NLMPC for Real Time Path Following and Collision Avoidance

Ugo Rosolia and Francesco Braghin
Politecnico di Milano

Andrew Alleyne
Univ of Illinois

Edoardo Sabbioni
Politecnico di Milano

INTRODUCTION

Real-time control of autonomous vehicles is an active field of research. The main aspects to take into account are: path following for a non-holonomic vehicle and collision avoidance.

Path following has been widely studied in the literature. A standard formulation is to use a moving coordinate system attached to the path: a Fernet reference frame [1] [2]. The in [1] is preferable, because the moving frame is uniquely determined by the vehicle position and it is not necessary to introduce an additional degree of freedom. Although a singularity problem arises when the lateral vehicle displacement coincides with the curvature radius of the path. This situation is not relevant in our application: a highway curvature radius is at least an order of magnitude larger than the admissible lateral displacement.

Collision avoidance has been thoroughly studied in the past twenty years and many solutions were proposed. Early work in this field used potential functions and online searching [3] [4]. These methods do not take into account the dynamics of the system and the planning phase is independent on the model. To remove these weak points in the planning process, some extensions have been proposed in [2] [5] [6]. However, some fundamental problems concerning local minima and grid discretization are still present [2] [7]. In this paper a model predictive controller (MPC) is presented that successfully overcomes these problems [7] [8]: in fact, in this control approach an optimal control problem is solved over a finite time moving window. Unfortunately, nonlinear model predictive control (NLMPC) is still an open research field and there is an exiguous number of techniques to find a numerical solution [9]. Among those available, an explicit technique, the MGRES/Continuation, has been chosen: since, conversely to implicit solvers, it uses forward steps to compute the solution and no iteration is required. Therefore, the computational time is fixed which is a key feature in real-time applications [11].

Recently, predictive steering control for path following has been studied and efficiently solved in real time [12] [13] [14]. But this control logic requires a collision-free reference trajectory, which must be computed on-line and could lead to unfeasible real time implementations. Thus to overcome this issue, the authors in [15] [16] have proposed a two level hierarchical control scheme composed of a higher level NLMPC trajectory planner and a lower level NLMPC trajectory follower. In [15] the authors use a simple nonlinear system for the planning phase, but the longitudinal velocity of the simplified system is fixed and the obstacles are modelled using potential functions. In [16] a single track vehicle model (STVM) in spatial coordinates is used for the higher level controller. In [11] the same spatial approach is applied to a more complex model and the optimization is performed on a single layer NLMPC. However, in [11] [16] the obstacle is treated as a boundary on the state. Namely, the vision algorithm has to decide if the obstacle should be overtaken to the left or to the right. This approach, in cruise highway conditions, limits performances. For instance, when the vehicle is leaving a curve, the controller has to decide from which side to avoid the obstacle based on the lateral velocity, on the positon and on the gradient of the steering. Furthermore, for cruise driving, speed should be kept constant except during an overtake when the controller should accelerate gently to rapidly complete the maneuver as a human would do. To achieve this target a change of coordinate is proposed where the curvilinear abscissa is a state of the system, and not an independent variable as in [11] [16]. Therefore, the velocity along the path can be considered within the optimization process. This leads to the human-like behavior. Moreover, when dealing with a long and complex path, it is not always possible to explicitly compute the information necessary for the NLMPC offline. In this paper an approach is proposed that overcomes this problem.

Summing up, this paper assumes cruise highway conditions with the aim of following a reference path while avoiding obstacles, detected with a suitable preview. Thus, no safety maneuvers are considered. However, the presented method is able to detect unfeasibility within the time horizon. Therefore, the method might be coupled with soft collision control logic.

CONTROL ARCHITECTURE

The complexity of a model plays an important role in determining the feasibility of real-time problems. For this reason, the NLMPC optimization is performed on a simplified model: a Single Point Mass Model (SPMM). This approach is possible because the vehicle is driving under cruise highway conditions i.e. non critical conditions. Of course, there are differences between the dynamics of a real vehicle and the simple system; therefore it is required to adjust the input computed by the NLMPC using a correction factor based on a Single Track Vehicle Model (STVM). This quantity is computed in the preview distance block that is the novel control strategy presented in this paper. It is interesting to notice that the NLMPC and the preview distance block are at the same level, meaning they are discretized with the same sampling time.

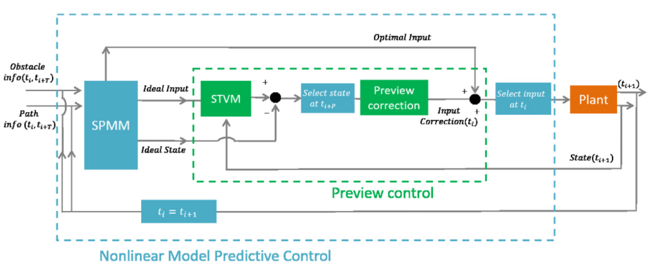


Figure 1. Control architecture.

The single point mass model (figure 1) is governed by the following equations:

$$\begin{cases} \dot{x}_c = V\cos\theta \\ \dot{y}_c = V\sin\theta \\ \dot{\theta} = u \end{cases} \tag{1}$$

where V and θ are the velocity vector magnitude and angle respectively. The chosen control inputs are V and u.

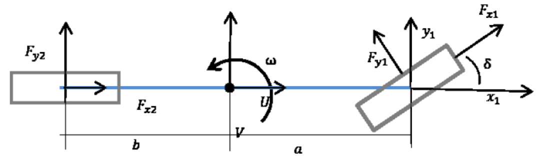


Figure 2. Free body diagram of the single track vehicle model.

Clearly, the dynamics of a real vehicle is not entirely captured by the single point mass model. A more accurate one is needed to test the solution of the NLMPC problem. To achieve this goal, a single track vehicle model is used which, under cruise condition, is considered to be sufficiently accurate to describe the real system dynamics. The equations of motion are derived from Newtons law.

$$\sum F_x = m\left(\dot{U} - \omega V\right) = x_1 + x_2 \tag{2}$$

$$\sum F_y = m\left(\dot{V} + \omega U\right) = y_1 + y_2 \tag{3}$$

$$\sum M_z = J\dot{\omega} = ay_1 - by_3 \tag{4}$$

where the variables x_i and y_i are a function of the longitudinal and lateral tire-road contact force:

$$x_1 = F_{x1}\cos\delta - F_{y1}\sin\delta \tag{5}$$

$$y_1 = F_{x1}\sin\delta + F_{y1}\cos\delta \tag{6}$$

$$x_2 = F_{x2} \tag{7}$$

$$y_2 = F_{y2} \tag{8}$$

The slip angle α is defined as the angle between the longitudinal axis of the wheel and its velocity:

$$\alpha_1 = \arctan\frac{V + a\omega}{U} + \delta \tag{9}$$

$$\alpha_2 = \arctan\frac{V - b\omega}{U} \tag{10}$$

The lateral wheel force is considered proportional to the slip angle as defined in [17]:

$$F_{y1} = C_1\alpha_1 \tag{11}$$

$$F_{y2} = C_2 \alpha_2$$

(12)

Finally the longitudinal force is considered proportional to the torque:

$$F_{xi} = \frac{T}{R}$$

(13)

R being the wheel radius. The input variables of the single track model are the torque *T* and the steering angle δ.

PATH FOLLOWING

In order to simplify the problem formulation for the NLMPC, the approach proposed by [1] is adopted: a moving point, called Fernet reference frame, is attached to the path. Accordingly, the system in 1 can be rewritten as:

$$\begin{cases} \dot{s} = \frac{V\cos(\theta - \gamma(s))}{1 - y\dot{\gamma}(s)} \\ \dot{y} = V\sin(\theta - \gamma(s)) \\ \dot{\theta} = u \end{cases}$$

(14)

where $\gamma(s)$ represents the angle of the tangent vector to the Fernet reference frame at the curvilinear abscissa *s* and *y* the distance from the vehicle to the path. Therefore, the problem of path following is reduced to minimizing *y*.

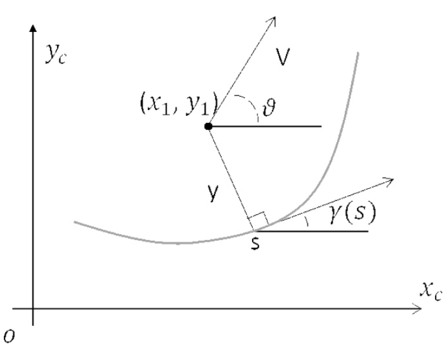


Figure 3. Fernet reference frame.

To compute an analytic expression for $\gamma(s)$ and $\dot{\gamma}(s)$, requested by equation 14, the path is approximated with Bezier curves. Namely, it is assumed that the control points of the fifth order Bezier curves that define the target path are known. This is not a strong assumption since these control points can be computed off-line before the navigation starts. It should be noted that, since the NLMPC evaluates the properties of a point on the path using one Bezier curve, the computational time of this approach is independent of the path length and complexity.

CONTROL ALGORITHM

The optimization problem is performed on a fixed time window which moves in time. Given a system described by:

$$\dot{x}(t) = f(x(t), u(t))$$

(15)

where $x(t)$ denotes the states and $u(t)$ the control input. The objective is to minimize, over a finite time horizon $[t, t+T]$, the functional

$$J(u(t)) = \int_{t}^{t+T} h(x(t), u(t))\,dt + m(x(t+T)) \rightarrow min$$

(16)

subject to the following constraint

$$C(x(t)) = 0$$

(17)

It has been shown that upcoming information about the road can significantly improve performances in vehicle path following. In [18] and [19], previewed road curvature information is used to compute a feedforward action that improves the behavior of the vehicle when entering or leaving a curve. The proposed controller also exploits this feedforward action for improving performances.

The information is provided by the NLMPC. In fact, the optimal control problem is solved over a finite horizon and a prediction of the input is available. This prediction is used to compute the estimated position of the real system ahead of time and it is compared with the optimal trajectory of the SPMM to compute an error vector.

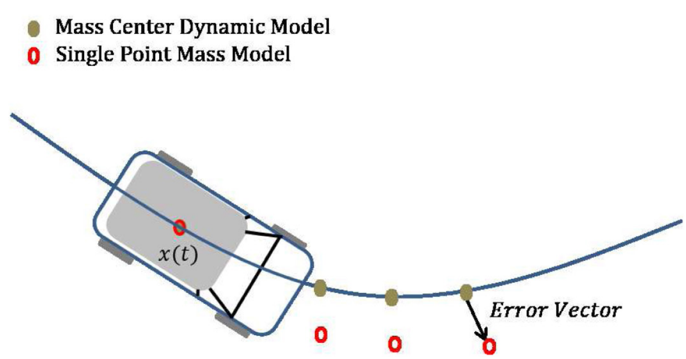


Figure 4. Computation of the error vector.

The predicted error is the result of the different dynamics of the two systems and it can be used to design a PI controller to correct the input. This control strategy, although simple, is sufficient to reduce the tracking error under cruise highway conditions.

To guaranty safety the infeasibility region of the state space are delimited by an ellipse. However, the Lagrangian multiplier associated with the inequality constraint, according to the KKT (Karush-Kuhn-Tucker) optimality conditions [21], is set to zero when the strict inequity is satisfied. Therefore, this procedure could cause the violation of the hard constraint, due to the discretization along the time axis. Moreover, there are some issues related with the reactive nature of the MGRES/Continuation algorithm (for more details see [9] and [20]). In order to overcome these problems, it is possible to convert the inequality constraints to an equality constraint with a slack variable Z [20]:

$$C\left(s,y\right)=\frac{\left(s-s_{obst}\right)^{2}}{A}+\frac{\left(y-y_{obst}\right)^{2}}{B}-1+Z^{2}=0. \tag{18}$$

The semi-axis of the ellipse, A and B, are taken equal to the length and the width of the vehicle, plus a safety factor. The position of the center of the ellipse is not taken exactly on the path to have a unique solution on a straight road.

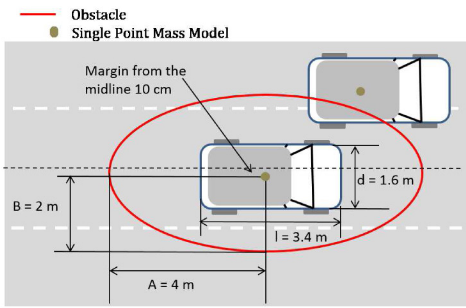


Figure 5. Obstacle modelling.

Finally, it is assumed that the obstacle is detected before entering the horizon so that no sudden change in the control input is required. This assumption is reasonable under cruise highway conditions. Moreover, it is a necessary condition to be able to apply the MGRES/ Continuation method.

It is now possible to design the cost function to achieve the controller goals. In the curvilinear reference system, it is sufficient to minimize the lateral distance, y, to follow the path. Furthermore, it is minimized the deviation from a target velocity. So the cost function J is defined as

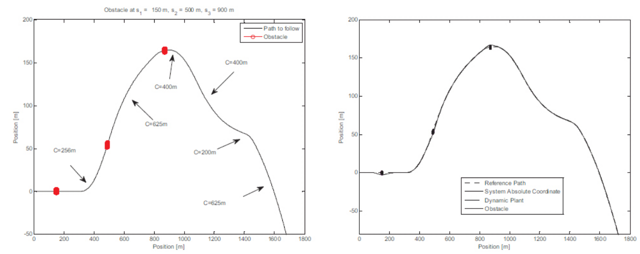
$$J=\int_{0}^{T}C_{s2}y^{2}+C_{d1}\left(\dot{y}\right)^{2}+C_{d2}\left(\dot{s}-V_{ref}\right)^{2}+ \\ +C_{d3}\left(\dot{\theta}\right)^{2}-C_{z}\ln\left(Z-\zeta\right)dt, \tag{19}$$

where the slack variable, Z, as to be included into the cost function to avoid bifurcation of the numerical solution [20].

RESULTS

The proposed algorithm has been tested on a 2000m long path. The curvature radius has been chosen according to the U.S. Department of transportation, i.e. the minimum radius allowed on an highway with no super elevation is 200 meters.

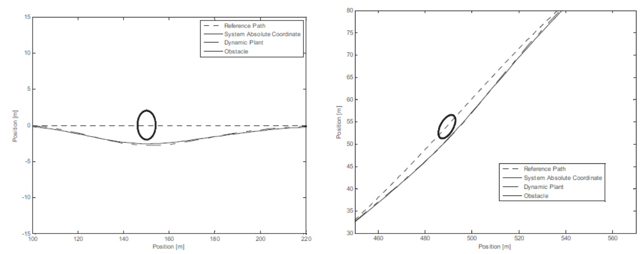
Obstacles are positioned randomly and their presence is detected and transmitted to the NLMPC block when the distance between the SPMM and the obstacle is smaller than 100 meters. Obstacles dimensions are illustrated in figure 5. Finally, figure 6a shows the imposed path and the obstacle location.



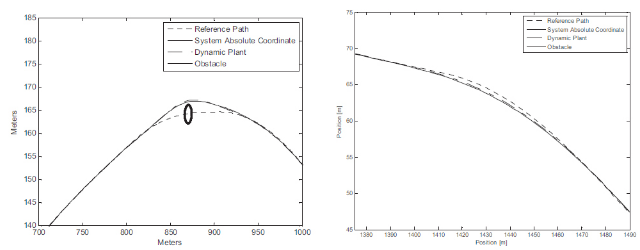
(a) Imposed path and obstacle position. (b) Overall path.

Figure 6. Test track.

Figures 6b-7c show the generated trajectory which minimizes the distance between the reference path and the system trajectory avoiding the obstacles along the track.



(a) First obstacle. (b) Second obstacle.

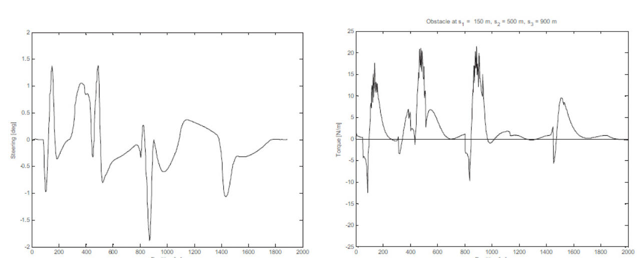


(c) Third obstacle. (d) Example of a severe curve.

Figure 7. Details of the test track.

It is interesting to notice that the generated trajectory tends to stay inside the curve and then widens to reduce the gradient of the steering (figure 7d). This is a clear advantage of using a model based control logic.

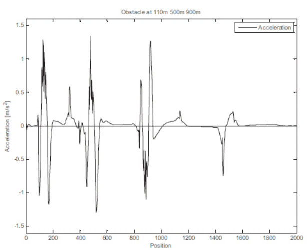
Figures 8a and 8b show the space history of the steering input and the torque input respectively. It can clearly be seen that the steering input is less than $2deg$ in absolute value, i.e. the vehicle is working in its linear region, and smooth actions on the steering wheel are applied. Same conclusions can be drawn for the torque that is below $20Nm$.



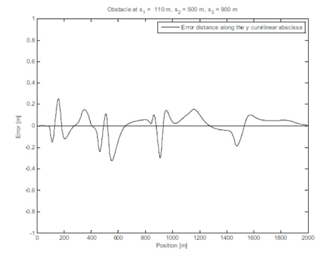
(a) Steering input. (b) Torque input.

Figure 8. Control inputs.

Finally, figures 9a and 9b shown the lateral acceleration and the tracking error respectively. The lateral acceleration is almost everywhere less than 1.35m/s^2 meaning that the drive is very comfortable and the lateral error is alway less than 0.3m which can definitley be considered as an acceptable error.



(a) Lateral acceleration.



(b) Lateral error between the optimal trajectory and the actual position of the single track model.

Figure 9. Control outputs.

CONCLUSIONS

The presented NLMPC is capable of driving autonomous during under cruise highway conditions. A simplified system is used to plan the ideal trajectory and to compute the optimal input. Then, a correction factor, compensating the differences in the dynamics of the system used in the planning phase and the real one, is computed based on the non-causal information of the NLMPC. The problem is formulated using a curvilinear reference system and the obstacle is taken into account into the optimization.

The proposed control logic has been successfully tested on a simulated highway track showing very good performances in terms of tracking errors and driver inputs. A sensitivity analysis on obstacle dimensions and an extinction for cooperative driving are envisaged for future works.

REFERENCES

1. Micaelli A. and Samson C., *Trajectory tracking for unicycle-type and two-steering-wheels mobile robots*, Rapport de Recherce, INRIA, 1993.
2. Lapierre L., Zapata R., and Lepinay P., *Simulatneous path following and obstacle avoidance control of a unicycle-type robot*, Proceedings of the IEEE International Conference on Robotics and Automation (ICRA'07), Roma, Italy, pp. 1014, 2007.
3. Rimon E. and Koditschek D. E., *Exact robot navigation using artificial potential functions*, IEEE Transactions on Robotics and Automation, Vol. 8, N. 5, pp. 501518, 1992.
4. Minguez J. and Montan L., *Nearness diagram navigation (nd): a new real time collision avoidance approach*, Proceedings of the 2000 IEEE/RSJ International Conference on Intelligent Robots and Systems, Takamatsu, Japan, pp. 20942100, 2000.
5. Pepy R., Lambert A., and Mounier H., *Path planning using a dynamic vehicle model*, Proceedings of the 2nd Conference on Information and Communication Technologies (ICTTA'06), Damascus, Syria, pp. 781-786, 2006.
6. Ulrich I. and Borenstein J., *VFH+: reliable obstacle avoidance for fast mobile robots*, Proceedings of the 1998 IEEE International Conference on Robotics and Automation. Leuven, Belgium, pp. 1572-1577, 1998.
7. Yoon Y., Shin J., Kim H. J., Park Y., and Sastry S., *Model-predictive active steering and obstacle avoidance for autonomous ground vehicles*, Control Engineering Practice, Vol. 17, N. 7, pp. 741750, 2009.
8. Park J.-M., Kim D.-W., Yoon Y.-S., Kim H. J., and Yi K.-S., *Obstacle avoidance of autonomous vehicles based on model predictive control*, Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, Vol. 223, N. 12, pp.1499-1516, 2009.
9. Diehl M., Ferreau H., and Haverbeke N., *Efficient numerical methods for nonlinear MPC and moving horizon estimation*, Nonlinear Model Predictive Control, Lecture Notes in Control and Information Sciences, Vol. 384, pp. 391-417, 2009.
10. Ohtsukat T. and Words-real-time K., *Real-time optimization algorithm for nonlinear receding-horizon control*, Automatica, Vol. 33, N. 6, pp. 11471154, 1997.
11. J. V Frasch, Gray A., Zanon M., Ferreau H. J., Sager S., Borrelli F., and Diehl M., *An auto-generated nonlinear MPC algorithm for real-time obstacle avoidance of ground vehicles*, Proceedings of the 2013 European Control Conference (ECC2013), Zurich, Switzerland, pp. 41364141, 2013.
12. Borrelli F., Falcone P., Keviczky T., Asgari J., and Hrovat D., *MPC-based approach to active steering for autonomous vehicle systems*, International Journal of Vehicle Autonomous Systems, Vol. 3, N. 2-4, pp. 265-291, 2005.
13. Falcone P., Borrelli F., Asgari J., Tseng H. E., and Hrovat D., *Predictive active steering control for autonomous vehicle systems*, IEEE Transactions on Control Systems Technology, Vol. 15, N. 3, pp. 566580, 2007.
14. Falcone P., Borrelli F., Asgari J., Tseng H. E., and Hrovat D., *A model predictive control approach for combined braking and steering in autonomous vehicles*, Proceedings of the IEEE 2007 Mediterranean Conference on Control & Automation (MED'07), Athens, Greece, pp. 1-6, 2007.
15. Elizabeth F. Q., Gray A., Gao Y., Lin T., Hedrick J. K., Tseng H. E., and Borrelli F., *Predictive control for agile semi-autonomous ground vehicles using motion primitives*, Proceedings of the 2012 American Control Conference (ACC2012), Montreal, Canada, pp. 42394244, 2012.
16. Gao Y., Gray A., Frasch J. V, Lin T., Tseng E., Hedrick J. K., and Borrelli F., *Spatial predictive control for agile semi-autonomous ground vehicles*, Proceedings of the 2012 Advanced Vehicle Control Conference (AVEC'12), Seoul, South Corea, pp.1-6, 2012.
17. Alleyne A., *A comparison of alternative intervention strategies for unintended roadway departure (URD) control*, Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, Vol. 27, N. 3, pp. 157-186, 1997.
18. Peng H. and Tomizuka M., *Optimal preview control for vehicle lateral guidance*, California Partners for Advanced Transit and Highways (PATH), 1991.
19. Peng H. and Tomizuka M., *Preview control for vehicle lateral guidance in highway automation*, Proceedings of the 1991 American Control Conference (ACC1991), Boston, USA, pp. 3090-3095, 1991.
20. Ohtsuka T., *A continuation/GMRES method for fast computation of nonlinear receding horizon control*, Automatica, Vol. 40, N. 4, pp. 563574, 2004.
21. Bryson A. E., Applied optimal control: *optimization, estimation and control*, CRC Press, 1975.