A new second-order method for branch contingency analysis and static voltage security

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1 Introduction

Power system steady-state security analysis requires the assessment of the power system behaviour in the presence of a reasonable set of perturbations, that may result in cascading events and blackouts. A significant part of the security assessment [1] is the *Contingency Analysis*, and in particular the evaluation of the $(n-1)$ *Security Criterion*, concerning the single outage of generators, transmission lines or transformers. When this analysis is performed in real-time power system operation, it is essential for the adopted computational tools to have a high degree of accuracy, speed and numerical robustness [2]. There is a trade-off between the above three aspects; the resulting procedure must produce solutions practical and useful for the system operator both to estimate post-contingency conditions and to define suitable preventive and/or corrective actions. Usually, contingency analysis is carried out by first selecting and ranking the most dangerous contingencies and by eventually performing more detailed studies on the first ranked contingencies. The identification of the more severe contingencies is carried out by assessing the violation level of the bus voltage magnitude and/or transmission lines power flow limits. Once critical contingencies have been identified, some cases require detailed dynamic simulation including all nonlinearities and control systems, depending on the dynamics involved. For these, dynamic simulation techniques must be integrated, in order to identify suitable countermeasures; they must be consistent with the time frame considered (day-ahead, real-time, etc.) [3]. The most challenging part of the static $(n-1)$ analysis is represented by the assessment of contingencies that involve the topological change of the network, i.e., branch (transmission lines, transformers) contingencies.

The methods used in steady-state contingency analysis can be divided into two classes: accurate methods and approximated methods. The most obvious and "brute force" method consists in carrying out a complete Power Flow (PF) computation for each contingency considered. Of course, as any nonlinearity is fully considered, the results are accurate; however, although some recent computational techniques can significantly improve the numerical performances [4], and some computing facilities can be used to speed-up computations [5], this approach might require a huge computational effort and consequently high computation time, as the number of branches considered may be high in practice, unless expensive investments in hardware are adopted.

Hence, in order to perform contingency analysis in real-time, a compromise between speed and accuracy must be achieved. A first step consists in adopting PF decoupled methods, which exploit $P\delta - QV$ decoupling, with the aim of determining fast solutions. The accuracy is the same as the complete power flow, with slight savings in computation times.

Approximated methods based on linearised PF equations (considering the results of a single PF iteration, for example) obtain a quicker solution, which is associated necessarily with loss of accuracy. Further simplification and time savings can be obtained by the exploitation of both linearisation and decoupling at the same time, e.g., performing a $P\delta$ iteration of the Fast Decoupled Load Flow, or adopting a DC PF. However, although this approach is very fast, it may lead to significant inaccuracy, especially when the reactive QV subproblem plays a significant role for the contingencies under consideration. Other contingency analysis methods are based on sensitivities, that can also be used in other power system analysis tools, like Security-Constrained Optimal Power Flows. The formulation of analytical methods based on the DC PF is presented in details in [1], including the definition of sensitivity relationships and distribution factors concerning the active power generation and power flows from the pre-contingency condition. These methods provide typically fast and accurate information about the real power flows, that are acceptable in a number of studies. One of the earlier works in this field [6] proposes a scheme named *Iterative Linear Power Flow*, which is aimed at finding an approximate AC power flow solution: non-linear aspects of the PF equations are taken into account in the study of the line outage. Sparsity techniques as well as strategies for ordering elimination are used to speed-up the computational process. This work was developed as a cooperative research with Bonneville Power Administration.

No detail about the numerical results obtained from the application of this technique is presented. In [7], the DC power flow is used to assess the outages of transmission lines, from the point of view of the real power flow. Although the proposed approach has good features in terms of computational efficiency, its drawbacks concerning the linear model and the exclusion of the effects of the QV aspects are well known. Reference [8] introduces the calculation of distribution factors for reactive power flows after contingencies in transmission lines. The proposed model represents analytically the power network through complex power and bus voltages, and therefore takes into account the non-linearity inherent to the PF equations. However, limits of generators are not included, and the implicit assumption of constant voltages limits the accuracy of the method. Many commercial software tools adopted by utilities are based on either the decoupled approach or DC approximation as well as on sensitivities, although the details are not always published. They are aimed at computing line outage distribution factors or generation-shift distribution factors, able to identify overloads consequent to possible branch contingencies. These methods are quite well-known and available on textbooks [9]. However, they do not consider the QV components, which can result in considerable errors, due to both non-linearity of the relationships involving these variables and hitting of generator capability limits [10].

Aiming at combining accuracy and computation speed, AC PF equations were considered in [11,12] and linearisation was applied to reduce the errors in the estimates. The inaccuracy of the estimates based on linear sensitivity relationships is related to two main aspects. The first one concerns the information brought by the Taylor series terms of order higher than the first, which are usually neglected: depending on the degree of severity of the contingency (unknown a priori), this information can be critical to obtain accurate estimates, especially for the very nonlinear QV equations. The second aspect deals with the explicit consideration of the limits of the equipments: contingencies generally require the adjustment of voltage control devices (generators, synchronous compensators, tap-changers, etc). In particular, when the generator capability limits are reached, voltage cannot be further controlled, thus resulting in a very important nonlinearity, which affects the accuracy of any linear model. Neglecting this aspect may lead to significant errors in the estimates, such as to make the results of the contingency analysis completely useless.

In [13,14], methodologies for the estimation of post-outage voltage profile conditions are presented with reference to a reduced optimisation problem, in which only the major components related to the transmission line under contingency (parameters, power flows, power injections in the terminal buses, etc) are considered; the method is based on linearisation, carried out to minimise the differences among the values (estimated and actual) of the reactive power injections at the terminal buses of the faulted branch. The reduced set of buses considered speeds up the contingency analysis but deteriorates the accuracy of the estimates. In [10], a methodology for estimating bus voltages, reactive power injections and reactive power flows is presented, including multiple contingencies and generation redispatching. This strategy is based on linearised relationships and linear segmentation to model both the contingency and the capacity limits of control devices. The results show a better accuracy, while the computational effort is moderate.

In order to improve the accuracy of the sensitivity, the use of second-order information is proposed in [15]. Branch and generator outages are modelled by fictitious power injections at the terminals of the outaged branch. The use of second-order information increases the accuracy but, as the PF equations are in polar coordinates, the sensitivity computation is burdensome. More recently, [16] also uses sensitivity relationships to evaluate the severity of a contingency involving transmission lines. The results obtained by this approach show a good accuracy. However, the limits of generation of reactive power are not taken into account, thus restraining the application of this method.

This paper presents a methodology to improve accuracy, in particular in the estimation of bus voltages after branch contingencies. Adopting rectangular coordinates for the formulation of PF equations makes it possible to explore two main properties of the rectangular formulation: a) The Taylor series expansion can be stopped at the second order terms without losing any information, that is, without any loss of accuracy; b) The Hessian of the PF equations is constant, i.e., it depends only on the network parameters and not on voltage magnitudes or angles. The paper exploits both properties: Property a) makes it possible to increase accuracy by fully exploiting second order information and to improve estimation of post-contingency power system state. Property b) is used in the derivation of the algorithm because each contingency can be studied by looking only at the change of the Hessian T.

A second significant contribution of the paper, which also increases the accuracy, is a methodology to take into account the capability limits of generators as a consequence of the contingency. This is particularly valuable also increases accuracy, especially in case of contingencies that result in system stressed from the reactive power point of view. In this paper, only changes in the transmission structure are assessed (and therefore the outage of the generation units is not considered) because both they are more challenging and the proposed method is particularly suited to face them with an excellent trade-off of accuracy and computation time. The features of the proposed method suggest its use in the evaluation of voltage profile security either in the day ahead framework, e.g., to evaluate reactive margins after day ahead electricity market sessions, or in real-time, due to its optimal trade-off between accuracy and computational speed. The algorithm might be used for filtering and ranking contingencies, preliminary to more detailed, analysis including dynamic simulation or specific voltage stability tools.

Numerical results with power systems of different sizes, including a model of the Italian system, illustrate the main features of the proposed methodology.

2 Theoretical background

The active and reactive power flows of the branch $i-j$ are given by,

$$P_{ij}(e,f) = g_{ij}\left(e_i^2+f_i^2\right) - g_{ij}\left(e_ie_j+f_if_j\right) - b_{ij}\left(e_if_j-e_jf_i\right) \tag{1}$$

$$Q_{ij}(e,f) = -\left(b_{ij}+b_{ij}^{sh}\right)\left(e_i^2+f_i^2\right) + b_{ij}\left(e_ie_j+f_if_j\right) + g_{ij}\left(e_if_j-e_jf_i\right) \tag{2}$$

where g_{ij}, b_{ij} and b_{ij}^{sh} are the branch series conductance, series susceptance and shunt susceptance, respectively, and e_i and f_i are the real and imaginary part of complex nodal voltage at bus i, respectively. Any quadratic form can be described in a compact matrix representation by means of a vector and a three-dimensional array. Here, the quadratic form of transmission line flows is given by the combination of $\mathbf{x}$, which holds the real and imaginary components of voltages, and the three-dimensional array $\mathbf{T}_s$, which depends only on the branch parameters g_{ij}, b_{ij} and b_{ij}^{sh}.

$$\mathbf{h}_S(\mathbf{x}) = \frac{1}{2}\mathbf{x}^t\mathbf{T}_s\mathbf{x} \tag{3}$$

It is worth noticing that $\mathbf{T}_s$ fully represents power system topology (and its possible changes).

According to Eqs. (1) and (2), bus power injections are given by the following quadratic expressions:

$$\begin{aligned} P_i(e,f) &= \sum_{k\epsilon\Omega_{0_i}} \left(g_{ik}\left(e_i^2+f_i^2\right) - g_{ik}\left(e_ie_k+f_if_k\right) - b_{ik}\left(e_if_k-e_kf_i\right)\right) \\ Q_i(e,f) &= \sum_{k\epsilon\Omega_{0_i}} \left(-\left(b_{ik}+b_{ik}^{sh}\right)\left(e_i^2+f_i^2\right) + b_{ik}\left(e_ie_k+f_if_k\right) + g_{ik}\left(e_if_k-e_kf_i\right)\right) \end{aligned} \tag{4}$$

where Ω_{0_i} represents the set of buses adjacent to bus i. The set of the PF equations is:

$$\begin{aligned} P_{g_i} - P_{d_i}^0 - P_i(e,f) &= 0 \quad \text{(PV and PQ buses)} \\ Q_{g_i} - Q_{d_i}^0 - Q_i(e,f) &= 0 \quad \text{(PQ buses)} \\ V_i^{ref^2} - e_i^2 + f_i^2 &= 0 \quad \text{(PV buses)} \end{aligned} \tag{5}$$

where P_{g_i} and Q_{g_i} are the real and reactive power generation; $P_{d_i}^0$ and $Q_{d_i}^0$ are the real and reactive power demand at the i-th bus. Equations (4) can be written in a compact form as [17]

$$\mathbf{g}_0(\mathbf{x}) = \frac{1}{2}\mathbf{x}^t\mathbf{T}\mathbf{x} \tag{6}$$

The set of PF equations can be written in a compact form as

$$\mathbf{g}(\mathbf{x}) = \mathbf{y}_0 - \mathbf{g}_0(\mathbf{x}) = \mathbf{y}_0 - \frac{1}{2}\mathbf{x}^t\mathbf{T}\mathbf{x} = \mathbf{0} \tag{7}$$

where $\mathbf{y}_0$ stands for the vector of specified variables of (5) and $\mathbf{T}$ is a constant three-dimensional array, with characteristics similar to $\mathbf{T}_s$, such that $\mathbf{g}_0(\mathbf{x})$ has derivatives expressed by

$$\frac{\partial\mathbf{g}_0(\mathbf{x})}{\partial\mathbf{x}} = \mathbf{x}^t\mathbf{T} \qquad \frac{\partial^2\mathbf{g}_0(\mathbf{x})}{\partial\mathbf{x}^2} = \mathbf{T} \tag{8}$$

According to [17], (8) shows the most interesting features of the formulation of PF equations in rectangular coordinates. The Jacobian matrix is linear with respect to $\mathbf{x}$, and the Hessian matrix $\mathbf{T}$ is constant: no residuals are present when the Taylor series is stopped at the term of order two. This does not occur when using Taylor series with polar formulation, even using terms of order higher than two: when the series is truncated, a residual is always present and this results in an error, more or less significant depending on the order adopted.

3 Branch contingency analysis

In order to carry out a branch contingency analysis, the following model is considered: branch $i - j$ under contingency is parameterised as suggested in [18,19], and shown in Figure 1, where β is a scalar that models the status of the branch: $\beta = 0$ for line in operation and $\beta = 1$ otherwise.

The PF injections are then re-written as:

$$\begin{aligned} P_i(e,f) = & \sum_{k\epsilon\Omega_{1_i}} \left(g_{ik}\left(e_i^2+f_i^2\right) - g_{ik}\left(e_ie_k+f_if_k\right) - b_{ik}\left(e_if_k - e_kf_i\right)\right) \\ & + (1-\beta)\left(g_{ij}\left(e_i^2+f_i^2\right) - g_{ij}\left(e_ie_j+f_if_j\right) - b_{ij}\left(e_if_j-e_jf_i\right)\right) \\ Q_i(e,f) = & \sum_{k\epsilon\Omega_{1_i}} \left(-\left(b_{ik}+b_{ik}^{sh}\right)\left(e_i^2+f_i^2\right) + b_{ik}\left(e_ie_k+f_if_k\right) + g_{ik}\left(e_if_k-e_kf_i\right)\right) \\ & + (1-\beta)\left(-\left(b_{ij}+b_{ij}^{sh}\right)\left(e_i^2+f_i^2\right) + b_{ij}\left(e_ie_j+f_if_j\right) + g_{ij}\left(e_if_j-e_jf_i\right)\right) \end{aligned} \tag{9}$$

where Ω_{1_i} represents the set of buses adjacent to bus i, except for bus j, such that Eq. (7) becomes:

$$\mathbf{g}(\mathbf{x},\beta) = \mathbf{y}_0 - \mathbf{g}_0(\mathbf{x},\beta) = \mathbf{y}_0 - \frac{1}{2}\mathbf{x}^t\mathbf{T}(\beta)\mathbf{x} = \mathbf{0} \tag{10}$$

where the three-dimensional array $\mathbf{T}(\beta)$ is now a function of the scalar β.

When the transmission line $i - j$ is operating, $\beta = 0$ and the equilibrium point $\mathbf{x}_e$, which represents the solution of the PF equations for the intact system, satisfies the condition

$$\mathbf{g}(\mathbf{x}_e, 0) = \mathbf{y}_0 - \frac{1}{2}\mathbf{x}_e^t\mathbf{T}_0\mathbf{x}_e = \mathbf{0} \tag{11}$$

where $\mathbf{T}_0 = \mathbf{T}(0)$. When the branch $i - j$ is out of service, $\mathbf{T}_0$ changes to $\mathbf{T}_1 = \mathbf{T}(1) = \mathbf{T}_0 - \mathbf{\Delta T}$, where $\mathbf{\Delta T}$ is a sparse three-dimensional array with elements representing the second derivatives of the real and reactive power flows in transmission line $i-j$ with respect to the real and imaginary components of the bus voltage at its terminal buses. Hence, if a single contingency is considered, it has only 16 nonzero elements. This change is related to the branch parameters only and fully represents the modification of the bus admittance matrix resulting from the outage. The PF equations are expressed as

$$\mathbf{g}(\mathbf{x}_e + \mathbf{\Delta x}, 1) = \mathbf{y}_0 - \frac{1}{2}(\mathbf{x}_e + \mathbf{\Delta x})^t\left(\mathbf{T}_0 - \mathbf{\Delta T}\right)(\mathbf{x}_e + \mathbf{\Delta x}) = \mathbf{0} \tag{12}$$

where the term $\mathbf{y}_0$, i.e., the specified power injections and squared PV-bus voltage magnitudes, is assumed to be constant.

Eq. (12) can be re-written as

$$\begin{aligned} \mathbf{y}_0 - \frac{1}{2}\Big(& \mathbf{x}_e^t\mathbf{T}_0\mathbf{x}_e + \mathbf{x}_e^t\mathbf{T}_0\mathbf{\Delta x} + \mathbf{\Delta x}^t\mathbf{T}_0\mathbf{x}_e + \mathbf{\Delta x}^t\mathbf{T}_0\mathbf{\Delta x} \\ & -\mathbf{x}_e^t\mathbf{\Delta T}\mathbf{x}_e - \mathbf{x}_e^t\mathbf{\Delta T}\mathbf{\Delta x} - \mathbf{\Delta x}^t\mathbf{\Delta T}\mathbf{x}_e - \mathbf{\Delta x}^t\mathbf{\Delta T}\mathbf{\Delta x}\Big) = \mathbf{0} \end{aligned} \tag{13}$$

and the re-arrangement of this equation provides

$$\begin{aligned}\mathbf{y}_0 - \frac{1}{2}\mathbf{x}_e^t\mathbf{T}_0\mathbf{x}_e - \frac{1}{2}\Big(\mathbf{J}_e\boldsymbol{\Delta}\mathbf{x} + \boldsymbol{\Delta}\mathbf{x}^t\mathbf{J}_e^t + \mathbf{J}_{dn}\boldsymbol{\Delta}\mathbf{x} - \mathbf{J}_t\mathbf{x}_e - \mathbf{J}_t\boldsymbol{\Delta}\mathbf{x} - \boldsymbol{\Delta}\mathbf{x}^t\mathbf{J}_t^t - \mathbf{J}_{dt}\boldsymbol{\Delta}\mathbf{x}\Big) &= \mathbf{0}\\ \mathbf{y}_0 - \frac{1}{2}\mathbf{x}_e^t\mathbf{T}_0\mathbf{x}_e - \mathbf{J}_e\boldsymbol{\Delta}\mathbf{x} - \frac{1}{2}\mathbf{J}_{dn}\boldsymbol{\Delta}\mathbf{x} + \frac{1}{2}\mathbf{J}_t\boldsymbol{\Delta}\mathbf{x} + \mathbf{b}_1 + \frac{1}{2}\mathbf{J}_{dt}\boldsymbol{\Delta}\mathbf{x} &= \mathbf{0}\end{aligned} \tag{14}$$

or, in compact form,

$$\mathbf{y}_0 - \frac{1}{2}\mathbf{x}_e^t\mathbf{T}_0\mathbf{x}_e - (\mathbf{J}_e - \mathbf{J}_t)\boldsymbol{\Delta}\mathbf{x} - \frac{1}{2}(\mathbf{J}_{dn} - \mathbf{J}_{dt})\boldsymbol{\Delta}\mathbf{x} + \mathbf{b}_1 = \mathbf{0} \tag{15}$$

where

- $\mathbf{J}_e = \mathbf{x}_e^t\mathbf{T}_0 = \left.\frac{\partial \mathbf{g}_0(\mathbf{x},\beta)}{\partial \mathbf{x}}\right|_{(\mathbf{x}_e,0)}$ is the Jacobian matrix of the conventional PF equations, calculated at the point of equilibrium (see Eq. (8));
- $\mathbf{J}_t = \mathbf{x}_e^t\boldsymbol{\Delta}\mathbf{T}$ is the Jacobian matrix (first derivative) of the power flows of the branch under contingency, computed at the pre-fault equilibrium point. Alternatively, this matrix can be interpreted as

$$\mathbf{J}_t = \frac{\partial}{\partial \mathbf{x}}\left.\left(\frac{\partial \mathbf{g}_0(\mathbf{x},\beta)}{\partial \beta}\right)\right|_{(\mathbf{x}_e,0)} = -\left.\frac{\partial^2 \mathbf{g}(\mathbf{x},\beta)}{\partial \mathbf{x}\partial\beta}\right|_{(\mathbf{x}_e,0)} \qquad \text{(see Eq. (10)).}$$

- $\mathbf{b}_1 = \frac{1}{2}\mathbf{x}_e^t\boldsymbol{\Delta}\mathbf{T}\mathbf{x}_e$ is a vector whose only non-zero elements are

$$\begin{aligned}\frac{\partial P_i(e,f)}{\partial\beta} &= P_{ij}(e,f) \qquad & \frac{\partial P_j(e,f)}{\partial\beta} &= P_{ji}(e,f)\\ \frac{\partial Q_i(e,f)}{\partial\beta} &= Q_{ij}(e,f) \qquad & \frac{\partial Q_j(e,f)}{\partial\beta} &= Q_{ji}(e,f)\end{aligned} \tag{16}$$

  Alternatively, this matrix can be interpreted as (see Eq. (9))

$$\mathbf{b}_1 = -\left.\frac{\partial \mathbf{g}_0(\mathbf{x},\beta)}{\partial\beta}\right|_{(\mathbf{x}_e,0)} = \left.\frac{\partial \mathbf{g}(\mathbf{x},\beta)}{\partial\beta}\right|_{(\mathbf{x}_e,0)}$$

- $\mathbf{J}_{dn} = \boldsymbol{\Delta}\mathbf{x}^t\mathbf{T}_0$ represents the Jacobian matrix of the PF equations, calculated in terms of the increments $(\Delta e_k, \Delta f_k)$;
- $\mathbf{J}_{dt} = \boldsymbol{\Delta}\mathbf{x}^t\boldsymbol{\Delta}\mathbf{T}$ is a Jacobian matrix that takes into account only the branch under contingency, calculated as a function of the increments $(\Delta e_k, \Delta f_k)$; its structure is similar to $\mathbf{J}_t$.

The Jacobian matrix after the contingency is expressed as

$$\begin{aligned}(\mathbf{x}_e + \boldsymbol{\Delta}\mathbf{x})^t(\mathbf{T}_0 - \boldsymbol{\Delta}\mathbf{T}) &= \mathbf{x}_e^t\mathbf{T}_0 - \mathbf{x}_e^t\boldsymbol{\Delta}\mathbf{T} + \boldsymbol{\Delta}\mathbf{x}^t\mathbf{T}_0 - \boldsymbol{\Delta}\mathbf{x}^t\boldsymbol{\Delta}\mathbf{T}\\ &= (\mathbf{J}_e - \mathbf{J}_t) + (\mathbf{J}_{dn} - \mathbf{J}_{dt})\end{aligned} \tag{17}$$

According to Eq. (11), the first two terms of Eq. (15) represent the solution of the pre-fault PF, thus resulting in

$$\mathbf{y}_0 - \frac{1}{2}\mathbf{x}_e^t\mathbf{T}_0\mathbf{x}_e = \mathbf{0} \tag{18}$$

such that Eq. (15) becomes

$$-(\mathbf{J}_e - \mathbf{J}_t)\mathbf{\Delta x} - \frac{1}{2}(\mathbf{J}_{dn} - \mathbf{J}_{dt})\mathbf{\Delta x} + \mathbf{b}_1 = \mathbf{0} \tag{19}$$

According to Eqs. (13) and (14), the Jacobian matrices $\mathbf{J}_{dn}$ and $\mathbf{J}_{dt}$ take into account the contingency considered, as they depend on $\mathbf{\Delta T}$. Moreover, they are function of the increment $\mathbf{\Delta x}$ and therefore Eq. (19) is quadratic and cannot be solved in one step. However, given the quadratic nature of the PF equations, all terms of the Taylor series expansion, as well as the change of the power system topology, are fully considered. Eq. (19) could be solved iteratively, e.g., by a Newton's method; at any rate, to save time and computations, an approximated solution of this equation can be obtained through the following procedure:

(1) Determine the power flow solution for the base case (intact system);
(2) Determine the matrix $\mathbf{J}_t$ and the vector $\mathbf{b}_1$, computed at $\mathbf{x}_e$;
(3) Solve the linear system $(\mathbf{J}_e - \mathbf{J}_t)\mathbf{d}_1 = \mathbf{b}_1$ for $\mathbf{d}_1$. Vector $\mathbf{d}_1$ represents the component of vector $\mathbf{\Delta x}$ corresponding to the first order information only.
(4) Using $\mathbf{\Delta x} = \mathbf{d}_1$, compute $\mathbf{J}_{dn}$, $\mathbf{J}_{dt}$ and the vector $\mathbf{b}_2 = -\frac{1}{2}(\mathbf{J}_{dn} - \mathbf{J}_{dt})\mathbf{d}_1$; in this way, the matrices and vector are updated to take into account the search direction modified by the contingency considered.
(5) Solve the linear system $(\mathbf{J}_e - \mathbf{J}_t)\mathbf{d}_2 = \mathbf{b}_2$. Vector $\mathbf{d}_2$ represents the component of $\mathbf{\Delta x}$ corresponding to the second order information. It is worth noticing that the same factorized matrix is used as in step (3).

Repeating the steps (2)-(5) above would allow solving Eq.(19) iteratively; however, in this paper, an approximate solution is determined directly after step (5), by combining the above directions directions $\mathbf{d}_1$ and $\mathbf{d}_2$ by means of the step factor α that minimizes the quadratic norm of Eq. (19), in the direction $\mathbf{\Delta x} = \mathbf{d}_1 + \alpha\mathbf{d}_2$. For this purpose, Eq. (19) is re-written as

$$\mathbf{F}(\alpha) = -(\mathbf{J}_e - \mathbf{J}_t)(\mathbf{d}_1 + \alpha\mathbf{d}_2) - \frac{1}{2}(\mathbf{J}'_{dn} - \mathbf{J}'_{dt})(\mathbf{d}_1 + \alpha\mathbf{d}_2) + \mathbf{b}_1 \tag{20}$$

where, from Eq. (17),

$$\begin{aligned} \mathbf{J}'_{dn} &= (\mathbf{d}_1 + \alpha\mathbf{d}_2)^t\mathbf{T}_0 = (\mathbf{d}_1^t\mathbf{T}_0 + \alpha\mathbf{d}_2^t\mathbf{T}_0) = \mathbf{J}_{dn} + \alpha\mathbf{J}_{dn1} \\ \mathbf{J}'_{dt} &= (\mathbf{d}_1 + \alpha\mathbf{d}_2)^t\mathbf{\Delta T} = (\mathbf{d}_1^t\mathbf{\Delta T} + \alpha\mathbf{d}_2^t\mathbf{\Delta T}) = \mathbf{J}_{dt} + \alpha\mathbf{J}_{dt1} \end{aligned} \tag{21}$$

and therefore,

$$(\mathbf{J}'_{dn} - \mathbf{J}'_{dt}) = (\mathbf{J}_{dn} - \mathbf{J}_{dt}) + \alpha(\mathbf{J}_{dn1} - \mathbf{J}_{dt1})$$

such that, from the previous definition of $\mathbf{b}_2$,

$$\mathbf{F}(\alpha) = \mathbf{b}_2 + \left[-\mathbf{b}_2 - \frac{1}{2}(\mathbf{J}_{dn} - \mathbf{J}_{dt})\mathbf{d}_2 - \frac{1}{2}(\mathbf{J}_{dn1} - \mathbf{J}_{dt1})\mathbf{d}_1\right]\alpha - \frac{1}{2}(\mathbf{J}_{dn1} - \mathbf{J}_{dt1})\mathbf{d}_2\alpha^2 \tag{22}$$

and, in compact form,

$$\mathbf{F}(\alpha) = \mathbf{a}_0 + \mathbf{b}_0\alpha + \mathbf{c}_0\alpha^2 \tag{23}$$

where

$$\begin{aligned}
\mathbf{a}_0 &= \ \mathbf{b}_2 \\
\mathbf{b}_0 &= \ \left[-\mathbf{b}_2 - \frac{1}{2}(\mathbf{J}_{dn} - \mathbf{J}_{dt})\mathbf{d}_2 - \frac{1}{2}(\mathbf{J}_{dn1} - \mathbf{J}_{dt1})\mathbf{d}_1\right] \\
\mathbf{c}_0 &= \ -\frac{1}{2}(\mathbf{J}_{dn1} - \mathbf{J}_{dt1})\mathbf{d}_2
\end{aligned} \tag{24}$$

The scalar α is determined by solving the problem

$$\text{Minimize } \frac{1}{2}\mathbf{F}(\alpha)^t\mathbf{F}(\alpha) = \frac{1}{2}\left(\mathbf{a}_0 + \mathbf{b}_0\alpha + \mathbf{c}_0\alpha^2\right)^t\left(\mathbf{a}_0 + \mathbf{b}_0\alpha + \mathbf{c}_0\alpha^2\right)$$

which requires to derive the function $\frac{1}{2}\mathbf{F}(\alpha)^t\mathbf{F}(\alpha)$ with respect to α and to equal the result to zero. This provides the cubic scalar equation given by,

$$g_0 + g_1\alpha + g_2\alpha^2 + g_3\alpha^3 = 0 \tag{25}$$

where

$$\begin{aligned}
g_0 &= \mathbf{a}_0^t\mathbf{b}_0 \\
g_1 &= \mathbf{b}_0^t\mathbf{b}_0 + 2\mathbf{a}_0^t\mathbf{c}_0 \\
g_2 &= \mathbf{c}_0^t\mathbf{b}_0 + 2\mathbf{b}_0^t\mathbf{c}_0 = 3\mathbf{b}_0^t\mathbf{c}_0 \\
g_3 &= 2\mathbf{c}_0^t\mathbf{c}_0
\end{aligned}$$

whose smaller real root is adopted as the step factor. At each iteration, the real and imaginary components of nodal voltages are updated by $\mathbf{x} = \mathbf{x}_e + \mathbf{\Delta x}$.

The estimates based on sensitivity relationships obtained by adopting this procedure have a high degree of accuracy, as a consequence of considering all terms of the Taylor series expansion of the quadratic PF equations expressed in rectangular coordinates. The factor α used for accuracy purposes is computed to minimize the Euclidean norm of the vector of mismatches: this brings to another interesting feature of the proposed method. In case of divergence of PF computations, often the PF output does not provide the system operator with any information about the physical problem. In such cases, a damping factor (or optimal multiplier, [20]) can be adopted to prevent the divergence of the iterative process. The concept of α is based on the same principles as the damping factor above mentioned. In case of non existing post-contingency PF solution, the proposed approach provides at any rate the system operator with useful information: it does not diverge, and the computation of the optimal α (in such cases, results show that it becomes very low or even negative), brings to the point that (although it is not a solution of the PF equations, which does not exist) minimizes the distance to the hypersurface of the PF solutions, giving at any rate the system operator information about the physical problem and possible countermeasures.

4 Handling capability limits of generators

In case of loaded systems, reactive limits of generators play an important role in guaranteeing suitable voltage profiles in the system. Lack of reactive/voltage support can result in voltage collapse [21] and hence the occurrence of some contingencies may be particularly dangerous, especially when the capability limit of one or more generators is hit. In these cases, voltage control might be lost. The correct assessment of the reactive power that can be injected by generators is therefore paramount in these cases, and its rough approximation might result in underestimation of contingency criticality. In this case, the sensitivity relationship based on Eq. (15) does not represent the effective modelling of the control equipment (after a conversion from PV to PQ bus, for example), and this would deteriorate the accuracy of the estimates. The procedure normally used to deal with the reactive power generation limits in PF problems consists of checking, during the iterative process, if any of these limits is violated, with consequent conversion of the PV bus into PQ bus. Here, an anticipating strategy is adopted, by a scheme aimed at identifying which PV bus eventually reaches the capability limit as a result of the contingency. The method is based on the quadratic form of the PF equations [22].

The set of reactive power generation limits which restrains the conventional PF equations stated by Eq. (5) is given by the following $2N_{PV}$ inequalities:

$$Q_{g_i}^m \le Q_{g_i}(\mathbf{x}, \beta) \le Q_{g_i}^M \quad \text{(PV buses)} \tag{26}$$

where the superscripts M and m refers to upper and lower limits, respectively; and

$$Q_{g_i}(\mathbf{x}, \beta) = Q_{d_i}^0 + Q_i(\mathbf{x}, \beta)$$

where $Q_{d_i}^0$ is the reactive load connected at bus i and $Q_i(\mathbf{x}, \beta)$ is the reactive power injection at the same bus. For each generator, the two different operating points corresponding to upper and lower limits, respectively, are characterized by:

$$\begin{aligned} Q_{g_i}^M - (Q_{d_i}^0 + Q_i(\mathbf{x}, \beta)) = 0 \\ Q_{g_i}^m - (Q_{d_i}^0 + Q_i(\mathbf{x}, \beta)) = 0 \end{aligned} \tag{27}$$

where for bus i the equations differ for their free term. Reactive power injections, analogously to Eq.(6) and (10), are

$$\mathbf{h}_q(\mathbf{x}, \beta) = \mathbf{Q}(\mathbf{x}, \beta) = \frac{1}{2}\mathbf{x}^t\mathbf{T}_{q0}(\beta)\mathbf{x}$$

where $\mathbf{T}_{q0}(\beta)$ is a constant sparse three-dimensional array with characteristics similar to $\mathbf{T}(\beta)$.

For the sake of simplicity, the first of Eqs.(27) is now considered, which represents the conditions for reaching the maximum reactive capability limit. In this framework, the goal of the proposed procedure is to identify the generator closest to its limit along the direction $\mathbf{\Delta x}$ solution of Eq.(19). Hence, the method determines, for each generator i, the step-length γ_i such that its limit is hit for $\gamma_i\mathbf{\Delta x}$: the critical generator is the one with the smallest

γ_i. For that, the Taylor series expansion of $\mathbf{h}_q(\mathbf{x},\beta)$ is carried out around the point $(\mathbf{x}_e, 0)$ along the generic direction $(\gamma\boldsymbol{\Delta}\mathbf{x}, \Delta\beta)$, (see Eq. (15)), where γ is a scalar

$$\mathbf{h}_q(\mathbf{x},\beta) = \frac{1}{2}\mathbf{x}_e^t\mathbf{T}_{q0}\mathbf{x}_e + [(\mathbf{J}_{q_e} - \mathbf{J}_{q_t})\boldsymbol{\Delta}\mathbf{x}]\gamma + [\frac{1}{2}(\mathbf{J}_{q_{dn}} - \mathbf{J}_{q_{dt}})\boldsymbol{\Delta}\mathbf{x}]\gamma^2 - \mathbf{b}_q \tag{28}$$

It is worth noticing that $\mathbf{b}_q$ has non-zero elements only if some of the buses connected by the branch under contingency are PV type. For each generator i, the corresponding expression from (28) is substituted into the first of Eqs. (27) resulting in a quadratic scalar equation. Solving for γ gives the step-length γ_i that makes generator i reach its upper limit. Taking into account all generators, the N_{PV} equations can be written in a compact form as:

$$\mathbf{Q}_g^M - \mathbf{Q}_d - \frac{1}{2}\mathbf{x}_e^t\mathbf{T}_{q0}\mathbf{x}_e - [(\mathbf{J}_{q_e} - \mathbf{J}_{q_t})\boldsymbol{\Delta}\mathbf{x}] \circ \boldsymbol{\gamma} - [\frac{1}{2}(\mathbf{J}_{q_{dn}} - \mathbf{J}_{q_{dt}})\boldsymbol{\Delta}\mathbf{x}] \circ (\boldsymbol{\gamma} \circ \boldsymbol{\gamma}) + \mathbf{b}_q = 0 \tag{29}$$

where now $\boldsymbol{\gamma}$ is a vector holding for each generator the step length value that corresponds to reaching its capability limit, and $\circ$ is the Hadamard or element-wise product. Eq.(29) is not a system of equations: it is a set of N_{PV} independent scalar quadratic equations where $\boldsymbol{\gamma}$ is the vector of unknowns. The minimum entry of $\boldsymbol{\gamma}$ identifies the generator that reaches its capability limit first and that must be converted into PQ-type. Observe that the equations can be written as

$$\mathbf{a}_0 + \mathbf{b}_0 \circ \boldsymbol{\gamma} + \mathbf{c}_0 \circ (\boldsymbol{\gamma} \circ \boldsymbol{\gamma}) = 0 \tag{30}$$

where

$$\begin{aligned}
\mathbf{a}_0 &= \mathbf{Q}_g^M - \mathbf{Q}_d - \frac{1}{2}\mathbf{x}_e^t\mathbf{T}_{q0}\mathbf{x}_e + \mathbf{b}_q \\
\mathbf{b}_0 &= -(\mathbf{J}_{q_e} - \mathbf{J}_{q_t})\boldsymbol{\Delta}\mathbf{x} \\
\mathbf{c}_0 &= -\frac{1}{2}(\mathbf{J}_{q_{dn}} - \mathbf{J}_{q_{dt}})\boldsymbol{\Delta}\mathbf{x} = -\frac{1}{2}(\boldsymbol{\Delta}\mathbf{x}^t\mathbf{T}_q(0)\boldsymbol{\Delta}\mathbf{x}) = -\mathbf{h}_q(\boldsymbol{\Delta}\mathbf{x}, \mathbf{0})
\end{aligned} \tag{31}$$

In case the minimum $\gamma_i > 1$, the contingency considered does not result in any capability limit reached.

Considering the set of equations relevant to the second equations (lower limits) in (27), that results in a further set of N_{PV} independent quadratic equations to be solved.

Therefore, after estimating the increment $\boldsymbol{\Delta}\mathbf{x}$, adjusted by the step factor α calculated through Eq. (25), which does not take into account capability limits, the $2N_{PV}$ quadratic equations are solved, and the minimum value $0 < \gamma_i \leq 1$ is selected, for the re-adjustment of the increments $\boldsymbol{\Delta}\mathbf{x}$. This ensures that no limit of reactive power generation will be violated in the current estimation step. Subsequently, the $(i - th)$ bus whose limit was reached is converted to PQ bus and it is necessary to solve Eq.(19) again, where J_e and J_{dn} must be updated, to compute a new increment $\boldsymbol{\Delta}\mathbf{x}$. Note that the update of the Jacobian matrices only requires to change the elements of the row corresponding to the squared voltage magnitude and the row corresponding to the reactive power balance of the converted PQ bus. This procedure is repeated until no reactive power limit is reached. The computational effort to determine the final estimates depends on the number of inequality constraints that become active in the estimation process. Note also that this treatment of the capacity limits, which is essentially a uni-dimensional search, is similar to that shown in reference [10], named *linear segmented estimation*. The main advantage of the strategy presented here is to envisage the bus whose limit will be reached, which is facilitated by the quadratic form of the PF equations.

Other nonlinear issues in power systems can be treated according to the proposed methodology, provided that the quadratic properties of the PF equations in rectangular coordinates are preserved. For long-term contingency analysis, for example, Load Tap Changing (LTC) transformers must be taken into account. If taps had been included according to the classical model, PF equations would have been no longer quadratic, which would have deteriorated the performance of the proposed method. This is why we applied the modeling presented in [23], which consists of including an additional node for each LTC, in order to keep the quadratic form of the PF equations. This allows the use of the proposed method without losing accuracy and efficiency.

5 Tests and results

This section presents some results obtained by the proposed methodology for the analysis of branch contingencies. Particular attention is paid to both the accuracy and the computation time required. Actually, exact nonlinear analysis could be carried out by complete PF computations. However, that would be time requiring for power system operators; on the contrary, many methodologies based on linear sensitivities and/or first-order approximations provide very quick results, but they lack accuracy, especially when generator limits are hit. The proposed method provides a significant improvement in term of trade-off of accuracy and computation times.

Although the proposed methodology is mostly fit to stressed systems with several generators close to their capability limits, in this paper standard test systems were used, so that it is possible to show and compare the results of the proposed methodology to those presented in the technical literature: many numerical simulations were performed on the IEEE tests systems ranging from 14 up to 300 busses. Moreover, the method has been tested on a model of the Italian EHV and HV system, characterized by about 1000 busses. The complex bus voltages and the branch power flows are estimated by the presented procedure are compared with the exact solutions obtained by solving an exact non-linear AC PF problem. In addition, some of these results are also compared, whenever possible, with the output of other numerical methods proposed in the literature that include linear and non linear piecewise methods in polar coordinates [10,13,16]. In all cases, the error was computed by taking the exact solution of the power flow as reference, that is,

$$\text{Error}_i = \left| z_i^{pf} - z_i^{sb} \right|$$

where z_i^{pf} and z_i^{sb} represent the variables z_i (magnitude of nodal voltages, power flows, etc.) calculated in the solution of the power flow and through the sensitivity relationships, respectively. In order to give an estimate of the computational burden, i.e., computation time, results are provided with reference to the time to find the AC PF solution, because this gives the possibility to compare computation times obtained on different computers.

5.1 IEEE 14-bus test system

Because of the small size of this system, even a single contingency may affect the branch power flows considerably. The results obtained on this system are shown in Tables 1 to 2. Since the proposed method is particularly suitable to solve the more nonlinear and challenging QV subproblem, the contingencies to be studied in detailed were selected on the basis of the loading of the transmission lines in the base case, in terms of reactive power flows. In this section, results relevant to outages of lines (TL) 2-4, 7-9, 2-3 and transformer (TR) 5-6 are shown. Here, some results shown in references [10,16] were reported in Table 1. It is worth noticing that [10] is a nonlinear method taking into account reactive capability limits, while [16] is a linear method, not considering them.

Table 1 presents the bus voltage magnitude obtained from the exact power flow solution, the estimates computed through both the first-order and the second-order sensitivity relationships and the corresponding errors, as well as the values from the technical literature references [10,16]. No voltage control device has reached its reactive power generation limit and this make errors generally low; however, it is worth noticing that the errors associated with the proposed method are very small (about 1.0×10^{-4} pu), one order of magnitude lower than for first-order estimates, i.e., better than first-order methods. The higher accuracy of these estimates can be attributed to the use of the second-order information in the variables computation.

This improvement in the accuracy brought by second-order information is more evident when looking at either the estimates of the reactive power generations and power flows or the corresponding errors with respect to the PF exact solution (Table 2 and Figure 2). From the analysis of the magnitude of the errors presented, it can be inferred that the proposed method is very accurate: errors, in the case of the linear estimates, are some orders of magnitude higher.

Tables 3 and 4 and Figure 3 show the similar results obtained for the outage of the TL 2-3 and TR 5-6. The reactive power generated in buses 3 and 6 have reached their upper limits in case of contingencies of TL 2-3 and the TR 5-6, respectively (results bolded in Tables). In these cases, the strategy described in Subsection 4 was adopted to calculate the estimates, as can be seen from Table 4, where for the second-order method the enforcement of reactive power generated is clear for generators at busses 3 and 6 in the two cases presented, while of course it is not present for the linear approximations. Nevertheless, Table 3 shows that the level of accuracy of the estimates using second-order information remains high enough: this method provides better results than those found in the literature. In case of the outage of the transformer 5-6, the inaccuracies corresponding to buses 12, 13 and 14 (column 11) are due to the proximity of these buses to bus 6, which is directly affected by this line trip. At any rate, they are much lower than using traditional linear sensitivities.

5.2 IEEE 57-bus test system

The numerical results obtained with the IEEE 57-bus test-system are shown in Tables 5 and 6. This system was also used in references [14,10]. The cases presented in reference [10] were simulated with the proposed methodology. In the contingency of the transmission line 12-13, the generator of bus 9 reaches its reactive power generation limit, while after the outage of the transformer 13-49 no reactive power generation limit is reached. In both cases, the results in Tables 5 and 6 show that second-order estimates are accurate and result in small errors, particularly in the calculation of reactive power generations and reactive flows of power transmission lines. In particular, Table 5 demonstrates that the errors for the proposed method are 1-2 orders of magnitude lower than for linear sensitivities. Values taken from [10] are shown in those Tables marked as Ref for comparison.

5.3 Italian system

To demonstrate the features of the proposed method, it has been adopted in the *(n-1)* security assessment of a model of the Italian system. The real model considered is relevant to the 380 and 220 kV network including 959 buses, 1099 transmission lines and a total real and reactive power load level of 32615.58 MW and 7637.14 Mvar. The contingency analysis was performed for a contingency set of 110 transmission lines. Table 7 summarizes the performance of the proposed method for 15 selected contingencies. It depicts the number of the transmission line under contingency, the buses connected by the transmission line, the number of generators at their limit after the contingency, the step factor α and the maximum absolute value of the pu voltage magnitude error with respect to the exact AC PF solution. It is worth noticing that α remains in the same range shown in the previous tables. The last column of this table shows the maximum error in the voltage magnitude and indicates the highly satisfactory performance of the proposed technique in terms of accuracy.

5.4 Numerical Performance

To end this section, some important issues related to the numerical performance of the proposed technique are presented in Table 8, with reference to some tests performed on the following IEEE test systems: 14-bus, 57-bus, 118 bus, 300-bus respectively. Table 8 shows a few results for some transmission line contingencies, the number of reactive power devices whose reactive power generation reaches the limit after the contingency and the CPU times required by both the proposed approach (*2nd Order*) and the full Newton-Raphson PF ($N-R$). Since the conditions for measuring CPU times (programming language, type of computer, some features of the power flow solution, etc.) are usually unknown for comparison, we have taken PF computation times as a reference, according to other papers in the technical literature. From previous results, it is clear that the proposed procedure exploiting second-order and taking suitably into account generator reactive power limits is characterized by very good levels of accuracy, comparable with the exact PF solution. As for the computational efficiency, CPU times for the proposed methodology are in this case lower than those for the N-R full power flow, which has the same level of accuracy. For example, for the IEEE 300 bus system, the time required by the proposed methodology is less than one fourth of the corresponding N-R method; this confirms the desirable features of the proposed methodology, not only in terms of accuracy but also with respect to the reduced computational requirements. CPU times required by linear approach are lower, but as shown the accuracy resulting is also lower; this is particularly true when reactive capability limits are hit, because linear methods can not catch this phenomenon.

Finally, Table 9 provides some information about the computational effort corresponding to the analysis of a complete set of contingencies: the total number of contingencies assessed, the CPU times (total and average per contingency) obtained by the PF and by the proposed sensitivity-based method. In these tests, capability limits of generators are taken into account. The very good CPU times of the proposed methodology are more evident as the size of the power system increases. Observe that in case of the IEEE 300-bus system, the mean CPU time per contingency is approximately 16% of that corresponding to the use of the nonlinear Newton-Raphson PF. The methodology presented in [10] can be compared to the proposed methodology: in that paper, CPU times are ranging from 39% to 46% of the time required for a complete AC PF.

For what the Italian system is concerned, the overall computational effort required by the proposed method is about 30% of that required by the nonlinear solution of the AC PF. In case of many contingencies the CPU times are about 16% of the time required by the full PF; the worst case found was for the outage of line 283, when the saving is 32%.

6 Conclusions

This work presents a methodology to estimate the power flow variables after contingencies in the transmission system. Similarly to other methods found in the literature, the proposed estimation is based on sensitivity relationships. However, rectangular coordinates are used to model analytically the power flow equations and this allows the straightforward exploitation of the effects of contingency taking into account all the terms of the Taylor series expansion of the power flow equations. This reduces the errors compared with the sensitivity model inherent to the use of the linear term only, and makes it possible to include the treatment of capability limits of the QV control devices, which improves the accuracy of the estimation considerably.

The results presented here indicate further potential of the proposed method for studies of security analysis in large power systems involving the selection of contingencies. Future work in this research topic will include the analysis of multiple contingencies and the use of this methodology in contingency selection. Moreover, the computation of factors γ for the enforcement of capability limits provides sensitivity of each generator reactive reserve with respect to every contingency; this feature, not exploited in the present paper, will be further investigated.

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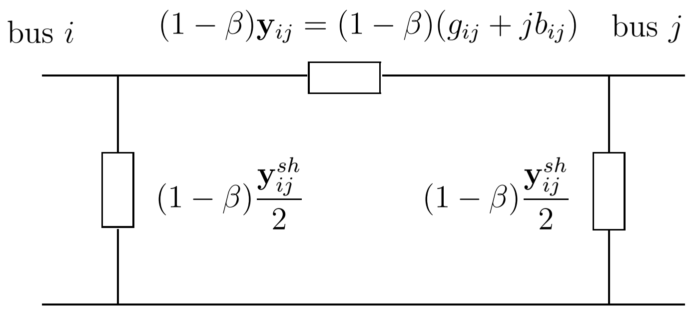

Fig. 1. Parameterised π circuit representing the transmission line $i - j$

Table 1
Voltage magnitude: IEEE 14-bus test system, outage of TL's 2-4 and 7-9

	Outage TL 2-4 Voltage Magnitude (pu)						Outage TL 7-9 Voltage Magnitude (pu)					
	Exact	1^{st} order		2^{nd} order		Ref.	Exact	1^{st} order		2^{nd} order		Ref.
Bus	Solution	Estimate	Error	Estimate	Error	Value	Solution	Estimate	Error	Estimate	Error	Value
1	1.0600	1.0600	0.0000	1.0600	0.0000	1.0600	1.0600	1.0600	0.0000	1.0600	0.0000	1.0600
2	1.0450	1.0450	0.0000	1.0450	0.0000	1.0450	1.0450	1.0450	0.0000	1.0450	0.0000	1.0450
3	1.0100	1.0103	0.0003	1.0100	0.0000	1.0100	1.0100	1.0100	0.0000	1.0100	0.0000	1.0100
4	1.0070	1.0091	0.0021	1.0070	0.0000	1.0090	1.0169	1.0180	0.0011	1.0169	0.0000	1.0169
5	1.0114	1.0129	0.0014	1.0115	0.0000	1.0129	1.0177	1.0185	0.0008	1.0177	0.0000	1.0174
6	1.0700	1.0709	0.0009	1.0700	0.0000	1.0700	1.0700	1.0709	0.0009	1.0700	0.0000	1.0700
7	1.0570	1.0586	0.0016	1.0570	0.0000	1.0573	1.0676	1.0709	0.0033	1.0675	0.0001	1.0671
8	1.0900	1.0912	0.0012	1.0900	0.0000	1.0900	1.0900	1.0921	0.0021	1.0899	0.0001	1.0900
9	1.0513	1.0528	0.0015	1.0514	0.0000	1.0513	1.0289	1.0349	0.0060	1.0290	0.0002	1.0291
10	1.0471	1.0485	0.0014	1.0472	0.0000	1.0471	1.0280	1.0331	0.0051	1.0282	0.0002	1.0282
11	1.0549	1.0561	0.0011	1.0549	0.0000	1.0548	1.0446	1.0476	0.0030	1.0447	0.0001	1.0446
12	1.0549	1.0558	0.0009	1.0549	0.0000	1.0549	1.0534	1.0547	0.0012	1.0535	0.0001	1.0535
13	1.0497	1.0506	0.0010	1.0497	0.0000	1.0496	1.0459	1.0475	0.0016	1.0460	0.0001	1.0459
14	1.0326	1.0338	0.0013	1.0326	0.0000	1.0325	1.0178	1.0219	0.0041	1.0179	0.0001	1.0179

Table 2
Reactive power generation of the PV buses: IEEE 14-bus test system, outage of TL's 2-4 and 7-9

	Outage TL 2-4 Reactive Power Generation (Mvar)					Outage TL 7-9 Reactive Power Generation (Mvar)				
	Exact	1^{st} order		2^{nd} order		Exact	1^{st} order		2^{nd} order	
Bus	Solution	Estimate	Error	Estimate	Error	Solution	Estimate	Error	Estimate	Error
2	36.776	35.687	1.088	36.749	0.027	44.829	43.657	1.171	44.823	0.005
3	31.141	30.149	0.991	31.144	0.003	25.559	24.912	0.646	25.560	0.001
6	18.979	18.511	0.467	18.979	0.000	21.261	19.413	1.847	21.200	0.060
8	20.429	20.237	0.191	20.431	0.002	13.860	13.113	0.747	13.877	0.016

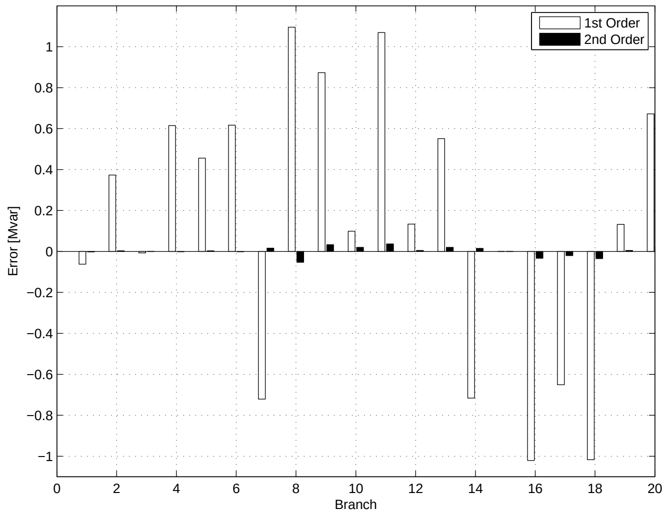

Fig. 2. Reactive power flows in the transmission lines: IEEE 14-bus test system, outage of TL 7-9

Table 3
Voltage magnitude: IEEE 14-bus test system, Outage of TL 2-3 and TR 5-6

	Outage TL 2-3 Voltage Magnitude (pu)					Outage TL 5-6 Voltage Magnitude (pu)				
	Exact	1^{st} order		2^{nd} order		Exact	1^{st} order		2^{nd} order	
Bus	Solution	Estimate	Error	Estimate	Error	Solution	Estimate	Error	Estimate	Error
1	1.0600	1.0600	0.0000	1.0600	0.0000	1.0600	1.0600	0.0000	1.0600	0.0000
2	1.0450	1.0450	0.0000	1.0450	0.0000	1.0450	1.0450	0.0000	1.0450	0.0000
3	**0.9534**	**0.9915**	**0.0381**	**0.9533**	**0.0001**	1.0100	1.0100	0.0000	1.0100	0.0000
4	0.9980	1.0080	0.0100	0.9981	0.0000	1.0137	1.0175	0.0038	1.0138	0.0001
5	1.0056	1.0122	0.0066	1.0056	0.0000	1.0241	1.0269	0.0028	1.0242	0.0000
6	1.0700	1.0710	0.0010	1.0701	0.0001	**1.0069**	**1.0525**	**0.0456**	**1.0083**	**0.0013**
7	1.0530	1.0582	0.0052	1.0531	0.0001	1.0532	1.0633	0.0101	1.0537	0.0004
8	1.0900	1.0914	0.0014	1.0901	0.0001	1.0900	1.0922	0.0022	1.0902	0.0002
9	1.0475	1.0525	0.0050	1.0475	0.0001	1.0439	1.0617	0.0178	1.0447	0.0008
10	1.0439	1.0482	0.0043	1.0440	0.0001	1.0302	1.0529	0.0227	1.0311	0.0009
11	1.0533	1.0560	0.0027	1.0534	0.0001	1.0156	1.0495	0.0339	1.0167	0.0011
12	1.0546	1.0559	0.0013	1.0547	0.0001	0.9934	1.0373	0.0440	0.9948	0.0015
13	1.0491	1.0507	0.0016	1.0492	0.0001	0.9942	1.0361	0.0419	0.9956	0.0014
14	1.0301	1.0336	0.0035	1.0302	0.0001	1.0026	1.0312	0.0286	1.0038	0.0012

Table 4
Reactive power generation of the PV buses: IEEE 14-bus test system, Outage of TL 2-3 and TR 5-6

	Outage TL 2-3 Reactive Power Generation (Mvar)					Outage TL 5-6 Reactive Power Generation (Mvar)				
	Exact	1^{st} order		2^{nd} order		Exact	1^{st} order		2^{nd} order	
Bus	Solution	Estimate	Error	Estimate	Error	Solution	Estimate	Error	Estimate	Error
2	42.271	31.813	10.457	42.195	0.075	44.513	40.215	4.298	44.415	0.098
3	**40.000**	**52.910**	**12.910**	**40.114**	**0.114**	27.329	25.095	2.234	27.266	0.063
6	22.617	19.003	3.614	22.623	0.005	**24.000**	**32.564**	**8.564**	**24.044**	**0.044**
8	22.891	20.580	2.311	22.896	0.005	22.767	17.889	4.878	22.597	0.170

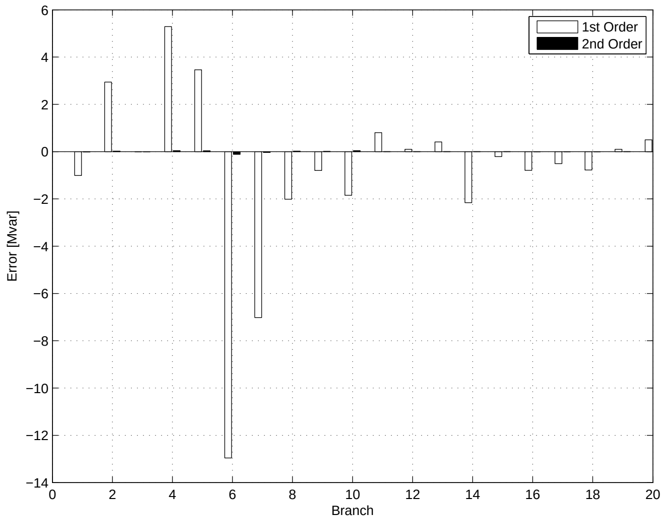

Fig. 3. Reactive power flows in the transmission lines: IEEE 14-bus test system, Outage of TL 2-3

Table 5
Reactive power generation of PV buses: IEEE 57-bus test system, Outage of TL 12-13 and TR 13-49

	Outage TL 12-13 Reactive Power Generation (Mvar)						Outage TR 13-49 Reactive Power Generation (Mvar)					
	Exact	1^{st} order		2^{nd} order		Ref.	Exact	1^{st} order		2^{nd} order		Ref.
Bus	Solution	Estimate	Error	Estimate	Error	Value	Solution	Estimate	Error	Estimate	Error	Value
2	-0.713	-0.724	0.011	-0.713	0.000	-0.72	-0.671	-0.693	0.022	-0.671	0.000	-0.70
3	22.160	21.409	0.751	22.151	0.008	20.95	3.407	2.580	0.826	3.407	0.000	2.01
6	2.725	2.669	0.056	2.724	0.001	2.28	3.015	2.863	0.152	3.013	0.002	2.48
8	79.023	78.422	0.601	79.017	0.005	78.02	64.210	64.064	0.145	64.208	0.002	63.71
9	9.000	8.952	0.048	9.000	0.000	9.00	2.224	1.353	0.871	2.230	0.005	0.83
12	76.661	76.376	0.284	76.657	0.004	75.73	126.159	125.034	1.124	126.170	0.010	124.61

Table 6
Reactive power in the transmission lines: IEEE 57-bus test system, outage of TR 13-49

Transm. line		Outage TL 12-13 Reactive Power Generation (Mvar)						Outage TR 13-49 Reactive Power Generation (Mvar)					
From		Exact	1^{st} order		2^{nd} order		Ref.	Exact	1^{st} order		2^{nd} order		Ref.
Bus	Bus	Solution	Estimate	Error	Estimate	Error	Value	Solution	Estimate	Error	Estimate	Error	Value
1	2	74.806	74.858	-0.052	74.807	-0.000	74.86	74.616	74.713	-0.097	74.615	0.000	74.75
1	15	47.396	46.987	0.409	47.392	0.003	46.83	35.454	35.063	0.391	35.454	0.000	34.87
3	15	3.978	3.307	0.670	3.971	0.007	2.99	-15.221	-15.875	0.653	-15.221	-0.000	-16.26
13	14	6.719	7.160	-0.440	6.721	-0.001	5.84	36.461	35.881	0.580	36.438	0.023	35.86
13	15	-11.977	-11.515	-0.461	-11.973	-0.003	-12.45	10.897	10.824	0.072	10.885	0.011	10.63
10	12	-24.922	-24.791	-0.130	-24.920	-0.002	-24.39	-24.974	-24.609	-0.365	-24.967	-0.007	-24.47
11	13	7.263	6.936	0.326	7.260	0.002	7.59	-8.244	-8.393	0.148	-8.235	-0.008	-8.26
12	13	-0.000	-0.000	0.000	-0.000	0.000	0.00	52.606	52.030	0.575	52.624	-0.018	51.88
12	16	9.111	9.082	0.029	9.110	0.000	9.01	8.759	8.681	0.077	8.759	-0.000	8.66
12	17	9.457	9.427	0.029	9.456	0.000	9.36	9.107	9.029	0.077	9.107	-0.000	9.01
14	46	26.271	26.320	-0.049	26.268	0.002	25.49	42.437	41.538	0.898	42.415	0.022	41.93
50	51	-9.868	-9.785	-0.083	-9.867	-0.001	-9.40	-13.339	-12.778	-0.560	-13.328	-0.010	-13.12
38	48	-16.028	-16.100	0.071	-16.028	-0.000	-16.58	-17.500	-17.6407	0.1404	-17.499	-0.001	-18.35

Table 7
Results for 959-bus Italian system

TL number	Contingency From-To	No. of $Q_{g_i} = Q_{g_i}^{lim}$	Step factor α	Max Error
5	174-311	8	1.0053	2.30×10^{-3}
9	176-264	8	0.9813	3.58×10^{-5}
11	177-214	8	1.0025	2.28×10^{-7}
17	181-217	5	1.2160	4.66×10^{-2}
43	190-327	8	1.0200	3.30×10^{-6}
83	206-211	8	1.0100	7.64×10^{-7}
283	296-317	15	0.9967	8.88×10^{-3}
295	301-490	8	1.0109	8.60×10^{-7}
483	339-570	8	1.0066	5.91×10^{-7}
473	404-574	8	1.0000	4.38×10^{-6}
596	228-229	8	1.0228	9.60×10^{-6}
637	264-265	8	1.0212	2.26×10^{-6}
747	479-785	8	1.0000	2.28×10^{-7}
803	370-584	8	1.0371	2.65×10^{-4}
923	505-600	8	1.0284	7.47×10^{-6}

Table 8
Some features of the iterative process (a)

Test System	Contingency TL	No. of $Q_{g_i} = Q_{g_i}^{lim}$	Step factor α	2nd Order time (s)	N-R time (s)
IEEE-14 bus	2-3	1	1.1768	0.0425	0.0637
	5-6	1	1.1113	0.0434	0.0655
IEEE-57 bus	12-13	1	1.0570	0.0845	0.1636
	13-49	0	1.1119	0.0675	0.1207
IEEE-118 bus	79-80	3	1.0387	0.0924	0.2160
	75-77	3	1.0122	0.0796	0.2612
IEEE-300 bus	154-158	1	1.0357	0.1874	0.7725
	79-83	3	1.0055	0.2130	0.8394

Table 9
Some features of the iterative process (b)

Test System	Number of contingencies	N-R CPU (s)		2nd Order CPU (s)	
		total	per cont.	total	per cont.
IEEE-14 bus	20	0.098	0.004	0.063	0.003
IEEE-57 bus	80	1.089	0.013	0.559	0.007
IEEE-118 bus	186	8.784	0.047	2.719	0.014
IEEE-300 bus	411	315.021	0.766	48.795	0.118