 (18.75)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE BAUD RATE</th>
<th>VARIABLE MOD. FORMAT</th>
<th>YES</th>
<th>NO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GROOM.</td>
<td>REGEN.</td>
<td>YES</td>
</tr>
<tr>
<td>YES</td>
<td>RSA-RG</td>
<td>Mi-Br</td>
<td>RSA-R</td>
</tr>
<tr>
<td>NO</td>
<td>RSA-G</td>
<td>Mi-Br</td>
<td>RSA-R</td>
</tr>
</tbody>
</table>

Fig. 3. The considered network settings.
Objective Function

\[
\min \alpha_1 \sum_{i,j \in N, k \in K} (2h_{ijk}^z + \Gamma z b_{ijk}^h) + \alpha_2 \sum_{i,j \in N, k \in K, i \neq j} L_{ijk}(\sum_{h \in H, z \in Z} b_{ijk}^h F_h + y_{ijk} G) \tag{1}
\]

Constraints

\[
\sum_{k \in K} x_{ijk}^{sd} \leq 1 \quad \forall i, j, s, d \in N : i \neq j, s \neq d \quad \tag{2}
\]

\[
\sum_{k \in K} x_{ijk}^{sd} \leq \sum_{h \in H} r_{ijk}^h b_{ijk}^h \quad \forall i, j \in N, k \in K : i \neq j \quad \tag{3}
\]

\[
\sum_{k \in K} x_{ijk}^{sd} = \sum_{h \in H} x_{ijk}^{sd} = \begin{cases} 
1 & \text{if } s = i \\ -1 & \text{if } d = i \\
0 & \text{otherwise}
\end{cases} \quad \forall i, j, s, d \in N : s \neq d \land t^{sd} > 0 \tag{6}
\]

\[
f_{ijk} + \sum_{h \in H} b_{ijk}^{h} F_h + G - f_{i'j'k'} \leq (S+G)(1-d_{ijk'}k'ijk + 2-y_{ij} - y_{ij'}k') \quad \forall i,j,i',j' \in N, k,k' \in K : E_k(ij) \cap E_{k'}(i'j') \neq \emptyset \tag{7}
\]

\[
f_{ijk} + \sum_{h \in H} b_{ijk}^{h} F_h + G - f_{i'j'k'} \leq (S+G)(1-d_{ijk'}k'ijk + 2-y_{ij} - y_{ij'}k') \quad \forall i,j,i',j' \in N, k,k' \in K : E_k(ij) \cap E_{k'}(i'j') \neq \emptyset \tag{11}
\]

A. RSA-RG Slice-based Integer Linear Programming Formulation

Parameters:

- \( N \): set of nodes of the ring network
- \( L \): set of physical links \((m, n)\) of the ring network
- \( K = \{1, 2\} \): set of directions (1=counter-clockwise, 2=counter-clockwise)
- \( H \): set of possible transceiver baud rates
- \( F \): spectral width of a frequency slice
- \( F_i \): optical bandwidth of transceiver with baud rate \( h \), expressed as integer multiple of \( F \)
- \( Z = \{0, \ldots, |N| - 2\} \): set of possible cardinalities of the set of regenerators placed along a lightpath
- \( S \): total available spectrum width on each link\(^3\)
- \( G \): width of the guardband (GHz) used to separate adjacent spectrum paths, expressed as integer multiple of \( F \)
- \( r_{ijk}^h \): Spectral Efficiency (SE) along the lightpath between node \( i \) and node \( j \) in direction \( k \) using transceivers with optical bandwidth \( h \) and \( z \) regenerators (expressed as multiple of the basic SE of BPSK and computed according to the lightpath length and the reach limitations of the various modulation formats)
- \( C_h \): capacity of a transceiver with baud rate \( h \) associated to the BPSK modulation format (Gbps)
- \( T^{sd} \): traffic matrix between source-destination pairs (Gbps)
- \( L_{ijk} \): number of links traversed by the lightpath between node \( i \) and node \( j \) in direction \( k \)

Variables:

- \( x_{ijk}^{sd} \): boolean variable, indicates whether a lightpath is established between \((i, j)\) node pair in direction \( k \) is used to serve the connection request between \((s, d)\)
- \( b_{ijk}^h \): integer variable, indicates the number of transceiver pairs with baud rate \( h \) used to serve the lightpath between \((i, j)\) node pair in direction \( k \) with \( z \) regenerators
- \( y_{ijk} \): boolean variable, is 1 if a lightpath is established between \((i, j)\) node pair in direction \( k \)
- \( f_{ijk} \): positive variable, indicates the starting frequency of the lightpath between \((i, j)\) node pair in direction \( k \)
- \( d_{ijk'}k'ijk \): boolean variable, is 0 if \( f_{i'j'k'} < f_{ijk} \), 1 otherwise

Objective function: By varying the parameters \( \alpha_1 \) and \( \alpha_2 \), the objective function (1) allows minimizing either \( i) \) the number of transceivers and regenerators to be installed (in case \( \alpha_1 > \alpha_2 \)) or \( ii) \) the overall spectrum occupation (in case \( \alpha_1 \leq \alpha_2 \). Note that in case \( ii) \) if we assign a small positive value to \( \alpha_2 \), the model will still minimize the number of installed transceivers but in case of multiple solutions, the model will choose the one that occupies the lowest amount of optical spectrum. Similar considerations hold for case \( i) \), in which in case multiple configurations minimizing the spectrum occupation exist, the one requiring the lowest amount of transceivers/regenerators is selected.

Note also that the parameter \( \Gamma \) defines the regenerator-to-transceiver normalized cost, which depends on the technological characteristics of the devices.

Constraints: Constraint (2) ensures that no bifurcation of traffic flows occurs by imposing that each traffic request is routed along a single direction (either clockwise or counter-clockwise, as this formulation is for a ring topology), while

Please note that whenever traffic grooming is performed in electronic layers (RSA-G), also regeneration occurs at the electronic layer, but in our RSA-RG approach we consider in addition the possibility to perform regeneration at the optical layer.

Note that in this paper we set \( S \) to values which ensure that the blocking probability is null, meaning that optical resources are always sufficient to serve all the traffic requests.
Constraint (4) imposes coherence of the values of $y_{ijk}$ and $x_{ijk}$ variables by forcing the lightpath indicator $y_{ijk}$ to be 1 if the lightpath $(i, j)$ in direction $k$ is used to serve at least one traffic request. Even if constraint (8) and (9) by ensuring that either $f_{ijk} < f_{i'j'k'}$ or $f_{ijk} > f_{i'j'k'}$. Moreover, the starting frequencies are forced to assume a value in the available range of spectrum by Constraint (5). The correct placement of the guardbands between adjacent optical (super)channels is imposed by Constraints (10) and (11): these are activated only in case $y_{ijk}$ and $y_{i'j'k'}$ are both set to 1 and are mutually exclusive, depending on the value of $d_{ijk}(i'j'k')$. In particular, when Constraint (10) is activated, it becomes $f_{i'j'k'} + \gamma > f_{ijk}$, thus ensuring that the frequency slices allocated for the two lightpaths $(i, j, k)$ and $(i', j', k')$ do not overlap. Similar considerations can be drawn for Constraint (11).

1) RSA-G Slice-based Adaptation: The model provided in Section IV-A can be reduced to RSA-G by setting $Z = \{0\}$ and maintaining variables and constraints unaltered.

2) RSA-R Slice-based Adaptation: Grooming can be eliminated by suppressing variable $x_{ijk}$ and Constraints (2), (3), (4). Moreover, Constraint (6) must be replaced by the following equation:

$$\sum_{k \in K} y_{ijk} = \begin{cases} 1 & \text{if } T^{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j \in N$$

Note also that the usage of a single transceiver baud rate can be imposed by defining $H$ as a singleton set, while the usage of single/multiplier modulation formats can be completely captured by properly defining the capacity multiplier $r_{ijk}$ as in [3].

B. RSA-RG Channel-based Integer Linear Programming Formulation

In this subsection, we formulate the RSA-RG problem using a channel-based approach [4]. As mentioned above, in flexi-grid networks the spectrum is divided in frequency slices of the same size $F$, which we denote here as $[a_1, a_2, ..., a_S]$, where $S$ is the total number of slices available on each link. A channel is defined as a set of contiguous (adjacent) slices. In more detail, if a lightpath needs exactly $n$ slices the possible channels for allocation of spectrum for this lightpath are $[a_1, a_2, ..., a_n], [a_2, a_3, ..., a_{n+1}], [a_2-n+1, a_2, ..., a_S]$. The main motivation behind the channel approach is to enable easy control of the spectrum overlapping in flexi-grid networks. Let $C$ denote a set of all defined channels. Each channel $c \in C$ is described by a constant $n_c$, denoting the number of contiguous slices in channel $c$, and constant $\gamma_{ca}$, that is 1 if channel $c$ uses slice $a$, and 0 otherwise. In the optimization, the channel for each lightpath is selected according to a binary variable $y_{jkc}$ that is 1, if channel $c$ is used to realize lightpath $(i, j)$ in direction $k$ and 0 otherwise. Values of $y_{jkc}$ enable also controlling the slice overlapping. In particular, the term $\sum_{i,j \in N, k \in K, c \in C} y_{jkc} z_{ijk}(0) \gamma_{ca}(i) y_{jkc}$ can be used to check how many lightpaths use slice $a$ in link $l$, where the binary constant $\delta_{jkl}$ determines whether link $l$ belongs to a path realizing lightpath $(i, j)$ in direction $k$. To avoid overlapping of slices, this term cannot exceed 1.

Parameters (additional):

- $\delta_{jkl}$: it is 1 if link $l$ belongs to lightpath $(i, j)$ in direction $k$; 0, otherwise
- $A$: set of slices available in the network
- $C$: set of channels, each channel contains a set of contiguous slices
- $n_c$: number of contiguous slices in channel $c$
- $\gamma_{ca}$: it is 1 if channel $c$ uses slice $a$; 0, otherwise

Variables (additional):

- $y_{jkc}$: 1 if channel $c$ is used for lightpath $(i, j)$ in direction $k$, 0 otherwise

Objective function: Equation (13) is very similar to (1), however the spectrum occupation (the second term) is defined according to the channel approach, i.e., $y_{jkc} n_c$ provides the number of spectrum slices allocated to provision lightpath $(i, j)$ in direction $k$ using channel $c$.

Constraints: Equations (2), (3) and (6) are the same as in the previous model. Constraint (14) is introduced in the model to ensure that at most one spectrum channel is used for a lightpath $(i, j)$ in direction $k$. Constraint (15) imposes coherence of the values of $y_{jkc}$ and $x_{sdij}$ variables by forcing the lightpath indicator $y_{jkc}$ to be 1 if the lightpath $(i, j)$ in direction $k$ is used to serve at least one traffic request. To assure that a channel selected for a lightpath $(i, j)$ in direction $k$ provides enough slices in the network we use constraint (16). In more detail, the left-hand side of (16) is equal to number of slices required by the transceivers $\sum_{i,j \in H, c \in C} b_{ijk} x_{ij}$. The right-hand side is equal to the number of slices provided by channel $c$ selected for a lightpath $(i, j)$ in direction $k$ minus $G$ slices for the guardband. To guarantee that a slice on a particular link can be allocated to at most one lightpath, i.e., the slices are not overlapped in the network, we add to the model Constraint (17). Similarly to the previous model, also in this case the channel-based RSA-RG can be adapted to solve RSA-G and RSA-R. In more detail, the channel-based model can be reduced to RSA-G by setting $Z = \{0\}$ and keeping all variables and constraints unaltered. In the case of the RSA-R version of the model, we must consider two cases. First, we assume that variable baud rate (Br) is used. Therefore, the number of transceivers required between node $i$ and node $j$, and consequently the number of spectrum slices required between these nodes is not constant, due to the variable $b_{ijk}$. Therefore, channels $c \in C$ that can be selected for node pair $(i, j)$ must be of various size $n_c$, which significantly increases the number of channels to be considered in the model for each node pair. To obtain RSA-R, we must remove from the RSA-RG model variable $x_{sdij}$ and Constraints (2), (3), (6). Moreover, we must replace Constraint (14) with:
The number of slices required for a particular lightpath \((i, j)\) in direction \(k\) only to those that provide the number of slices required for a particular lightpath \((i, j)\) in direction \(k\). Let \(C(i, j, k)\) denote a set of channels for lightpath \((i, j)\) in direction \(k\) providing the required number of slices. Using this notation, we can replace constraints (14) and (17), with:

\[
\sum_{k \in K, c \in C(i, j, k)} y_{ijkc} = \begin{cases} 
1 & \text{if } T^{ij} > 0 \\
0 & \text{otherwise} 
\end{cases} \quad \forall i, j \in N 
\]

(19)

\[
\sum_{i, j \in N, k \in K, c \in C(i, j, k) : i \neq j} \gamma_{ca} \delta_{ijkl} y_{ijkc} \leq 1 \quad \forall l \in L, a \in A 
\]

(20)

### V. Complexity Evaluation

We now compare the complexity of the two models described in Section IV in terms of number of variables and constraints. The comparison benchmark is the complexity of the model proposed in [24] to solve the classical Routing and Wavelength Assignment (RWA) problem in a fixed-grid ring network. Table II reports the number of variables and constraints of the ILP formulations for each setting in case of slice-based modeling: the models related to flexible grid constraints. The comparison benchmark is the complexity similarly to Table II. Comparing Table II to Table III, we can notice that comparison between slice-based model and channel-based model in terms of the number of variables is determined mostly by a relation \(N^2\) versus number of channels \((S, S^2\) or \(A\) depending on the considered case). While the corresponding comparison in terms of number of constraints is based on a relation \(N^4\) versus number of links \(L\) multiplied by number of channels \((S, S^2\) or \(A\)). Therefore, if the number of channels used in the modelling significantly exceeds \(N^2\), the channel-based model experiences higher complexity than the slice-based model.
TABLE II
COMPLEXITY OF SLICE-BASED ILP FORMULATIONS (N=NUMBER OF NODES, W=NUMBER OF WAVELENGTHS OF THE WDM GRID, H=NUMBER OF TRANSCIEVER’S BAUD RATES)

<table>
<thead>
<tr>
<th>Setting</th>
<th>Fixed Grid Fx/Mf</th>
<th>Flexi Grid Fx/Mf</th>
<th>Fixed Grid Br/Mf-Br</th>
<th>Flexi Grid Br/Mf-Br</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSA</td>
<td>$\propto 2N^3(W+2)$</td>
<td>$\propto N^2(2W+5)$</td>
<td>$\propto 4N^4$</td>
<td>$\propto 10N^4$</td>
</tr>
<tr>
<td>RSA-R</td>
<td>$\propto 2N^3$</td>
<td>$\propto N^2(2W+5)$</td>
<td>$\propto 4N^4$</td>
<td>$\propto 10N^4$</td>
</tr>
<tr>
<td>RSA-G</td>
<td>$\propto 2N^4$</td>
<td>$\propto N^4$</td>
<td>$\propto 6N^4$</td>
<td>$\propto 13N^4$</td>
</tr>
<tr>
<td>RSA-RG</td>
<td>$\propto 2N^4$</td>
<td>$\propto N^4$</td>
<td>$\propto 6N^4$</td>
<td>$\propto 13N^4$</td>
</tr>
</tbody>
</table>

TABLE III
COMPLEXITY OF CHANNEL-BASED ILP FORMULATIONS (N=NUMBER OF NODES, L=NUMBER OF LINKS, S=NUMBER OF SLICES OF THE FLEXIGRID, A=NUMBER OF SPECTRUM CHANNELS, H=NUMBER OF TRANSCIEVER’S BAUD RATES)

<table>
<thead>
<tr>
<th>Setting</th>
<th>Fixed Grid Fx/Mf</th>
<th>Flexi Grid Fx/Mf</th>
<th>Fixed Grid Br/Mf-Br</th>
<th>Flexi Grid Br/Mf-Br</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSA</td>
<td>$\propto 2N^3(S+1)$</td>
<td>$\propto 4N^2+LS$</td>
<td>$\propto 2N^4(H+S^2)$</td>
<td>$\propto 4N^2+LS^2$</td>
</tr>
<tr>
<td>RSA-R</td>
<td>$\propto 2N^2(S+N)$</td>
<td>$\propto 4N^2+LS$</td>
<td>$\propto 2N^2(HN+S^2)$</td>
<td>$\propto 4N^2+LS^2$</td>
</tr>
<tr>
<td>RSA-G</td>
<td>$\propto 2N^2(S^2+N^2)$</td>
<td>$\propto 3N^2+LS^2$</td>
<td>$\propto 2N^2(N^2+S^2)$</td>
<td>$\propto 3N^4+LS^2$</td>
</tr>
<tr>
<td>RSA-RG</td>
<td>$\propto 2N^2(A+N^2)$</td>
<td>$\propto 3N^2+LA$</td>
<td>$\propto 2N^2+(N^2+A)$</td>
<td>$\propto 3N^4+LA$</td>
</tr>
</tbody>
</table>

VI. RESULTS

We consider a ring topology as described in Section III with $N = 8$ nodes and 500 km radius. The offered traffic is 1-to-all, i.e., one node is elected as gateway and it establishes bidirectional communications with all the other nodes, or all-to-all, meaning that every node communicates to each other node of the ring. The spectrum is divided in frequency slices (FS) of $F = 6.25$ GHz in the flexi grid, and 50 GHz in the fixed case. The total available optical spectrum per link is 1 THz, from which it follows that in the fixed-grid case the total number of wavelength is 20, whereas in the flexigrid case the number of slices is 160. MF transceivers support dual-polarization QPSK and $n$-QAM, with $n = 8, 16, 32, 64$. Br transceivers can work at two baud rates ($H_1 = 14$ or $H_2 = 28$ Gbaud, occupying an optical bandwidth of $3F = 18.75$ GHz and $5F = 31.25$ GHz, respectively). The transmission reaches for these transceivers are reported in Table I. In case of superchannel formation, the superchannel bandwidth is computed as an integer multiple of the channel bandwidth of each transceiver. Guardband spacing is $G = F = 6.25$ GHz.

For the flexi grid, we consider the following transceivers: (1) Fx DP-QPSK transceivers operating at 28 Gbaud; (2) MF transceivers operating at 28 Gbaud; (3) Br DP-QPSK transceivers (either 14 or 28 Gbaud); (4) Br-MF transceivers. In the fixed grid transceivers can be Fx or MF, always at 28 Gbaud. The objective function to be minimized is $N_t$ (note that in case of regeneration, we set $G_t = 1.2$), and, among the solutions minimizing $N_t$, we chose the one with minimal $S_o$.

In Fig. 4 we can see that grooming allows a small reduction of $N_t$ only for low traffic in both fixed and flexi grid (such reduction is even smaller in case of regeneration). Note that, since in the flexi grid the signal reaches are shorter than in the fixed grid (as the ratio $c/B$ is higher), flexi grid requires a slightly larger $N_t$ than fixed grid, especially for high traffic. The advantages of flexi grid emerge if we consider $S_o$ (Fig. 5) that is always significantly reduced using flexi grid. Results with mixed values (14 and 28 Gbaud) of baud rates are not reported as they do not affect the $N_t$ (but a minor gain in terms of $S_o$ can be observed).

The computational time required to obtain the optimal solution using the CPLEX solver is plotted in Fig. 6. We can see that introducing traffic grooming has a significant impact for low traffic volumes, where numerous grooming alternatives must be considered, thus dominating the complexity increase due to the flexible grid. Conversely, for higher traffic, grooming capabilities become less useful (as offered traffic easily saturates the transceiver capacity) and the additional complexity due to managing the flexi grid emerges (on average one order of magnitude higher computational times w.r.t. the fixed grid). Finally, note that including variable modulation formats, regeneration and multiple baud rates does not lead to remarkable increases in the computational times (less than one order of magnitude) w.r.t. the benchmark RSA scenario with single QPSK modulation format and 28 Gbaud.
TABLE IV
Spectrum occupation, transceiver utilization and computational time (order of magnitude) for $R = 500$ km with 28 Gbaud Mf transceivers and all-to-all traffic matrix, when minimizing $N_t$. The width of the frequency slices in the flexi grid is $F = 6.25$ GHz.

<table>
<thead>
<tr>
<th>Traffic</th>
<th>Fixed Grid Mf</th>
<th>Flexi Grid Mf</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 Gb/s</td>
<td>150 Gb/s</td>
</tr>
<tr>
<td>Setting</td>
<td>$S_o$ $N_t$</td>
<td>$S_o$ $N_t$</td>
</tr>
<tr>
<td>RSA</td>
<td>6400 112</td>
<td>128 8000</td>
</tr>
<tr>
<td>RSA-R</td>
<td>6400 112</td>
<td>128 8000</td>
</tr>
<tr>
<td>RSA-G</td>
<td>2000 50</td>
<td>126 6150</td>
</tr>
<tr>
<td>RSA-RG</td>
<td>2000 50</td>
<td>126 6150</td>
</tr>
</tbody>
</table>

TABLE V
Spectrum occupation, transceiver utilization and computational time (order of magnitude) for $R = 500$ km with 28 Gbaud Mf transceivers and 1-to-all traffic matrix, when minimizing $S_o$. The width of the frequency slices in the flexi grid is $F = 6.25$ GHz.

<table>
<thead>
<tr>
<th>Traffic</th>
<th>Fixed Grid Mf</th>
<th>Flexi Grid Mf</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 Gb/s</td>
<td>150 Gb/s</td>
</tr>
<tr>
<td>Setting</td>
<td>$S_o$ $N_t$</td>
<td>$S_o$ $N_t$</td>
</tr>
<tr>
<td>RSA</td>
<td>1600 28</td>
<td>2000 32</td>
</tr>
<tr>
<td>RSA-R</td>
<td>1600 28</td>
<td>1600 32.8</td>
</tr>
<tr>
<td>RSA-G</td>
<td>700 20</td>
<td>1500 36</td>
</tr>
<tr>
<td>RSA-RG</td>
<td>700 20</td>
<td>4800 140.8</td>
</tr>
</tbody>
</table>

TABLE VI
Nr. of transceivers and computational time for the European-like network (only fixed grid).

<table>
<thead>
<tr>
<th>Traffic (Gb/s)</th>
<th>RSA</th>
<th>RSA-R</th>
<th>RSA-G</th>
<th>RSA-RG</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>55</td>
<td>9.14</td>
<td>16</td>
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<tr>
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<td>55</td>
<td>11.54</td>
<td>43</td>
<td>1512</td>
</tr>
</tbody>
</table>

Fig. 5. Spectrum occupation for $R = 500$ km with 28 Gbaud Mf transceiver and 1-to-all traffic matrix, when minimizing $N_t$. The width of the frequency slices in the flexi grid is $F = 6.25$ GHz.

Fig. 6. Computational time (order of magnitude) for $R = 500$ km with 28Gbaud Mf transceivers and 1-to-all traffic matrix, when minimizing $N_t$. The width of the frequency slices in the flexi grid is $F = 6.25$ GHz.

transceivers. Similar results obtained for an all-to-all traffic matrix are summarized in Table IV. With such traffic matrix, the placement of regenerators is much more frequent in the RSA-R and RSA-RG settings, especially for high bit rates. As for the one-to-all scenario, traffic grooming has the heaviest
impact on the problem complexity, which in turn increases the computational time required by the CPLEX solver. Note that in absence of traffic grooming, the spectrum occupancy in case of 500 Gbd traffic demands exceeds the overall spectrum availability, thus leading to infeasible network configurations.

Table V summarizes the results obtained for the same scenario, in case the objective function to be minimized is $S_o$ (again, among the solutions minimizing $S_o$, we chose the one with minimal $N_o$). With this optimization goal it is worth noting that, when grooming is enabled, large superchannels spanning only one physical link are privileged, since short distances allow for the utilization of higher modulation formats (32-QAM for 28 Gbd transceivers and 64-QAM for 14 Gbd transceivers), which ensures high spectrum efficiency. Due to the preference for such network configurations, the order of magnitude of the computational times is generally lower than the transceiver minimization case. The drawback is that multiple short superchannels require the installation of a high number of transceivers.

We expect similar considerations to hold for mesh networks. Some preliminary results are reported in Table VI for a European-like fixed grid mesh network [25] with average link length of 550 km. Mesh networks will be object of a follow-up complexity analysis also in the case of flexi-grid.

A. Slice-based model vs. channel-based model

We now compare the channel-based model to the slice-based one, assuming the same 8-nodes ring topology described above. We report results related to two most distant cases: pure RSA with fixed transceiver type (Table VII) and RSA-RG with variable modulation format and baud rate (Table VIII). Performance of both cases is analyzed for two objective functions: transceiver utilization and spectrum occupation.

The execution time of CPLEX is limited to 3 hours and therefore not all presented results are optimal. First, we discuss the simpler RSA problem. As we can see in Table VII, both ILP modeling approaches provide the same optimal results in relatively short time. However, the execution time of the channel-based model is about 8 times the one required by the slice-based to find the optimal results. In the case of the RSA-RG model, scalability issues in the channel-based model are even more pronounced. As mentioned in the complexity analysis, in case of grooming, the width of channel for a particular lightpath can be of various size. However, if all possible channels are to be considered, we need to generate up to about $S^2$ possible channels, where $S$ denotes the number of available spectrum slices. Thus, the size of the model grows significantly and as a consequence CPLEX for the considered cases returns out-of-memory error. Therefore, we decided to limit the slice granularity of the channels and we consider channels having size defined as a multiple of 5 slices (i.e., 5 slices, 10 slices, 15 slices, etc.) and a multiple of 3 slices (3 slices, 6 slices, 9 slices, etc.). This approach significantly reduces the number of candidate channels in the modeling and according to our experiments, only with this approach, the CPLEX is able to return some results. However, we must underline that changing the slice granularity of channels means, that the channel-based model is not optimal in a global sense as the slice-based model is, since this assumption can lead to overprovisioning a channel with more slices than required by the number of transceivers used for this lightpath. In Table VIII, we show the results concerning RSA-RG model. We can easily notice that in terms of transceiver utilization both models provide the same results, even though the channel-based model uses only limited set of possible channels due to fixed slice granularity. The execution time of the channel-based model with 5-slice granularity is comparable to the slice-based model, in 3 of 6 reported cases the channel-based model is even faster. The channel-based model with 3-slice granularity is less efficient in terms of the execution time. Performance of both analyzed ILP models in terms of spectrum utilization varies considerably. First, due to changing the channel granularity, the channel-based model yields worse results and the gap with respect to the slice-based model grows when increasing the channel granularity from 3-slice channels to 5-slice channels. In terms of the execution time the gap is large — the slice-based model provides results in few seconds, while the channel-based model in half of the reported cases cannot find optimal result in 3 hours. The presented results confirm the observations provided above, i.e., when introducing additional constraints (grooming, regeneration, variable modulation format and baud rate) the complexity of the channel-based model increases much faster than the slice-based model, especially when the spectrum utilization is the optimization criterion.

VII. CONCLUSION

This paper analyzes the complexity trade-offs in the design of optical flexigrid networks supporting electronic traffic grooming, regeneration, modulation format and baud rate assignment. Integer linear programs exploiting two different modeling approaches (slice-based and channel-based) are provided and adapted to multiple network settings. First, we have shown that in presence of traffic grooming the additional complexity due to the flexible grid has a minor impact on problem complexity. Similarly, in all the considered scenarios, regeneration and modulation/baud rate assignment do not relevantly impact on the problem complexity. Then by comparing the channel based and slice based models, the analysis of the model complexity and the results of numerical experiments demonstrate that the slice-based model is more scalable than the channel-based one. The slice-based model generally solves problems faster and requires significantly less memory resources, especially when traffic grooming is applied.

VIII. ACKNOWLEDGMENT

The work of Roza Goscien and Krzysztof Walkowiak was supported by the Polish National Science Centre (NCN) under Grant DEC-2012/07/B/ST7/01215. The work of Massimo Tornatore and Krzysztof Walkowiak was supported by the European Commission under the 7th Framework Programme, Coordination and Support Action, Grant Agreement Number 316097, ENGINE - European research centre of Network intelligence for INnovation Enhancement (http://engine.pwr.wroc.pl/).

REFERENCES

TABLE VII
SLICE-BASED MODEL VS. CHANNEL-BASED MODEL FOR RSA WITH FIXED (FX) TRANSCIEVER TYPE, RING NETWORK WITH 8 NODES AND 1000 KM RADIUS AND ALL-TO-ALL TRAFFIC

<table>
<thead>
<tr>
<th>Traffic per request (Gbps)</th>
<th>Frequency slice (GHz)</th>
<th>Result Time [s]</th>
<th>Spectrum utilization</th>
<th>Slicebased model</th>
<th>Channel-based model</th>
<th>Slicebased model</th>
<th>Channel-based model</th>
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<tbody>
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<td>6.62</td>
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</table>

TABLE VIII
SLICE-BASED MODEL VS. CHANNEL-BASED MODEL FOR RSA-RG WITH VARIABLE MODULATION FORMAT AND BAUD RATE (MF-BR), RING NETWORK WITH 8 NODES AND 500 KM RADIUS, 1-TO-ALL TRAFFIC

<table>
<thead>
<tr>
<th>Traffic per request (Gbps)</th>
<th>Frequency slice (GHz)</th>
<th>Result Time [s]</th>
<th>Spectrum occupation (GHz)</th>
<th>Slicebased model</th>
<th>Channel-based model</th>
<th>Slicebased model</th>
<th>Channel-based model</th>
</tr>
</thead>
<tbody>
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<td>738</td>
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</tr>
</tbody>
</table>

*Not optimal results


